Azimuthal angle dependence of the longitudinal spin polarization in relativistic heavy ion collisions

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Azimuthal angle dependence of longitudinal spin polarization in relativistic heavy ion collisions

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The longitudinal quark spin polarization along the beam direction in non-central relativistic heavy ion collisions is studied in the chiral kinetic approach. Its azimuthal angle dependence in the transverse plane of the collision is found to have a quadrupole structure as in studies from hydrodynamic and transport models based on the relativistic spin-vorticity coupling in a thermodynamically equilibrated hadronic matter at kinetic freeze-out. The direction of the quark longitudinal spin polarization is, however, opposite to that from these models, and this is due to the axial charge current induced by the transverse component of vorticity field in the chiral kinetic approach. Our finding is consistent with the azimuthal angle dependence of the longitudinal spin polarization of Λ hyperons, which is mainly determined by that of the strange quark, recently measured in the experiments by the STAR Collaboration at the Relativistic Heavy Ion Collider (RHIC).

Keywords: longitudinal spin polarization, vorticity field, chiral kinetic theory, AMPT

I. INTRODUCTION

In non-central relativistic heavy ion collisions, a strong vorticity field is generated in the produced matter as a result of the large orbital angular momentum that is brought into the system. This vorticity field can lead to the polarization of particles of non-zero spin along its direction due to their spin-orbit [1, 2] or spin-vorticity coupling [3–6]. Measurements of the global spin polarization of Λ hyperons by the STAR Collaboration [7, 8] have confirmed the existence of the most vortical fluid ever known, with an average vorticity of more than 10^{21} s^{-1}, in these collisions [9–11]. Theoretical studies based on the assumption of local thermal equilibrium [12–18] or using the non-equilibrium chiral kinetic approach [19] have reproduced the experimental results.

Besides its large global value along the direction of the total angular momentum, vorticity field in the matter produced in relativistic heavy ion collisions also shows local structures [11, 16, 20], such as the longitudinal one along the beam direction and the circular one in the transverse plane of a collision. Furthermore, according to Ref. [20] based on the transport approach, the distribution of the longitudinal component of vorticity field in the transverse plane of a heavy ion collision shows a quadrupole structure as a result of the elliptic flow in non-central relativistic heavy ion collisions. As suggested in this study as well as in Ref. [21, 22] based on the hydrodynamic approach, the local variation of vorticity field can be studied by measuring the azimuthal angle distribution of the spin polarization of Λ hyperons. Although preliminary experimental results from the STAR Collaboration at RHIC [23] on the longitudinal spin polarization of Λ hyperons in the transverse plane of heavy ion collisions have indeed shown a quadrupole struture, its magnitude and sign are not consistent with those predicted in Refs. [20, 21]. In particular, the measured longitudinal spin polarization of Λ hyperons points at an opposite direction from that predicted in these models.

In addition to possible effects due to the initial conditions used in hydrodynamic approach, decays of higher lying states, and non-equilibration of spin degrees of freedom at kinetic freeze-out as suggested in Ref. [24], the discrepancy between the predictions from these theoretical studies and the experimental observation may also be due to the neglect of effects from the transverse component of vorticity field on quark propagations and scatterings. Taking into account the redistribution of axial charges, defined as the difference between the numbers of right-handed (positive helicity) and left-handed (negative helicity) quarks, after propagations and scatterings of quarks in vorticity field can lead to non-vanishing local axial chemical potential in the transverse plane, which would then affect the polarization of quarks through their scatterings [19] and thus the resulting Λ hyperons. As illustrated in the next Section, the transverse component of vorticity field can lead to a quark longitudinal spin polarization that has a different azimuthal angle dependence from that of the longitudinal component of vorticity field. As a result, this will modify the direction of the spin polarization of quarks or Λ hyperons predicted in models that assume local thermal equilibrium of the spin degrees of freedom in vorticity field. This effect is further studied quantitatively in the chiral kinetic approach, which takes into account the different effects of vorticity field on quarks with positive and negative helicities, to show that it can indeed describe the observed azimuthal angle dependence of the longitudinal spin polarization of Λ hyperons in experiments.

