Photometric supernovae redshift systematics requirements

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Imaging surveys will find many tens to hundreds of thousands of Type Ia supernovae in the next decade, and measure their light curves. In addition to a need for characterizing their types and subtypes, a redshift is required to place them on a Hubble diagram to map the cosmological expansion. We investigate the requirements on redshift systematics control in order not to bias cosmological results, in particular dark energy parameter estimation. We find that additive and multiplicative systematics must be constrained at the few $\times 10^{-3}$ level, effectively requiring spectroscopic followup for robust use of photometric supernovae. Catastrophic outliers need control at the subpercent level. We also investigate sculpting the spectroscopic sample.

I. INTRODUCTION

Type Ia supernovae (SN Ia) are standardizable distance measures, whose use led to the discovery of cosmic acceleration [1, 2], and still provide the most stringent constraints on the nature of dark energy [3–7]. In the next decade, the number of SN Ia discovered and imaged in multiple photometric wavelength bands will increase by a factor of ~ 100, driven by surveys such as the Zwicky Transient Factory (ZTF [8]) and the Large Synoptic Survey Telescope (LSST [9]). If those SN can be fully utilized for cosmology they will provide powerful leverage on uncovering the nature of cosmic acceleration.

However, even with current samples systematics contribute at least equally to statistical uncertainty in the cosmological use of SN Ia. These systematics can be addressed through careful characterization of the supernova properties, through enhanced wavelength coverage into the infrared [10, 11] and ultraviolet [12, 13], and in particular spectroscopic data [14–16]. Spectroscopy not only confirms the source to be a true SN Ia but also give subtyping, e.g. through line ratios, high vs low velocities, etc. This then permits matching of similar SN Ia at different redshifts, greatly ameliorating systematics, through "like vs like" [17–19] or more detailed "twinning" [20, 21] methods.

The next decade imaging surveys (and the recently completed Dark Energy Survey [22]) rely on spectroscopic follow up to obtain the detailed information, as well as accurate redshifts. This limits the most robust sample to a few hundred in the case of Dark Energy Survey or a few thousand for next decade surveys, due to the time requirements for making the spectroscopic measurements. While there will be next decade multiobject spectroscopic instruments such as the Dark Energy Spectroscopic Instrument (DESI [23, 24]) and 4-metre Multi-Object Spectroscopic Telescope (4MOST [25]), the relatively low multiplexing of SN Ia observations means that many will not have spectroscopic data.

This has led to an extensive literature exploring whether purely photometric measurements can robustly place SN Ia on the Hubble diagram (see, e.g., [26–37]

among others). Issues include contamination by non-SN Ia, lack of subtyping, and selection effects. These all can distort the Hubble diagram vertically, by misestimating the source distance. Here we focus on redshift errors, which biases the Hubble diagram horizontally.

Note that a common practice is to obtain the source redshift by measuring the spectroscopic redshift of the host galaxy. If successful, this is adequate, but such a measurement still requires telescope time and becomes increasingly expensive at higher redshifts where leverage on dark energy may be greater. Galaxy catalog redshifts will also become more incomplete as one goes to the higher redshifts accessed by the next decade surveys. Moreover, not all SN Ia will have readily (or uniquely) identified hosts [38–40]. Photometric redshifts give a rough indication of the source redshift, but face challenges in use for accurate cosmology.

The redshift requirements for the Hubble diagram were investigated in the pioneering article of [41] (and later [42] for high redshift). They propagated redshift uncertainties into cosmological parameter uncertainties, i.e. how a finite prior on the mean redshift within a bin of supernovae nearly at the same distance increased the cosmology uncertainty. Their conclusion is that the redshift must be known to 0.002 or better to limit the increase in uncertainty on a constant dark energy equation of state to less than 10%. Here we investigate the complementary issue of systematic bias – shift in derived cosmology – rather than increase in statistical uncertainty.

In Sec. II we present the redshift systematic and cosmological parameter bias formalism. We assess the impact, and derive the requirements on systematic control in Sec. III, for additive, multiplicative, and catastrophic systematics. In Sec. IV we discuss the results and conclude.

