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Estimate of the CME signal in heavy-ion collisions from measurements relative to the participant and spectator flow planes

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An interpretation of the charge dependent correlations sensitive to the Chiral Magnetic Effect (CME) – the separation of the electric charges along the system magnetic field (across the reaction plane) – is ambiguous due to a possible large background (non-CME) effects. The background contribution is proportional to the elliptic flow \( v_2 \); it is the largest in measurements relative to the participant plane, and is smaller in measurements relative to the flow plane determined by spectators, where the CME signal, on opposite, is likely larger. In this note I discuss a possible strategy for corresponding experimental measurements, and list and evaluate different assumptions related to this approach.

I. INTRODUCTION

The search for the chiral magnetic effect (CME) \[1, 2\] – the separation of the electrical charges along the magnetic field in a chirally asymmetric medium – is a very active topic in the field of heavy ion collisions for more than 10 years (for recent reviews, see \[3, 4\]). The CME states that particles originating from the same “P-odd domain” are preferentially emitted either along or opposite to the magnetic field direction depending on the particle charge. As only a few particles (originating from the same domain) are correlated, the signal is expected to be small and one has to suppress other charge-dependent correlations, such as due to the resonance decays, charge ordering in jets, etc.. The so-called “gamma” correlator suggested in Ref. \[5\] was designed to do just that – to suppress non-CME correlation at least by a factor \( \sim v_2 \) – the typical value of elliptic flow.

\[
\gamma_{\alpha\beta} = \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi) \rangle = \langle \cos(\phi_\alpha - \Psi) \cos(\phi_\beta - \Psi) \rangle - \langle \sin(\phi_\alpha - \Psi) \sin(\phi_\beta - \Psi) \rangle,
\]

where \( \phi_\alpha \) and \( \phi_\beta \) are the azimuthal angles of two charged particles, \( \alpha \) and \( \beta \) taking values “+” or “−” denote the charge. \( \Psi \) denotes the azimuth of the plane across which the charge separation is measured. For measurements relative to the reaction plane (perpendicular to the direction of the magnetic field) only “sin-sin” term has contribution from the CME, while all other non-CME sources contribute to both, “sin-sin” and “cos-cos” terms and thus largely cancel. The remaining difference between “in-plane” (“cos-cos”) and “out-of-plane” (sin-sin) correlations constitutes the background to the CME measurements via gamma correlator. The background is zero in case of
no elliptic flow present in the system.

The experimental measurements \([6–8]\) are in qualitative agreement with the theoretical expectations, but a reliable separation of the CME signal from background effects is still missing. As already mentioned, the background correlations depend on the magnitude of elliptic flow and as such are largest in the measurements performed relative to the so-called participant plane, and should be smaller in measurements relative to the spectator flow plane. On opposite, the CME signal, driven by the magnetic field, is likely larger in measurements relative to the spectator plane, as the magnetic field is mostly determined by spectator protons. This idea was recently and independently used in Ref. \([9]\), where the authors attempted to estimate the CME signal from the existing measurements as well as make prediction for the future isobar collision measurements at RHIC. In this short note I discuss an evaluation of the CME signal based on the same general idea from a different perspective. In particular, I discuss in detail the role of flow fluctuations in measurements relative to different flow planes and by different methods, as well as explicitly list different assumptions required in this approach, some of which are more important than others.

II. DEFINITIONS AND THE MAIN IDEA

I start with more definitions and recalling the derivation of the background contribution to the gamma correlator. The correlator defined in Eq.1 includes contributions from charge independent effect (e.g. dipole flow). These are poorly known and not very important for the CME search. Due to this only the charge dependent part is discussed here

\[
\Delta \gamma = \gamma_{\text{opposite}} - \gamma_{\text{same}}. \tag{2}
\]