This paper is organized as follows. In the next section, we show how the transverse component of vorticity

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field produced in relativistic heavy ion collisions can lead to the redistribution of axial charges, and how this can give rise to a quadrupole structure for the distribution of quark longitudinal spin polarization in the transverse plane. A brief description of the chiral kinetic approach is given in Sec. III. In Sec. IV, with the initial phase-space distribution of quarks taken from the event generator AMPT model [25] and assuming that their helicities are randomly assigned, we solve the chiral kinetic equations of motion for quarks and also take into account their scatterings in the presence of a self-consistently calculated local vorticity field to study quantitatively the effect of the transverse component of vorticity field on the quark longitudinal spin polarization. Finally, a summary is given in Sec. V.

II. EFFECT OF TRANSVERSE VORTICITY FIELD ON LONGITUDINAL SPIN POLARIZATION

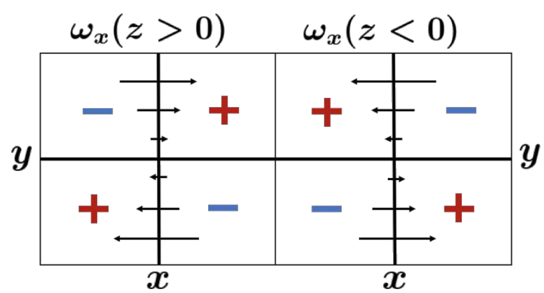

FIG. 1: (Color online) Distributions of the in-plane x component ω_x of vorticity field (arrows) [20] and the induced axial charge distribution (plus and minus signs) in the transverse plane of a heavy ion collision.

Following the usual convention for describing heavy ion collisions, we take the z-axis along the beam direction, the x-axis along the direction of the impact parameter, and the y-axis perpendicular to the $x-z$ or reaction plane. Since the matter produced in relativistic heavy ion collisions is mostly located around the center of the collisions, i.e., $z=0$ on the z-axis, the velocity in the transverse or $x-y$ plane, which is driven by the pressure in the matter, has a maximum value at $z=0$ and decreases with increasing $|z|$. As shown in Ref. [20], this then results in a vorticity field $\boldsymbol{\omega}_\perp = \frac{1}{2}\partial_z v_\perp(r,z)\mathbf{e}_\phi$ in the transverse plane that is clockwise in the forward region ($z>0$) and anti-clockwise in the backward region ($z<0$) of the collision. In the above, $v_\perp(r,z)$ and $\mathbf{e}_\phi$ are, respectively, the velocity field and the unit vector along the azimuthal direction in the transverse plane. For the in-plane x component (ω_x) of the transverse vorticity field, which is shown by arrows in Fig. 1, it leads to not only the spin polarization of quarks along the x direction but also an axial charge current in that direction as a result of the so-called gravitational anomaly [26].

In the forward region of the collision shown in the left panel of Fig. 1, the axial charge current is along the positive x direction above the reaction plane ($y>0$) and along the negative x direction below the reaction plane ($y<0$). This then leads to more right-handed than left-handed quarks in the first and third quadrants ($xy>0$) and more left-handed than right-handed quarks in the second and fourth quadrants ($xy<0$), resulting thus a quadrupole structure for the axial charge distribution in the transverse plane as shown by the "+" and "−" symbols in the left panel of Fig. 1. Since the momenta of quarks in the forward region point preferentially along the positive z direction, and their longitudinal spin polarizations are thus positive in the first and third quadrants and negative in the second and fourth quadrants, leading also to a quadrupole structure for the distribution of quark longitudinal spin polarization in the transverse plane. For quarks in the backward region, their axial charge distribution in the transverse plane is shown in the right panel of Fig. 1, and they show a similar quadrupole structure as that in the forward region since the momenta and axial charges of quarks in the backward region are opposite to those in the forward region. This azimuthal angle dependence of quark longitudinal spin polarization due to the in-plane component of transverse vorticity field thus has an opposite direction from that due to the longitudinal z component (ω_z) of vorticity field and is therefore similar to that observed in the experiments by the STAR Collaboration for the longitudinal spin polarization of Λ hyperons.

The above effect due to the in-plane x component of vorticity field is, however, reduced by the out-of-plane y component (ω_y) of vorticity field [20]. This is because in the forward region, the latter is along the negative y direction for $x>0$ and along the positive y direction for $x<0$, which thus leads to a quadrupole structure for the axial charge distribution in the transverse plane that has an opposite sign to that due to the in-plane component of vorticity field. Because of the azimuthal anisotropy of produced quark matter and vorticity field in non-central heavy ion collisions, there is only partial cancellation between the contributions to the quark longitudinal spin polarization from the in-plane and out-of-plane components of transverse vorticity field, resulting in an additional contribution to the quark longitudinal spin polarization from the transverse component of vorticity field besides its longitudinal component. As in the case for quark global spin polarization [19], which is dominated by the out-of-plane component of vorticity field, the effect of the transverse component of vorticity field on the local quark spin polarization will be further enhanced if one includes the effect of the spin-vorticity coupling in the scattering of quarks. This effect is studied quantitatively in the next Section by using the chiral kinetic approach to include the different dynamics of right-handed and left-handed quarks under the influence of a self-consistently calculated vorticity field with their initial distributions taken from the AMPT model [25], which is an event generator for heavy ion collisions at relativistic energies.