II. REDSHIFT SYSTEMATICS AND COSMOLOGY BIAS

When the measurement of an observable is systematically offset from its true value, the cosmology estimation following from the data will be biased. For small

offsets, the Fisher bias formalism provides a straightforward technique for investigating the size and impact of this effect. The bias on a cosmological parameter is given by [43, 44]

$$\delta p_i = \left(F^{-1}\right)_{ij} \sum_k \frac{\partial \mathcal{O}_k}{\partial p_j} \frac{1}{\sigma_k^2} \Delta \mathcal{O}_k \ , \qquad (1)$$

where F^{-1} is the inverse Fisher matrix, $\mathcal{O}_k$ is the observable, σ_k its uncertainty, and $\Delta \mathcal{O}_k$ is the offset in the observable due to the systematics. The expression here is given for a simple diagonal noise matrix σ_k^2; see [44] for a general expression.

For the SN Ia case, the observable is the apparent magnitude m (really derived from observed photometry and redshift), and the offset is due to systematic misestimation of the redshift, so

$$\Delta \mathcal{O}(z) = \frac{\partial m}{\partial z} \delta z \ . \qquad (2)$$

The partial derivative involves two components, the change in the distance modulus or luminosity distance d_L with redshift, and how the change in redshift alters the relation between the apparent magnitude and distance modulus. The latter part is due to the data analysis procedure for standardization based on light curve width and color or dust extinction.

From [41] we see that the wavelength dependent color (k-correction) and extinction factor varies rapidly and with spikes in the redshift as the rest frame SN flux moves through the various photometric survey bands. This does not resemble the smooth variation from a cosmological parameter and so will not significantly bias cosmology estimation [19, 45–47]. (We have verified this numerically with a spiking toy model.) The light curve width does contain a component from time dilation, proportional to $1 + z$, and so the standardization procedure with an incorrect redshift will cause a shift in m. Thus we take

$$\frac{\partial m}{\partial z} = \frac{\partial m}{\partial d_L} \frac{\partial d_L}{\partial z} + \frac{\partial m}{\partial \text{width}} \frac{\partial \text{width}}{\partial z} \ . \qquad (3)$$

Since $m \sim 5 \log[d_L(z)]$, the factor

$$\frac{\partial m}{\partial d_L} = \frac{5}{\ln 10} \frac{1}{d_L(z)} \ . \qquad (4)$$

Recall that $d_L(z) = (1 + z) \int_0^z dz'/[H(z')/H_0]$, for a flat universe as we will assume, with H the Hubble parameter. In common light curve width standardization methods, the apparent magnitude is linearly proportional to the rest frame light curve width, and so

$$\frac{\partial m}{\partial \text{width}} = \text{const} \ . \qquad (5)$$

For example, in stretch standardization $m \sim -\alpha(s - 1)$ and in SALT2 [48] light curve fitting $m \sim -\alpha' X_1$,

where α and α' are constants. Since the rest frame width equals the observer frame width divided by $1 + z$, then $\partial \text{width}/\partial z = -dz/(1 + z)$.

Putting this all together we have

$$\Delta m = \frac{5}{\ln 10} \ln \left[\frac{d_L(z + \delta z)}{d_L(z)} \right] + \frac{C \, \delta z}{1 + z} \ . \qquad (6)$$

For $\delta z \ll z$ one could expand the distance ratio by evaluating the derivative $d \, d_L/dz$; we do not do this since for low redshift SN we may not have $\delta z \ll z$, and in any case the derivative would still leave integrals to be evaluated and so does not save much effort. However we can note that we expect $\Delta m \propto \delta z$ to a good approximation. The constant $C \approx \alpha s \approx \alpha'(X_1 + 1) \approx 1.4$ averaging over supernovae [48].

The cosmological parameters p_i are the matter density Ω_m in units of the critical density, the dark energy equation of state parameter today w_0 and a measure of its time variation w_a, and the combination $\mathcal{M}$ of the SN absolute magnitude and Hubble constant. When quoting constraints on one parameter we marginalize over the other parameters.

