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An ultra-relativistic electron emits Cherenkov radiation in vacuum with an induced by strong electromagnetic wave refraction index larger than unity. During the interaction with this wave the electron also radiates photons via the Compton scattering. Synergic Cherenkov-Compton process can be observed by colliding laser accelerated electrons with a high intensity electromagnetic pulse. Extremely high energy photons cannot be emitted via the Cherenkov radiation because the vacuum refraction index tends to unity at these energies. Experiments on studying these phenomena will reveal the properties of vacuum predicted by nonlinear quantum electrodynamics.

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In quantum electrodynamics (QED) the photons interact with each other via virtual electron-positron pairs, which gives rise to a broad range of processes, in particular, related to vacuum polarization, vacuum birefringence [1], and other processes. The study of the photon-photon interactions is considered one of the most important applications of the high power laser facilities from the point of view of the fundamental science [2–8]. Among these interactions there are those that attract particular interest due to the fact that their description lies outside the framework of the perturbation theory [6, 9–11]. Arguably, the electron-positron pair production from vacuum by strong electromagnetic (EM) field is the most well known one. The characteristic field associated with his interaction is the QED critical field. It is also known as the Schwinger field [1]. It is equal to $E_S = m_e^2 c^3/e\hbar$, where e and m_e are the electron charge and mass, c equal to the speed of light in vacuum, and $\hbar$ is the Planck constant. This field strength corresponds to the light intensity of the order of 10^{29} W/cm^2. Although the EM field intensity needed to observe pair production can be lowered by using special configurations of EM fields [12, 13] down to 10^{27} W/cm^2, such intensities are well beyond the reach of the next generation of laser facilities. However, the physics at the QED critical field level can be probed by high energy electron beams colliding with intense EM waves [5]. For high enough electron (and photon) energy as well as for enough strong electromagnetic field such processes of electromagnetic interaction cannot be described within the framework of perturbation theory [14, 15] making them attractive for studies with high power lasers and accelerators of charged particles [16–19].

In what follows we consider the interaction of a high energy electron beam with 10 PW class lasers, focused to a one micron spot. In this case the laser intensity can reach 10^{24} W/cm^2, which corresponds to the laser EM field normalized amplitude of $a_0 = 10^3$. Here $a_0 = eE_0/m_e\omega_0 c$, E_0 and ω_0 are the laser electric field strength and frequency respectively.

The electron interacting with the field of the amplitude a_0 emits the photons with the energy $\hbar\omega_\gamma \approx \hbar\omega_0 a_0 \gamma_e^2$, where γ_e is the elecron Lorentz factor. Estimating the electron quiver energy as $\gamma_e \approx a_0$ (this assumption corresponds to the case when the size of the laser-electron interaction region is approximately equal to the wavelength, and gives lower value for the electron energy), we find that the photon energy for $a_0 \approx 10^3$ and $\hbar\omega_0 = 1$ eV is in the γ-ray range. As we see, an electron interacting with the electromagnetic wave emits high order harmonics. The maximum harmonic number could be approximately equal to a_0^3. In quantum physics, this corresponds to absorption of $N_{ph} = a_0^3$ laser photons by an electron in order to emit one high energy photon. The quantum effects come into play when $\hbar\omega_\gamma \approx m_e c^2 \gamma_e$ and $a_0 \gg 1$, and the photon emission occurs in the multi-photon Compton scattering process (see Ref. [20] and review article [21] and references cited therein).

The Lorentz invariant parameters, $\chi_e = \sqrt{p_\mu F^{\mu\nu} F_{\nu\rho} p^\rho}/m_e c E_S$ and $\chi_\gamma = \hbar\sqrt{k_\mu F^{\mu\nu} F_{\nu\rho} k^\rho}/m_e c E_S$, characterize the QED processes for electrons and photons interacting with the electromagnetic field. Here p_μ is the electron 4-momentum, k_μ is the photon 4-wave vector, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and A_μ is the electromagnetic 4-potential. If the parameter χ_e is equal to unity, in the electron rest frame, the electric field is equal to the critical QED field E_S.