III. THE CHIRAL KINETIC APPROACH

For the equations of motion in the chiral kinetic approach to massless quarks and antiquarks in a vorticity field $\boldsymbol{\omega}$, we follow Refs. [19, 27, 28] and use

$$\dot{\mathbf{r}} = \frac{\hat{\mathbf{p}} + 2\lambda p(\hat{\mathbf{p}}\cdot\mathbf{b})\boldsymbol{\omega}}{1+6\lambda p(\mathbf{b}\cdot\boldsymbol{\omega})}, \quad \dot{\mathbf{p}} = 0. \tag{1}$$

In the above, $\mathbf{r}$ and $\mathbf{p} = p\hat{\mathbf{p}}$ are the position and momentum of a quark or antiquark, $\lambda = \pm 1$ is its helicity, and $\mathbf{b} = \frac{\mathbf{p}}{2p^3}$ is the Berry curvature that results from the adiabatic approximation of taking the spin of a massless quark or antiquark to be always parallel or anti-parallel to its momentum. The above chiral vortical equations of motion are derived in Refs. [29, 30] from the three-dimensional reduction of the covariant four-dimensional Wigner function for massless quarks. Corrections to above equations due to finite quark masses ($m_u = 3$ MeV, $m_d = 6$ MeV, and $m_s = 100$ MeV) can be included by replacing $\hat{\mathbf{p}}$, p and $\mathbf{b}$ with $\frac{\mathbf{p}}{E_p}$, E_p and $\frac{\hat{\mathbf{p}}}{2E_p^2}$, respectively, as in Ref. [31]. Since the chiral equations of motion in the vorticity field do not depend on the charges and baryon numbers of quarks, we thus use quarks to denote both quarks and antiquarks in the present study.

The factor $\sqrt{G} = 1 + 6\lambda p(\mathbf{b}\cdot\boldsymbol{\omega})$ in the denominator of Eq.(1) modifies the phase-space distribution of quarks and ensures the conservation of vector or baryon charges. The modified quark equilibrium distribution can be achieved from quark scatterings by requiring the quark momenta $\mathbf{p}_3$ and $\mathbf{p}_4$ after a two-body scattering, which is determined by their total scattering cross section, with the probability $\sqrt{G(\mathbf{p}_3)G(\mathbf{p}_4)}$ [19]. As shown in Ref. [19] and also mentioned in the Introduction and Section II, the vorticity field through the phase-space factor $\sqrt{G}$ has a larger effect on the spin polarization of quarks from their scatterings than propagations. This is because including $\sqrt{G}$ in scattering makes quarks of momentum along the direction of vorticity field more right-handed and quarks of momentum opposite to the direction of vorticity field more left-handed, leading to a fast appearance of quark spin polarization. Although quark propagations in vorticity field can result in an axial current in the quark matter, it takes additional times for quarks to have their momenta pointing either along or opposite to the direction of vorticity field before they can acquire a finite spin polarization [19].

IV. QUARK LONGITUDINAL SPIN POLARIZATION

For the application of the chiral kinetic equations of motion to heavy ion collisions, we use the phase-space distributions of quarks and antiquarks from the AMPT model without the partonic and hadronic evolutions as the initial conditions. Assuming that there are no local topological charges in the produced quark matter, the helicity of each quark is randomly assigned with equal probabilities to be positive and negative. The local vorticity field is calculated from the velocity field $\mathbf{u} = \gamma\mathbf{v} = \frac{1}{\sqrt{1-v^2}}\mathbf{v}$ according to $\boldsymbol{\omega} = \frac{1}{2}\boldsymbol{\nabla}\times\mathbf{u}$. For the velocity field, it is obtained from averaging the velocities of quarks in a local cell of size $\Delta x = \Delta y = 0.5$ fm and $\Delta\eta = 0.2$, where $\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$ is the space-time rapidity. Since quarks from the AMPT model are produced at different formation times, those that are not formed at a given time are propagated back to that time and included in the evaluation of the velocity field. As to the quark scattering cross section, we use a constant value of 10 mb and an isotropic angular distribution for simplicity. Results given below are obtained from the phase-space distributions of quarks after they stop scattering in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and the impact parameter $b = 8.87$ fm, corresponding to the 30-40% centrality in the STAR experiments.

