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Phys. Rev. D **101**, 063010 — Published 10 March 2020

DOI: 10.1103/PhysRevD.101.063010

Resonant interaction between dispersive gravitational waves and scalar massive particles.

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The Klein-Gordon equation is solved in the curved background spacetime created by a dispersive gravitational wave. Unlike solutions of perturbed Einstein equations in vacuum, dispersive gravitational waves do not travel exactly at the speed of light. As a consequence, the gravitational wave can resonantly exchange energy with scalar massive particles. Some details of the resonant interaction are displayed in a calculation demonstrating how relativistic particles (modeled by the Klein-Gordon equation), feeding on such gravitational waves, may be driven to extreme energies.

PACS numbers:
Keywords:

Gravitational waves, propagating perturbations of spacetime, travel at the speed of light (c) in vacuum. In certain media however [1, 2], they become dispersive and their phase and group velocities can differ from c, in analogy with electromagnetic waves. Pushing forward the gravitational-electromagnetic waves analogy, one may wonder if the gravitational waves could, then, resonantly transfer energy to the particles of the medium. The wave-particle resonant energy exchange is a highly investigated phenomenon in plasmas, for instance, the Landau damping or growth [3] of electromagnetic waves on electrons moving with the phase speed of the wave. Naturally, this process is possible only when the electromagnetic waves disperse, and do not travel quite at the speed of light. In an analogous fashion, and depending on the gravitational wave polarization, cyclotron resonances, Alfvén wave resonances, or plasma wave resonances between gravitational waves and relativistic magnetized plasmas may be triggered [1]; different nonlinear mechanisms can also convert energy of the gravitational waves into electromagnetic energy [4–6].

In this paper, we explore effects, similar to Landau processes, in the context of dispersive gravitational waves interacting with spinless massive particles. We show that under well-defined conditions, the energy of particles can be resonantly boosted up to very high values, several orders of magnitude higher than the rest–mass energy of the particle. The studied mathematical model is based on the Klein–Gordon equation in the curved spacetime background created by the gravitational wave; we will assume that the particle does not self–gravitate. The process of resonant energization of Klein-Gordon particle/waves through arbitrary amplitude dispersive electromagnetic waves has been recently reported and serves as the first paper in the exploration of this class of phenomena [7]. This paper, dealing with the resonant energization of Klein-Gordon particle/waves by the dispersive gravitational waves, develops the subject further.

We start by noticing that gravitational waves propagating in a dispersive medium does not travel at the speed of light [1, 2, 8–17]. The same effect can occur if the graviton mass [18–21] is assumed to be non-zero. In both these cases, the gravitational wave is dispersive. Independent of the origin of dispersion, propagation of (non-vacuum like) gravitational waves can be modeled in a very general fashion. Without loss of generality, we assume a gravitational wave propagating in a z-direction. We consider a spacetime interval $ds^2 = g_{\mu\nu}dx^\mu dx^\mu$, with the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu} = (-1, 1, 1, 1)$ is the flat spacetime metric, and $h_{\mu\nu}$ ($h_{\mu\nu} \ll \eta_{\mu\nu}$) is the perturbation caused by the gravitational wave (from now $c = 1$). The nonzero components of the perturbation metric are $h_{22} = -h_{33} = h_+(\chi)$, and $h_{23} = h_{32} = h_\times(\chi)$, where $\chi = \omega\, t - k\, z$. Here, $\omega = \omega(k)$ is the gravitational wave frequency depending on the wavenumber k, that propagates in time t and spatial direction z. For a dispersive wave, the phase velocity must depart from the $\omega = k$ condition. A model dispersion for a gravitational wave (propagating in a general medium) will be of the type

$$\omega^2 - k^2 \equiv \omega_G^2 \neq 0\,, \tag{1}$$

where ω_G signifies a "response" frequency characteristic of the medium. Depending on the nature of the medium, this response frequency may or may not be constant. For specific forms of ω_G, one may consult Refs. [1, 2, 8–21]. For a constant response frequency, $\omega^2 > \omega_G^2 > 0$, and the group velocity of the gravitational wave is always less than the speed of light, $\partial\omega/\partial k = k/(k^2 + \omega_G^2) < 1$.