We now have to specify the survey properties, i.e. the number and distribution of SN Ia and their magnitude uncertainty. We do not attempt to model a next decade survey, with all its real world selection effects; rather we adopt a simple model that should be a reasonable approximation. To a statistical dispersion of $\sigma_{\text{stat}} = 0.15 \, \text{mag}$ per SN Ia (reasonable for a photometric survey), we add in quadrature a systematic measurement floor of $\sigma_{\text{sys}} = 0.01 \, (1 + z)$ per redshift bin of width 0.1. That is, infinite numbers of SN Ia will not give infinite accuracy, but rather uncertainties will be limited by the floor, representing, e.g., photometric band calibration zeropoint uncertainties, light curve model uncertainties, survey selection effects, etc.

Table I summarizes the number of SN Ia used in each redshift bin, and the ratio of the systematic to the total magnitude error, showing that more statistics will not help significantly with reducing the magnitude error. (And indeed if the error is lowered, any bias will become more severe in a relative sense. Note though that use of SN for other cosmological probes, e.g. peculiar velocity surveys, could profit from an increased sample size.) We take a redshift range of $z = 0 - 1.2$ (though the $z < 0.1$ SN may come from a separate survey). One can regard this as a reasonable, if rough, approximation to a next decade SN survey.

Finally, we need to specify a model for δz. We take it to be systematic among all supernovae at redshift z and consider three types of redshift systematics: additive, multiplicative, and catastrophic errors. The first two are simply described by

$$\delta z = d_0 + d_1 z \ . \qquad (7)$$

As long as $\delta z \ll z$ one can show that the parameter biases δp_i are linear in δz and so one can explore the

z	n	σ_{tot}	sys/total
0.05	300	0.014	0.77
0.15	300	0.014	0.80
0.25	300	0.015	0.82
0.35	300	0.016	0.84
0.45	300	0.017	0.86
0.55	300	0.018	0.87
0.65	300	0.019	0.89
0.75	300	0.020	0.90
0.85	300	0.020	0.91
0.95	300	0.021	0.91
1.05	150	0.024	0.86
1.15	150	0.025	0.87

TABLE I. Survey characteristics adopted as an approximation of a next decade survey in terms of total magnitude uncertainty σ_{tot}. This is given in magnitudes and is the most important property. The last column shows that the total uncertainty is predominantly systematics dominated, so the number n of SN Ia in each redshift z bin is mostly moot.

impact of d_0 (additive) and d_1 (multiplicative) separately. One can then add the δp_i afterward if desired, or take the individual effects as a lower limit on the systematics control required.

Catastrophic redshift errors are more sensitive to the complicated survey characteristics so we only adopt three toy models to give a rough estimation of their effects. For each, we assume a fraction f of the SN Ia in each redshift bin are affected, and derive the control needed on f. The first model puts misestimated SN from each redshift at $z = 0.1$, and the second model puts them at $z = 1$, regardless of their true redshift. This scattering of outliers to the survey extremes is reminiscent of certain actual outlier behavior, but we make no claims for it beyond a toy model. The third model puts SN from true redshifts $z < 0.6$ at $z + 0.2$ and SN from true redshifts $z > 0.6$ at $z - 0.2$. That is, we examine the effect of narrowing the redshift distribution.

We emphasize that these redshift errors are to be regarded as the residuals after calibrating the photometric redshift distributions. Surveys have active, extensive efforts to do precisely this, using both their data and external information, and deriving these in detail is beyond the scope of this article.

Given all the elements we propagate the redshift systematics into cosmological parameter biases. Since the covariance between parameter shifts is important – i.e. a modest shift orthogonal to the degeneracy direction can place the derived values well outside the true joint confidence contour – we quantify this by evaluating the change in likelihood due to the bias [49, 50],

$$\Delta\chi^2 = \boldsymbol{\delta p}\, \boldsymbol{F}^{\text{sub}}\, (\boldsymbol{\delta p})^T \;, \tag{8}$$

where we define a subspace of interest, e.g. the dark energy w_0–w_a plane, and convolve the Fisher submatrix (marginalized over other parameters) with the parameter bias vectors. In all calculations we include a Planck prior on the distance to CMB last scattering.

