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Cosmological attractors to general relativity and spontaneous scalarization with disformal coupling

Hector O. Silva[1, 2] and Masato Minamitsuji[3]

[1]Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA
[2]eXtreme Gravity Institute, Department of Physics, Montana State University, Bozeman, Montana 59717, USA
[3]Center for Astrophysics and Gravitation (CENTRA), Instituto Superior Técnico, University of Lisbon, Lisbon 1049-001, Portugal.

The canonical scalar-tensor theory model which exhibits spontaneous scalarization in the strong-gravity regime of neutron stars has long been known to predict a cosmological evolution for the scalar field which generically results in severe violations of present-day Solar System constraints on deviations from general relativity. We study if this tension can be alleviated by generalizing this model to include a disformal coupling between the scalar field φ and matter, where the Jordan frame metric $\tilde{g}_{\mu\nu}$ is related to the Einstein frame one $g_{\mu\nu}$ by $\tilde{g}_{\mu\nu} = A(\varphi)^2(g_{\mu\nu} + \Lambda\,\partial_\mu\varphi\,\partial_\nu\varphi)$. We find that this broader theory admits a late-time attractor mechanism towards general relativity. However, the existence of this attractor requires a value of disformal scale of the order $\Lambda \gtrsim H_0^{-2}$, where H_0 is the Hubble parameter of today, which is much larger than the scale relevant for spontaneous scalarization of neutron stars $\Lambda \sim R_s^2$ with $R_s (\sim 10^{-22} H_0^{-1})$ being the typical radius of these stars. The large values of Λ necessary for the attractor mechanism (i) suppresses spontaneous scalarization altogether inside neutron stars and (ii) induces ghost instabilities on scalar field fluctuations, thus preventing a resolution of the tension. We argue that the problem arises because our disformal coupling involves a dimensionful parameter.

I. INTRODUCTION

Einstein's theory of general relativity (GR) has passed all experimental tests to date, ranging from the weak-field, low-velocity regime from of the Solar System to the strong-field, low-velocity regime of binary pulsars [1]. With the advent of gravitational-wave astronomy a new frontier for testing GR has opened, providing us with the first glimpses of relativistic gravity in its strong-field, high-velocity, nonlinear regime and to directly probe the radiative properties of the theory [2–4].

To make the most out of this new arena for experimental gravity, it is important not only to confront the predictions of GR against observations, but also to embed it in a large *theory space*, obtained by relaxing one (or more) of the fundamental pillars of GR and then let experiments guide us towards the region of this theory space which is most favorable by observations [5].

In the vast landscape of extensions to GR, scalar-tensor theories stand out as one of the simplest and most well-motivated [6, 7]. In their simplest variant, they introduce a new scalar degree of freedom (φ), violating the fundamental pillar of GR that gravity is mediated by a single spin-2 field. A simple scalar-tensor theory can be described (in the Einstein-frame) by the action:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,(R+4X) + \int d^4x\sqrt{-\tilde{g}}\,\mathscr{L}_m\left[\tilde{g}_{\mu\nu},\Psi\right], \tag{1}$$

where $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are respectively the Einstein and Jordan frame metrics, $g \equiv \det(g_{\mu\nu})$ and $\tilde{g} \equiv \det(\tilde{g}_{\mu\nu})$, R is the Ricci scalar curvature associated with $g_{\mu\nu}$, $\kappa \equiv (8\pi G)/c^4$ where G is the gravitational constant in the Einstein frame and c the speed of light. Finally, $X \equiv -(1/2)g^{\mu\nu}\varphi_\mu\varphi_\nu$, where $\varphi_\mu \equiv \nabla_\mu\varphi$ is the covariant derivative of the scalar field associated the metric $g_{\mu\nu}$ and $\mathscr{L}_m$ is the Lagrangian density of matter fields Ψ which couple minimally to $\tilde{g}_{\mu\nu}$.

In Ref. [8], it was shown that these theories can not only pass Solar System constraints, but also allow for large deviations relative to GR in the strong-field regime found in neutron star (NS) interiors, through a process known as *spontaneous scalarization*. In the simplest case where the two metrics are related by a conformal transformation

$$\tilde{g}_{\mu\nu} = A(\varphi)^2 g_{\mu\nu}, \tag{2}$$

the scalar field can become tachyonic unstable if $(\ln A)_{,\varphi\varphi} < 0$, resulting in a NS which supports a nontrivial scalar field configuration [9, 10]. For an exponential coupling $A(\varphi) = \exp(\gamma_\alpha\varphi^2/2)$, spontaneous scalarization of static and spherically symmetric NSs can happen below the threshold $\gamma_\alpha \lesssim -4.35$ [10, 11], depending weakly on the NS equation of state (EOS) and fluid properties [12, 13]. On the experimental side, binary-pulsar observations (see e.g. [14–17]) have placed the bound $\gamma_\alpha \gtrsim -4.5$. These two results confine γ_α to a very limited range, in which the effects of scalarization on isolated NSs are bound to be small.

It was soon realized in [18, 19] that the parameter space region in which the tachyonic instability of the scalar field ($\gamma_\alpha < 0$) can happen for NSs, would also affect the scalar field's *cosmological evolution*, leading to large violations of present day Solar System constraints unless significant fine-tuning is imposed at the time of matter-radiation equality. Conversely, when $\gamma_\alpha > 0$, the GR solution with $\varphi = 0$ is an attractor of the theory, just after the matter-radiation equality time, making the theory consistent with present day observations, but then preventing spontaneous scalarization from happening.

While scalar-tensor theories which exhibit spontaneous scalarization can still be used as toy-models to explore strong-field gravity phenomenology, ideally one would like to find a model which reconciles its cosmology with present-day physics. Considerable effort has been placed on this issue recently. For instance, Ref. [20] considered higher-order polynomial corrections to the quadratic conformal coupling $\ln A = \gamma_\alpha \varphi^2/2 + \delta\varphi^4/4 + \cdots$ (with $\delta > 0$), where the higher-order terms make it possible to satisfy the Solar System constraints, but weakening considerably scalarization. Another possibility to solve this issue was presented in Ref. [21] where, during inflation, φ gets a larger effective mass through a coupling to the inflaton (ψ) of the form $g^2\psi^2\varphi^2$. This coupling suppresses exponentially the amplitude of φ by the end of inflation and thus realizes the otherwise *ad hoc* fine-tuning previously mentioned. Then, even if φ grows after inflation, its amplitude at present day could still be small enough to satisfy Solar System constraints.

Here we explore whether this issue can be resolved by introducing a *disformal coupling* between matter and the scalar field. More specifically, we consider a more general form for $\tilde{g}_{\mu\nu}$ [appearing in Eq. (1)], now related with $g_{\mu\nu}$ by a disformal transformation:

$$\tilde{g}_{\mu\nu} = A^2(\varphi)\left[g_{\mu\nu} + \Lambda B(\varphi)^2 \varphi_\mu \varphi_\nu\right], \qquad (3)$$

where Λ is a constant with dimensions of $(\text{length})^2$. Disformal transformations were originally introduced by Bekenstein as the most general metric transformation constructed from the metric $g_{\mu\nu}$ and the scalar field φ (and the first order derivative φ_μ) that respects causality and the weak equivalence principle [22]. They have been studied mainly in cosmology [23–34] and have also been shown to allow for spontaneous scalarization of NSs [35, 36]. Modern scalar-tensor theories such as Horndeski gravity [37–39] allow for conformal/disformal couplings to matter fields [22, 40, 41] and they also preserve the mathematical structure of the theory [41].