A. Time evolution of local vorticity field

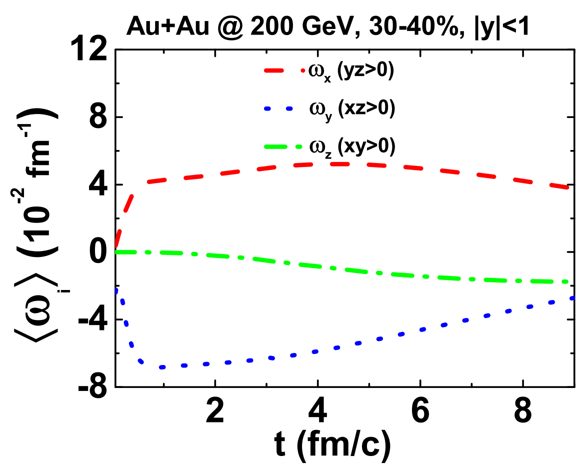

FIG. 2: (Color online) Time evolution of different components of average vorticity field in midrapidity ($|y| < 1$) for selected regions of transverse plane.

The local structure of vorticity field discussed qualitatively in the previous Section can be seen from the average value of its three components ω_x, ω_y, and ω_z in appropriate spatial regions according to

$$\langle\omega_i\rangle = \frac{\sum_j \omega_i(x_j, y_j, z_j)}{\sum_j 1} \tag{2}$$

with $i = x, y, z$ and j running over all midrapidity ($|y| < 1$) quarks in the selected spatial region. Shown in Fig. 2 are the time evolution of $\langle\omega_x\rangle$ in the region $yz > 0$, $\langle\omega_y\rangle$

in the region $xz > 0$, and $\langle\omega_z\rangle$ in the region $xy > 0$. These regions correspond to the first and third quadrants of the $y-z$, $z-x$ (reaction) and $x-y$ (transverse) planes, respectively. It is seen that $\langle\omega_x\rangle$ is positive and $\langle\omega_y\rangle$ and $\langle\omega_z\rangle$ are negative as discussed in Section II. Also, the magnitudes of $\langle\omega_x\rangle$ and $\langle\omega_y\rangle$ are seen to increase faster with time and are much larger than the magnitude of $\langle\omega_z\rangle$. Their final values are $\langle\omega_x\rangle \approx 4\times10^{-2}$ fm^{-1}, $\langle\omega_y\rangle \approx -3\times10^{-2}$ fm^{-1}, and $\langle\omega_z\rangle \approx -2\times10^{-2}$ fm^{-1}, which are similar to those from Ref. [20] using the AMPT model after including both partonic and hadronic evolutions. We note that the initial non-zero value of $\langle\omega_y\rangle$ is due to the large global orbital angular momentum in the y direction in non-central heavy ion collisions.

B. Time evolution of quark longitudinal spin polarization

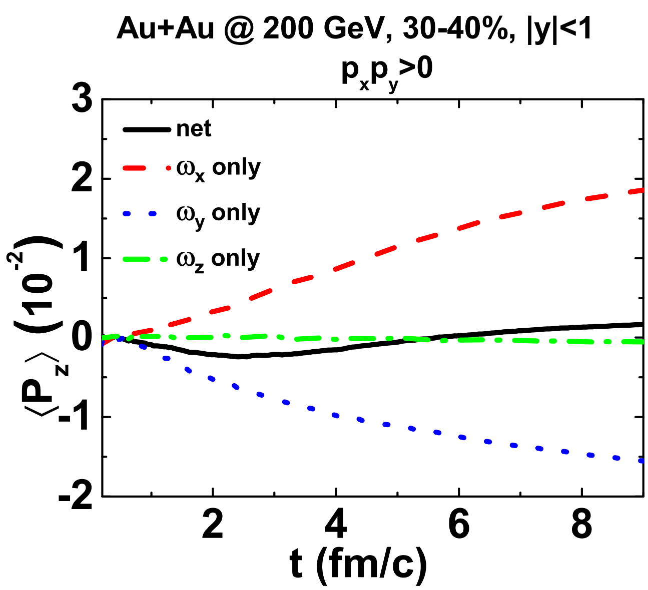

FIG. 3: (Color online) Time evolution of average longitudinal spin polarization of midrapidity quarks with momenta satisfying $p_xp_y > 0$.