III. SYSTEMATICS REQUIREMENTS

For each of the forms of the redshift systematics we calculate the cosmological parameter biases, and the $\Delta\chi^2$ in the w_0–w_a plane, i.e. the offset of dark energy properties relative to the true joint likelihood (marginalized over the other parameters). Due to the linearity of δp_i with respect to δz we can estimate the systematics control necessary, in terms of limiting d_0, d_1, or f, so as to ensure bias is not significant. Note that a condition such as, say, $\delta p_i < \sigma(p_i)/2$ is not sufficient to prevent a large shift in terms of $\Delta\chi^2$, since that involves a nonlinear combination of various parameters p_i and their covariances. Therefore we evaluate $\Delta\chi^2$ and impose $\Delta\chi^2 < 2.30$, i.e. limiting the misestimation of the dark energy properties to stay within the 1σ joint confidence contour of w_0–w_a. This gives the requirements on the systematics control.

A. Additive Systematic

For the additive redshift systematic we take $d_0 = 0.01$, $d_1 = 0$, i.e. $z \to z + 0.01$ for illustration. We calculate the cosmological parameter bias induced by this systematic, applied to each of the 12 redshift bins individually, and to all of them. Figure 1 shows the results in the w_0–w_a dark energy plane. Each red box shows the shift from the fiducial ΛCDM cosmology with $w_0 = -1$, $w_a = 0$ (black dot at the center of the blue 68.3% joint confidence contour ellipse). The lowest redshift bin $z = [0, 0.1]$ is marked with orange fill and the highest redshift bin $z = [1.1, 1.2]$ is marked with blue fill; all bins are connected in order of redshift by the red curve.

The largest individual bias occurs for the lowest bin but substantial bias is evident for several redshift bins. The green arrow gives the total bias for the systematic applied to all redshifts. Note the bias for $d_0 = 0.01$ is so large that it extends well beyond the 1σ joint confidence contour, as shown by the inset figure. We can best quantify the overall dark energy bias by using the $\Delta\chi^2$ statistic: such a redshift systematic deliver $\Delta\chi^2 = 505$, some 20σ off. To determine the systematic control requirement we can solve numerically for the condition $\Delta\chi^2 = 2.3$ (or use that $\Delta\chi^2 \sim [\delta z]^2$), to find the requirement $d_0 \lesssim 0.0006$. This is quite severe, and while later we will investigate ways of easing this, we see that photometric redshifts will be greatly challenged to give robust cosmology.

B. Multiplicative Systematic

For the multiplicative redshift systematic, we take $d_0 = 0$, $d_1 = 0.01$, i.e. $z \to (1 + 0.01)\, z$ for illustration. The analysis for bias induced by multiplicative systematics follows that of the previous subsection on the additive case. In Fig. 2 we can see that the total bias due to redshift systematic applied to all redshift bins is significantly
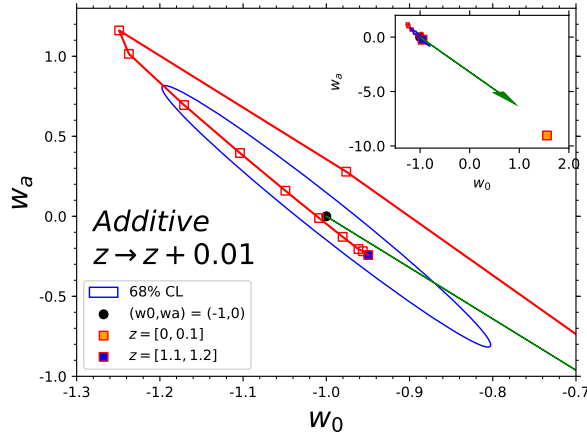
FIG. 1. The dark energy parameter shifts from an additive redshift systematic, $(d_0, d_1) = (0.01, 0)$, are plotted in the w_0–w_a plane, along with the statistical 1σ joint confidence contour. The red curve indicates the shifts as the systematic is applied individually to each redshift, from lowest bin ($z = [0, 0.1]$: solid orange box) to highest ($z = [1.1, 1.2]$: solid blue box), with open red squares every 0.1 in redshift. Applying the systematic at all redshifts shifts the fiducial cosmology from the black dot (ΛCDM) to the end of the green arrow. We show the full extent of the shift in the inset plot.

smaller than the additive case. There is a coincidental cancellation among the (lesser) biases of individual redshift bins such that the sum is small, lying within the 1σ joint confidence contour.

