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Denoising Weak Lensing Mass Maps with Deep Learning

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Weak gravitational lensing is a powerful probe of the large-scale cosmic matter distribution. Wide-field galaxy surveys allow us to generate the so-called weak lensing maps, but actual observations suffer from noise due to imperfect measurement of galaxy shape distortions and to the limited number density of the source galaxies. In this paper, we explore a deep-learning approach to reduce the noise. We develop an image-to-image translation method with conditional adversarial networks (CANs), which learn efficient mapping from an input noisy weak lensing map to the underlying noise field. We train the CANs using 30000 image pairs obtained from 1000 ray-tracing simulations of weak gravitational lensing. We show that the trained CANs reproduce the true one-point probability distribution function (PDF) of the noiseless lensing map with a bias less than 1σ on average, where σ is the statistical error. We perform a Fisher analysis to make forecast for cosmological parameter inference with the one-point lensing PDF. By our denoising method using CANs, the first derivative of the PDF with respect to the cosmic mean matter density and the amplitude of the primordial curvature perturbations becomes larger by $\sim 50\%$. This allows us to improve the cosmological constraints by $\sim 30 - 40\%$ with using observational data from ongoing and upcoming galaxy imaging surveys.

I. INTRODUCTION

Gravitational lensing is a relativistic effect that causes characteristic distortion of the images of distant astrophysical sources. The degree of distortion is determined by the gravitational potential of intervening mass (lens) and the geometry between the lens and source objects [1]. Although the induced distortion is tiny for individual sources, averaging over a large number of sources can reveal the gravitational lensing effect due to large-scale mass distribution in the Universe. In the literature, the lensing effect caused by the large-scale structure is referred to as weak lensing effect. The great advantage of measuring the weak lensing effect is that it enables us to study the matter density distribution in a physical and unbiased manner in principle. Statistical analysis of weak lensing effect is one of the most important studies in modern cosmology; one can extract rich information of gravitational clumping of dark matter and cosmic expansion (see, Refs [2–4] for reviews).

The two-point angular correlation function of shapes of sources, or its Fourier-space counterpart known as power spectrum, are commonly used to characterize weak lensing maps. Although the power spectrum provides a complete statistical description of a random Gaussian field, numerical simulations of weak lensing effect have shown that weak lensing maps have non-Gaussian properties. Hence the power spectrum alone cannot fully describe

a weak lensing map [5–7]. Various statistical methods have been proposed to study the non-Gaussian features [8–24]. Most of these proposed methods consider two-dimensional maps, i.e., images.

Distortion of galaxy shape is commonly used as a measure of weak lensing effect. The weak lensing effect of individual galaxies is expected to be much smaller than the intrinsic shape in practice [4], and the shape measurement is affected by various observational effects [25], rendering the measurement and the resulting statistics uncertain. These are altogether called “shape noises”, which make weak lensing maps estimated from galaxy shape to be intrinsically noisy. Often the original cosmological information imprinted in the map is obscured. There exist several approaches to reduce the shape noise in weak lensing maps in the literature [26–31], but it is still challenging to obtain a completely noiseless weak lensing map from observations with an angular resolution of ~ 1 arcmin. Noise reduction in weak lensing maps would be crucial for various cosmological analyses including robust search for clusters of galaxies [32–35] and detection of diffuse components such as filaments in a cosmic mass density field [36]. It is also known that non-Gaussianity of noiseless lensing maps can be utilized to avoid the degeneracy of cosmological parameters effectively, and hence to allow precise measurement of time evolution in dark energy density (e.g., Ref. [37]) and neutrino masses (e.g., Refs. [21, 38, 39]).

In this paper, we explore a deep-learning approach to generate a high-resolution weak lensing map from noisy, observed one. The procedure is essentially an image-to-image translation from a noisy input map to the under-

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lying noise field. We adopt the conditional adversarial networks for image-to-image translation used in Ref. [40]. We train and validate the networks by using 30000 noisy weak lensing maps based on cosmological weak lensing simulations. The training is performed with effective angular resolution of ~ 1.5 arcmin, and we assume that the root-mean-square error of the input noisy maps is totally dominated by the shape noise. We then test the trained networks by using additional 1000 data sets and investigate if our deep-learning method can reproduce two summary statistics of weak lensing maps in absence of noise: one is the power spectrum and the other is one-point probability distribution function (PDF). We also aim at reconstruction of noise-free weak lensing map on pixel-by-pixel basis. We finally examine if our denoising method is a valid approach for two common weak-lensing analyses of detection of massive galaxy clusters and precise determination of cosmological model.

The rest of the present paper is organized as follows. In Section II, we summarize the basics of gravitational lensing. In Section III, we describe the conditional adversarial networks adopted in this paper and provide how to produce the data set for training, validating, and testing the networks. In Section IV, we show the performance of our trained networks when applying them to test data set. We also study the applicability of our denoising method in cosmological analyses in Section V. Future prospects in upcoming surveys with a higher source number density are discussed in Section VI. Concluding remarks and discussions are given in Section VII.

II. WEAK GRAVITATIONAL LENSING

A. Basics

Weak lensing effect is commonly characterized by the distortion of image of a source object (galaxy) by the following 2×2 matrix between the observed position of a source object $\boldsymbol{\theta}_{\text{obs}}$ and the true (unlensed) position $\boldsymbol{\theta}_{\text{true}}$:

$$A_{ij} = \frac{\partial \theta_{\text{true}}^i}{\partial \theta_{\text{obs}}^j} \equiv \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 - \omega \\ -\gamma_2 + \omega & 1 - \kappa + \gamma_1 \end{pmatrix}, \quad (1)$$

where κ is the convergence, γ is the shear, and ω is the rotation. In the weak lensing regime ($\kappa, \gamma \ll 1$), the convergence can be expressed as the integral of the density contrast of underlying matter density field $\delta_m(\mathbf{x})$ with a weight over redshift [1],

$$\kappa(\boldsymbol{\theta}) = \int_0^\infty d\chi W_\kappa(\chi) \delta_m(r(\chi)\boldsymbol{\theta}, \chi), \quad (2)$$

$$W_\kappa(\chi) = \frac{3}{2} \left(\frac{H_0}{c} \right)^2 \Omega_{m0} (1 + z(\chi)) r(\chi) \times \int_\chi^\infty d\chi' p(\chi') \frac{r(\chi' - \chi)}{r(\chi')}, \quad (3)$$

where H_0 is the present-day Hubble constant, Ω_{m0} is the matter density parameter at present, $\chi(z)$ is the radial

comoving distance to redshift z , $r(\chi)$ is the angular diameter distance, and $p(\chi)$ represents the source distribution normalized to $\int d\chi p(\chi) = 1$. Throughout this paper, we assume that the source galaxies are located at a single plane at redshift of $z_{\text{source}} = 1$, i.e. $p(\chi) = \delta(\chi - \chi_1)$ where $\delta(x)$ is the Dirac delta function and $\chi_1 = \chi(z = 1)$ for simplicity.

B. Estimator of convergence field

The lensing convergence κ is of our primary interest since it contains rich cosmological information. In the following, we summarize how to estimate κ from observables in modern galaxy imaging surveys. We first define the smoothed convergence map (field) as

$$\hat{\kappa}(\boldsymbol{\theta}) = \int d^2\phi \kappa(\boldsymbol{\theta} - \boldsymbol{\phi}) U(\boldsymbol{\phi}), \quad (4)$$

where U is the filter function to be specified below. We can calculate the same quantity by smoothing the shear field γ as

$$\hat{\kappa}(\boldsymbol{\theta}) = \int d^2\phi \gamma_+(\boldsymbol{\phi} : \boldsymbol{\theta}) Q_+(\boldsymbol{\phi}), \quad (5)$$

where γ_+ is the tangential component of the shear at position $\boldsymbol{\phi}$ relative to the point $\boldsymbol{\theta}$. The filter function for the shear field Q_+ is related to U by

$$Q_+(\boldsymbol{\theta}) = \int_0^{\theta_o} d\theta' \theta' U(\theta') - U(\boldsymbol{\theta}). \quad (6)$$

We consider a filter function Q_+ that has a finite extent. In such cases, one can write

$$U(\boldsymbol{\theta}) = 2 \int_{\boldsymbol{\theta}}^{\boldsymbol{\theta}_o} d\theta' \frac{Q_+(\boldsymbol{\theta}')}{\theta'} - Q_+(\boldsymbol{\theta}), \quad (7)$$

where θ_o is the outer boundary of the filter function. Note that the filter function U should be compensated because the smoothed field $\hat{\kappa}$ does not depend on undetermined constant [41].