As both, the CME signal and the background correlations are small, one can safely assume that

\[
\Delta \gamma = \Delta \gamma^{BG} + \Delta \gamma^{CME}, \tag{3}
\]

neglecting, in principle possible, interplay between the two effects. The background contribution to \(\Delta \gamma\) very generally can be described as that due to “flowing clusters” \([5]\), when both particles, \(\alpha\) and \(\beta\) belong to the same “cluster”:

\[
\Delta \gamma^{BG} = \Delta \langle \cos(\alpha + \beta - 2\Psi) \rangle = \Delta \langle \cos(\alpha + \beta - 2\phi_{\text{clust}}) \cos(2\phi_{\text{clust}} - 2\Psi) \rangle_{\alpha,\beta \in \text{clust}}, \tag{4}
\]

where to simplify notations the symbols \(\alpha\) and \(\beta\) are used instead of \(\phi_{\alpha}\) and \(\phi_{\beta}\). Note that the mean of the product of two cosines in general does not factorize. The mean can be non-zero either
in the case of non-zero elliptic flow of clusters \(\langle \cos(2\phi_{\text{clust}} - 2\Psi) \rangle\) (see, for example, [10]), or due to the fact that the “kinematic” factor \(\langle \cos(\alpha + \beta - 2\phi_{\text{clust}}) \rangle\) varies with the cluster emission angle, or both (as in the case of the so-called “local charge conservation” background [11]). The first assumption about background is

\[
\Delta \gamma_{BG} \propto \tilde{v}_{2,\text{clust}},
\]

where I used “tilde” to denote the fact that there might be no factorization in Eq. 4 in which case this flow coefficient also accounts for the emission angle dependence of the “kinematic factor”.

The assumption (A1a) by itself is not very useful without further assumption on \(\tilde{v}_{2,\text{clust}}\):

(A1b) \(\tilde{v}_{2,\text{clust}} \propto v_2\), where \(v_2\) is the average (over some rapidity and \(p_T\) ranges) elliptic flow of charged particles. One can combine (A1a) and (A1b) into one assumption

\[
\Delta \gamma_{BG} = b v_2,
\]

where \(b\) is the proportionality constant.

This is the assumption employed almost in any attempt to disentangle background effects from the CME signal, e.g. used by ALICE and CMS Collaborations [12, 13] in the estimates of the CME signal with the Event Shape Engineering technique [14]. Reiterate, that (A1a) assumes linear dependence of the background contribution to \(\Delta \gamma\) on \(\tilde{v}_{2,\text{clust}}\), and (A1b) assumes the proportionality of the latter to the elliptic flow of charged particles.

Due to the initial state fluctuations, the elliptic flow, as well as the elliptic flow fluctuations, measured relative to different flow symmetry planes, are different. Then it becomes convenient to modify the correlator, namely consider, \((\Delta \gamma/v_2)\) with \(v_2\) calculated in the same way as the \(\gamma\) itself:

\[
(\Delta \gamma/v_2) = \langle \cos(\alpha + \beta - 2\psi) \rangle / \langle \cos(2\alpha - 2\psi) \rangle,
\]

where, for simplicity, the sign \(\Delta\) in the numerator is omitted (here and everywhere below in the expressions involving particles \(\alpha\) and \(\beta\) the difference between opposite and same charge combinations is assumed); \(a\) stands for the same set of particles as \(\alpha\) and \(\beta\), and the average is performed inclusively of all charges. In the denominator measurement it is assumed that the non-flow contribution is eliminated/suppressed. Note that the calculations of this ratio does not involve any explicit correction for the so-called reaction plane resolution. To emphasize this, here and below all the flow planes that include statistical fluctuations (due to finite number of particles used for their determination [15]) are denoted with lower case \(\psi\), and the angles that do not include statistical fluctuations (depend only on specific initial configuration) with upper case \(\Psi\).

An important feature of the ratio Eq. 5 is that in the case of zero CME-signal (pure background) this ratio is the same irrespective of what is used for the \(\psi\) and how strongly (or weakly) elliptic flow fluctuates relative to this plane. Namely, in the no-CME case this ratio equals \(b\) – the
proportionality coefficient in the assumption A1. For example, if instead of $\psi$ the azimuthal angle of a particle $c$ is used, this ratio equals

$$\langle \Delta \gamma / v_2 \rangle_c = \frac{\langle \cos(\alpha + \beta - 2c) \rangle}{\langle \cos(2a - 2c) \rangle} = b \frac{\langle v_{2,pp}^2 \rangle}{\langle v_{2,pp}^2 \rangle} = b,$$  \hfill (6)$$

where again for shorter notations the particle symbol is used to denote the particle azimuthal angle.

Note, that this case corresponds to elliptic flow measured with respect to the participant plane, and $\langle v_{2,pp}^2 \rangle = v_2^2 \{2\}$. For simplicity it is also assumed that the flow of both particles, $a$ and $c$ are the same.