This gives a small shift $\Delta\chi^2 = 0.4$ for $d_1 = 0.01$, meaning that $d_1 \lesssim 0.024$ would satisfy the $\Delta\chi^2 < 2.3$ criterion. However we emphasize that this cancellation is somewhat fine tuned, as we explore in the following subsection, and so the requirements on multiplicative systematics control would be better regarded as $d_1 \lesssim 0.01$.
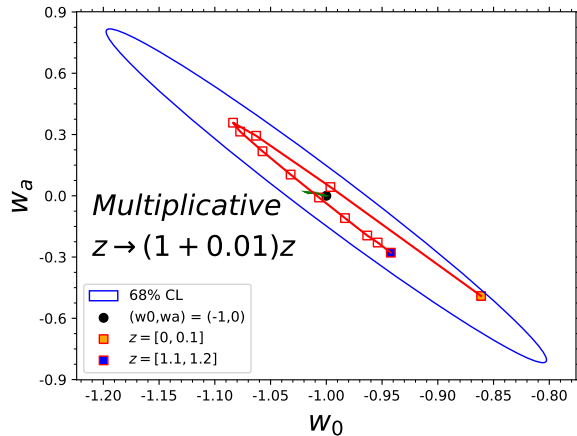

FIG. 2. As Fig. 1 but for a multiplicative redshift systematic with $(d_0, d_1) = (0, 0.01)$.

C. Systematics Control

Table II summarizes the additive and multiplicative redshift systematics cases. We see the additive case is much more severe, with spectroscopic level requirements on the redshifts. Allowing for both additive and multiplicative systematics tightens the control requirements. (Note that a multiplicative systematic in $1 + z$ corresponds to a combination of additive and multiplicative systematics in z.) In all cases, photometric precision is insufficient for robust cosmology determination.

We have also explored the cosmology biases that arise in a more restricted model with constant equation of state w, i.e. fixing $w_a = 0$. The results are substantially similar, qualitatively and mostly quantitatively, to what we present, e.g. $\delta z_{\rm req} = 0.0008$ for the additive case and the multiplicative case is essentially unchanged.

Since we expect a small redshift shift to affect low redshifts more, due to the sensitivity of the SN apparent magnitude to redshift at low z (going roughly as $1/z$), then we also consider the case where spectroscopic redshifts exist for all local ($z < 0.1$) SN and so systematics there vanish. This helps substantially (but not enough) for the additive case, while for the multiplicative case it actually worsens the effect.

Model	$\delta\Omega_m$	δw_0	δw_a	$\Delta\chi^2$	$\delta z_{\rm req}$
Additive	0.065	1.92	-6.14	505	0.0006
Additive (no local)	0.041	-0.64	2.90	33	0.003
Mult.	-0.004	-0.02	0.021	0.4	0.024
Mult. (no local)	-0.005	-0.16	0.51	3.4	0.008

TABLE II. Cosmology biases due to additive and multiplicative redshift systematics at the 0.01 level. The requirement on the systematic level to control bias to $\Delta\chi^2 < 2.3$ (1σ joint confidence) is given by $\delta z_{\rm req}$ for each case (where $\delta z_{\rm req}$ is to be interpreted as either d_0 or d_1). The case with no systematics for $z < 0.1$ SN is shown by "(no local)"; note for the multiplicative case this removes a cancellation and tightens the requirement.

It is worthwhile pursuing the question further as to whether systematics control can be concentrated in particular redshift bins. As mentioned, the lowest redshift bin systematic gives substantial cosmology bias in both cases. Note that a similar characteristic was found in [41]. Indeed, if we eliminate the systematic in the $z = [0, 0.1]$ bin then for the additive case the $\Delta\chi^2$ drops from 505 to 33. Of course this is still far more biased than we can accept. Note however that for the multiplicative case, $\Delta\chi^2$ actually worsens from 0.4 to 3.4, because the bias from the lowest bin canceled some of the bias from higher bins.