In this paper, we consider the truncated Gaussian filter (for U):

$$U(\boldsymbol{\theta}) = \frac{1}{\pi \theta_G^2} \exp\left(-\frac{\theta^2}{\theta_G^2}\right) - \frac{1}{\pi \theta_o^2} \left[1 - \exp\left(-\frac{\theta_o^2}{\theta_G^2}\right)\right], \quad (8)$$

$$Q_+(\boldsymbol{\theta}) = \frac{1}{\pi \theta^2} \left[1 - \left(1 + \frac{\theta^2}{\theta_G^2}\right) \exp\left(-\frac{\theta^2}{\theta_G^2}\right)\right], \quad (9)$$

for $\theta \leq \theta_o$ and $U = Q_+ = 0$ elsewhere. Throughout this paper, we set $\theta_o = 30$ arcmin and $\theta_G = 1.5$ arcmin. The choice of $\theta_G = 1.5$ arcmin is found to be an optimal smoothing scale for the detection of massive galaxy clusters using weak lensing for $z_{\text{source}} \sim 1$ [33, 34].

In actual observations, ellipticity of galaxies is used as an indicator of the shear field. In the weak-field limit

($\kappa, \gamma \ll 1$), the observed ellipticity of galaxy can be decomposed into two parts as

$$\epsilon_{\text{obs}} = \epsilon_N + \gamma, \quad (10)$$

where ϵ_{obs} and ϵ_N are the observed ellipticity and shape noise, respectively. In typical galaxy imaging surveys, the shape noise term is mostly contributed by the intrinsic ellipticity of source galaxies and the shape measurement inaccuracy. Since we expect both are independent of weak lensing shear γ , the smoothed convergence estimated by Eq. (5) can be expressed as

$$\kappa_{\text{obs}} = \kappa_N + \kappa, \quad (11)$$

where the left-hand side represents the observed convergence, while κ_N in the right-hand side is the noise convergence from the shape noise. The primary purpose of this paper is to estimate the underlying convergence field κ from the observed map κ_{obs} . To this end, we adopt an image-to-image translation based on conditional adversarial networks.

III. METHOD

A. Image-to-Image Translation

We use conditional adversarial networks developed in Ref [40], referred to as **pix2pix**, for denoising the weak lensing convergence field. **pix2pix** is designed so as to learn mapping from input to output, but also to learn a loss function associated with this mapping. Therefore, it provides a generic approach to image-to-image translation problems, and is free from the formulation of loss function in the networks on a problem-by-problem basis.

The networks have two main pieces, a generator and a discriminator. The generator applies some transform to the input image to get an output image. The discriminator compares the input image to an unknown image (either a target image from the data set or an output image from the generator) and tries to guess if it is produced by the generator.

The generator in **pix2pix** uses a U-Net which is a kind of encoder-decoder structure [42]. On the encoder part in the generator, an input image is progressively compressed with eight convolution layers. In this compression process, the generator tries to learn the important features of the input at different scales. Each convolution layer consists of convolution with a kernel size of 5×5 , the batch normalization, and the application of activation function of leaky ReLU with a leak slope of 0.2. When arriving at the final convolution layer, the generator performs the inverse operation of convolution layer and combines the simplified feature from the final convolution layer into more and more complicated representations. In addition, the U-Net has additional skip connections between mirrored layers in the encoder and

decoder stacks, allowing to propagate the small-scale information that would be lost as the size of the images decreases in the encoder.

In the discriminator, a given set of target and input images are reduced with 4 convolution layers and then all responses are averaged out to provide the ultimate output. The final output in the discriminator is used to determine if the input is real or fake. Note the target image in the discriminator can be randomly selected from the ground truth or the output of the generator.

In this paper, we adopt the structure of the generator and the discriminator as shown in Figure 1. To obtain underlying convergence from an input noisy map, we first perform an image-to-image translation from a noisy lensing map κ_{obs} to the underlying noise field κ_N ¹. Once the noise field is predicted by the networks, we then estimate the underlying convergence κ by subtracting the predicted noise from the observed map κ_{obs} .

B. Ray-tracing simulation

To construct a large data set for deep learning, we utilize ray-tracing simulations of gravitational lensing in Ref [43]. The simulations are based on 200 realizations of high-resolution N -body simulations with a box size of $240 h^{-1} \text{Mpc}$ on a side. The N -body simulations are performed with the number of particles of 256^3 for the concordance ΛCDM model². The mass of N -body particle is set to $5.3 \times 10^{10} h^{-1} M_\odot$. We work with a single source redshift at $z_{\text{source}} = 1$. The ray-tracing simulations are performed on 2048^2 pixels with the pixel size of 0.15 arcmin. From the 200 N -body simulations, they have produced 1000 realizations of 5×5 squared-degrees lensing fields by rotating and shifting the structures in N -body simulations randomly. Details of the ray-tracing simulations are found in Ref [43].

When producing a training set for deep learning, we first degrade the lensing shear fields on 512^2 pixels by averaging over nearby 4 by 4 pixels (the degraded pixel size is then 0.6 arcmin). We then randomly subtract the contiguous area consisting of 256^2 pixels to increase the number of training sets. We perform the random subtraction 100 times for individual simulations with 512^2 pixels. For the main training data set, we use 600 realizations of lensing shear among 1000 parent realizations,

¹ In Appendix A, we also examine the training so that the generator can predict the underlying noiseless convergence field from noisy input, but such training is found to be less effective for denoising.

² The following cosmological parameters are adopted in the simulations: the matter density $\Omega_{\text{m}0} = 0.238$, the baryon density $\Omega_b = 0.042$, the dark energy density $\Omega_\Lambda = 1 - \Omega_{\text{m}0} = 0.762$, the equation of state parameters of dark energy $w = -1$, the scalar spectral index $n_s = 0.958$, the amplitude of curvature perturbations $A_s = 2.35 \times 10^{-9}$ at $k = 0.002 \text{Mpc}^{-1}$, Hubble parameter $h = 0.732$, and the variance of the present-day density fluctuation in a sphere of radius $8 h^{-1} \text{Mpc}$ $\sigma_8 = 0.76$.

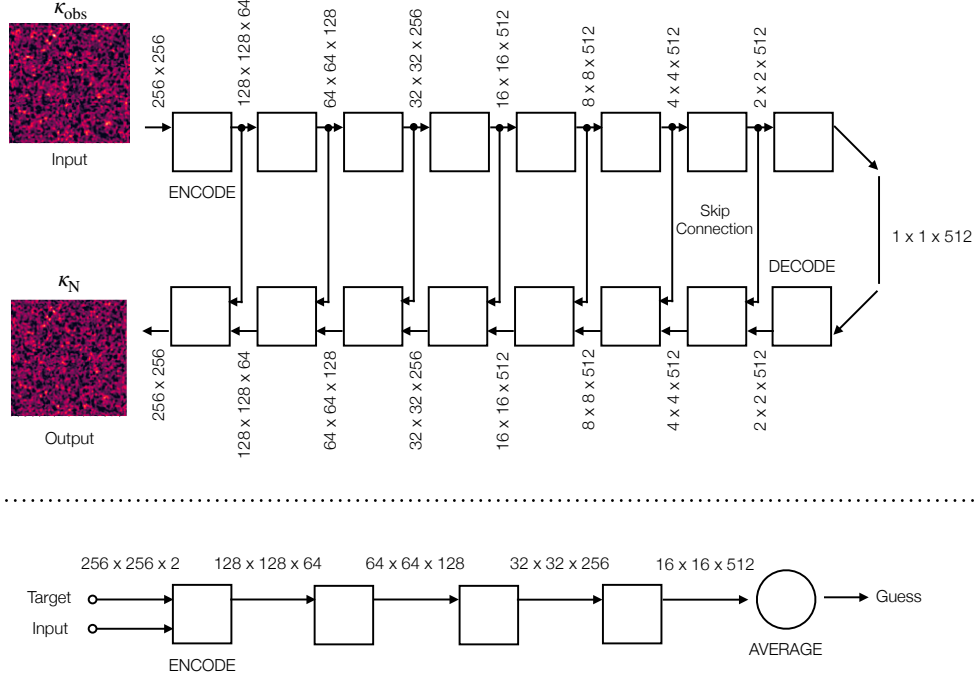


FIG. 1. The structure of conditional adversarial networks in this paper. The upper portion represents the generator that tries to translate an input noisy weak lensing map κ_{obs} to the underlying noise field κ_N . The lower shows the discriminator which guesses if the input image has been produced by the generator or not. Both of the generator and discriminator consist of a series of (de)convolution layers and the resulting number of model parameters in the networks is close to 400000.