Is there any reason to expect that a disformal coupling could remedy the issue outlined above? Let us introduce the functions which control the interaction strength between scalar field and matter arising from the purely conformal (A) and purely disformal (B) terms of Eq. (3):

$$\alpha(\varphi) \equiv \frac{d\log A(\varphi)}{d\varphi}, \quad \beta(\varphi) \equiv \frac{d\log B(\varphi)}{d\varphi}. \qquad (4)$$

The value of $\beta(\varphi_0)$, where φ_0 is the cosmological value of the scalar field at the present time, is poorly constrained [42], because in the nonrelativistic regime, where the pressure is negligible and the scalar field is slowly-varying relative to cosmological time scales the disformal coupling becomes negligibly small.[1] However, since the scalar field φ varies on a cosmological time scale, the disformal interaction is expected to impact the cosmic expansion history, potentially as important as the conformal contribution. This opens the possibility that the disformal interaction *may quench* the growth of the scalar field in the regime $\gamma_\alpha < 0$ in which scalarization happens [35] and at the same time make the model consistent with Solar System constraints. Indeed, as we will show later, the presence of the simplest disformal coupling $B = 1$ is sufficient for the existence of a late-time attractor mechanism to GR, in which $\varphi = 0$, However, the presence of the GR attractor requires very large magnitudes of disformal coupling $\Lambda < 0$ – *so large that scalar field fluctuations suffer from ghost instability.*

The existence of the late-time attractor mechanism can be qualitatively understood as follows. From Eq. (3), assuming $\varphi \sim 1$, $B \sim 1$, $\varphi_\mu \sim \varphi/R_s$ (with $R_s \sim 10$ km being the typical NS radius) the disformal coupling could be as important as the conformal coupling in NSs when $\Lambda \sim 100\,\text{km}^2 = 10^{12}\,\text{cm}^2$ [35]. On the other hand, as our quantitative analysis will show, the effective force which drives the cosmological evolution of the scalar field in the presence of disformal term [see Eq. (51) for the precise definition] is given by $-M_{\text{eff}}^2\varphi$, where $M_{\text{eff}}^2 \sim \gamma_\alpha H^2/(1 + k\Lambda H^2)$ is the effective mass of φ, k is a dimensionless constant of $\mathcal{O}(1)$, and H is the Hubble expansion rate at the given moment of time. Starting with an initial condition $\dot{\varphi} = 0$ (where $\dot{\varphi}$ is the derivative of φ with respect to the cosmological proper time) the amplitude of φ remains constant during the matter-dominated phase when $M_{\text{eff}}/H \ll 1$, When $M_{\text{eff}}/H \sim 1$, the effective force starts to act of φ, driving the scalar field towards $\varphi = 0$ for $\gamma_\alpha < 0$ as long as $\Lambda < 0$. The existence of the GR attractor for $\gamma_\alpha < 0$ requires that φ starts to feel the effective force in the vicinity of present day, $M_{\text{eff},0}/H_0 \sim 1$, where $H_0 \sim 10^{-28}\,\text{cm}^{-1}$ is the Hubble parameter of today, and therefore $\Lambda \sim -H_0^{-2} \sim -10^{56}\,\text{cm}^2$. Thus, the magnitude of Λ which is necessary for the existence of the GR attractor is larger than the Λ for scalarization of NSs by 44 order of magnitude, a prohibitively large value for the theory to even allow for the existence of scalarized relativistic stars [35].

In the rest of this work we present the details which led to these conclusions. In Sec. II we present the theory's field equations and derive the equations which describe cosmology in this theory. In Sec. III we study analytically the existence of GR-attractor solutions when $\gamma_\alpha < 0$ and verify their existence numerically in Sec. IV, also relating our results with spontaneous scalarization of NSs. Finally, in Sec. V we present our conclusions. Hereafter we use geometrical units where $c = G = 1$.

[1] When the scalar field time dependence is negligible, the disformal term contributes only past the second post-Newtonian (PN) order and therefore does not affect the parametrized post-Newtonian (PPN) parameters γ_{PPN} and β_{PPN} which are identical to those of 'conformal' scalar-tensor gravity.

II. COSMOLOGICAL EQUATIONS

Let us start by describing the field equations of the theory given by the action (1) with the disformal coupling (3). Variation of the action with respect to the Einstein frame metric $g_{\mu\nu}$ results in the Einstein field equations

$$G^{\mu\nu} = \kappa \left(T^{\mu\nu}_{(m)} + T^{\mu\nu}_{(\varphi)} \right), \tag{5}$$

where the energy-momentum tensors of matter fields Ψ and scalar field φ are given by

$$T^{\mu\nu}_{(m)} \equiv \frac{2}{\sqrt{-g}} \frac{\delta \left(\sqrt{-\tilde{g}} \mathcal{L}_m \left[\tilde{g}(\varphi), \Psi \right] \right)}{\delta g_{\mu\nu}}, \tag{6}$$

and

$$\begin{aligned} T^{\mu\nu}_{(\varphi)} &\equiv \frac{4}{\kappa} \frac{1}{\sqrt{-g}} \frac{\delta \left(\sqrt{-g}\, X \right)}{\delta g_{\mu\nu}} \\ &= \frac{2}{\kappa} \left(\varphi^\mu \varphi^\nu - \frac{1}{2} g^{\mu\nu} \varphi^\alpha \varphi_\alpha \right), \end{aligned} \tag{7}$$

respectively, where $\varphi^\mu \equiv g^{\mu\nu}\varphi_\nu$.

Variation of the action (1) with respect to φ results in the scalar field equation of motion

$$\Box \varphi = (\kappa/2) \mathcal{Q}, \tag{8}$$

where the function $\mathcal{Q}$ characterizes the strength of the coupling of matter to the scalar field [35]

$$\begin{aligned} \mathcal{Q} \equiv & -\alpha(\varphi) T_{(m)} + \Lambda \nabla_\rho \left(B(\varphi)^2 T^{\rho\sigma}_{(m)} \varphi_\sigma \right) \\ & - \Lambda B(\varphi)^2 \left[\alpha(\varphi) + \beta(\varphi) \right] T^{\rho\sigma}_{(m)} \varphi_\rho \varphi_\sigma, \end{aligned} \tag{9}$$

where $T_{(m)} \equiv g^{\rho\sigma} T_{(m)\rho\sigma}$ is the trace of $T_{(m)\rho\sigma}$, and $\alpha(\varphi)$ and $\beta(\varphi)$ were defined in Eq. (4). Observe that terms proportional to Λ in (9) are *nonzero even for the trivial choice* $B = 1$. By taking the divergence of (5), employing the contracted Bianchi identity $\nabla_\rho G^{\rho\sigma} = 0$, and using the scalar field equation of motion (8), we obtain

$$\nabla_\rho T^{\rho\sigma}_{(m)} = -\nabla_\rho T^{\rho\sigma}_{(\varphi)} = -\mathcal{Q} \varphi^\sigma. \tag{10}$$

Therefore, the coupling strength $\mathcal{Q}$ can be rewritten as

$$\mathcal{Q} = \Lambda B(\varphi)^2 \left(\nabla_\rho T^{\rho\sigma}_{(m)} \right) \varphi_\sigma + \mathcal{Y}, \tag{11}$$

where we have introduced

$$\begin{aligned} \mathcal{Y} \equiv \Lambda B(\varphi)^2 & \left\{ \left[\beta(\varphi) - \alpha(\varphi) \right] T^{\rho\sigma}_{(m)} \varphi_\rho \varphi_\sigma + T^{\rho\sigma}_{(m)} \varphi_{\rho\sigma} \right\} \\ & - \alpha(\varphi) T_{(m)}. \end{aligned} \tag{12}$$

