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Phys. Rev. C **98**, 054910 — Published 28 November 2018

DOI: 10.1103/PhysRevC.98.054910

Landau Pomeranchuk Midgal Effect and Charm Production in pp Collisions at Large Hadron Collider Energies using the Parton Cascade Model

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We study the impact of the Landau Pomeranchuk Midgal (LPM) effect on the dynamics of parton interactions in proton proton collisions at the Large Hadron Collider energies. For our investigation we utilize a microscopic kinetic theory based on the Boltzmann equation. The calculation traces the space-time evolution of the cascading partons interacting via semihard pQCD scatterings and fragmentations. We focus on the impact of the LPM effect on the production of charm quarks, since their production is exclusively governed by processes well described in our kinetic theory. The LPM effect is found to become more prominent as the collision energy rises and at central rapidities and may significantly affect the model's predicted charm distributions at low momenta.

I. INTRODUCTION

Studies of relativistic collisions of heavy nuclei underway at the Relativistic Heavy Ion Collider at Brookhaven and the Large Hadron Collider at CERN have provided ample evidence for a deconfining transition of strongly interacting matter into a (strongly coupled) Quark Gluon Plasma (QGP) expected from lattice QCD calculations (see e.g., Refs. [1–3] and references therein). These studies, both on the theoretical and the experimental fronts, have now reached a high level of sophistication and the quantitative determination of QGP properties [4–7] is now in progress. Very often the results for heavy-ion collisions are compared with those for proton proton collisions at the same center of mass energy ($\sqrt{s_{NN}}$) in order to arrive at some of these conclusions, with the rationale that no QGP is likely to be formed in pp collisions. This simple expectation is now under strain as more and more indications of formation of an interacting system, emerge in pp collisions, especially for events having a large particle multiplicity (see e.g., Refs. [8, 9]).

Is an interacting system formed in pp collisions? Recently we have explored this question within Parton Cascade Model (PCM) [10]. The PCM is a transport model based on the relativistic Boltzmann equation for the time evolution of the parton density in phase-space due to semi-hard perturbative QCD interactions including scattering and radiations [11, 12] within a leading logarithmic approximation [13]. Our study indicated the formation of a medium driven by a substantial amount of multiple parton interactions, including fragmentation of partons after scattering. These aspects were found to be more strongly prevalent for collisions at small impact parameters or with large parton multiplicities and at higher incident beam energies. Even though the precise number of collisions and fragmentations are dependent on the $p_T^{\text{cut-off}}$ and μ_0 used to regularize the pQCD cross-sections and the fragmentations respectively, the results are sufficiently general.

Based on these previous findings it is opportune to investigate the importance of quantum coherence effects in parton-parton interactions, such as the Landau Pomeranchuk Midgal (LPM) effect [14]. The LPM effect is known to be important for large collision systems with lifetimes of multiple fm/c, but has commonly been neglected in the microscopic study of the proton-proton system, due to its small size and short lifetime.

Here we focus on the investigation of the LPM effect on charm quark production in proton proton collisions. Charm production is particularly well suited in this context, since it only occurs via hard processes calculable in pQCD and charm is conserved throughout the reaction. The PCM was recently extended to treat the production and medium-evolution of heavy quarks [15].

Consider a parton traversing a cloud of quarks and gluons and undergoing multiple scatterings. If the separation between consecutive scatterings suffered by the parton is sufficiently large so that the radiations off these collision centers can be treated as an incoherent sum of radiation spectra resulting from individual scatterings, we reach what is known as the Bethe-Heitler limit [16]. If on the other hand, the scattering centers are too closely located to each other, the observed radiation has to be evaluated within what is known as the factorization limit, and is a product of a single scattering spectrum from the sum of the individual small momentum transfers from all the individual scatterings.