and then have 60000 shear fields with a total sky coverage of 2.5×2.5 squared degrees. We also produce another 1000 shear fields for testing the networks from remaining 10 realizations. To create a noisy field, we include galaxy shape noise ϵ_N in our simulation by adding to random ellipticities which follow the two-dimensional Gaussian distribution as

$$P(\epsilon_N) = \frac{1}{\pi \sigma_N^2} \exp\left(-\frac{\epsilon_N^2}{\sigma_N^2}\right), \quad (12)$$

where $\epsilon_N = \sqrt{\epsilon_{N,1}^2 + \epsilon_{N,2}^2}$ and $\sigma_N^2 = \sigma_\epsilon^2 / (n_{\text{gal}} \theta_{\text{pix}}^2)$ with the pixel size of $\theta_{\text{pix}} = 0.6$ arcmin. In this paper, we set $\sigma_\epsilon = 0.35$ and $n_{\text{gal}} = 20 \text{ arcmin}^{-2}$. These correspond to the typical value of the current-generation ground-based imaging survey [44]. We prepare 60000 and 1000 realizations of the random ellipticities for training and testing, respectively.

For a given data of shear, we perform the smoothing as in Section. II B to produce a smoothed convergence field. Note that we produce the smoothed fields from noiseless shear as well as noisy shear on realization-by-realization basis.

C. Setup of Training and Validation

The objective of our networks is expressed as

$$\arg \min_G \max_D \left\{ \mathcal{L}_{\text{cGAN}}(G, D) + \lambda \mathcal{L}_{\text{L1}} \right\}, \quad (13)$$

where G is the generator and D is the discriminator. We here introduce two loss functions as

$$\mathcal{L}_{\text{cGAN}}(G, D) = \mathbb{E}_{x,y} \log D(x, y) + \mathbb{E}_{x,z} \log \{1 - D(x, G(x, z))\}, \quad (14)$$

$$\mathcal{L}_{\text{L1}}(G) = \mathbb{E}_{x,y,z} \sum_{\text{map}} |y - G(x, z)|, \quad (15)$$

where x is the input vector, y is the output vector, and z is a random noise vector at the bottom layer of the generator. In Eq. (15), the summation runs over all the pixels in a map. In the training, we alternate between one gradient descent step on D , then one step on G . As suggested in Ref. [45], we train to maximize the term of $\log D(x, G(x, z))$. In addition, we divide the objective by 2 while optimizing D , which slows down the learning rate of D relative to G .

When training the networks, we use the minibatch Stochastic Gradient Descent (SGD) method and apply the Adam solver [46], with learning rate 0.0002, momentum parameters $\beta_1 = 0.5$ and $\beta_2 = 0.9999$. We also set $\lambda = 100$ in Eq. (13). All the networks in this paper are

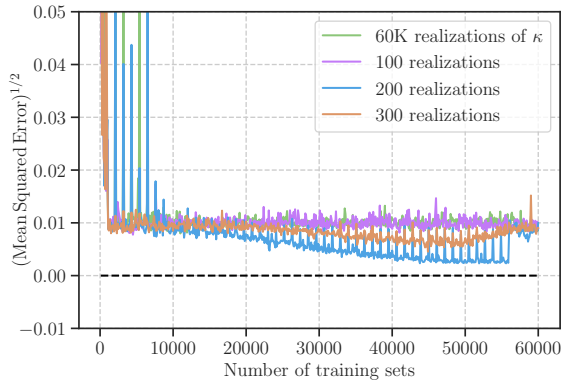


FIG. 2. The mean squared error of the lensing convergence maps in the training process. We measure the mean squared error of the validation data set at different epochs in training. In this figure, the colored lines show the training processes based on different realizations of the true convergence: *Green*: the number of realization of true maps is 60000. *Purple*: Among 60000 realizations (green), we select 100 realizations before training. *Blue*: Similar to purple one, but we select 200 realizations. *Orange*: Similar to purple one, but we select 300 realizations. This figure shows that ~ 200 realizations of noiseless convergence is suitable to learn the image-to-image translation in an efficient way.

trained with a batch size of 1. We initialize the model parameters in the networks from a Gaussian distribution with mean 0 and standard deviation 0.02. We train our networks using the TensorFlow implementation³ of `pix2pix` on a single NVIDIA Quadro P5000 GPU. While processing, we randomly select training and validation data from the input data sets. Each network is validated every time it learns 100 image pairs. The training procedure takes 0.5 – 2 seconds per an image pair, and about a few hours for learning 10000 image pairs.

Producing a realization of noisy data set κ_{obs} is non-trivial for our training task, because we use two different independent realizations of the noise κ_N and lensing field of interest, κ . To optimize the training, we test four cases as:

1. The input data set consists of 60000 independent realizations of κ_N . When adding the lensing convergence κ to produce a noisy data, we use different realizations of κ from 60000 different realizations.
2. Similar to case 1, but we restrict the number of realization of κ to be 300. When adding κ to the noise κ_N , we choose the map from 300 realizations at random.

3. Similar to case 2, but we use the number of realization of κ to be 200.
4. Similar to case 3, but we use the number of realization of κ to be 100.

In Case 1, we train the networks using 60000 independent noisy maps. We expect that there is some optimal number of realizations of κ in the training process, since too many realizations will make an efficient learning difficult for the networks, while fewer realizations can not provide a sufficient information for the image-to-image translation. From this point of view, Case 1 corresponds to oversampling of κ for training.

To find the optimal number of realizations of κ , we measure the mean squared error (MSE) in image space for validation sets. Figure 2 shows the MSEs for four different input data sets as a function of the number of training sets. Note that the figure illustrates the proficiency level of networks for denoising, not the convergence check in the training process. Clearly, efficient learning of image-to-image translation is achieved when we use ~ 200 realizations of κ to produce noisy data set. In the following, we use 200 realizations to produce noisy data for training. The predicted noise images by the generator with $\gtrsim 40000$ training sets show unrealistic fluctuations between nearby pixels, while the predicted noise images are smooth enough if the network uses 20000 – 40000 training sets.

In general, the optimal number of training sets depends on a number of factors. For our purpose of cosmology study here, it is important to train so that the estimation of summary statistics is unbiased. Hence, we try to find the optimal number by using two common summary statistics in cosmological studies. One is power spectrum defined as

$$\langle \tilde{\kappa}(\ell_1) \tilde{\kappa}(\ell_2) \rangle \equiv (2\pi)^2 \delta^{(2)}(\ell_1 - \ell_2) C_\kappa(\ell_1), \quad (16)$$

where $\tilde{\kappa}$ is the Fourier transform of a noiseless convergence field, $\delta^{(n)}(\mathbf{x})$ is the Delta function in n -dimensional space, and $C_\kappa(\ell_1)$ is the power spectrum. The other statistic is the one-point probability distribution function (PDF) \mathcal{P} of lensing convergence in real space. Note that the power spectrum is a measure of Gaussian information in κ fields, while the PDF can capture some non-Gaussian information such as skewness and kurtosis.