Multiplying Eq. (10) by φ_σ and solving it with respect to $(\nabla_\rho T^{\rho\sigma}_{(m)})\varphi_\sigma$, we obtain

$$\chi (\nabla_\rho T^{\rho\sigma}_{(m)}) \varphi_\sigma = 2X\mathcal{Y}, \quad \chi \equiv 1 - 2\Lambda B(\varphi)^2 X. \tag{13}$$

Then, substituting Eq. (13) in (11), using $\mathcal{Q} = \mathcal{Y}/\chi$, and finally eliminating $\mathcal{Q}$ from (8), we obtain the reduced scalar field equation of motion

$$\begin{aligned} \Box\varphi = \frac{\kappa}{2\chi(X,\varphi)} & \Big[\!\Big[\Lambda B(\varphi)^2 \Big\{ \left[\beta(\varphi) - \alpha(\varphi) \right] T^{\rho\sigma}_{(m)} \varphi_\rho \varphi_\sigma \\ & + T^{\rho\sigma}_{(m)} \varphi_{\rho\sigma} \Big\} - \alpha(\varphi) T_{(m)} \Big]\!\Big]. \end{aligned} \tag{14}$$

We consider the spatially flat Friedmann-Lemêitre-Roberton-Walker (FLRW) spacetime in the Einstein frame

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \tag{15}$$

where t, x^i are the coordinates of the time and the three-dimensional space and assume that the scalar field is only a function of time, i.e. $\varphi = \varphi(t)$ [18]. The Jordan-frame metric is given by the FLRW line element above by replacing the proper time $t \to \tilde{t}$ and the scale factor $a \to \tilde{a}$. These quantities are related as $d\tilde{t} \equiv A\sqrt{\chi} dt$ and $\tilde{a} \equiv Aa$.

We will describe matter by a multi-component perfect fluid, with energy-momentum tensor in the Einstein and Jordan frames denoted as ${T_{(m)}}^\mu{}_\nu = \sum_a {T_{(m)a}}^\mu{}_\nu$ and ${\tilde{T}_{(m)}}{}^\mu{}_\nu = \sum_a {\tilde{T}_{(m)a}}{}^\mu{}_\nu$, respectively. The fluid variables [pressure (p) and energy density (ρ)] in the two frames are related by

$$\tilde{\rho}_a = \frac{\sqrt{\chi}}{A^4} \rho_a, \quad \tilde{p}_a = \frac{1}{A^4 \sqrt{\chi}} p_a, \tag{16}$$

where, from Eq. (13), $\chi = 1 - \Lambda B^2 \dot{\varphi}^2$, with an overdot denoting derivatives with respect to t. The EOS parameters of the (a)-th component of the fluid in the Jordan and Einstein frames are defined by $\tilde{w}_a \equiv \tilde{p}_a/\tilde{\rho}_a$ and $w_a \equiv p_a/\rho_a$ respectively and are related by

$$w_a = \chi\, \tilde{w}_a. \tag{17}$$

Similarly, the EOS parameter for the whole fluid is defined as $\tilde{w} = \tilde{p}/\tilde{\rho} = \sum_a \tilde{p}_a / \sum_a \tilde{\rho}_a$ and $w = p/\rho = \sum_a p_a / \sum_a \rho_a$, which are also related by $\tilde{w} = w/\chi$. The physically measured EOS parameter is that of the Jordan frame and thus we should specify e.g. $\tilde{w}_a = 0,\ 1/3,\ -1$ to describe matter (i.e., dust), radiation, and cosmological constant, respectively. Here, by cosmological constant, we also include the equivalent vacuum energy.

Using Eq. (15), we find that the (t,t)-component of the gravitational equations in the Einstein frame (5) reduces to

$$H^2 = \frac{\kappa}{3}\rho + \frac{1}{3}\dot{\varphi}^2 = \frac{\kappa}{3}\sum_a \rho_a + \frac{1}{3}\dot{\varphi}^2, \tag{18}$$

where we have defined the Hubble parameter in the Einstein frame $H \equiv \dot{a}/a$. From Eq. (10), the energy conservation law of the (a)-th component yields

$$\dot{\rho}_a + 3H\left(\rho_a + p_a\right) = \frac{\mathcal{Y}_a}{\chi}\dot{\varphi}, \tag{19a}$$

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\kappa}{2}\frac{\mathcal{Y}}{\chi}, \tag{19b}$$

where $\mathcal{Y} = \sum_a \mathcal{Y}_a$ and

$$\mathcal{Y}_a \equiv \Lambda B^2 \left[(\beta - \alpha)\rho_a \dot{\varphi}^2 + \rho_a \ddot{\varphi} - 3H\dot{\varphi} p_a\right] + \alpha(\rho_a - 3p_a). \quad (20)$$

It is convenient to work with a *rescaled time coordinate* $d\tau \equiv H dt$ [18], where $\tau = 0$ corresponds to the matter-radiation equality and $\tau = \tau_0$ denotes the present day, which can be integrated as $a(\tau) = a_0 \exp(\tau - \tau_0)$. Hence τ describes the cosmic e-folding time, where a_0 is the size of the Universe today. In terms of τ, the Friedmann equation (18) becomes

$$H^2 = \frac{\kappa\rho}{3 - \varphi'^2}. \quad (21)$$

We can then use Eqs. (17) and (21) and simplify Eqs. (19) (recast in terms of τ) to the forms

$$\rho_a' + 3\rho_a(1 + \chi\tilde{w}_a) = \frac{\mathcal{Y}_a}{\chi}\varphi', \quad (22a)$$

$$\left[1 + \frac{3}{2}\lambda B^2 \rho \frac{1 - \varphi'^2}{3 - \varphi'^2}\right] \frac{2\varphi''}{3 - \varphi'^2} + \left\{1 - \chi\tilde{w} - \frac{\lambda B^2 \rho}{2(3 - \varphi'^2)}\left[3(1 + 3\chi\tilde{w}) + 3(1 - \chi\tilde{w})\varphi'^2 + 2(\alpha - \beta)\varphi'\right]\right\}\varphi' = -\alpha(1 - 3\chi\tilde{w}), \quad (22b)$$

where $\lambda \equiv \kappa\Lambda$, and

$$\mathcal{Y}_a = \rho_a \left[\!\left[\frac{\lambda B^2 \rho}{3 - \varphi'^2}\left\{(\beta - \alpha)\varphi'^2 + \varphi'' - \frac{1}{2}\left[3(1 + 3\chi\tilde{w}_a) + (1 - \chi\tilde{w}_a)\varphi'^2\right]\varphi'\right\} + \alpha(1 - 3\chi\tilde{w}_a)\right]\!\right], \quad (23)$$

where primes indicate derivatives with respect to τ and now

$$\chi = 1 - (\lambda\rho B^2 \varphi'^2)/(3 - \varphi'^2). \quad (24)$$

We note that in the radiation-dominated universe where $\tilde{w}_r = 1/3$ and $\rho_r \gg \rho_a$ ($a \neq r$), a nonzero constant constant scalar field φ_* is a solution of the equations of motion (see Sec. III). Eqs. (22) are our main results from this section and whose solutions will be studied Secs. III and IV. In the particular limit of purely conformal coupling ($\lambda = 0$) these equation reduce to those of Refs. [9, 18].