Because of the transverse expansion of produced quark matter in relativistic heavy ion collisions, there is a correlation between the positions and momenta of quarks. For example, quarks of momentum components $p_xp_y > 0$ are likely in the region $xy > 0$ of the transverse plane. From the local structure of vorticity field shown in Fig. 2, one expects quarks of momenta $p_xp_y > 0$ to acquire a longitudinal spin polarization in the z direction. Including only the ω_x, we find that the average longitudinal spin polarization of these quarks is along the positive z direction and increases with time, reaching a value of about 2×10^{-2} as shown by the red dashed line in Fig. 3, which confirms the expectation discussed in Sec. II. The longitudinal spin polarization of quarks of momentum $p_xp_y > 0$ due to ω_y also increases with time and reaches a value of about 1.5×10^{-2} along the negative z direction, again agreeing with the expectation discussed in Sec. II that it does not complete cancel the contribution from ω_x. For ω_z, which is along the negative z direction in the region $xy > 0$, it leads to a longitudinal spin polarization in the negative z direction for quarks of momenta $p_xp_y > 0$, as shown by the green dash-dotted line in Fig. 3. Although its magnitude slowly increases with time, the final value is only of the order of 10^{-3} and is much smaller than those predicted in Ref. [20] based on the transport model and Ref. [21] based on the hydrodynamic approach by assuming that the spin degrees of freedom are in equilibrium with the thermal vorticity tensor field.

Including all components of vorticity field, which is shown by the black solid line in Fig. 3, we find that the total longitudinal spin polarization of quarks of momenta $p_xp_y > 0$ is initially along the negative z direction, as a result of the larger effect of ω_y than that of ω_x. After about 2.5 fm/c, the effect of ω_x becomes more important than that of ω_y, and this makes the longitudinal spin polarization of these quarks less negative. Finally, the sign of the longitudinal polarization is along the positive z direction after 5 fm/c when the effect of ω_x dominates over the combined effects of ω_y and ω_z.

C. Rapidity dependence of longitudinal spin polarization

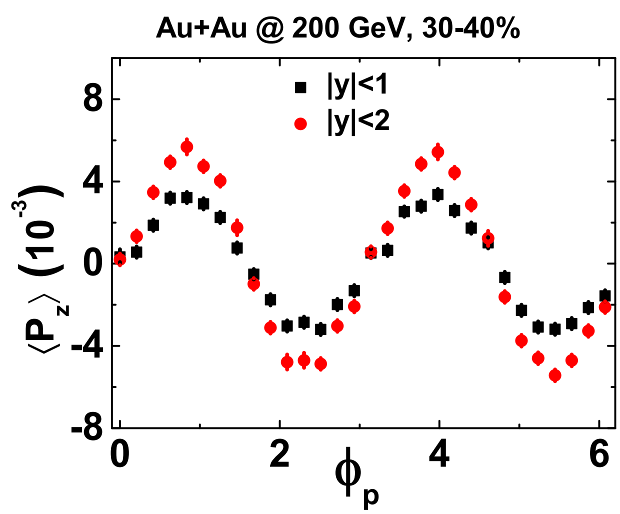

FIG. 4: (Color online) Average longitudinal spin polarization of quarks as a function of azimuthal angle ϕ_p of the quark momentum in the transverse plane for different rapidity ranges.

In Fig. 4, we show the longitudinal spin polarization of quarks as a function of the azimuthal angle ϕ_p of the quark momentum in the transverse plane of heavy ion collisions for different rapidity ranges of $|y| < 1$ and $|y| < 2$. It is seen that the longitudinal spin polariza-