Strong electromagnetic wave induces the vacuum polarization. As a result, in the long wavelength limit, the QED vacuum behaves as a medium with a refraction index larger than unity [10, 11, 14, 15], i. e. the speed of propagation of the interacting electromagnetic waves is below speed of light in vacuum. One of the consequences of this fact is a possibility of the Cherenkov radiation of the high-energy electrons traversing the electromagnetic field [14, 15, 22–26]. In Refs. [22, 23] the electromagnetic field was considered to be generated by exteremely high power lasers.

Below we analyze the properties of Synergic Cherenkov-Compton Radiation and Scattering (SCCRS) and discuss a way for observing this phenomenon with extreme high power lasers by colliding laser-accelerated electrons with high intensity electromagnetic pulse.

When ultra-relativistic electron collides with the EM wave it undergoes radiation losses which can prevent the electron from reaching the the high intensity EM field region. To describe the one-dimensional relativistic electron dynamics in the EM field, following to approach formulated in Ref. [27], we use a system of differential equations for the distribution functions of electrons, positrons and photons with the the differential probabilities of a photon emission by an electron/positron, and a photon decay into electron-positron pair (for details see Ref. [27]). It is emphasized here that they are the functions of initial electron/positron/photon energy and the instantaneous (at time t) value of the EM field. Here we assume that the electron, positron, and photon dynamics is dominated by the longitudinal motion.

For a 10 PW laser focused to a one-lambda spot the normalized amplitude approximately equals to $a_0 = 10^3$. Here we assume that the characteristic size of the high intensity region is equal to one lambda. As the energy of the laser wake field accelerated (LWFA) electrons according to the LWFA scaling [28] is $\mathcal{E}_e = 10\,(P/1\text{ PW})$ GeV, then for a 10 PW laser it can reach 100 GeV, i.e. $\gamma_{in} = 2\times 10^5$. Solving a system of equation for the electron, positron and photon distribution functions for these initial parameters and assuming the electron beam is monoenergetic before the interaction, we obtain a broad spectrum of electrons after the interaction. The electron beam lost approximately half of its energy, however 37% of electrons has energy above 40 GeV. The situation at the maximum of the EM field is slightly different. Approximately 7% of the initial electrons reach the maximum of the EM field without emitting a photon. Moreover, around 60% of electrons have energy higher than 40 GeV. The solution shows that a significant number of electrons with very high energies can reach the maximum of the EM field of the tightly focused laser pulse.

Here, we consider the kinematics of an inverse multiphoton scattering process when an ultra-relativistic electron collides with an electromagnetic wave. Before and after the collision the electron has the momentum $\boldsymbol{p}_0$ and energy $m_e c^2\gamma_0 = \sqrt{\boldsymbol{p}_0^2c^2 + m_e^2c^4}$ and the momentum $\boldsymbol{p}$ and energy $m_e c^2\gamma_e = \sqrt{\boldsymbol{p}^2c^2 + m_e^2c^4}$, respectively. The scattered photons have the frequency ω_0 before collision and ω_γ after it. By using the energy and momentum conservation in the electron-photon system, we can find the scattering photon frequency dependence on the electron energy, the wave amplitude, and the scattering angle. As the electron interacts with s photons from the wave and emits high energy photon, the energy and momenta of all particles are connected by the conservation laws, which yield:

$$m_e c^2\gamma_0 + s\hbar\omega_0 = m_e c^2\gamma_e + \hbar\omega_\gamma \tag{1}$$

and

$$\boldsymbol{p}_0 + s\,\hbar\boldsymbol{k}_0 = \boldsymbol{p} + \hbar\boldsymbol{k}_\gamma\,, \tag{2}$$

where $\hbar\boldsymbol{k}_0$ is the EM wave photon momentum and $\hbar\boldsymbol{k}_\gamma$ is the emitted photon momentum.

We consider the case when photon-electron interaction occurs in the (x,y) plane, i.e., $\boldsymbol{p}_0 = p_{\parallel,0}\boldsymbol{e}_{x,0} + p_{\perp,0}\boldsymbol{e}_y$ and $\boldsymbol{p} = p_{\parallel}\boldsymbol{e}_x + p_{\perp}\boldsymbol{e}_y$. For the perpendicular component of the momentum $p_{\perp,0}$ we may assume that it is equal to the quiver electron momentum in the electromagnetic field of the amplitude a_0 , i.e. the electromagnetic wave is linearly polarized with the electric field along the y axis.