Figure 3 presents the results of systematic redshift control, e.g. by use of a SN spectroscopic sample, to make all redshifts $z < z_{\rm free}$ free from additive systematics. Figure 4 shows the multiplicative systematics case. We show the effect of such control both on $\Delta\chi^2$ and on the requirement $\delta z_{\rm req}$ on the remaining redshifts $z > z_{\rm free}$ (compare

Table II). To avoid substantial cosmology bias we need to either eliminate systematics through use of a spectroscopic sample out to $z_{\text{free}} \gtrsim 0.9$ (with the higher redshift photometric sample having a systematic of 0.01), or use of a spectroscopic sample out to z_{free} and a systematic level at higher redshifts below the δz_{req} curve.
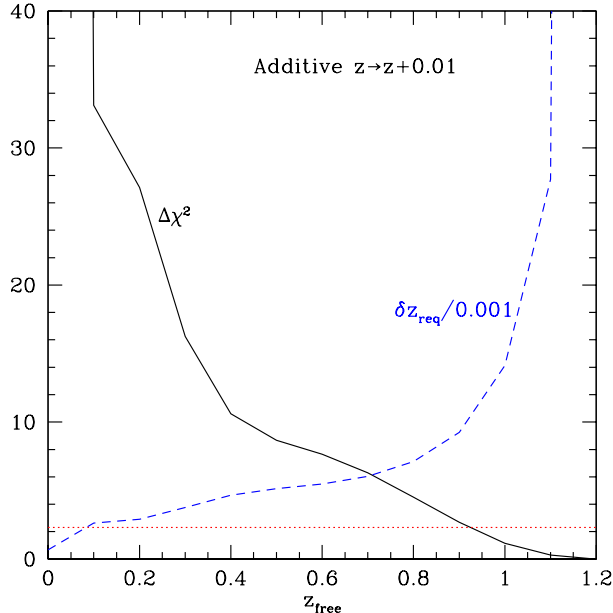

FIG. 3. The relaxation of the cosmology bias $\Delta\chi^2$ (solid black curve) and required additive redshift systematic control δz_{req} (dashed blue curve) is shown as a function of the redshift z_{free} out to which the systematic is eliminated, e.g. due to use of a spectroscopic sample. The dotted red line shows $\Delta\chi^2 = 2.3$; to avoid substantial cosmology bias we need to work in the region where the solid black curve lies below the dotted red curve.

D. Catastrophic Outliers

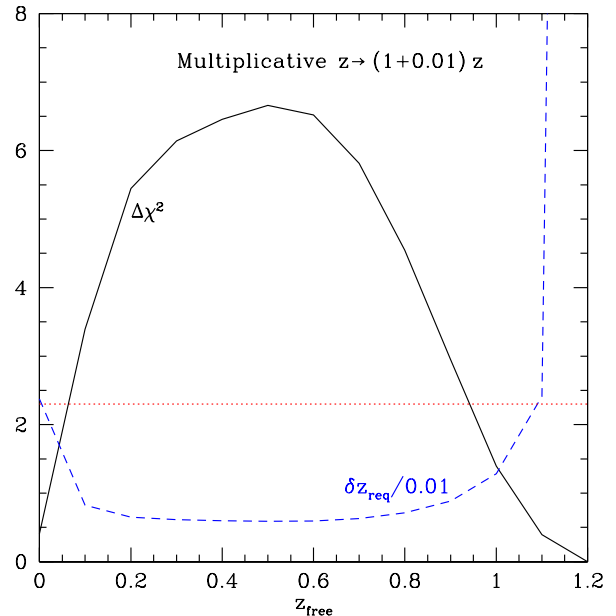The three catastrophic outlier toy models can also give rise to a bias on cosmology. In Table III we summarize

FIG. 4. As Fig. 3 but for the multiplicative systematic. Note the δz_{req} curve is divided by 0.01 not 0.001 as in the additive case.

the three cases, showing the bias induced for a catastrophic outlier fraction of 1%, and also the requirement on the fraction f in order to keep $\Delta\chi^2 < 2.3$. Recall the values δp_i will scale nearly linearly with f, and the $\Delta\chi^2$ will scale nearly as its square.

Model	$\delta\Omega_m$	δw_0	δw_a	$\Delta\chi^2$	f_{req}
$z \to 0.1$	0.026	0.19	-0.36	21	0.003
$z \to 1$	0.028	0.20	-0.35	25	0.003
$z \pm 0.2$	0.017	0.099	-0.14	8.1	0.005

TABLE III. Cosmology biases due to various catastrophic redshift models with 1% outliers. The requirement on the outlier fraction to control bias to $\Delta\chi^2 < 2.3$ (1σ joint confidence) is given by f_{req} for each case.