We examine three cases with 20000, 30000, and 40000 sets when training our networks. After training, we input the test data set to the generator, in order to have lensing convergence predictions by the networks, denoted as κ_{DL} . We use 1000 independent realizations of noisy lensing convergence for testing the network. Note that the data set in testing process is independent of the ones used in training. We then measure the power spectrum and the PDF of κ_{DL} . We also measure them for the corresponding true map κ_{true} . We normalize the lensing convergence so as to have a zero mean and a unit variance. This normalization is necessary to discern a small difference of summary statistics between κ_{DL} and

³ <https://github.com/yenchenlin/pix2pix-tensorflow>

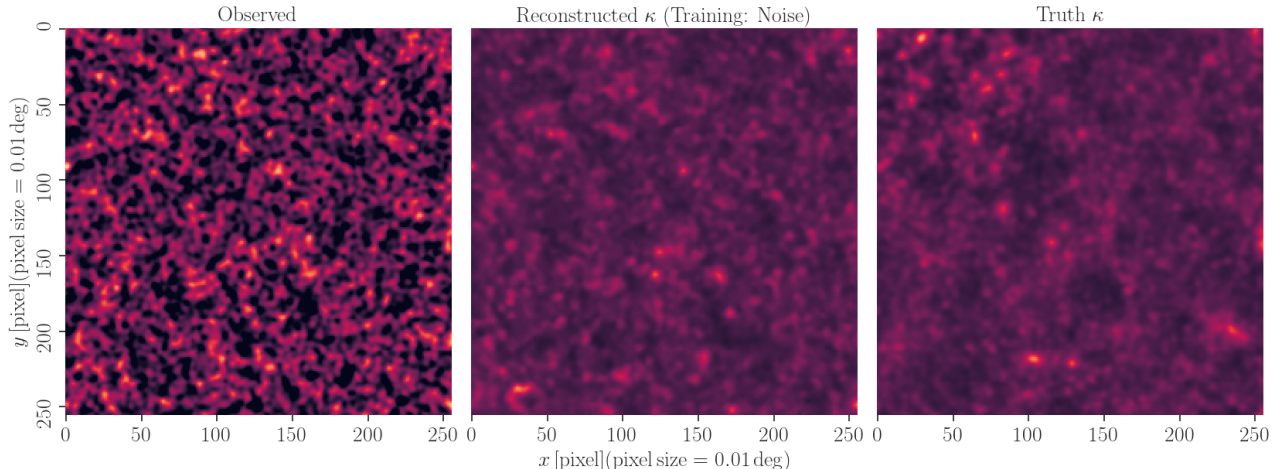


FIG. 3. An example of image-to-image translation by our networks. The left panel shows input noisy convergence map, while the right is for true (noiseless) convergence. The medium represents the reconstructed map by our networks. For the reconstructed map, we first obtain the underlying noise field from the generator in our networks and then derive the convergence map by the residual between the input noisy map and the predicted noise. In this figure, we use the networks trained by 30000 image pairs.

κ_{true} . We employ 20 bins logarithmically spaced in the range of $\ell = 100$ to 10^5 when measuring the power spectra. We measure the PDF in 100 linear spaced bins in the range of $(\kappa - \mu)/\sigma = [-5, 15]$, where μ is the mean and σ is the standard deviation of lensing convergence over a sky coverage of 2.5×2.5 squared degrees. Comparing the summary statistics of the predicted and true convergence fields, we will find how many realizations of input data sets are required to minimize the difference of statistics. Furthermore, we study the dependence of model prediction by the networks on different realizations of training sets by constructing 10 bootstrap subsamples of 20000 – 40000 realizations from parent 60000 realizations.

IV. RESULTS

In this section, we summarize the performance of denoising weak lensing maps by our deep-learning approach.

A. Visual impression

We first show a visual comparison among three lensing fields, an input noisy convergence κ_{obs} , the predicted field by our networks κ_{DL} , and the underlying noiseless convergence κ_{true} . Figure 3 compares the lensing fields for a given realization. The predicted convergence κ_{DL} is obtained by the networks trained with 30000 train-

ing sets. As seen in the figure, the predicted field has a similar smoothness to the true κ_{true} , and significant errors in the κ_{DL} field. We also find some filamentary structures extending over a few degree lengths that look similar between κ_{DL} and κ_{true} , while more compact structures such as peaks are found to be different. Since such peaks in noiseless convergence are generated by massive galaxy clusters [32], the deep-learning denoising in this paper is probably less effective for searching for the massive objects in the Universe (we discuss this point in Section IV C in more detail). Nevertheless, the reconstructed convergence by the networks shows a similar level of clumpiness and morphology to the true convergence over arcmin to degree scales. From the comparison, we expect our networks can be good at estimating summary statistics including non-Gaussian information, but they may not work for reconstruction of highly compact and rare features in the image.

B. Reconstruction of summary statistics

We discuss reconstruction of summary statistics. Figure 4 shows the average of one-point PDFs and that of power spectra for the predicted field κ_{DL} over 1000 realizations of input noisy κ_{obs} . In this figure, the red points show the average statistics for κ_{DL} and the error bars represents the standard deviation over 1000 realizations. For comparison, we also show the 1σ variance of statistics as green filled region in the figure. This confidence region is defined by the rms of PDF and power spectrum for true

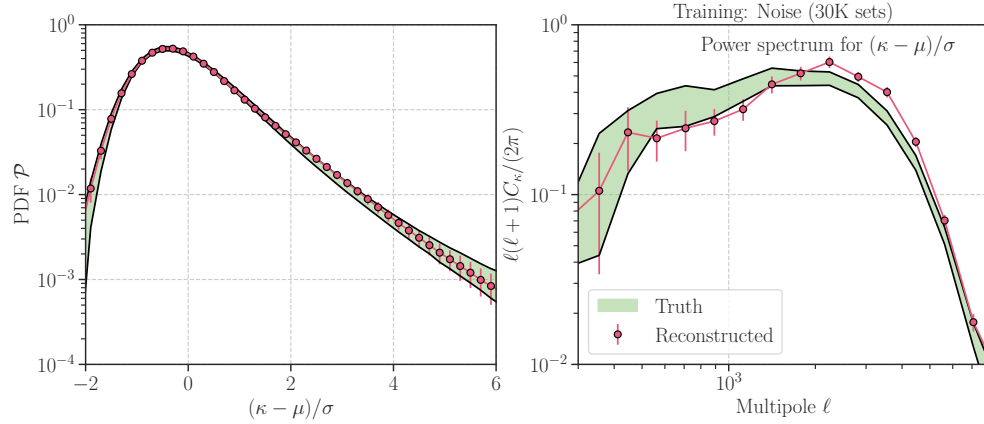


FIG. 4. Comparison of weak-lensing summary statistics. The left panel shows the one-point probability distribution functions, and the right panel compares the power spectra. In each panel, we normalize the convergence so as to have zero mean and unit variance. The red points with error bars represent the summary statistics for the reconstructed field by our deep-learning method (the points and error bars are for the average and the rms over 1000 realizations for testing, respectively), while the green shaded regions show the 1σ variance of the true values. We evaluate the variance by using 1000 noiseless weak lensing maps with a sky coverage of 2.5×2.5 squared degrees.

underlying fields κ_{true} over 1000 realizations. Note that the results shown in Figure 4 are obtained by one of our networks with 30000 training sets (we have 10 networks that are trained with varying the number of training sets based on bootstrap sampling). Figure 4 demonstrates that our deep-learning method can reconstruct the shape of the one-point PDF of underlying convergence field on average, while the reconstructed power spectrum has a systematic bias.

We also examine the variation of the averaged PDF and that of power spectrum over 10 networks with different realizations. Now we define the bias of a given statistic \mathcal{S} at i -th bin as

$$\text{Bias}[\mathcal{S}_i] \equiv \frac{\bar{\mathcal{S}}_i(\kappa_{\text{DL}}) - \bar{\mathcal{S}}_i(\kappa_{\text{true}})}{\text{Var}[\mathcal{S}_i(\kappa_{\text{true}})]^{1/2}}, \quad (17)$$

where \mathcal{S}_i is the statistic at i -th bin, $\bar{\mathcal{S}}_i(\kappa_{\text{DL}})$ is the averaged statistic for the predicted field by our networks, $\bar{\mathcal{S}}_i(\kappa_{\text{true}})$ is the average for underlying noiseless field, and $\text{Var}[\mathcal{S}_i(\kappa_{\text{true}})]$ represents the variance of \mathcal{S}_i for noiseless field.

Figure 5 shows the bias in averaged PDF over 10 different networks. The top, middle and bottom panels in the figure show the bias in PDF based on networks with 20000, 30000, and 40000 training sets, respectively. In each panel, we show the result of 10 networks by bootstrap sampling of training sets. The gray lines show the biases obtained by 10 different networks. The red points and error bars are the average and rms over 10 realizations of network. According to this figure, we find ~ 30000 training sets are sub-optimal to have a less biased estimate of PDF on average. The average bias level in reconstruction of PDF is found to be $\lesssim 1\sigma$ over the

wide range of $|(\kappa - \mu)/\sigma| \lesssim 1$. It would be worth noting that our networks will be able to reconstruct the PDF of noiseless lensing convergence at $(\kappa - \mu)/\sigma \sim 0$ where the noise completely dominates in the original noisy map. We also find the training of networks can depend on the realization of input data sets. Even if we set the number of training data sets to be 30000, we find one of 10 networks shows a bias in PDF at $|(\kappa - \mu)/\sigma| \lesssim 1$ with a level of $\sim 2\sigma$. Figure 5 clearly demonstrates that the network will have a variety due to the statistical fluctuations in input data sets. This suggests that training based on single realization of training sets is not sufficient for denoising weak lensing maps⁴.