The purpose of this work is to show that the energy of a relativistic massive quantum particle/wave, can be resonantly boosted in the presence of such a dispersive gravitational wave through the dynamics of Klein-Gordon field evolving in the background of this perturbed metric. For the above spacetime metric associated with the gravitational wave, the Klein-Gordon equation $\Box\Phi = m^2\Phi$ may

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be written as

$$0 = -\frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial^2 \Phi}{\partial z^2} - \omega f(\chi)\frac{\partial \Phi}{\partial t} - k f(\chi)\frac{\partial \Phi}{\partial z} - m^2 \Phi\,, \quad (2)$$

where the curved d'Alembert operator is defined as $\Box = (1/\sqrt{-g})\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)$, and $g^{\mu\nu}$ is the inverse metric (with determinant g). We have also assumed that the Klein-Gordon field, just like the gravitational wave, depends only on z and t, i.e, $\Phi = \Phi(t,z)$. In Eq. (7), $f(\chi) = (\ln\sqrt{-g})'$, and $'$ means the derivative with respect to χ. At $\mathcal{O}(h^2_{+,\times})$ order (the first relevant order), we have $f(\chi) \approx -h_+ h'_+ - h_\times h'_\times$.

Of all possible solutions of Eq. (2), we will focus only on those that can become "resonant" with the gravitational wave, i.e., those in which the Klein-Gordon field shares with the gravitational wave the z and t dependence strictly through χ. Thus, the field propagates with the gravitational wave and a resonance effect can be produced. After expressing the z and t variation in terms of χ, and using the dispersion relation (1), Eq. (2) becomes

$$0 = \Phi'' + f\,\Phi' + \frac{m^2}{\omega_G^2}\Phi\,. \quad (3)$$

By defining $\Phi(\chi) = (-g)^{-1/4}\,\varphi(\chi)$, Eq. (3) transforms to

$$0 = \varphi'' + \zeta^2\,\varphi\,, \quad (4)$$

where

$$\zeta(\chi) = \sqrt{\frac{m^2}{\omega_G^2} - \frac{f'}{2} - \frac{f^2}{4}}\,. \quad (5)$$

Eq. (4) mimics the equation of motion for a harmonic oscillator with time-dependent frequency [22, 23]. It can be readily shown to have the following WKB-type solution [22],

$$\varphi(\chi) = \frac{1}{\sqrt{2W(\chi)}}\exp\left(-i\int W(\chi)d\chi\right)\,, \quad (6)$$

with the function W defined through the equation

$$W^2 = \zeta^2 - \frac{W''}{2W} + \frac{3(W')^2}{4W^2}\,. \quad (7)$$

Let us now assume a slowly-varying spacetime, such that in Eq. (7) the derivatives of W are small compared to m/ω_G [22]. Also, consider a regime in which ω_G remains essentially constant such that $m \gg \omega_G$, and m/ω_G is larger than any possible spacetime variation of the gravitational wave. We could, then, approximate at lowest order

$$W \approx \frac{m}{\omega_G}\,. \quad (8)$$

Solution (6) signifies that the Klein-Gordon field behaves, to the leading order, as an harmonic oscillator in a dispersive gravitational wave background (8). This simplification pertains as long as $\omega, k \gg \omega_G$, for a dispersive gravitational wave moving almost at the speed of light. It further simplifies the solution for the Klein–Gordon field

$$\Phi(\chi) \approx \sqrt{\frac{\omega_G}{2m}}\exp\left(-i\frac{m}{\omega_G}\chi\right) \quad (9)$$

with its associated energy E and momentum P of the particle

$$E \approx \frac{m\omega}{\omega_G} \gg m\,, \qquad P \approx \frac{mk}{\omega_G}\,. \quad (10)$$

Since $\omega \gg \omega_G$, the energy (10) is much larger than the rest-mass energy of the massive Klein-Gordon particle; the Klein-Gordon field is in resonance with the gravitational wave. Notice that this effect cannot occur when the gravitational wave is not dispersive (when $\omega = k$) [24]. Thus, resonance of a massive particle-field cannot take place with a gravitational wave propagating at the speed of light.