Instead of $\psi$ in Eq. 5 one can use the “event plane” angle $\psi_{2,EP}$ (the azimuth of the flow vector in another subevent), or, what is more relevant for this discussion, the spectator flow angle $\psi_{1,SP}$

$$\langle \Delta \gamma / v_2 \rangle_{SP} = \frac{\langle \cos(\alpha + \beta - 2\psi_{1,SP}) \rangle}{\langle \cos(2a - 2\psi_{1,SP}) \rangle},$$  \hfill (7)$$

Under the “background scenario” all these ratios equal one to another. If two different measurements yield different ratios this would immediately indicate a contribution different from that of “background”, namely, the CME. Note that in calculations of the denominators (flow with respect to different angles) “non-flow” contribution should be eliminated/suppressed (e.g. by imposing a rapidity gap in measurements or by any other technique). If two ratios differ one can try to estimate the CME signal. This will rely on further assumptions, but as discussed below, the requirement to “accuracy” of those is lower.

In the case of a non-zero CME signal the ratios Eq. 5 calculated relative to different angles can be different. For concreteness let us consider the double ratio

$$\frac{\langle \Delta \gamma / v_2 \rangle_{SP}}{\langle \Delta \gamma / v_2 \rangle_c} = \frac{\langle \cos(\alpha + \beta - 2\psi_{1,SP}) \rangle}{\langle \cos(2a - 2\psi_{1,SP}) \rangle} \frac{\langle \cos(\alpha + \beta - 2c) \rangle}{\langle v_{2,pp}^2 \rangle},$$  \hfill (8)$$

where as above it is assumed the same elliptic flow of particles $a$ and $c$. Recall also, that the particles $a$ and $c$ flowing in the participant plane. For the discussion of the CME contribution, I introduce the angle $\Psi_{2,B}$ for the orientation of the plane perpendicular to the magnetic field (across which the maximum charge separation occurs). This angle is not measurable, and one needs to make further assumptions to relate the obtained expressions to the experimental measurements.

Then, decomposing the correlators in background and the signal parts similarly to Eq. 8

$$\langle \cos(\alpha + \beta - 2c) \rangle = \langle \cos(\alpha + \beta - 2c) \rangle_{BG} + \langle \cos(\alpha + \beta - 2c) \rangle_{CME} = b \langle v_{2,pp}^2 \rangle + \Delta \gamma_{CME} v_2 \{\Psi_{2,B}\},$$  \hfill (9)$$

where $\Delta \gamma_{CME} = \langle \cos(\alpha + \beta - 2\Psi_{2,B}) \rangle_{CME}$ and $v_2 \{\Psi_{2,B}\} = \langle \cos(2c - 2\Psi_{2,B}) \rangle$. In a similar way

$$\langle \cos(\alpha + \beta - 2\psi_{1,SP}) \rangle = b \langle \cos(2a - 2\psi_{1,SP}) \rangle + \Delta \gamma_{CME} \langle \cos(2\Psi_{2,B} - 2\psi_{1,SP}) \rangle$$  \hfill (10)$$
Combining everything together

\[
\frac{(\Delta \gamma/v_2)_{SP}}{(\Delta \gamma/v_2)_c} = 1 + f_{PP}^{CME} \left( \frac{\langle \cos(2\Psi_{2,B} - 2\Psi_{1,SP}) \rangle}{\langle \cos(2a - 2\Psi_{1,SP}) \rangle} \frac{v_2^2}{v_2 \{\Psi_{2,B}\}} - 1 \right)
\]

where

\[
f_{PP}^{CME} = \frac{\langle \cos(\alpha + \beta - 2c) \rangle^{CME}}{\langle \cos(\alpha + \beta - 2c) \rangle}
\]

is the fraction of the CME signal in 3-particle correlator measured relative to the second harmonic participant plane. The angle \(\psi_{1,SP}\) fluctuates around the spectator plane \(\Psi_{1,SP}\), but one can see that in the expression Eq. 11 the corresponding event plane resolution factors cancel out and

\[
\frac{(\Delta \gamma/v_2)_{SP}}{(\Delta \gamma/v_2)_c} = 1 + f_{PP}^{CME} \left( \frac{\langle \cos(2\Psi_{2,B} - 2\Psi_{1,SP}) \rangle}{\langle \cos(2a - 2\Psi_{1,SP}) \rangle} \frac{v_2^2}{v_2 \{\Psi_{1,SP}\} v_2 \{\Psi_{2,B}\} - 1 \right),
\]

where \(v_2 \{\Psi_{1,SP}\} = \langle \cos(2a - 2\Psi_{1,SP}) \rangle\).