Figure 6 shows the bias in averaged power spectrum over 10 different networks. Similar to the one-point PDF, we also find a non-negligible scatter over 10 different networks and ~ 40000 realizations will be sub-optimal for an unbiased estimate of power spectrum. Note that the lensing power spectrum can be estimated without denoising procedure and we already have some unbiased estimators of lensing power spectrum in practice [47–49]. Therefore, we expect the biases shown in Figure 6 will not be relevant to actual cosmological analyses. Rather, the figure indicates that our network may not use the Fourier decomposition to separate the noise and underlying lensing fields. To see the reconstruction in Fourier space, we compute the cross correlation between the predicted and

⁴ In Appendix B, we examine the variation of the rms of PDF over 10 different realizations. We find that the variation coming from the network-wise fluctuation is less important in the rms of PDF for denoised maps.

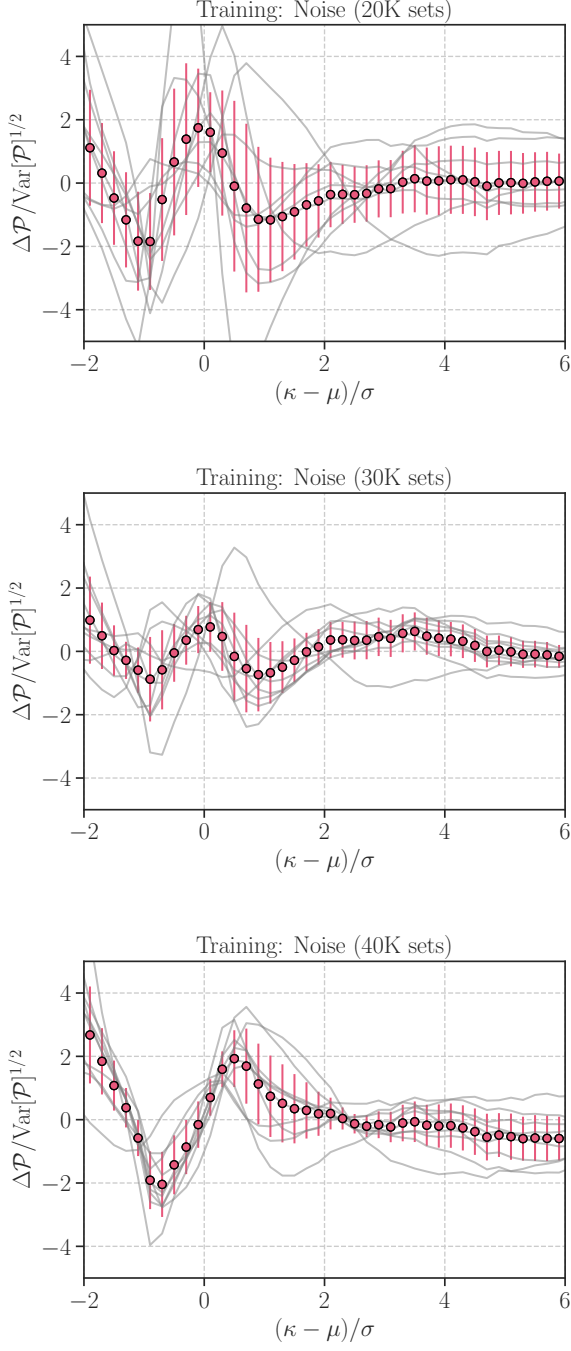


FIG. 5. The dependence of the bias in one-point PDF of the convergence field on the number of training sets. Three different panels show the results by the network based on different numbers of training sets, 20000, 30000, and 40000 image pairs from top to bottom. In each panel, the vertical axis shows the bias in convergence PDF with respect to the true noiseless counterpart and the horizontal axis is the value of convergence (normalized so as to have zero mean and unit variance). The gray lines show the results over 10 bootstrap realizations, while the red points and error bars are the average and standard deviation over 10 realizations. We note that the vertical axis is normalized by the statistical uncertainty of true PDF at each bin assuming a sky coverage of 2.5×2.5 squared degrees. The bootstrap scatter is found to be similar to the statistical uncertainty.

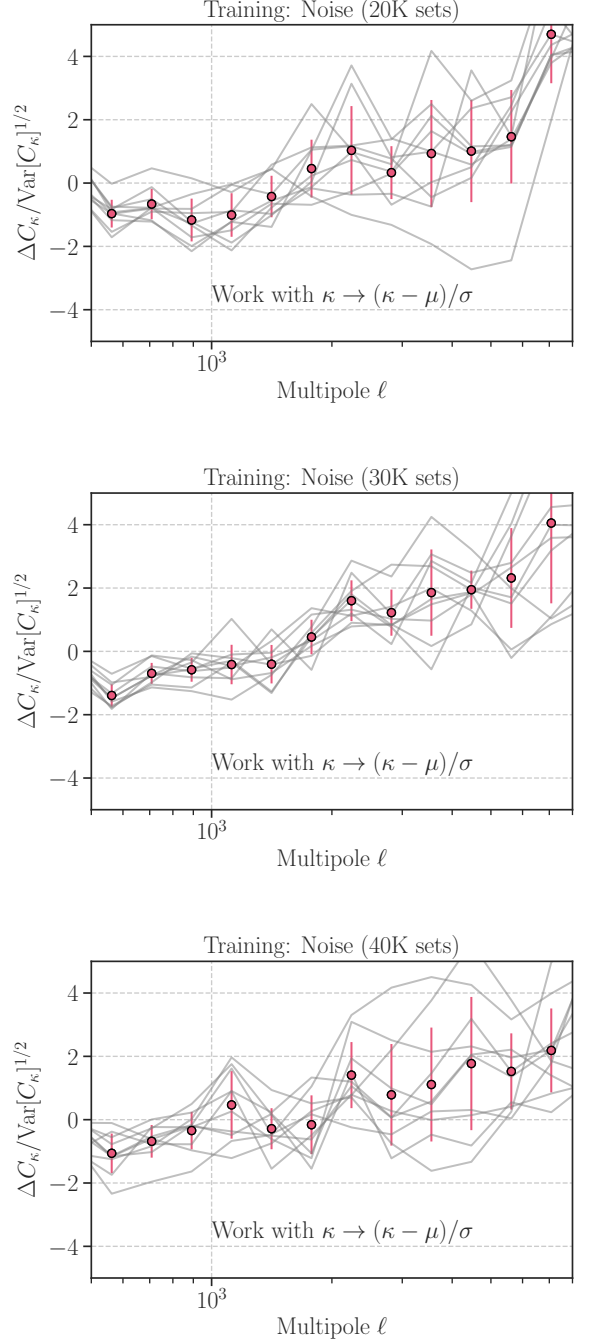


FIG. 6. Similar to Figure 5, but this figure is for the power spectrum.

true noiseless maps. We find the ratio between the cross power spectrum and the true auto power spectrum to be ~ 0.5 at $\ell = 500$ and ~ 0.1 at $\ell = 5000$. Hence, the denoising with our networks looks less efficient in Fourier space.

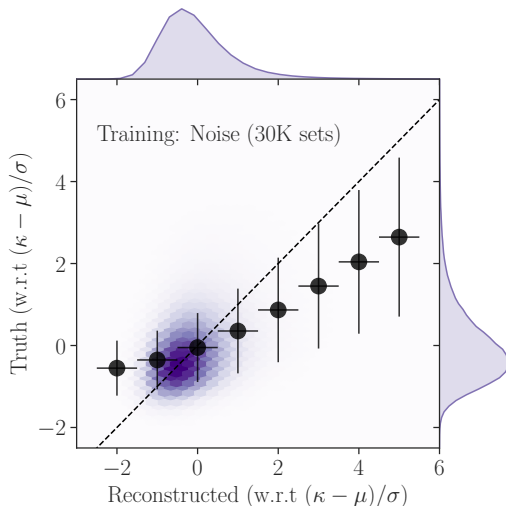


FIG. 7. An example of scatter plot between the reconstructed convergence and true noiseless convergence. The dashed line shows the one-to-one correspondence. In this figure, we use the networks trained by 30000 image pairs and then adopt it to 1000 test data.