In the case of classical electrodynamics described by the Maxwell equations, the electromagnetic wave frequency and wave vector are related to each other as $\omega^2 = \boldsymbol{k}^2c^2$. In quantum vacuum the polarization effects result in the dispersion equation [14, 15]

$$\omega^2 - \boldsymbol{k}^2c^2 - \mu_\pm^2\,(\chi_e)\,c^2\hbar^{-2} = 0\,. \tag{3}$$

Here $\mu_\pm$ is the invariant photon mass [14, 15]. The signs $\pm$ in a subscript of $\mu_\pm$ correspond to the parallel and perpendicular polarizations of the colliding electromagnetic waves. The invariant mass depends on the photon frequency (it is the photon energy expressed in terms of the quantum parameter χ_γ). Its square is given asymptotically by

$$\mu_\pm^2 = -\alpha m_e^2\left[\frac{11\mp 3}{90\pi}\chi_\gamma^2 + i\sqrt{\frac{3}{2}}\frac{3\mp 1}{16}\chi_\gamma \exp\left(-\frac{8}{3\chi_\gamma}\right)\right] \tag{4}$$

for $\chi_\gamma \ll 1$ and

$$\mu_\pm^2 = -\alpha m_e^2\left[\frac{5\mp 1}{28\pi^2}\sqrt{3}\Gamma^4\left(\frac{2}{3}\right)\left(1 - i\sqrt{3}\right)(3\chi_\gamma)^{2/3}\right] \tag{5}$$

for $\chi_\gamma \gg 1$. Here $\Gamma(x)$ is the Euler gamma function. The imaginary part of $\mu_\pm^2$ gives a probability of the electron-positron pair creation. As noted in Ref. [15], for large χ_γ, at $\alpha\chi_\gamma^{2/3} \sim 1$, the photon mass becomes of the order of the electron mass. In this limit, the perturbation theory becomes inapplicable.

Eqs. (3-5) yield that, in the limit $\chi_\gamma \ll 1$, a difference between the refraction index value and unity, $n_\pm - 1 = \Delta n_\pm$. It is

$$\Delta n_\pm = \alpha\frac{11\mp 3}{45\pi}\left(\frac{E}{E_S}\right)^2\,. \tag{6}$$

From this expression it follows that in the low photon energy limit, $\chi_\gamma \ll 1$ the normalized phase velocity (and equal to it group velocity), $\beta_\pm = v_\pm/c$, of the linearly polarized counter-propagating electromagnetic wave with the electric field parallel or perpendicular to the y axis equals $\beta_{ph,\pm} = 1 - \varepsilon_\pm (E_0/E_S)^2$, where the coefficients $\varepsilon_\pm$ are $\varepsilon_\pm = \alpha\,(11 \mp 3)/45\pi \approx 10^{-4}$.

Now for the sake of brevity we assume that the transverse component of electron momentum before interaction vanishes, i. e. $\boldsymbol{p}_0 = p_{\|,0}\boldsymbol{e}_x$. From Eqs. (3-6) follows that, as a consequence of the vacuum polarization, the relationship between wave number and frequency of the electromagnetic wave corresponding to the wave propagating in a medium with the refraction index not equal to unity can be written as

$$\boldsymbol{k} = \frac{\boldsymbol{k}}{|\boldsymbol{k}|c} n_\pm\,\omega\,, \tag{7}$$

with $\boldsymbol{k} = |\boldsymbol{k}|\,(\boldsymbol{e}_x \cos\theta + \boldsymbol{e}_y \sin\theta)$, where θ is the angle between the scattered photon wave number and the x axis. Using this relationship to solve Eqs. (1) and (2), we obtain for the energy of scattered photon

$$\hbar\omega_\gamma = g \pm \sqrt{g^2 + 2s\,\hbar\omega_0\left(\frac{m_e c^2\gamma_0 + p_{\|,0}c}{n_\pm^2 - 1}\right)}, \tag{8}$$

where

$$g = \frac{\left(p_{\|,0}c - s\,\hbar\omega_0\right) n_\pm \cos\theta - m_e c^2 \gamma_0 - s\,\hbar\omega_0}{n_\pm^2 - 1}. \tag{9}$$

Only positive solution for the photon frequency ω_γ is relevant to our problem.