Before proceeding, we observe that the Hubble parameters in both frames are related by

$$\tilde{H} = \frac{1 + \alpha(\varphi)\varphi'}{A\sqrt{\chi}} H. \quad (25)$$

Using Eq. (16), the Friedmann equation Eq. (21) can be rewritten as

$$\tilde{H}^2 = \frac{[1 + \alpha(\varphi)\varphi']^2}{3 - \varphi'^2} \frac{A(\varphi)^2}{\chi^{3/2}} \kappa\tilde{\rho}. \quad (26)$$

Using that Newton's constant in the Jordan frame at present day is

$$G_{\text{eff}} = \left[1 + \alpha(\varphi_0)^2\right] A(\varphi_0)^2, \quad (27)$$

where φ_0 is the present day value of the scalar field, the Friedmann equation can be written as

$$\tilde{H}^2 = \frac{3[1 + \alpha(\varphi)\varphi']^2}{(3 - \varphi'^2)\chi^{3/2}} \frac{1}{1 + \alpha(\varphi_0)^2} \left[\frac{A(\varphi)}{A(\varphi_0)}\right]^2 H_{\text{GR}}^2, \quad (28)$$

where H_{GR} is the expansion rate in the standard cosmology in GR. Assuming that φ remains constant[2] during Big-Bang nucleosynthesis (BBN), say $\varphi = \varphi_R$, the ratio the between Jordan-frame Hubble rates $\zeta(\tau) \equiv \tilde{H}/H_{\text{GR}}$, reduces to

$$\zeta(\tau_R) = \frac{1}{\sqrt{1 + \alpha(\varphi_0)^2}} \frac{A(\varphi_R)}{A(\varphi_0)}. \quad (29)$$

In order to be consistent with the observational BBN data, $\zeta(\tau_R)$ has to satisfy $|1 - \zeta(\tau_R)| \leq 1/8$ [43], which combined with the Solar System constraint $\alpha(\varphi_0) \ll 1$, gives

$$|1 - A(\varphi_R)/A(\varphi_0)| \leq 1/8. \quad (30)$$

III. THE GR ATTRACTOR

So far we have worked with a general scalar-tensor theory, keeping A and B as free functions. In this section, we focus on model which supports spontaneous scalarization studied in [35] and consider

$$A(\varphi) = e^{\gamma_\alpha \varphi^2/2}, \qquad B(\varphi) = e^{\gamma_\beta \varphi^2/2}, \quad (31)$$

and examine under which conditions Eqs. (22) admit a cosmological GR attractor, which forces the scalar field to evolve towards $\varphi = 0$. As we will see in this section, the choice of γ_β does not affect the existence of the GR attractor and their stability at all.

To gain some understanding on the existence of this attractor, let us first consider the simplest case of a single component of the fluid and a fixed scalar field $\varphi = \varphi_* =$ const., in which Eqs. (22) reduce to

$$\rho = \rho_* e^{-3(1+\tilde{w})\tau}, \qquad \gamma_\alpha \varphi_* (1 - 3\tilde{w}) = 0. \quad (32)$$

For $\tilde{w} \neq 1/3$, the existence of the GR solution requires $\varphi_* = 0$, while for $\tilde{w} = 1/3$ (i.e. radiation) an arbitrary value of φ_* is a solution.

Since $\tilde{w} \neq 1/3$ in general, let us consider a small homogeneous perturbation with respect to the GR solution

[2] In Sec. III we will show that an arbitrary constant φ is a solution in the radiation-dominated era $\tilde{w} = 1/3$ even in presence of the disformal coupling.

$\rho = \rho_* \exp[-3(1+\tilde{w})\tau] + \delta\rho(\tau)$ and $\varphi = \varphi_* + \delta\varphi(\tau)$. Since $\delta\rho \propto \exp[-3(1+\tilde{w})\tau]$, the density perturbation behaves as the background solution and can therefore be absorbed into it. The perturbation for the scalar field satisfies

$$\begin{aligned}&\frac{1}{3}\left(2+\lambda\rho_* e^{-3(1+\tilde{w})\tau}\right)\delta\varphi'' \\ &+\left[1-\tilde{w}-\rho_*\left(\lambda/2\right)(1+3\tilde{w})e^{-3(1+\tilde{w})\tau}\right]\delta\varphi' \\ &+\gamma_\alpha\left(1-3\tilde{w}\right)\delta\varphi = 0. \end{aligned} \tag{33}$$

A late-attractor to GR exists if the solution to Eq. (33) decays with time.

In the limit of a purely conformal coupling ($\lambda = 0$), we find that the solution to $\delta\varphi$ is given by

$$\delta\varphi \propto \exp\left[-\frac{3\tau}{4}\left(1-\tilde{w}\pm\sqrt{(1-\tilde{w})^2-\frac{8}{3}\gamma_\alpha(1-3\tilde{w})}\right)\right]. \tag{34}$$

Assuming that $1-\tilde{w} > 0$, for $\gamma_\alpha(1-3\tilde{w}) > 0$ both the solutions of Eq. (34) decay with time $\tau > 0$, while for $\gamma_\alpha(1-3\tilde{w}) < 0$ the 'minus'-branch solution grows with time. Thus, in the former case, the GR solution is an attractor. For $\tilde{w} < 1/3$, the condition for an GR attractor reduces to $\gamma_\alpha > 0$, consistent with the findings of [18, 19].

Now let us include the disformal coupling ($\lambda \neq 0$). For $\tilde{w} > -1$, the disformal contribution decays with time τ, due to the exponentials appearing in Eq. (33). Hence, the disformal contribution is negligible with respect to the conformal one, and $\delta\varphi$ can be approximately given by Eq. (34) at late times. Consequently, the condition for the GR solution to be an attractor is the same as in the purely conformal case. On the other hand, for $\tilde{w} \leq -1$, the disformal contribution is as important as the conformal one. More specifically, for a cosmological constant ($\tilde{w} = -1$) we have the equation

$$(2+\lambda\rho_*)\left(\delta\varphi''+3\delta\varphi'\right)+12\gamma_\alpha\delta\varphi = 0, \tag{35}$$

which can be solved analytically:

$$\delta\varphi \propto \exp\left[-\frac{3\tau}{2}\left(1\pm\sqrt{1-\frac{4\bar{M}^2}{9}}\right)\right], \tag{36}$$

where $\bar{M}^2 = 12\gamma_\alpha/(2+\lambda\rho_*)$. For $\bar{M}^2 > 0$, the solution decays with $\tau > 0$, and then the GR solution (i.e. a de Sitter Universe) is the late-time attractor, if $\gamma_\alpha > 0$ and $2+\lambda\rho_* > 0$, or if $\gamma_\alpha < 0$ and $2+\lambda\rho_* < 0$. Thus, in the latter case, the theory admits the existence of the GR attractor even if $\gamma_\alpha < 0$. The stability of the GR attractor will be discussed in Appendix A. We note that the negative sign of the kinetic term signals the appearance of the ghost instability. However, we expect that during the matter-dominated phase with the vanishing pressure $p_* = 0$ both the gradient term and the effective mass term in the equation for the scalar field fluctuations (A3) are suppressed by the large factor $|2+\lambda\rho_*\exp[-3(1+\tilde{w})\tau]| \gg 1$ (see Sec. IV), and hence the growth of instability would also be strongly suppressed and consequently proceed slowly compared to the cosmological time scales. During the dark energy (de Sitter) phase, both the gradient and kinetic terms of the perturbations in the equation for the scalar field fluctuations (A2) flip signs (see Appendix A) and hence there would be no exponential growth of the scalar field fluctuations. However, the issue of the ghost instability may be significant at present day and we will come back to it in Sec. IV.

These conclusions can easily be extended for a multi-component fluid. For the constant scalar field, $\varphi = \varphi_*$, the energy equation for the (a)-th component of the fluid and the scalar field equation of motion are given by

$$\rho_a' + 3(1+w_a)\rho_a = 0, \qquad \gamma_\alpha\varphi_*\left(1-3w\right) = 0. \tag{37}$$

where

$$1-3w = \frac{\sum_a(1-3w_a)\rho_a}{\sum_a\rho_a}. \tag{38}$$

Thus, unless $\sum_a(3w_a-1)\rho_a = 0$ at all the moments of time, GR solution is realized only for $\varphi_* = 0$. However, in the radiation-dominated universe, the second equation in Eq. (37) can be approximated satisfied for a constant scalar field $\varphi = \varphi_* \neq 0$.