C. Comparison on pixel-by-pixel basis

So far, we have focused on the statistical properties of weak lensing convergence predicted by our networks. In this subsection, we test and validate our denoising method on a pixel-by-pixel basis. Figure 7 is a simple scatter plot of lensing convergence in $\kappa_{\text{DL}} - \kappa_{\text{true}}$ plane, where κ_{DL} is the reconstructed one and κ_{true} is the noiseless counterpart. For this figure, we show the results from 1000 realizations of κ_{DL} and κ_{true} from single networks trained by 30000 data sets. The dashed line in the figure represents the one-to-one correspondence, while the deeper color shows denser region in the $\kappa_{\text{DL}} - \kappa_{\text{true}}$ plane. In Figure 7, the black points with error represent the mean and standard deviation of κ_{true} for a given range of κ_{DL} . The figure shows that the predicted field by our networks follows some probability distribution with the width of $\sim 1 - 3\sigma$ for a given κ_{true} , where σ is the rms of κ_{true} in a 2.5×2.5 squared-degrees sky, while the width slightly changes as the input value of κ_{true} . As seen in the figure, the mean relation in the $\kappa_{\text{DL}} - \kappa_{\text{true}}$ plane deviates from the one-to-one correspondence at $(\kappa - \mu)/\sigma \lesssim -1$ and $\gtrsim 2$.

In addition, we examined the corresponding positive peaks between noiseless and reconstructed (denoised) maps. Note that previous numerical studies have shown that single massive galaxy clusters often present along the direction of peaks with a peak height larger than $\kappa/\sigma > 3 - 4$ [32]. We first identify the local maxima in both of noiseless field κ_{true} and its denoised counterpart κ_{DL} . For selection of peaks, we set the threshold of peaks to be $(\kappa - \mu)/\sigma > 3$. For a given position of lensing peak

in κ_{DL} , we search for the matched peaks in κ_{true} within a radius of 3 arcmin. This search radius is set to be larger than the smoothing scale but still smaller than the angular size of massive haloes at $z = 0.1 - 0.7$ [32]. Among 1000 test data sets, we find 33563 and 44624 peaks in κ_{DL} and κ_{true} , respectively. The number of matched peaks is found to be 12,329. The corresponding completeness of lensing peaks in κ_{DL} is given by $12329/33553 = 36.7\%$ and the purity to be $12329/44624 = 27.6\%$. We confirmed these results are almost unaffected even if the threshold of peaks is changed to $(\kappa - \mu)/\sigma > 4, 5$, or 6. According to these results, our denoising method with CANs cannot always predict noiseless lensing convergence in unbiased way and it would be less effective for collecting less massive galaxy clusters and/or completing a search of massive galaxy clusters in weak lensing maps.

V. APPLICATION TO COSMOLOGICAL ANALYSIS

Although our denoising method does not perfectly reproduce the true density distribution, it may still be used to estimate cosmological parameters accurately. Figure 7 clearly shows positive correlation between the denoised map κ_{DL} and its noiseless counterpart κ_{true} . We thus expect that statistical analysis of denoised maps can be utilized for cosmological parameter inference.

To study quantitatively the possibility of extracting cosmological information from denoised lensing maps, we perform a Fisher analysis with one-point PDF. We argue that the one-point PDF as a function of $(\kappa - \mu)/\sigma$ should contain information of non-Gaussianity. For a given noisy lensing map, we generate 10 realizations of denoised lensing map from 10 different networks using 10 bootstrap sampling of training data sets. We then construct an estimator of the denoised one-point PDF as

$$\hat{\mathcal{P}}(\mathcal{K}) = \frac{1}{N_{\text{CAN}}} \sum_{i=1}^{N_{\text{CAN}}} \mathcal{P}_i(\mathcal{K}_{\text{DN}}), \quad (18)$$

where $\mathcal{K} = (\kappa - \mu)/\sigma$ is the normalized lensing convergence so as to have zero mean and unit variance, $N_{\text{CAN}} = 10$ is the number of available networks, and $\mathcal{P}_i(\mathcal{K}_{\text{DN}})$ represents the one-point PDF of predicted lensing maps by i -th CAN.

We compute the Fisher matrix of the estimator given by Eq. (18) as

$$F_{ij} = \text{Tr} \left[C^{-1} \left(\frac{\partial \mathcal{P}}{\partial p_i} \right) \left(\frac{\partial \mathcal{P}}{\partial p_j} \right) \right], \quad (19)$$

where C is the covariance matrix, \mathcal{P} is the assumed model of one-point PDF, and p_i describes the parameters of interest. The Fisher matrix provides an estimate of the error covariance for two parameters as $\langle \Delta p_\alpha \Delta p_\beta \rangle = (F^{-1})_{\alpha\beta}$, where Δp_α represents the statistical uncertainty of parameter p_α . Here, we consider two cosmological parameters as an illustrative example:

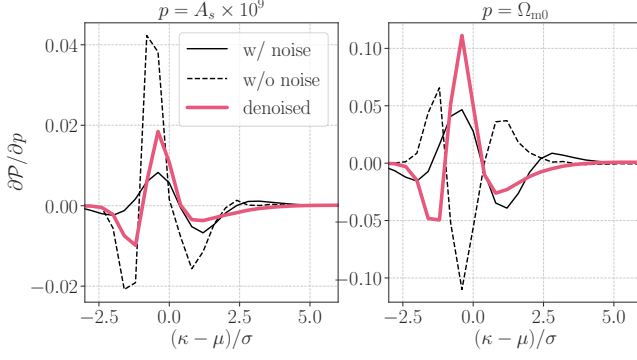


FIG. 8. The parameter dependence of one-point PDF \mathcal{P} . Left and right panels show the first derivative of \mathcal{P} with respect to the amplitude in curvature perturbations A_s and the mean matter density Ω_{m0} , respectively. In each panel, thick red line shows the parameter dependence of \mathcal{P} for denoised lensing maps, while thin dashed and solid lines are for noiseless and noisy maps. We use the networks trained using 30000 lensing maps and perform denoising. Note that we normalized the field as $(\kappa - \mu)/\sigma$ on realization-by-realization basis and this procedure can make the cosmological dependence of statistics much complicated (see, e.g. Ref. [50]).

$p = (A_s \times 10^9, \Omega_{m0})$, where A_s is the amplitude of the curvature perturbation and Ω_{m0} is the mean cosmic mass density. We set the fiducial values to be $A_s \times 10^9 = 2.35$ and $\Omega_{m0} = 0.238$ in the Fisher analysis. We calculate the cosmological dependence of one-point PDF \mathcal{P} , by using 40 realizations of ray-tracing simulations with different cosmological models from our fiducial model [51]. We consider four different models by varying $A_s \times 10^9$ or Ω_{m0} with $\pm 10\%$ in logarithmic space. These additional simulations are designed so as to cover the same sky coverage of 5×5 squared degrees with the source redshift of 1. We produce 1000 realizations of noisy convergence from the additional simulations in the same way as in Section III B. Once we get the noisy convergence maps for the set of cosmological models, we compute the estimator of Eq. (18) from all noisy maps and evaluate the first derivative of one-point PDF with respect to the i -th parameter as

$$\frac{\partial P(\mathcal{K})}{\partial p_i} = \frac{\langle \hat{\mathcal{P}}(\mathcal{K}; p_{i+}) \rangle - \langle \hat{\mathcal{P}}(\mathcal{K}; p_{i-}) \rangle}{p_{i+} - p_{i-}}, \quad (20)$$

where p_{i+} (p_{i-}) represents a higher (lower) parameter value than the fiducial one. In Eq. (20), $\langle \dots \rangle$ is the average over 1000 realizations of noisy maps. The data covariance in Eq. (19) is also estimated from 1000 realizations of the estimator $\hat{\mathcal{P}}$ for our fiducial setup. Because the covariance is estimated for a sky coverage of 2.5×2.5 square degrees, we simply assume that the covariance matrix would scale with the inverse of sky coverage when making forecast for parameter constraints. For comparison, we also compute the Fisher matrices of one-point

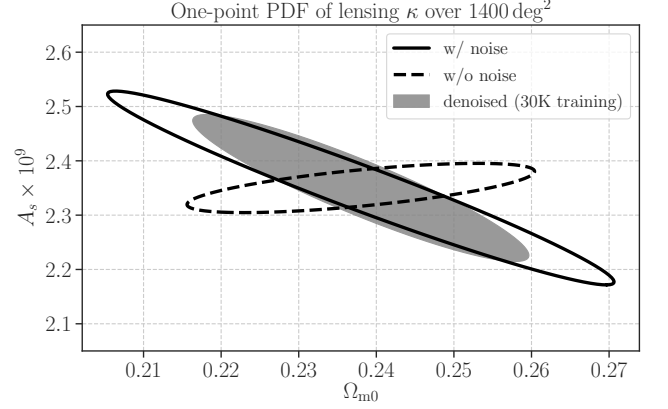


FIG. 9. The expected 1σ constraints by one-point PDF of lensing maps in a hypothetical galaxy imaging survey with a sky coverage of 1400 square degrees. To make a fair comparison, we restrict the range of $(\kappa - \mu)/\sigma$ in the Fisher analysis; we use only the one-point PDF of \mathcal{P} greater than 0.01.