In the light of (10), let us examine the scenario when the Klein-Gordon system is treated as a quantum field [22, 25]. We write the scalar field as $\Phi(\chi) = \sum_{\mathbf{k}}\left[a_{\mathbf{k}}u_{\mathbf{k}}(\chi) + a^\dagger_{\mathbf{k}}u^*_{\mathbf{k}}(\chi)\right]$, in terms of annihilation $a_{\mathbf{k}}$ and creation $a^\dagger_{\mathbf{k}}$ operators (with their respective commutation relations). Thus, fields $u_{\mathbf{k}}$ and $u^*_{\mathbf{k}}$ will satisfy Eq. (2) with solution (6). The energy-momentum tensor for the quantized field Φ is given by $T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - (1/2)g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi + (1/2)g_{\mu\nu}m^2\Phi^2$, from which we deduce that $T_{00} = (\omega^2+k^2)\Phi'^2/2 - m^2\Phi^2/2$, and $T_{0i} = -\omega k\Phi'^2$. At the leading order approximation (8), the normal order Hamiltonian and momentum are derived to be [22]

$$:\!H\!: = \sum_{\mathbf{k}}\frac{m\omega}{\omega_G}a^\dagger_{\mathbf{k}}a_{\mathbf{k}}\,, \qquad P = \sum_{\mathbf{k}}\frac{mk}{\omega_G}a^\dagger_{\mathbf{k}}a_{\mathbf{k}}\,. \quad (11)$$

From examining both the classical and the quantized expressions (Eqs. (10) and (11) respectively), we conclude the energy-momentum of a scalar field can be strongly boosted when it is in resonance with a dispersive gravitational wave. The condition for this enormous gain is that, to leading order, the phase velocity v of the scalar field is equal to the phase velocity of the gravitational wave (see Ref. [7])

$$v \sim \frac{E}{P} \sim \frac{\omega}{k}. \quad (12)$$

This condition ensures that the energy of the scalar massive field can increase under the bound imposed by the energy-momentum conservation $E^2 - P^2 = m^2$, which is ensured by the dispersion relation (1), and the energy-momentum solution (10). This is closely related with the adiabatic invariant $E/\zeta \approx \omega$ [23] of Eqs. (3) and (4).

Although much more sophisticated calculations are needed to calculate the relevant time scale τ for energization, we would present here a simple estimate exploiting the fact that it is a resonant process controlled by how close the gravitational wave speed ω/k is to the particle velocity v. One could expect, using the Landau damping analogy, that the time taken for the particle to achieve an energy that corresponds to the velocity v is

$$\tau \sim \frac{1}{\mathcal{H}\,(\omega - kv)}\,, \tag{13}$$

where ω is given by dispersion relation (1), and $\mathcal{H}$ is a normalized function that depends on the gravitational wave amplitude. If the gravitational wave amplitude were very large compared to the one of Klein-Gordon field (much larger than the maximum energy transfer), $\mathcal{H}$ will be essentially a constant. But for all finite energy waves, $\mathcal{H}$ will become a function of time, its value will decrease as the gravitational wave loses its energy to the particles. The time scale τ, thus, is determined by linear as well as nonlinear processes: the former affects it through the gravitational wave energy, allowing that τ increases with decreasing amplitude. In the following, we will simply focus in the linear resonant phenomena and assume a constant $\mathcal{H}$ (i.e. gravitational wave energy is large). The dispersion relation (1) gives $\omega \sim k + \omega_G^2/2\omega$, when $\omega_G \ll \omega$. In addition, for a high–energy relativistic particle with $v \sim 1$, its Lorentz factor $\gamma^{-2} = 1 - v^2 \sim 2(1-v)$. Combining the two simplifications coverts Eq. (13) into

$$\tau \sim \frac{2\omega\gamma^2}{\mathcal{H}\,(\omega^2 + \gamma^2\omega_G^2)}\,. \tag{14}$$

One immediately notices that when $\gamma \to \infty$, the limiting time for achieving the highest energy permissible given in (10), takes the simple form

$$\tau(\gamma \to \infty) \sim \frac{2\omega}{\mathcal{H}\,\omega_G^2}\,, \tag{15}$$

which is very large for diluted media with $\omega_G \ll \omega$. For a finite time interval, if the particle is boosted up from γ_0 to $\gamma_1 > \gamma_0$, the time interval for energization is

$$\Delta\tau = \tau(\gamma_1) - \tau(\gamma_0) \sim \frac{2\omega^3\left(\gamma_1^2 - \gamma_0^2\right)}{\mathcal{H}\,(\omega^2 + \gamma_1^2\omega_G^2)\,(\omega^2 + \gamma_0^2\omega_G^2)}\,. \tag{16}$$

For most relevant region where $\omega/\omega_G \gg \gamma_0, \gamma_1 \gg 1$, $\Delta\tau$ takes the revealing simple form

$$\Delta\tau \sim \frac{2}{\mathcal{H}\,\omega}\left(\gamma_1^2 - \gamma_0^2\right)\,. \tag{17}$$

These estimations for the time scale for energization do not consider the effective dependence upon the gravitational wave amplitude. The proper treatment will be left for future investigations.