If the Universe is evolving towards the GR attractor, the theory can (in principle) satisfy all the experimental bounds on the parametrized post-Newtonian (PPN) parameters of today [1]:

$$\gamma_{\rm PPN} - 1 < 2.3\times10^{-5}, \tag{39a}$$
$$\beta_{\rm PPN} - 1 < 8\times10^{-5}, \tag{39b}$$

where

$$\gamma_{\rm PPN} \equiv \frac{1-\alpha(\varphi_0)^2}{1+\alpha(\varphi_0)^2}, \tag{40a}$$

$$\beta_{\rm PPN} - 1 \equiv \frac{\alpha_2(\varphi_0)}{2}\frac{\alpha(\varphi_0)^2}{\left[1+\alpha(\varphi_0)^2\right]^2}, \tag{40b}$$

and φ_0, recall, is present-day cosmological background value of the scalar field. We also introduced

$$\alpha_2(\varphi_0) \equiv \frac{\partial^2\ln A(\varphi)}{\partial\varphi^2}\Big|_{\varphi=\varphi_0}. \tag{41}$$

In deriving Eqs. (40a) and (40b), we have followed the standard procedure for calculating the PPN parameters in our local Universe, and ignored the cosmological time dependence of φ, since the cosmological scalar field varies with the cosmological time scale 10^{10} yr, while the weak-field tests of gravity are done within the light-crossing time in the Solar System $30\,{\rm au}/c \sim 5\times10^{-4}$ yr, where 30 au is the approximated orbital radius of Neptune.

Thus, the corrections from the time dependence of φ are suppressed by some powers of the ratio of these time scales. Within these approximations, corrections due to a nonzero disformal coupling appear only via pressure effects, which are subdominant in the weak gravity regime.

IV. COSMOLOGICAL VALUE OF THE SCALAR FIELD AND SPONTANEOUS SCALARIZATION

With intuition built on the existence of GR-attracted solutions of Eqs. (22), we now numerically evolve the scalar field in a realistic cosmology.

We assume that after inflation that the cosmological expansion is driven by the three components of the fluid in turns, namely: radiation ρ_r, matter ρ_m, and the cosmological constant ρ_v.

In the radiation-dominated phase just after inflation during which $\rho_r \gg \rho_m, \rho_v$, the cosmological expansion can be very well approximated by that in the standard cosmology based on GR with a fixed amplitude of the scalar field φ_* which in general is nonzero. Strictly speaking, since even in the radiation-dominated phase there is still very small contribution of nonrelativistic particles, the force on the scalar field in the right-hand side of Eq. (22b) does not vanish and φ would evolve in time very slowly. Nevertheless, $\varphi \approx \varphi_*$ is a good approximation during the radiation-dominated phase.

Next, ρ_m eventually catches up with ρ_r and $\rho_m = \rho_r$ at the matter-radiation equality defined to happen at $\tau = 0$. As $\rho_m > \rho_r$, the scalar field φ starts to roll away from $\varphi = \varphi_*$ and the subsequent dynamics requires the numerical integration of Eqs. (22). Since $\varphi = \mathcal{O}(1)$ during most of the evolution, as long as $\gamma_\beta = \mathcal{O}(1)$ the γ_β dependence does not become significant for cosmological dynamics. Thus, we set $\gamma_\beta = 0$ in the rest of the paper, although nonzero values may be important in other contexts [24, 25, 35, 44].

To do our numerical integration, we start from $\tau = 0$ (the matter-radiation equality) and we neglect ρ_r in the matter-dominated phase $\tau > 0$, reducing our dynamical variables to φ, ρ_m, and ρ_v. From Eqs. (22) we obtain the set of the evolution equations:

$$\rho_m' + 3\rho_m = (\mathcal{Y}_m/\chi)\varphi', \tag{42}$$

$$\rho_v' + 3\rho_v(1-\chi) = (\mathcal{Y}_v/\chi)\varphi', \tag{43}$$

$$\begin{aligned}&\left[1 + \frac{3\lambda\rho}{2}\frac{1-\varphi'^2}{3-\varphi'^2}\right]\frac{2\varphi''}{3-\varphi'^2} \\ &+\left\{1 - \chi\tilde{w} - \frac{\lambda\rho}{2(3-\varphi'^2)}\left[3(1+3\chi\tilde{w}) + 3(1-\chi\tilde{w})\varphi'^2\right.\right. \\ &\left.\left.+2\gamma_\alpha\varphi\varphi'\right]\right\}\varphi' = -\gamma_\alpha\varphi(1-3\chi\tilde{w}),\end{aligned} \tag{44}$$

where

$$\begin{aligned}\mathcal{Y}_m = \rho_m &\left\{\frac{\lambda\rho}{3-\varphi'^2}\left[-\gamma_\alpha\varphi\varphi'^2 + \varphi'' - \frac{1}{2}\left(3+\varphi'^2\right)\varphi'\right]\right. \\ &\left.+\gamma_\alpha\varphi\right\},\end{aligned} \tag{45}$$

$$\begin{aligned}\mathcal{Y}_v = \rho_v &\left[\!\!\left[\frac{\lambda\rho}{3-\varphi'^2}\left\{-\gamma_\alpha\varphi\varphi'^2 + \varphi''\right.\right.\right. \\ &\left.\left.\left.-\frac{1}{2}\left[3(1-3\chi) + (1+\chi)\varphi'^2\right]\varphi'\right\} + \gamma_\alpha(1+3\chi)\varphi\right]\!\!\right],\end{aligned} \tag{46}$$

$$\chi = 1 - \frac{\lambda\rho\varphi'^2}{3-\varphi'^2}, \tag{47}$$

with

$$\rho = \rho_m + \rho_v, \qquad \tilde{w} = \frac{\sum_a \tilde{w}_a\rho_a}{\sum_a \rho_a} = -\frac{\rho_v}{\rho_m + \rho_v}. \tag{48}$$

As initial conditions, we impose

$$\rho_m(0) = \rho_{m,e}, \quad \rho_v(0) = \rho_{v,e}, \tag{49}$$

$$\varphi(0) = \varphi_0, \qquad \varphi'(0) = 0, \tag{50}$$

where the subscript "e" denotes the quantities evaluated at the matter-radiation equality. It is convenient to identify an *effective force* due to the disformal contribution $\mathscr{F}$ that acts on φ. This force is given by the right-hand side of (44) divided by $1 + (3\lambda/2)(1-\varphi'^2)/(3-\varphi'^2)$ i.e.:

$$\mathscr{F} \equiv -\frac{\gamma_\alpha(1-3\chi\tilde{w})}{1 + (3\lambda\rho/2)\,(1-\varphi'^2)(3-\varphi'^2)^{-1}}\,\varphi. \tag{51}$$

In standard cosmology in which $\varphi = 0$, matter and radiation energy densities evolve according to $\rho_m(\tau) = \rho_{m,e}\exp(-3\tau)$ and $\rho_r(\tau) = \rho_{m,e}\exp(-4\tau)$, where we have used the definition $\rho_{m,e} = \rho_{r,e}$. We can then relate the proper time τ with ratio between matter and radiation density as $\tau = \ln(\rho_m/\rho_r)$. At present day ($\tau \equiv \tau_0$) the ratio $\rho_m(\tau_0)/\rho_r(\tau_0)$ is approximately 3450, and hence $\tau_0 = \ln(3450) \approx 8.15$. Moreover, $\rho_{v,0} \approx 0.69\rho_{\rm crit}$ and $\rho_{m,0} \approx 0.31\rho_{\rm crit}$, where $\rho_{\rm crit}(\approx 1.88\times10^{-29}\,h^2\,{\rm g/cm^3})$ is the critical energy density of today in standard cosmology. If the cosmological evolution follows that of standard cosmology, we can rewrite the initial conditions Eq. (49) as:

$$\begin{aligned}&\rho_m(0) \approx \rho_{m,0}e^{3\tau_0} \approx 0.31\,\rho_{\rm crit}e^{3\tau_0} \approx 1.27\times10^{10}\rho_{\rm crit}, \\ &\rho_v(0) \approx 0.69\rho_{\rm crit},\end{aligned} \tag{52}$$

which we will also use for our integration in scalar-tensor theory.