Lensing maps	$\Delta(A_s \times 10^9)$	$\Delta\Omega_{m0}$
With Denoising	8.96×10^{-2} (3.58×10^{-2})	1.43×10^{-2} (5.73×10^{-3})
Noisy	1.18×10^{-1} (2.92×10^{-2})	2.16×10^{-2} (5.36×10^{-3})
Noiseless	3.00×10^{-2} (2.22×10^{-2})	1.48×10^{-2} (1.09×10^{-2})

TABLE I. Summary of the marginalized error of cosmological parameters by one-point PDF of lensing map. Second and third columns list the expected statistical error assuming 1400 square degrees, and the number with bracket shows the unmarginalized error in the Fisher analysis.

PDF for noisy and noiseless lensing maps.

Before discussing the Fisher analysis result, we first look into the parameter dependence of \mathcal{P} for denoised maps. Figure 8 summarizes the parameter dependence evaluated by Eq. (20). There, the thick red lines show the parameter dependence of \mathcal{P} for denoised maps. By comparing with the counterparts for noisy maps, we find that our denoising method with CANs can increase the sensitivity on the cosmological parameters at $(\kappa - \mu)/\sigma \sim 0$, where the galaxy shape noise usually dominates in observations. This highlights the possibility of our denoising method to extract cosmological information from realistic observational data.

Figure 9 shows the expected parameter constraints for a sky coverage of 1400 square degrees. To perform the Fisher analysis, we adjust the the range of $(\kappa - \mu)/\sigma$ in noisy and denoised maps independently so that we can use the range with $\mathcal{P} \geq 0.01$. This threshold is found to correspond to $-2.5 < (\kappa - \mu)/\sigma < 2.5$ in noisy lensing maps, and $-2 < (\kappa - \mu)/\sigma < 4$ in denoised and noiseless ones. We find that the one-point PDF in denoised lensing maps can actually improve the constraints of A_s and Ω_{m0} by 30 – 40%, compared to the case using noisy lensing

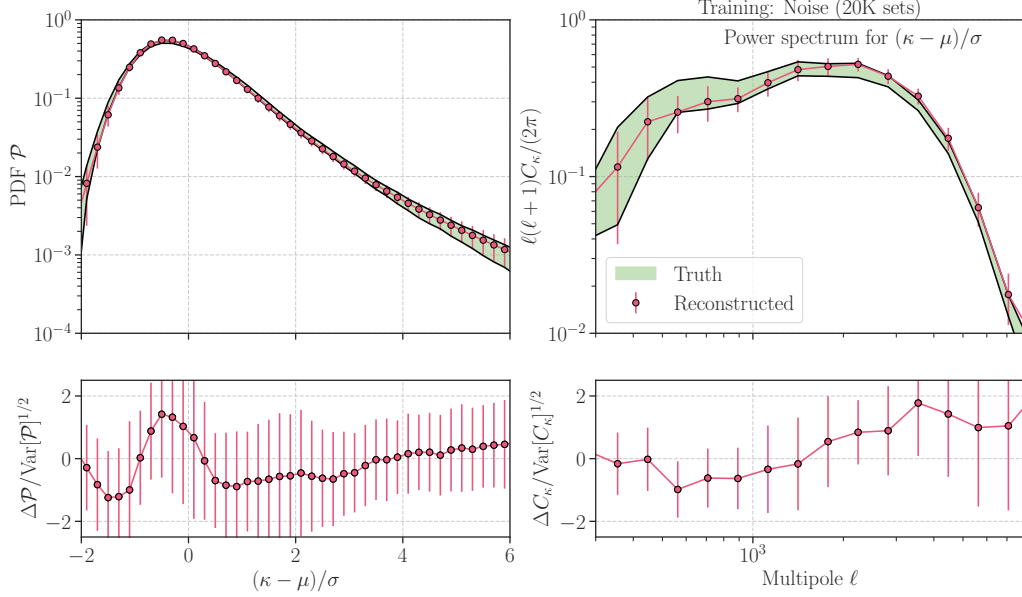


FIG. 10. Performance for denoising in a hypothetical survey with the high source number density of 40 arcmin^{-2} . We plot the summary statistics as in Figure 4. The bottom left panels show the average bias in the reconstructed PDF normalized to the rms of the noiseless PDF, while the bottom right panel is for the power spectrum. Note that the red error bars include the statistical error over 1000 test data as well as the bootstrap scatter over 10 different networks.

maps. We note that the improvement is not significantly affected even if we vary the range of $(\kappa - \mu)/\sigma$ when performing the Fisher analyses. Table I summarizes the marginalized errors in cosmological parameters.

VI. FUTURE PROSPECTS IN UPCOMING SURVEY

Upcoming galaxy imaging surveys are aimed at generating accurate lensing maps by increasing the source number density and the area coverage. These include the Wide Field Infrared Survey Telescope (WFIRST⁵), the Large Synoptic Survey Telescope (LSST⁶), and the Euclid satellite⁷.

Since the noise rms of weak lensing maps approximately scales with the inverse of the source galaxy number density, we expect that the upcoming surveys with a higher source number density can provide less noisy maps. We here examine if our denoising method works for such accurate observations in the near future. As a representative example, we assume the source number density to be 40 arcmin^{-2} , and fix the rms of shape noise for individual galaxies $\sigma_\epsilon = 0.35$ and source redshift $z_{\text{source}} = 1$. In this case, the rms of noisy and noiseless

map are found to be 1.7×10^{-2} and 8.6×10^{-3} , respectively. This set up roughly corresponds to the aforementioned upcoming imaging surveys. Following the method in Section III B and setting $n_{\text{gal}} = 40 \text{ arcmin}^{-2}$ in Eq. (12), we produce new 60000 and 1000 realizations of input data set for training and testing networks, respectively. We then train the networks with 20000 realizations of these new data sets. We obtain 10 different networks for denoising using 10 bootstrap sampling in total.

Figure 10 summarizes the performance of our networks for denoising. In this figure, left and middle panels show the comparison of two summary statistics, one-point PDF and power spectrum. The red points represent the average statistics over 10 bootstrap realizations and 1000 test data sets, while the error bars are defined as

$$\text{Var}[\mathcal{S}(\kappa_{\text{DL}})]_{\text{stat}} + \text{Var}[\bar{\mathcal{S}}(\kappa_{\text{DL}})]_{\text{bootstrap}}. \quad (21)$$

Here the first term is the standard deviation of statistics \mathcal{S} of reconstructed κ_{DL} over 1000 realizations of test data, and the second term is the scatter in the average $\bar{\mathcal{S}}$ over 10 bootstrap networks. If we increase the source number density, our networks perform well for both of the one point PDF and the power spectrum.

As in Section V, we perform the Fisher analyses of one-point PDF assuming the source number density of 40 arcmin^{-2} and the sky coverage of 20000 square degrees. For this observational spec, our denoising method allows us to constrain $A_s \times 10^9$ and $\Omega_{\text{m}0}$ with a marginal-

⁵ <https://wfirst.gsfc.nasa.gov/>

⁶ <https://www.lsst.org/>

⁷ <https://www.euclid-ec.org/>

ized fractional error of 0.67% and 0.95%, while the PDF of noisy maps can provide the constraints of $A_s \times 10^9$ and Ω_{m0} with 0.85% and 1.47%. Note that the improvement of cosmological parameters by denoising is equivalent to the increase in sky coverage by $\sim 50\%$ in the analysis of noisy lensing map.