The equation (8) describes kinematics of the SCCRS process in QED vacuum, whose optical properties are modified by the strong electromagnetic wave.

In the case, when the modification of the vacuum refraction index is weak enough, i.e. when $0 \le n_\pm^2 - 1 \ll 1$, the dependence on the parameters of the emitted photon frequency ω_γ given by Eq, (8) can be presented as a combination of two modes with a continous transition between them. When the function g given by Eq. (9) is positive and $s\,\hbar\omega_0 \ll m_e c^2/\gamma_e$ the photon energy can be found to be

$$\hbar\omega_{Ch} \approx 2g + s\,\hbar\omega_0\left[\frac{m_e c^2\gamma_0 + p_{\|,0}c}{g(n_\pm^2 - 1)}\right]. \tag{10}$$

This expression corresponds to the Cherenkov radiation with a notation ω_{Ch} used for the frequency of the photon emitted in this regime.

In the opposite limit, when the function g given by Eq. (9) is negative, in the limit $s\,\hbar\omega_0 \ll m_e c^2/\gamma_e$, Eq. (8) can be rewritten as

$$\hbar\omega_C \approx -\frac{s\,\hbar\omega_0 (m_e c^2\gamma_0 + p_{\|,0}c)}{m_e c^2\gamma_0 + s\,\hbar\omega_0 - \left(p_{\|,0}c - s\,\hbar\omega_0\right) n_\pm \cos\theta}\,, \tag{11}$$

corresponding to the Compton scattering mode with the photon frequency equal to ω_C. In the limit $s \ll s_m$, where

$$s_m = \frac{m_e c^2}{4\hbar\omega_0\,\gamma_0} \tag{12}$$

with $\gamma_0 = \sqrt{1 + p_{\|,0}^2/m_e^2c^2}$. The photon energy equals $\hbar\omega_C \approx 4s\,\hbar\omega_0 m_e c^2\gamma_0^2$. At $s \ge s_m$ we have $\hbar\omega_C \approx m_e c^2\gamma_0$.

The condition $s \le s_m$ shows when the recoil effects due to the photon emission become dominant. Eq. (12) gives the photon number absorbed by the electron with the energy $m_e c^2\gamma_0$ from the electromagnetic wave for radiating one high energy photon.

Assumption $0 \le n_\pm^2 - 1 \ll 1$ in Eq. (10) yields a condition of Cherenkov radiation,

$$n_\pm > \frac{\sqrt{m_e^2c^2 + p_{\|,0}}}{p_{\|,0}}\,, \tag{13}$$

imposing the requirement on the electron energy [22, 26]. We note that this condition is written assuming that

$$s\,\hbar\omega_0 \ll \frac{\sqrt{m_e^2c^4 + p_{\|,0}^2c^2} - p_{\|,0}c n_\pm \cos\theta}{1 + n_\pm\cos\theta}. \tag{14}$$

The electron energy should be large enough to have

$$\gamma_0 > \gamma_{Ch} = \frac{1}{\sqrt{2\Delta n_\pm}} = \sqrt{\frac{45\pi E_S^2}{\alpha(11 \mp 3)E_0^2}} \approx 30\sqrt{\frac{I_S}{I_0}}\,. \tag{15}$$

Here the laser intensity $I_0 = cE_0^2/4\pi$, which in the focus region of 10 PW laser approximately is equal to 10^{24} W/cm^2 and $I_S = cE_S^2/4\pi \approx 10^{29}$ W/cm^2, i. e. the Cherenkov radiation threshold is exceeded for the electron energy above 10 GeV.