In scalar-tensor cosmology, the ratio ρ_m/ρ_r evolves differently from that in standard cosmology. Since the coupling between the scalar field and radiation is negligible whenever $\varphi' \ll 1$, the evolution of the radiation energy density follows closely that of the standard cosmology $\rho_r(\tau) = \rho_{m,e}\exp(-4\tau)$. On the other hand, the evolution of the matter energy density $\rho_m(\tau)$ is in general nontrivial, even in the presence of only the conformal coupling. Thus, in the presence of the nontrivial conformal/disformal couplings to the scalar field, matter and radiation energy densities at the present day τ_0 have to satisfy

$$\frac{\rho_m(\tau_0)}{\rho_r(\tau_0)} \approx \frac{\rho_m(\tau_0)}{\rho_{m,e}e^{-4\tau_0}} = \frac{\rho_{m,0}}{\rho_{r,0}} \approx 3450, \tag{53}$$

which we will use to define τ_0 in scalar-tensor cosmology.

If we assume that $\varphi_0 \ll 1$ at the present day [consistent with the bounds (39)] and using the identification of $\varphi_R = \varphi(0)$, the BBN constraint (30) can be rewritten as $7/8 \le \exp[(1/2)\gamma_\alpha\varphi(0)^2] \le 9/8$. For $\gamma_\alpha > 0$ this yields

$$0 < \gamma_\alpha\varphi(0)^2 \lesssim 0.2355, \tag{54}$$

while for $\gamma_\alpha < 0$ we have

$$-0.2670 \lesssim \gamma_\alpha\varphi(0)^2 < 0, \tag{55}$$

which can be used to fix a range of allowed scalar field amplitudes $\varphi(0)$ consistent with BBN constraints.

Before studying the impact of the disformal coupling, we first consider the case of the pure conformal coupling ($\lambda = 0$). In Fig. 1, we show the results of integrating the equations for $\gamma_\alpha = 5$ (dashed curves) and $\gamma_\alpha = -5$ (solid curves), using initial condition $\varphi(0) = 0.5$ which satisfies Eq. (54) in both examples. In the top-left panel we show the phase space portrait of $\varphi(\tau)$. For $\gamma_\alpha < 0$, we see that the scalar field is attracted towards GR ($\varphi = 0$), while for $\gamma_\alpha > 0$ the scalar field drifts away from GR and asymptotes to infinity with constant 'velocity' ≈ 1.6. The top-right panel shows the evolution for ρ_m/ρ_v. For $\gamma_\alpha < 0$, the present day observed density $\rho_m/\rho_v \approx 0.455$ is reached at $\tau = 8.246$, which is indicated by the circle in the top-right panel and the vertical lines in the other panels. For $\gamma_\alpha > 0$, ρ_m/ρ_v evolves inconsistently with observations. The contrasting behavior of the scalar field, depending on the sign of γ_α, also reflects on the evolution of the PPN parameters $\beta_{\rm PPN}$ and $\gamma_{\rm PPN}$. As shown in the bottom row, for $\gamma_\alpha < 0$ the value of these parameters evolves towards being consistent with present day PPN constraints (39), while for $\gamma_\alpha > 0$ these constraints are strongly violated. These results are consistent with those of [18, 19].

We now consider how the inclusion of the disformal coupling changes this picture. In the case of a pure disformal coupling ($\gamma_\alpha = 0$), we observe that all the terms in the scalar field equation of motion Eq. (44) are proportional to the derivatives φ' or φ''. Therefore, for the initial condition $\varphi'(0) = 0$, φ remains constant, and consequently the cosmological evolution will be the same as that in GR.

Now let us consider the more interesting case in which both conformal and disformal terms contribute to the scalar field dynamics. More specifically, we want to examine if this case now admits an attractor to GR when $\gamma_\alpha < 0$. To do this, it is convenient to use the effective

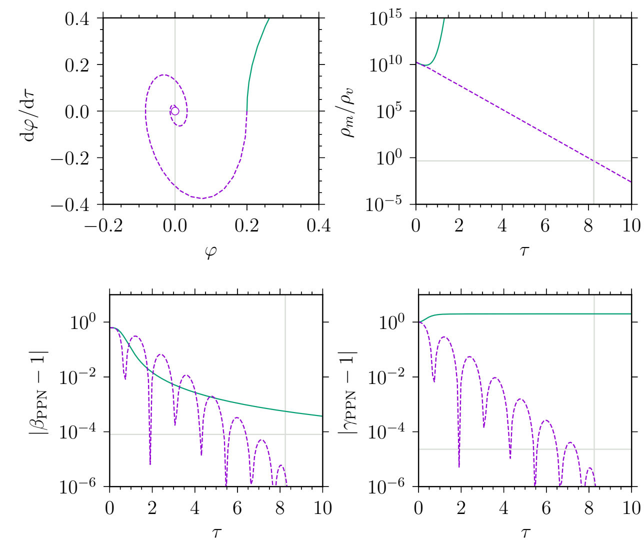

FIG. 1. Scalar-tensor cosmology with only the conformal coupling. In all panels, the solid curves correspond to $\gamma_\alpha = -5$, while the dashed curves to $\gamma_\alpha = 5$. In both cases we used $\varphi(0) = 0.2$ as initial condition, which satisfies Eq. (54). *Top-left:* the phase space portrait of the scalar field's evolution (left-panel) clearly shows the attractor mechanism in action for $\gamma_\alpha > 0$. For $\gamma_\alpha < 0$ the scalar field asymptotes to infinity with constant 'velocity' $\varphi' \approx 1.6$. *Top-right:* the evolution of the ratio between matter and cosmological constant densities. For $\gamma_\alpha > 0$, the present day $\rho_m/\rho_v = 0.455$ ratio happens around $\tau = 8.246$ which is indicated by the circle in the top-left panel and by the vertical lines in the other panels. For $\gamma_\alpha < 0$, the ratio evolves to dramatically violate observations. *Bottom row:* the evolution of the PPN parameters $\beta_{\rm PPN}$ (left) and $\gamma_{\rm PPN}$ (right). For $\gamma_\alpha > 0$, scalar field evolves as to satisfy the PPN constraints (39) (horizontal lines), while for $\gamma_\alpha < 0$ both constraints are violated at present day and in future.

force $\mathscr{F}$ defined in Eq. (51). Since $\tilde{w} \le 1$ and $|\varphi'| < 1$, we see that the effective force can be attractive as long as $\lambda < 0$ and $|\lambda|\rho > 1$ even when $\gamma_\alpha < 0$. When $|\lambda|\rho \gg 1$, $\mathscr{F}$ becomes of order $\mathcal{O}(\gamma_\alpha\varphi/(|\lambda|\rho)) \ll \gamma_\alpha\varphi$, and hence the effective force on the scalar field is suppressed in comparison with the (pure) conformal case and φ stays at the nearly constant amplitude $\varphi(0)$.