These results hold for catastrophic redshift outliers in every bin. However, it is useful to break this down to investigate the contribution from each individual redshift bin. Figures 5 and 6 show an interesting effect. For the $z \to 0.1$ case, although all the redshift outliers at each redshift bin give small shifts lying within the 68% joint confidence contour (i.e. $\Delta\chi^2 < 2.3$), more of them lie to the lower right so the summed cosmological parameter shift amounts to $\Delta\chi^2 = 21$, or approximately 4.5σ. (Recall that the $\Delta\chi^2$ do not sum linearly.) For the $z \to 1$ case, all but the lowest redshift bin do not give large parameter shifts. However the lowest redshift bin gives a very strong dark energy shift (which to some extent is canceled by the shift in the opposite direction by the other redshift bins). If the first bin were systematics free, then $\Delta\chi^2$ drops from 24.6 to 9.5, meaning the require-

Note the multiplicative case has a different behavior than the additive case in the shape of the $\Delta\chi^2$ curve. Elimination of systematics for the lowest redshift bin controls cosmology bias, but this is due to a fine tuned cancellation. The $\Delta\chi^2$ curve increases initially, rather than monotonically decreasing as in the additive case. So we cannot depend on removing systematics only from the lowest bin; if we removed systematics from the first two or three bins, instead the cosmology would be strongly biased. Only by removing systematics out to $z_{\text{free}} \gtrsim 0.9$ can we guarantee robust cosmology results. In all other cases we see that photometric redshift systematic control at the 0.01 level is insufficient; spectroscopy for the majority of the SN is required.

ment on f for the other bins loosens to 0.005. (For the $z \to 0.1$ case the lowest z bin systematic does not give a strong effect.)
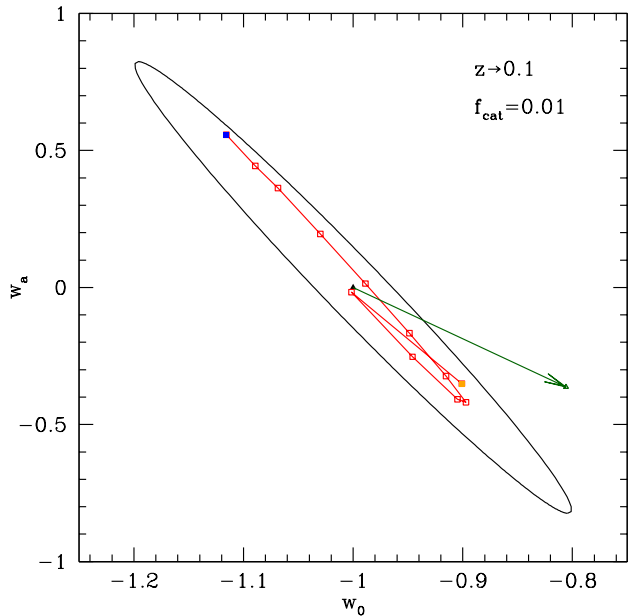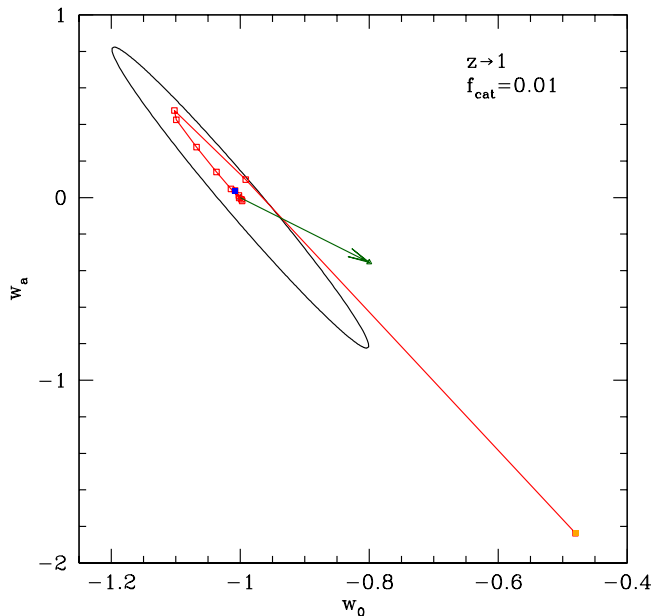
For the third model, with $z \to z \pm 0.2$, again we find that catastrophic outliers in the lowest redshift bin are the most damaging, and again it is partially controlled by opposite shifts from the other bins, as seen in Fig. 7. If the first bin were systematics free, then $\Delta\chi^2$ drops from 8.1 to 3.2 (and $f_{\rm req}$ rises to 0.008).