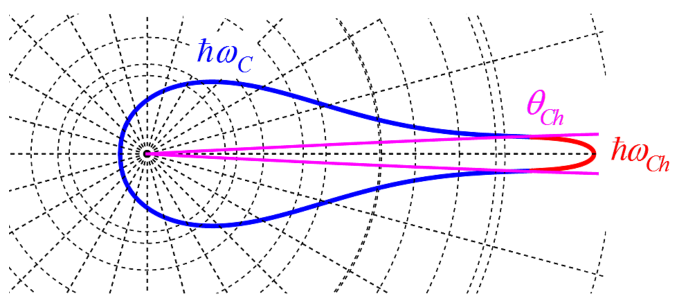

FIG. 1: Angle distribution of the energy logarithm for photon radiated by the SCCRS mechanism. Blue color used for Compton scattering, $\hbar\omega_C(\theta)$, and red color for the Cherenkov radiation, $\hbar\omega_{Ch}(\theta)$. Magenta lines $\theta = \pm\theta_{Ch}$ show the Cherenkov cone.

We can see from Eqs. (10) and (11) that photons emitted via Cherenkov and Compton mechanisms have different angular distribution. The photons emitted via the Cherenkov radiation process are confined within the Cherenkov cone with the angle $\theta_{Ch} = 2\sqrt{\varepsilon_\pm\, I_0/I_S}$. In the focus of 10 PW laser it is approximately equal to 2×10^{-5}. The Compton scattered photons are within the cone with the angle $\theta_C \approx 1/\gamma_0$. Although characteristic angle values for these process are of the same order, the dependence of the photon energy on the angle is different as it is seen from Fig. 1, where we show the angle distribution of the photon energy logarithm for Cherenkov radiation and for Compton scattering. Photons of substantially low energy emitted via the Compton scattering are localized within the relatively wide cone (blue curve). The more narrow beam of the Cherenkov radiation photons with higher energy is shown with the red color. The Cherenkow cone θ_{Ch} is shown by the lines $\theta = \pm\theta_{Ch}$.

According to the Cherenkov radiation theory [29] the rate of the energy loss due to the Cherenkov radiation friction force along the electron trajectory is

$$\frac{d\mathcal{E}_e}{dx} = -\frac{e^2}{c^2} \int\limits_{v_e n/c>1} \left(1 - \frac{c}{v_e n_\pm}\right) \omega d\omega$$

$$\approx -\frac{e^2}{\lambda_C^2} \varepsilon_\pm \left(\frac{E_0}{E_s}\right)^2 . \qquad (16)$$

Integration is done over the region where $v_e n/c > 1$ with $v_e = cp_{||,0}/\sqrt{m_e^2c^2 + p_{||,0}^2}$ is the electron velocity.

The formation length [30] in the case of Cherenkov radiation is given by

$$\ell_{Ch} \approx \gamma_e \gamma_{Ch} \lambda_{Ch} \approx \lambda_C \gamma_e \, . \qquad (17)$$

Here $\lambda_C = \hbar/m_e c \approx 3.8 \times 10^{-11}$ cm is the Compton scattering wavelength [1], $\lambda_{Ch} \approx \lambda_C/\gamma_{Ch}$ is the wavelength of radiation emitted via the Cherenkov radiation mechanism, and γ_{Ch} is the threshold energy determined by expression (15). For 10 PW laser radiation the electron threshold energy is approximately equal to 10 GeV. According to Eq. (17) the radiation formation length is $\ell_{Ch} \approx 5 \times 10^{-5}$ cm, i. e. it is comparable in magnitude but less than the laser wavelength. Traversing the laser focus region the electron emits 0.2 photons. Assuming the electric charge of the LWFA electron bunch of 100 pC we obtain 10^4 photons (the efficiency of the electron energy conversion to the Cherenkov radiation is $\approx 10^{-4}$). Photons produced in the Compton scattering have approximately the same frequency. Interaction of the Compton scattering photons with the laser field will result in the Breit-Wheeler electron-positron pair plasma generation [31, 34].

As one may see from the expression for the photon invariant mass (4) at the high photon energy end, when the parameter χ_γ becomes larger than unity, the vacuum polarization effects weaken. In this limit the Cherenkov radiation does not occur. As a result the photons with the energy above $\hbar\omega_\gamma \approx m_e c^2 E_S/E_0$ are not present in the high frequency spectrum of the radiation. For 10 PW laser parameters this energy is approximately equal to 100 MeV.