VII. CONCLUSION AND DISCUSSION

We have studied denoising weak lensing maps using the conditional adversarial networks (CANs) developed in Ref [40]. We have developed a training strategy for the networks to denoise noisy lensing maps. Our findings are summarized as follows. The networks learn efficiently mapping from input noisy maps to the underlying noise field. About 30000 realizations of image pairs are found to be sub-optimal for efficient training of networks to denoise weak lensing with a sky coverage of 2.5×2.5 squared degrees and with a similar noise level to the ongoing ground-base galaxy surveys [44]. The trained networks reproduce the one-point PDF of noiseless weak lensing maps with deviations within a 1σ level. Although the networks do not reproduce the noiseless power spectrum equally well if we assume the noise level in the ongoing survey, we find that the performance can be improved in upcoming survey with higher source number density. When assuming the source number density of 40 arcmin^{-2} , the one-point PDF and power spectrum are in good agreement with the true counterparts.

We have studied reconstruction of noiseless lensing maps on a pixel-by-pixel basis and have found that denoising with CANs produces a biased estimate of local convergence. Nevertheless, we find that a clear positive correlation between noiseless and denoised maps remains. We thus argue that cosmological information imprinted in noiseless maps can be extratrated at least partially with our denoising method. We have demonstrated that our deep-learning denoising can improve the cosmological constraints by increasing the sensitivity of one-point PDF to a few cosmological parameters. Assuming that the noise level is similar to the ongoing ground-base galaxy survey by Subaru Hyper-Suprime-Cam, we find the improvement by denoising can reach the level of 30 – 40% in two primary cosmological parameters. Our denoising approach enables us to tighten cosmological constraints by one-point PDF using data from upcoming galaxy imaging surveys. We emphasize that the improvement by denoising is equivalent to the increase in the survey area by a factor of 1.5. We thus conclude that the denoising method developed in the present paper effectively maximizes the science output from ongoing and upcoming galaxy imaging surveys.

Denoising with deep-learning networks can be a powerful tool to probe the cosmic large-scale structure with weak lensing. It is yet to be investigated how the method works for real data sets. In this paper, we have worked with an idealized situation while ignoring several effects

in real observations. These include inhomogeneous angular distribution of source galaxies, masking around bright stars, photometric redshift uncertainty, biases in galaxy shape measurement, and the correlation between intrinsic ellipticity and lensing-induced shear (see, Ref [52] and the references therein for systematics in weak lensing measurement). Further studies are necessary to examine the applicability to real data sets.

In addition, more studies are desirable for reducing the bias in summary statistics induced by denoising. As found in Section. IV B, our denoising procedure cannot reproduce the two-point correlation of noiseless lensing field with a reasonable accuracy. This would be partly because the objective of our networks does not include any information about the two-point correlation. It is interesting to develop appropriate way providing the information of spatial correlations on the networks. We expect that there are still rooms for optimizing the training strategy for cosmological analysis. As a brute-force approach, one can make use of a large set of numerical simulations for gravitational lensing and emulate the bias of statistics induced by denoising as a function of cosmological parameters. Although this is straightforward, one need reasonable prior information in the parameter space of interest in practice.

Nevertheless, the denoising with deep learning is a unique method at present to reconstruct the true weak lensing map with an angular resolution of $\sim 1 \text{ arcmin}$. It is worth exploring this approach further for cosmological analyses. In principle, the image-to-image translation with deep learning can be applied for multi-dimensional data sets. Interesting examples in modern cosmology include reconstruction of the three-dimensional matter density field from the observed spatial distribution of galaxies and the component separation in intensity mapping observations over a wide range of frequencies of photons. It is a challenging task to extract cosmological information in an efficient way from future cosmology surveys. Deep leaning is among the promising approaches and is expected to play a crucial role in future statistical analyses. Our method proposed in this paper may be one of the most potent forces that drive big-data analyses in the era of precision cosmology.

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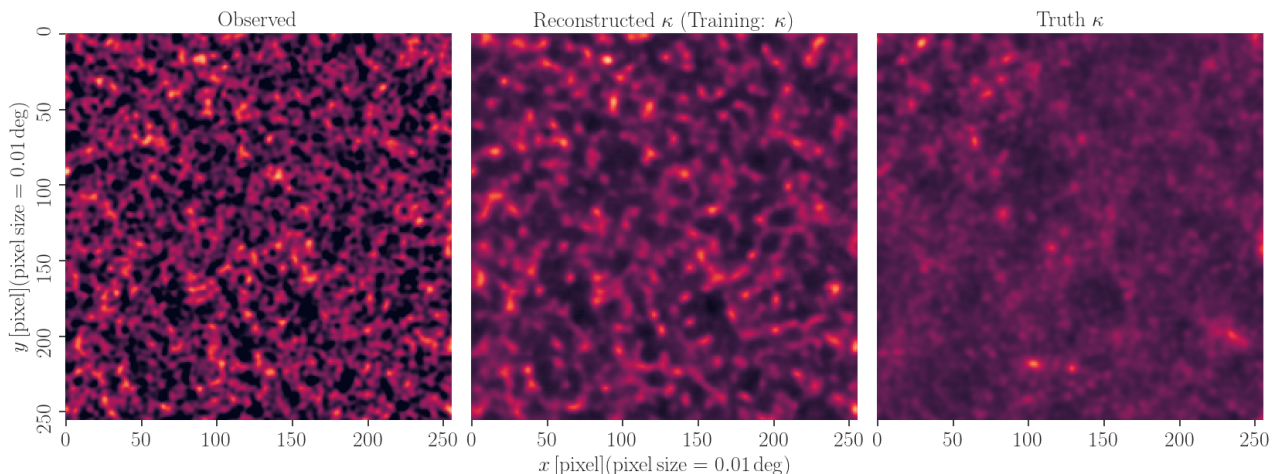


FIG. 11. Similar to Figure 3, but the networks have been trained so that the generator can predict the lensing convergence.

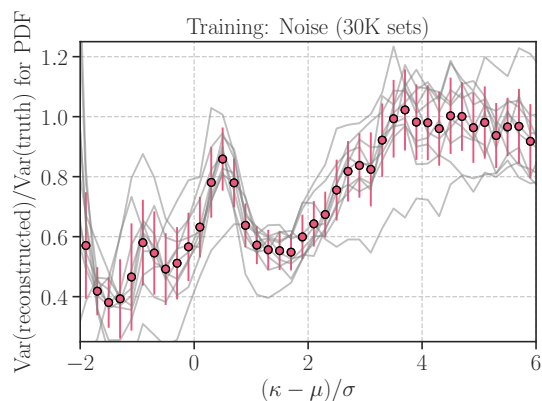


FIG. 12. The scatter of one-point PDF of denoised map divided by one for the noiseless counterpart. The gray lines represent the results over 10 bootstrap realizations of training datasets, while the red points and error bars are the average and standard deviation over 10 realizations.

Appendix A: Training to generate lensing convergence

In this appendix, we study another way of training the network for denoising weak lensing map. We train the networks so that the generator will produce the underlying lensing convergence from an input noisy map. Similar to the method in Section III C, the training is most efficient when noisy data sets consist of 20000 – 40000 independent realizations of noiseless convergence maps combined with 100 noise realizations. After some trials, we find that the summary statistics predicted by the net-

works do not agree with the true counterparts within a $\sim 3\sigma$ level.

Figure 11 shows the comparison between the predicted convergence by the networks and the true noiseless map. Apparently the networks fail to reproduce the smoothness of convergence with the same level as the truth, and there are too many clumpy structures in the predicted convergence.

Appendix B: Variance of lensing statistics after denoising

We here study the variance in two summary statistics of reconstructed convergence. Figure 12 shows the scatter of one-point PDF for reconstructed fields, divided by the scatter for noise-free lensing maps. For this figure, we used the networks trained by 30000 image pairs and adopted them to 1000 realizations of test dataset to produce denoised maps. We then evaluated the variance of one-point PDF over 1000 denoised maps as well as noiseless true maps. The gray line in the figure represents the scatter in variance for 10 different bootstrap realizations of deep-learning networks. In practice, the scatter in variance of reconstructed PDF (shown in the red error bar) can be reduced when one work with the average PDF over bootstrap realizations of networks.

We find the variance in one-point PDF for reconstructed field is close to that for true noiseless field at $(\kappa - \mu)/\sigma > 3$, but the variance for PDF of κ_{DL} can be smaller by a factor of 0.4 – 0.8 at $(\kappa - \mu)/\sigma \lesssim 3$. This underestimation in lower $(\kappa - \mu)/\sigma$ bins can be mitigated when we decrease the number of training sets. This implies that the variance of reconstructed PDF is subject to overfitting in our networks. Also, the variance in power

spectrum for κ_{DL} is found to range from 75 – 100% of the true variance.

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