**III. DISCUSSION**

To proceed further one has to make assumptions about the relative orientations of three angles, \(\Psi_{2,PP}, \Psi_{1,SP}\) and \(\Psi_{2,B}\). A few “reasonable” scenarios are discussed below. First, it is instructive to compare the centrality dependence of \(v_2\{2\}, v_2\{4\},\) and \(v_2 \{\Psi_{1,SP}\}\) [16]. Recall also that to a good approximation (exact in the so-called Gaussian model of eccentricity fluctuations [15]), \(v_2\{4\}\) measures the flow relative to the true reaction plane. Experimentally in midcentral collisions, centrality \(\approx 40 - 50\%\), \(v_2 \{\Psi_{1,SP}\}\) is very close to \(v_2\{4\};\) it is much closer to \(v_2\{2\}\) in central, \(< 10\%\), collisions. A possible interpretation of that would be that the spectator plane is close to the reaction plane in midcentral collisions and close to the participant plane in central collisions.

Having this in mind, one of the assumption would be

(A2) in midcentral collisions, both, the spectator plane and the magnetic field plane, coincide with the reaction plane. In this case

\[
\frac{(\Delta \gamma/v_2)_{SP}}{(\Delta \gamma/v_2)_c} = 1 + f_{PP}^{CME} \left( \frac{\langle v_2^{PP} \rangle}{\langle v_2 \{\Psi_{1,SP}\} \rangle^2} - 1 \right)
\]

Note that this relation really requires only coincidence of \(\Psi_{1,SP}\) and \(\Psi_{2,B}\), not necessarily coincidence with \(\Psi_{RP}\). Then Eq. 14 is also true even if

(A3) in central collision \(\Psi_{2,B}\) deviates from \(\Psi_{RP}\) but coincides with \(\Psi_{1,SP}\).

It is interesting that one has the same relation event under quite different assumption that

(A4) in central collision the spectator plane coincides with participant plane but, \(\Psi_{2,B}\) coincides...
with $\Psi_{RP}$. In this case

$$\frac{v_2\{\Psi_{2,B}\}}{\langle \cos(2\Psi_{2,B} - 2\Psi_{1,SP}) \rangle} = v_2\{\Psi_{1,SP}\}$$  \hspace{1cm} (15)$$

and one again arrives to Eq. 14.

Although in general it is difficult to get the exact value of the expression in parenthesis in Eq. 14 based on the above assumptions (A2)-(A4), and having in mind that experimentally $v_2\{2\}$ is larger than $v_2\{\psi_{1,SP}\}$ by about 15%, one can conclude that for an estimate of the CME fractional contribution to the gamma correlator $f_{\text{CME}}^{\text{PP}}$ at the level of 5%, the ratio Eq. 5 should be measured with an accuracy better than 1%.

Finally I make two short remarks on the experimental selection of the angles $\psi_{1,SP}$ and its relation to $\Psi_{2,B}$. Experimentally $\psi_{1,SP}$ is usually measured with zero degree calorimeters (ZDC), most often capturing only neutrons. Then (a) an additional decorrelations between $\psi_{1,SP}$ and $\Psi_{2,B}$ can arise due to difference in plane determined by spectator neutrons and spectator protons. If two ZDC are used, then (b) the result might depend on how the angles from two detectors are used in the analysis. For example using only one of ZDCs might yield $\psi_{1,SP}$ which is stronger correlated with the participant plane, while combining two angle might eliminate this bias.

\section*{IV. SUMMARY}

In conclusion, it is shown that measuring the ratios Eq. 5 relative to the participant and spectator planes can be used to determine the fraction of the CME signal in the gamma correlator measurements. If the double ratio, Eq. 8 deviated from unity it will indicate a non-zero CME contribution that can be further quantified under reasonable assumptions. On order to measure the fractional CME signal at the level of about 5% one would need to measure the ratio Eq. 8 free from non-flow effect at the level better than 1%.

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