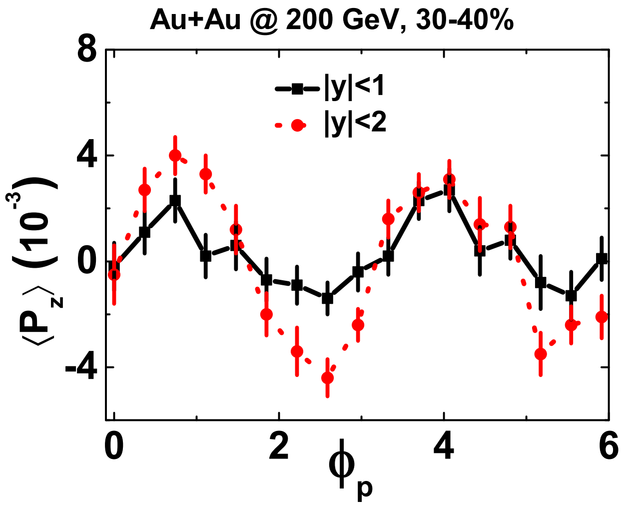

FIG. 5: (Color online) Same as Fig. 4 for strange quarks.

tion indeed has a quadrupole structure and is positive for quarks $p_x p_y > 0$, similar to that of Λ hyperons measured in experiments [23] and different from the longitudinal polarization calculated from assuming local thermal equilibrium of the spin degrees of freedom. Furthermore, the amplitude of the azimuthal dependence, which can be expressed as $\sin(2\phi_{\rm p})$, is larger for the larger rapidity, and this is due to the larger values of longitudinal and transverse vorticities at larger rapidity [11, 15].

We also show the longitudinal spin polarization of strange quarks in Fig. 5, which is expected to be almost identical to that of Λ hyperons [1, 19, 32]. It is seen that the amplitude of the azimuthal angle dependence of the longitudinal spin polarization of strange quarks is smaller than that of light quarks, but is still comparable to the experimental results [23]. The reason for this is because of the mass effect in the chiral kinetic approach and the different spatial and temporal distributions between initial strange and light quarks from the AMPT model.

We further find that with a smaller quark cross section, the quark longitudinal spin polarization decreases and can even change the sign of the quadrupole structure in the longitudinal spin polarization. This thus indicates that taking into account the non-equilibrium effect, which is included in the chiral kinetic approach, is important for understanding the local spin polarization of quarks and thus that of Λ hyperons.

V. SUMMARY

Using the chiral kinetic approach with initial phase-space distributions of quarks taken from the AMPT model and their helicities randomly assigned, we have studied the effect of the transverse components of local vorticity field in the transverse plane of a heavy ion collision on the longitudinal spin polarization of quarks along the beam direction. We have found that the quark longitudinal spin polarization depends not only on the longitudinal component of vorticity field but also on its transverse components due to the resulting axial charge redistribution. Using a constant quark scattering cross section of 10 mb, we have obtained a quark longitudinal spin polarization that has an azimuthal angle dependence and amplitude similar to those measured in experiments for Λ hyperons, as a result of the dominant effect of the in-plane component ω_x over those of the out-of-plane component ω_y and the longitudinal component ω_z of local vorticity field. We have also found that decreasing the quark scattering cross section leads to a reduction of the quark longitudinal spin polarization. Our study thus demonstrates the importance of non-equilibrium effects as well as the local structure of vorticity field and its time evolution on the spin polarizations of quarks in relativistic heavy ion collisions.

In the present study, we have only considered the spatial components of vorticity field on quark longitudinal spin polarization. According to Refs. [21, 22] based on the relativistic spin-vorticity coupling in a thermodynamically equilibrated medium, the azimuthal angle dependence of longitudinal spin polarization is largely determined by the temperature gradient and its time derivative at the kinetic freeze-out of relativistic heavy ion collisions. If this is the case, this may explain the negligible quark longitudinal spin polarization in our study from the longitudinal component of vorticity field compared to that in Ref. [20], which is obtained from the AMPT transport model by assuming that the spin of Λ hyperon at the kinetic freeze-out of hadronic matter is in thermal equilibrium with the relativistic vorticity tensor field. To include the time component of vorticity field is expected to cancel the effect from the transverse component of vorticity field on the longitudinal spin polarization obtained in the present study. To study this effect requires the extension of the non-covariant chiral kinetic approach used in the present study to a covariant one, which to our knowledge is currently not available.

Also, we have neglected in the present study possible fluctuations in the initial axial charge distribution due to the chiral anomaly. How this effect would modify our results and conclusions need to be studied. Furthermore, although the Λ longitudinal spin polarization has a similar azimuthal angle dependence as that of strange quarks immediately after hadronization of the quark matter, how it is affected by scattering during subsequent hadronic evolution also needs to be studied before one can conclude that the observed opposite direction in the quark longitudinal spin polarization from that predicted by thermal models is indeed due to the effect of the transverse component of vorticity field described in the present study .

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