When $|\lambda|\rho \sim 1$, the effective force starts to be enhanced and φ is attracted towards $\varphi = 0$. However, in the case that $|\lambda|\rho \sim 1$ is reached during the matter dominated phase, since $\rho \propto \exp(-3\tau)$ decreases fast, the effective force term Eq. (51) changes sign within the short period and φ experiences a runaway growth after passing through $\varphi = 0$. On the other hand, in the case that $|\lambda|\rho \sim 1$ is reached during the cosmological constant dominated phase, the effective force term (51) does not change sign, since $\lambda\rho$ is approximately constant and $(1-\varphi'^2)(3-\varphi'^2)^{-1}$ varies only mildly, without changing its sign. Thus, the scalar field gradually approaches zero. However, since at the present day $\rho_v \sim \rho_m$, in general $\varphi(0) \sim 1$, which would easily be conflict with the

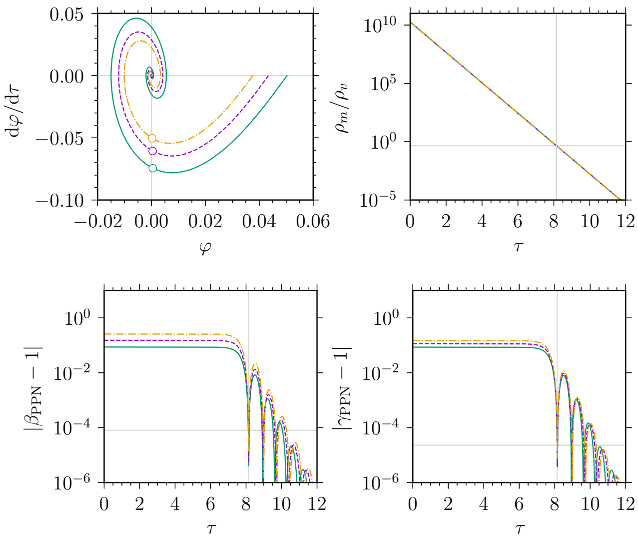

FIG. 2. Scalar-tensor cosmology with disformal coupling. In all panels, the solid curves corresponds $(\lambda_x, \gamma_\alpha) = (\lambda_{0.6}, -4.22)$, the dashed curves $(\lambda_{0.7}, -5.68)$ to and the dot-dashed curves to $(\lambda_{0.8}, -7.54)$. We used the initial conditions $\varphi(0) = 0.0503, 0.0434, 0.0376$, respectively, which satisfy the BBN constraint Eq. (55). The panels are similar to those of Fig. (1), but here we focus only on examples in which attractor mechanisms happens and therefore focus on $\lambda < 0$. *Top-left:* we show the phase space portrait of the scalar field. For γ_α we see that the attract towards GR persists, while it *can now also occur* for $\gamma_\alpha < 0$. *Top-right:* we show of the ratio ρ_m/ρ_v. In all three cases they are similar, visually indistinguishable. The present day ratio 0.455 is reached at $\tau_0 \approx 8.14$ in all three examples. The circles in the left panel and the horizontal line on the right panel are the present day $\tau_0 \approx 8.14$, at which $\rho_m/\rho_v = 0.455$.

Solar System test. Which of the two scenarios happens depends on the magnitudes of λ, γ_α and the value of φ at the matter-radiation equality time. These imply that for a viable cosmology $|\lambda|\rho \sim 1$ has to be reached near the present day, when ρ_v starts to catch up with ρ_m. Thus, it is convenient to normalize λ in terms of the energy density of the cosmological constant at the present day, $\lambda = \lambda_x \equiv -10^x/(0.69\rho_{\rm crit})$, where $0 < x < 1$ is a constant parameter.

In Fig. 2, the left panels show the evolution of $\varphi(\tau)$ and $\varphi'(\tau)$, while the right panels show that of ρ_m/ρ_v. The left panel shows the evolution of $\varphi(\tau)$ and $\varphi'(\tau)$ in the phase space, and the right panel shows that of ρ_m/ρ_v. The solid, dashed, and dot-dashed curves correspond to the cases of $(\lambda, \gamma_\alpha) = (\lambda_{0.6}, -4.22)$, $(\lambda_{0.7}, -5.68)$, and $(\lambda_{0.8}, -7.54)$, respectively. The corresponding initial conditions are $\varphi(0) = 0.0503, 0.0434, 0.0376$ respectively, which satisfy the BBN constraint Eq. (55). The circles in the left panel are the present day $\tau_0 \approx 8.14$, at which $\rho_m/\rho_v = 0.455$. The PPN parameters at $\tau = \tau_0$ are given by $(\gamma_{\rm PPN} - 1, \beta_{\rm PPN} - 1) \approx (7.82 \times 10^{-6}, -8.23 \times 10^{-6})$, $(1.26 \times 10^{-5}, -1.78 \times 10^{-5})$, and $(1.08 \times 10^{-5}, -2.03 \times 10^{-5})$, respectively, which satisfy the PPN constraints (39).

The attractor mechanism to GR in this scalar-tensor theory is reminiscent of the absence of spontaneous scalarization of NSs in this theory when Λ is negative and large in magnitude [35]. We have thus seen that the existence of a GR attractor and the compatibility with the bounds on the PPN parameters requires that $|\lambda|\rho_{\rm crit} \sim |\Lambda|(\kappa\rho_{\rm crit}) \sim |\Lambda| H_0^2$, $|\Lambda| \sim H_0^{-2} \sim 10^{56}\,{\rm cm}^2$. What are the effects of such 'disformal scale' on gravitating systems? Ref. [35] (cf. Sec. VIII there) argued that the kinetic part of the equation of motion for the scalar field in the presence of a perfect fluid behaves as

$$-\left[1 - (|\lambda|/2)\tilde{\rho}\right]\ddot{\varphi}, \tag{56}$$

in a linearized approximation where $\chi \approx B \approx 1$. Hence, assuming that $A \simeq 1$ and hence $\tilde{\rho} \simeq \rho$, the kinetic term can flip sign (i.e. cause a ghost instability) if $\rho \gtrsim 2/|\lambda|$. For the value $|\Lambda| \sim 10^{56}\,{\rm cm}^2$, this implies a threshold density $\rho_t \sim 10^{-29}\,{\rm g/cm}^3$ necessary to induce the instability. Moreover, the assumption $\chi \approx 1$ imposes $X \ll 1$ on the scalar field's kinetic energy. Therefore, this tremendously small density (of the same order of magnitude as the cosmic mean density) indicates that even though the Universe may be consistent with GR, small fluctuations of the permeating scalar field would necessarily be unstable. We note that in higher density regions ghost instabilities would proceed more slowly due to the presence of a larger coefficient $|1 - (|\lambda|/2)\tilde{\rho}|$, and hence the instability may be more significant on larger length scales.

V. DISCUSSIONS

We investigated whether in the presence of disformal coupling scalar-tensor theories with conformal coupling $\gamma_\alpha < 0$ allow the GR attractor in the late-time Universe, and if it is the case, whether the same coupling is compatible with spontaneous scalarization of NSs.

We showed that the effect of disformal coupling could make it possible to realize the GR attractor. The effective force on the cosmological scalar field is given by Eq. (51). Even if $\gamma_\alpha < 0$, the effective force becomes attractive, if $\lambda < 0$ and $|\lambda|\rho > 1$. As long as $|\lambda|\rho \gg 1$ the effective force is suppressed compared to the case of the pure conformal coupling and φ remains a nonzero constant, and when the energy density of the Universe becomes lower as such $|\lambda|\rho \sim 1$ the effective force starts to act and φ is attracted towards zero. In the case $|\lambda|\rho \sim 1$ during the matter-dominated phase, since ρ exponentially decreases with respect to τ the force becomes repulsive again when φ approaches zero, and φ grows again. On the other hand, in the case $|\lambda|\rho \sim 1$ during the cosmological constant-dominated phase, i.e., today or in the future, $|\lambda|\rho$ approaches and constant and φ approaches zero and hence the GR attractor after oscillations across $\varphi = 0$. However, since the GR attractor is reached in the future, the cosmological values of φ could satisfy the PPN constraints (39) unless the values of couplings and/or initial conditions are fine-tuned. Examples satisfying the bounds on the PPN parameters are shown in Fig. 2.