FIG. 5. As Fig. 1 but for a catastrophic redshift systematic with outlier fraction $f_{\rm cat} = 0.01$ misinterpreted as being at $z = 0.1$.
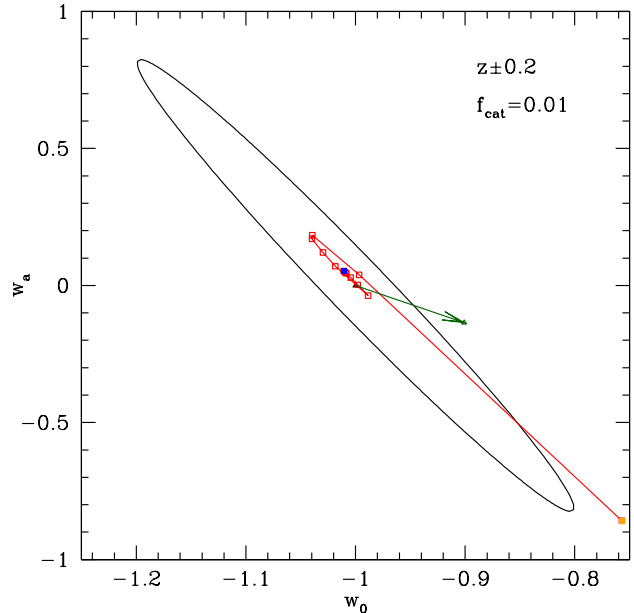

FIG. 7. As Fig. 1 but for a catastrophic redshift systematic with outlier fraction $f_{\rm cat} = 0.01$ misinterpreted as being at $z + 0.2$ for $z \le 0.6$ and $z - 0.2$ for $z > 0.6$, i.e. losing from the extremes. Note the large shift due to the lowest redshift bin outliers.

IV. CONCLUSION

Tens to hundreds of thousands of Type Ia supernovae will be discovered by 2020s wide area imaging surveys such as ZTF and LSST. While these provide some information on the SN, they do not provide spectral information which is useful for classification and subclassification, and redshifts. Follow up spectroscopy, even for host galaxy redshifts, is expensive in terms of telescope time. We investigate specifically the issue of cosmology bias due to systematically imperfect redshift determination, quantifying the cosmology bias and resulting requirements for systematic control.

We examine three classes of redshift systematics – additive, multiplicative, and catastrophic – and conclude that in all cases robust cosmology requires control of redshift systematics at the subpercent level. We show how cosmology bias evolves as the systematic enters at different redshifts, and generally the sum over the full sample leads to large offsets from the true cosmology. Investigating whether limited spectroscopy in the form of a focus

FIG. 6. As Fig. 1 but for a catastrophic redshift systematic with outlier fraction $f_{\rm cat} = 0.01$ misinterpreted as being at $z = 1$. Note the huge shift due to the lowest redshift bin outliers.

on particular redshift bins, e.g. low redshift, removes the issue gives the conclusion that it generally does not; systematic control throughout the sample is essential. This analysis is complementary to that of [41], which examined the "bloat" of uncertainties rather than bias, and came to similar accuracy requirement conclusions.

Additive systematics appear the most harmful, and then certain types of catastrophic outliers. Multiplicative redshift systematics initially appear relatively benign, but this is due to a fine tuned cancellation and small deviations from the model do impose subpercent control requirements. A spectroscopic sample, though more limited in numbers, is needed for robust cosmology determination. Many spectroscopic instruments will be operating during the 2020s, such as the Dark Energy Spectroscopic Instrument (DESI) and Wide Field

Infrared Survey Telescope (WFIRST), and could play useful roles in contributing to a well controlled sample of several thousand SN, capable of high accuracy constraints on dark energy and cosmology.

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