The LPM effect [14] describes the results between these two limiting regimes, by accounting for the suppression

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of the radiation relative to the Bethe-Heitler limit, when the formation time of the radiated gluon is large compared to the mean free path and thus destructive interference between the radiated spectra becomes important. The dynamics of LPM effect on the production of light partons (u, d, s, and g) and photons in collision of gold nuclei at RHIC energy, within the PCM, was discussed earlier [17–19]. That work also demonstrated that the inclusion of the LPM effect greatly improved the agreement of the scaling of multiplicity distributions in pp collisions up to 200 GeV.

We shall investigate the consequences of the LPM effect on charm production in pp collisions at $\sqrt{s_{\rm NN}}$ of 0.20, 2.76, 5.02, 7.00, and 13.00 TeV. The results at RHIC energy (0.20 TeV) are included to clearly bring out the abundance of parton production etc. at LHC energies.

There are several reasons for focusing on charm quarks. As pointed out above, charm quarks can be produced only from semi-hard scattering of gluons and annihilation of a quark-antiquark pair or from a splitting of a gluon which has a large virtuality following a semi-hard scattering. The corresponding scattering matrix elements are not singular because of the mass of the charm quark and thus do not need any $p_T^{\text{cut-off}}$. We do realize, though, that the momentum distribution of the charm quarks can be modified by radiation of gluons or by scattering with other partons, which will be affected by variation of the $p_T^{\text{cut-off}}$ used for regularizing the pQCD matrix elements and the μ_0 used for terminating the fragmentations. The number of charm quarks which are produced is very small and thus the probability that their number is depleted by charm-anticharm annihilation is limited. Finally, there is no production of charm quarks during the hadronic phase.

We briefly discuss the basic ingredients of the PCM model pertaining to this investigation in the next section, results are given in Section III, and finally we summarize our findings.

II. MODEL DESCRIPTION

The details of the parton cascade model, including its Monte Carlo implementation `VNI/BMS`, have been discussed in significant detail in Refs. [11, 12], while production of heavy quarks has been laid out in Ref. [15]. Presently, we shall just briefly summarize the features most important to our investigation:

The parton cascade model is a transport model for the time-evolution of an ensemble of quarks and gluons based on the Boltzmann equation. We include $2 \to 2$ scatterings between light quarks, heavy quarks and gluons, and the $2 \to 3$ reactions via time-like branchings of the final-state partons (see Refs. [12, 13]) following the well tested procedure adopted in `PYTHIA` [20].

In the PCM, the IR-singularities in these pQCD cross-sections are avoided by introducing a lower cut-off on the momentum transfer $p_T^{\text{cut-off}} = 2$ GeV (please note that