The disformal coupling which is necessary for the existence of the GR attractor is given by $\lambda\rho_0 \sim \Lambda H_0^2 \gtrsim 1$, and hence $\Lambda \gtrsim H_0^{-2}$. On the other hand, for spontaneous scalarization of NSs, the typical value of the disformal coupling is given by $\Lambda \gtrsim R_s^2$, where R_s is a typical radius of NSs. Since $H_0^{-1} \sim 10^{22} R_s$, the value of Λ necessary for the existence of the GR attractor is much larger than that of spontaneous scalarization of NSs. As argued in Ref. [35], such a huge value of disformal coupling prevents scalarization of NSs and even worse, induce ghost instabilities of matter present in all scales of the Universe. Therefore, the introducing a disformal coupling does not help reconciling spontaneous scalarization model of [8] with cosmological evolution of the scalar field. We expect that the problem is ubiquitous to any model with spontaneous scalarization induced by dimensionful coupling constant. Such a large disformal coupling parameter Λ might also affect local gravitational physics and modify the expression of the leading-order PPN parameters (40a) and (40b). Even if there would be a change of the PPN parameters, the Solar System constraints would be satisfied only for the particular initial conditions and our main results would not be affected.

Ultimately, the problem arises because Λ is a dimensionful coupling and hence the effective dimensionless coupling crucially depends on the environment. A conceptually similar problem was argued in the context of embedding the model for BH scalarization of [45, 46] into the inflationary cosmology [47], which involves a coupling to the Gauss-Bonnet term $\lambda^2\varphi^2(R^2 - 4R^{\alpha\beta}R_{\alpha\beta} + R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu})$, where the coupling λ has dimension of (length). In order to scalarize a BH with mass of $M = \mathcal{O}(M_\odot)$, where $M_\odot$ is the Solar mass, the coupling has to be $\lambda \sim GM \sim M_\odot/M_{\rm Pl}^2 \sim 10^{19}\,{\rm GeV}^{-1}$. Assuming that the scalar field φ is present at the beginning of inflation, it is quantized in a Bunch-Davies vacuum as the inflaton. It was suggested that for $\lambda > 0$ the same coupling induces a catastrophic production of the φ-particles within the time scale $(\lambda H_{\rm inf}^2)^{-1} \sim 10^{-32}(H_{\rm inf})^{-1}$, assuming that the Hubble rate during inflation is given by $H_{\rm inf} = 10^{13}\,{\rm GeV}$. Thus, quantum fluctuations of φ would rapidly grow and completely destroy the inflationary universe within the time scale much smaller than the Hubble time. This comes from the huge hierarchy between the two different curvature lengths $GM \sim 10^{19}\,{\rm GeV}^{-1}$ and $H_{\rm inf}^{-1} \sim 10^{-13}\,{\rm GeV}^{-1}$. In our case, the similar problem arises from the huge hierarchy between $R_s \sim 10^6\,{\rm cm}$ and $H_0^{-1} \sim 10^{28}\,{\rm cm}$.

At last, let us briefly comment on some possible extensions of our work and also place our results in perspective with other recent work of spontaneous scalarization. First, in the context of scalar-tensor theories, the disformal transformation (3) could be generalized by the inclusion of a X-dependence i.e., $A = A(X,\varphi)$ and $B = B(X,\varphi)$ [22]. This generalization maps the Lagrangian in (1) (after going to the Jordan frame) to a subclass of degenerate higher-order scalar-tensor theories [48–50]. How this generalized disformal coupling influences spontaneous scalarization of stars has not been investigated yet and it would be interesting to perform an analysis similar to that presented here for the cosmological evolution of the scalar field. Second, Ref. [36] recently isolated all the terms within Horndeski gravity which can potentially induce a tachyonic instability at the linear level (see also [51]). They are the original Damour-Esposito-Farèse model [8], the scalar-Gauss-Bonnet theory [45, 46, 52], and the model with disformal coupling to matter (related to [35]). A potential term of the scalar field, which cannot trigger scalarization on its own, can however influence the *onset* of the instability caused by the other three terms. Individually, each of the three 'instability trigger' terms have been shown to generically lead to violations of Solar System constraints, while the mass term can alleviate the tension depending on the scalar field's mass [53]. It would be interesting to study the cosmology of the full theory, combining these three terms and study if the combination of more than one dimensionful coupling parameters (e.g. arising from the scalar field's coupling to the Gauss-Bonnet term and disformally to matter) could resolve the tension. Moreover, other terms belonging to the Horndeski action (or beyond-Horndeski for more general models which satisfy the recent bounds on the speed of gravitational waves [54–60]) besides these four could be relevant for cosmological evolution at the *nonlinear* level, and could be able to make spontaneous scalarization compatible with cosmology.

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Appendix A: Stability of GR attractor

In this appendix, we briefly comment on the stability of GR solution against inhomogeneous perturbation of the scalar field $\varphi = \delta\varphi(t, x^i)$, which follows from the equation

$$(2+\lambda\tilde{\rho})\,\delta\ddot{\varphi} + (2-\lambda\tilde{\rho}\tilde{w})\left(3H\delta\dot{\varphi} - a^{-2}\Delta\delta\varphi\right) + \kappa\gamma_\alpha\tilde{\rho}\,(1-3\tilde{w})\,\delta\varphi = 0, \tag{A1}$$

where $\Delta \equiv \delta^{ij}\partial_i\partial_j$ is the Laplacian operator.

For $2+\lambda\tilde{\rho} < 0$ and $2-\lambda\tilde{\rho}\tilde{w} > 0$, $\delta\varphi$ suffers the ghost instability, while for $2+\lambda\tilde{\rho} > 0$ and $2-\lambda\tilde{\rho}\tilde{w} < 0$, $\delta\varphi$ suffers the spatial gradient instability. On the other hand, for $2+\lambda\tilde{\rho}>0$ and $2-\lambda\tilde{\rho}\tilde{w}>0$, or for $2+\lambda\tilde{\rho}<0$ and $2-\lambda\tilde{\rho}\tilde{w}<0$, $\delta\varphi$ does not suffer any instability arising from the modified kinetic term. In the case of the cosmological constant $\tilde{w}=-1$, Eq. (A1) reduces to

$$(2+\lambda\tilde{\rho})\left(\delta\ddot{\varphi}+3H\delta\dot{\varphi}-a^{-2}\Delta\delta\varphi\right)+4\kappa\gamma_\alpha\tilde{\rho}\delta\varphi=0. \quad \text{(A2)}$$

Since the coefficients for the second derivative terms are common, no ghost and gradient instability happen. On the other hand, for matter $\tilde{w}=0$,

$$(2+\lambda\tilde{\rho})\,\delta\ddot{\varphi}+2\left(3H\delta\dot{\varphi}-a^{-2}\Delta\delta\varphi\right)+\kappa\gamma_\alpha\tilde{\rho}\delta\varphi=0. \quad \text{(A3)}$$

When $2+\lambda\tilde{\rho}<0$, the ghost instability happens. We note that for the intermediate non-GR solutions nonzero $\dot{\varphi}$ and $\ddot{\varphi}$ would nontrivial contribute to the kinetic terms of cosmological perturbations, and the appearance of the ghost mode is unclear.

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