The maximum energy accessible to the particle (in terms of restored units) is

$$\frac{E}{mc^2} = \frac{\omega}{\omega_G} \sim \frac{c}{\lambda\,\omega_G}\,, \tag{18}$$

where λ is the wavelength of the gravitational wave. As an example, let us consider a gravitational wave with a characteristic wavelength $\lambda \sim 10^6$[m] (the kind detected in LIGO [26]) corresponding to frequencies of the order $\omega \sim 300$[Hz]. In order to estimate ω_G, let us assume the gravitational wave is propagating in a very dilute medium. The response frequency, then, can be estimated to be [13]

$$\omega_G \approx \sqrt{\frac{4\pi G\mathcal{E}}{c^2}} \sim 10^{-3}\sqrt{\rho}\,, \tag{19}$$

where G is the gravitational constant, and $\mathcal{E}$ and ρ are the energy and mass density of the medium. For media with 10^3 to 10^8 nucleons per cubic meter (interstellar gas), then we have mass densities ρ ranging from 10^{-27}[gr/cm^3] to 10^{-22}[gr/cm^3]. This produce response frequencies ω_G from $\sim 3 \times 10^{-17}$[Hz] to 10^{-14}[Hz]. For this medium, the increment in energy (18) of the Klein–Gordon particle under the above gravitational wave can range up from $E/mc^2 \sim 10^{19}$ to $\sim 3 \times 10^{16}$. If the mass density increases to $\rho \sim 10^{-12}$[gr/cm^3] (10^{18} nucleons per cubic meter), then $\omega_G \sim 10^{-9}$[Hz] and $E/mc^2 \sim 3\times 10^{11}$. For a proton (treated as a scalar massive particle), with $mc^2 \sim 900$[MeV], the previous estimation implies that its energy can reach values from $E \sim 10^{20}$[eV] to $E \sim 10^{28}$[eV], when it enters in resonance with this dispersive gravitational wave in the corresponding background media. Those energies are in the range peculiar to the most energetic cosmic rays.

We, thus, see that the resonance described in (6) and (8) can produce very energetic massive particles when the appropriate conditions are met, i.e., when the particle moves in a very dilute media ($\omega \gg \omega_G$) in consonance with the dispersive gravitational wave, allowing it to extract energy from the wave, as long as the self–gravitation of the scalar field is ignored. It must be stressed that the basic conclusions of resonant energy transfer hold even for the more general conformally invariant Klein-Gordon equation $\Box\Phi = m^2\Phi + \xi R\Phi$, where R is the Ricci scalar and ξ a constant. The extra term, modifying the right-hand side of Eq. (7), does not change the main result (8).

The proposed process of the energization of spinless massive particles is bound to be relevant in several astrophysical scenarios where gravitational waves are expected to be present. This process has to be considered in the context of creating high-energy particles in astrophysics. Besides, similar effects can be expected in the interaction of electromagnetic waves with dispersive gravitational waves. This kind of interactions have been studied using a background of gravitational waves in vaccum [16, 27] and viceversa [28]. A similar effect in the energization of photons by gravitational waves has been also reported [16].

We must point out that the gravitational waves considered in this paper ($h_{\mu\nu} \ll \eta_{\mu\nu}$) are basically linear. In order for them to have sufficient energy to efficiently catapult particles to high energies, they must be high intensity (see [7] for the electromagnetic case), and therefore essentially nonlinear. The current calculation, therefore, must be seen only as an important first step demonstrating a new possible process. To do a proper energy inventory, a more advanced model of the intense gravitational waves must be invoked. A detailed study of relativistic fields interacting, resonantly, with dispersive gravitational waves can bear highly promising results, and the authors are investigating several aspects of this problem.

Acknowledgments

F.A.A. thanks Fondecyt-Chile Grant No. 1180139. The work of SMM was partially supported by the US-DOE grants DE-FG02-04ER54742.

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