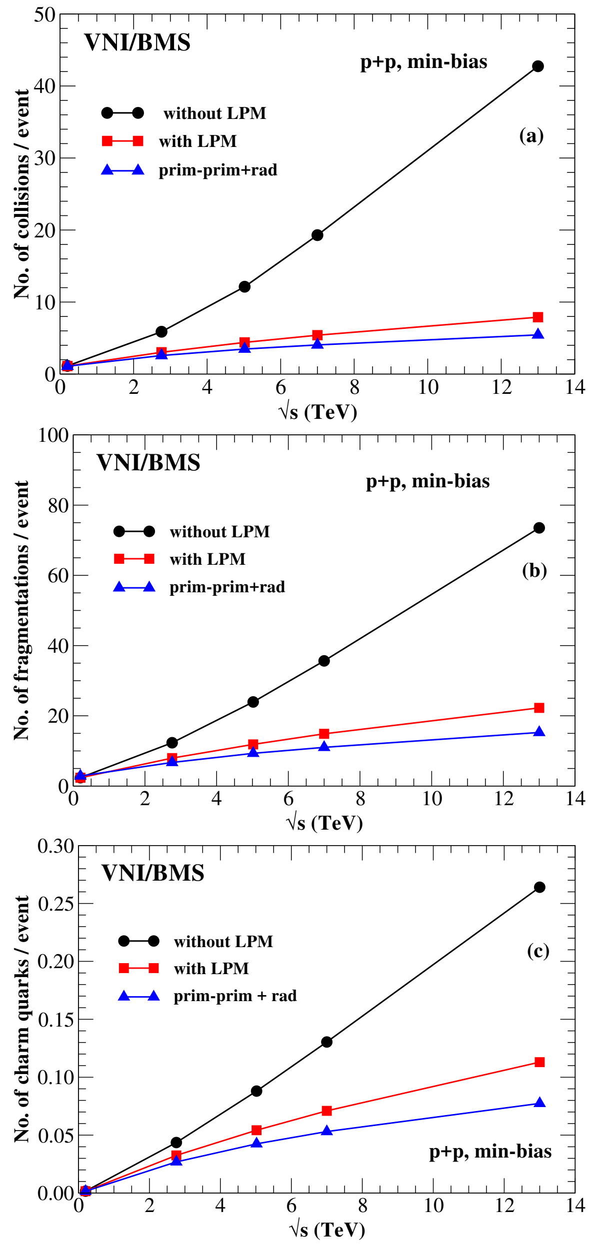

FIG. 1: (Color online) Number of collisions (a), number of fragmentations (b) and number of charm quarks produced per event (c) for minimum bias pp interactions as a function of center of mass energy. The three calculations involve multiple collisions among partons by neglecting and including the LPM effect and collisions only among primary partons with radiations off the scattered partons.

results discussed in Refs. [15, 17] were obtained by using a much smaller value for $p_T^{\text{cut-off}}$ of about 0.7 GeV, which increased the parton scatterings; see Fig. 4 later).

Most of the studies using `VNI/BMS` reported earlier were performed using a constant value of $\alpha_s = 0.3$. In the present work, we have taken $\alpha_s(Q^2)$, as we wish to study the momentum distribution of charm quarks for large values of transverse momenta. Details of the initial parton

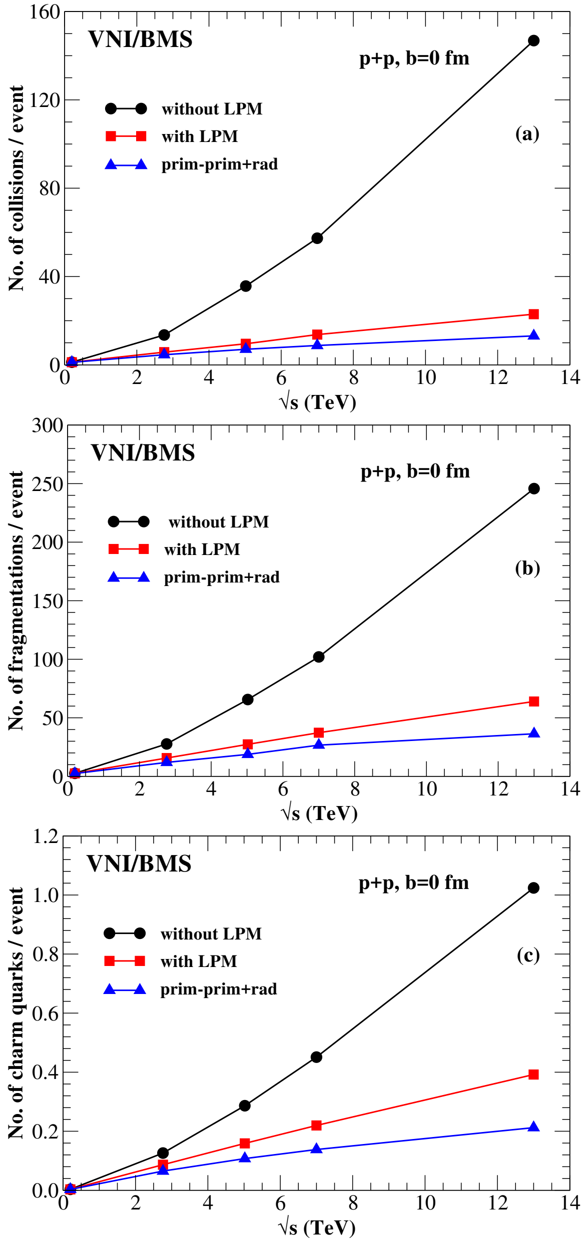

FIG. 2: (Color online) Number of collisions (a), number of fragmentations (b) and number of charm quarks produced per event (c) for pp interactions as a function of center of mass energy at impact parameter equal to zero fm. The three calculations involve multiple collisions among partons by neglecting and including the LPM effect and collisions only among primary partons with radiations off the scattered partons.