Considering kinematics of the process $e \to e\gamma$ in a strong electromagnetic field with taken into account the radiation correction of the “photon mass”, i. e. $\mu_\pm^2 \neq 0$ given by Eq. (3), we neglect the electron mass radiation correction. According to Ref. [15] the radiation correction to the electron mass scales as $\Delta m_e^2 \propto \alpha\chi_e^{2/3}$ in the limit $\chi_e^{2/3} \gg 1$. The Cherenkov radiation condition (15) can be written in the form $c - v_\gamma > c - v_e$, where v_γ and v_e are velocity of scattered photon and the electron velocity. In the right hand side of this inequality, the radiation correction of the electron mass changes the electron velocity approximately on $\alpha m_e^2 \chi_e^{2/3}$. Thus, the radiation correction of electron mass for the Cherenkov radiation process would become important when $\chi_e \gtrsim \alpha^{-3/2}$, whereas the Cherenkov radiation condition requires $\chi_e \gtrsim \alpha^{-1/2}$. Such relationship indicates that the first experiments on Sinergic Cherenkov-Compton process revealing modification of virtual electron-positron sea in a strong electromagnetic field are expected to be far from the conditions, when the radiation correction of the electron mass would become important because the electrons and the scattered Cherenkov photons have quite different energies.

In addition to ”photon mass” and ”electron mass” radiation corrections there is also the radiation correction to the Compton process, i.e., the radiation correction to the vertex diagram, where the virtual photon line connects the initial and final electron lines. This diagram was considered in Refs. [35, 36], where it was shown that this radiation correction scales as $\alpha^2\chi_e^{2/3}\ln\chi_e$. Thus the reasoning applied above for neglecting the ”electron mass” correction can also be applied to the radiation correction to the vertex diagram.

In conclusion, a scheme of the experiments on the laser accelerated electron interaction with focused EM wave aimed at studying such fundamental physics processes as the radiation friction effects, electron-positron pair creation, and vacuum polarization has been considered theoretically in Refs. [32–34]. Its principle setup was realized in the experiments whose results are presented in Refs. [34, 37, 39]. Here we pay attention to the fact that the electron undergoing multi-photon Compton scattering also emits the Cherenkov radiation in the QED vacuum, where a strong electromagnetic field induces a refraction index larger than unity, thus entering the regime of Synergic Cherenkov-Compton Radiation-Scattering. In the range of the parameters where the Cherenkov and Compton modes can be distinguished the angle and energy distributions for the gamma photons emitted by these two mechanisms are different. With extreme high power lasers (they are of the type of lasers build within the ELI project) the synergic Cherenkov-Compton process can be observed by colliding laser accelerated electrons with a high intensity electromagnetic pulse. At extremely high photon energy end, when the parameter χ_γ becomes larger than unity, as shown in Refs. [14, 15], the vacuum refraction index tends to unity thus quenching the Cherenkov radiation. Observation of these phenomena can shed light on the properties of nonlinear QED vacuum, allowing us to reveal the physical processes on a way towards the limit when $\alpha\chi_\gamma^{2/3}$, i.e. when the electromagnetic field interaction with charged particles develops according to unperturbative regime scenario.

We note that our consideration based on the energy-momentum conservation should be supplemented by calculation of matrix elements giving, in particular, the radiation amplitude dependence on the angle. The work on this problem is now in the progress and its results will be published in for-coming paper. Moreover, we want to note that the scaling $\alpha\chi_e^{2/3}$ for the radiation corrections were obtained for the constant field case, and according to the recent results presented in Refs. [19, 38] need to be treated with caution for other field configurations.

In the experiment under dicussion, a single laser pulse can be used within the framework of the all-optical scheme, to accelerate ultra-relativistic electrons in the laser-plasma interaction and then to be focused in the λ^3 region to achieve extreme high field amplitude providing conditions required for colliding the electrons with the electromagnetic field. Similar scheme is proposed in Ref. [40] for a Compton source based on the combination of a laser-plasma accelerator and a plasma mirror. Using this scheme, the Synergic Cherenkov-Compton Radiation-Scattering in the interaction of 100 GeV LWFA accelerated by 10 PW laser electrons with the electromagnetic field of 10^{24}W/cm^2 will reveal the vacuum properties predicted by nonlinear QED theory.

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