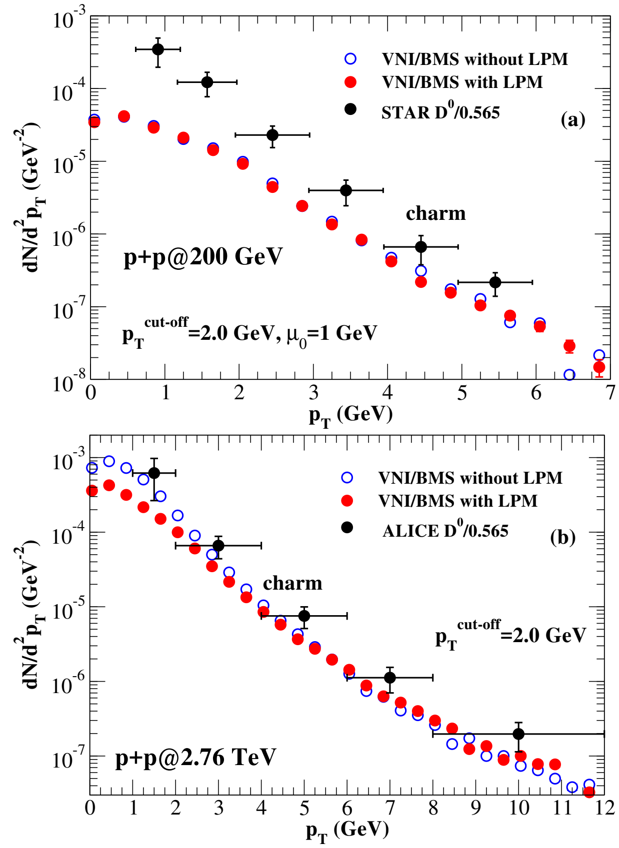

FIG. 3: (Color online) The transverse momentum spectra of charm quarks in pp collisions at 200 GeV (a) and 2.76 TeV (b) due to multiple collisions among partons and fragmentations off final state quarks, with and without inclusion of LPM effect.

distributions and the relevant matrix elements etc. have already been discussed repeatedly [10, 12, 15] which we closely follow.

For the sake of completeness, we recall that the $2 \to 3$ processes are accounted for by inclusion of radiative processes for the final state partons in a leading logarithmic approximation. The collinear singularities are then regularized by terminating the time-like branchings, once the virtuality of the parton drops to $M_0^2 (= m_i^2 + \mu_0^2)$, where m_i is the current mass of the parton (zero for gluons, current mass for quarks) and μ_0 has been kept fixed as 1 GeV. We have included $g \to gg$, $q \to qg$, $g \to q\bar{q}$, and $q \to q\gamma$ branchings for which the relevant branching functions $P_{a\to bc}$ are taken from Altarelli and Parisi [13]. The interference of soft gluons is included by angular ordering of radiated gluons as in PYTHIA.

Implementing the LPM effect in a semi-classical transport such as the PCM is not easy. First of all, the quarks and gluons are treated as quasi-particles in the PCM and thus a full quantum mechanical treatment for the process is out of question. We implement the LPM effect by assigning a formation time τ to the radiated particle:

$$\tau = \frac{\omega}{k_T^2}, \tag{1}$$

where ω is its energy and k_T is its transverse momentum with respect to the emitter. During the formation time, the radiated particle is assigned zero cross-section and thus it does not interact. The emitter, however continues to interact and if that happens, the radiated particle is removed from the list and does not participate in later

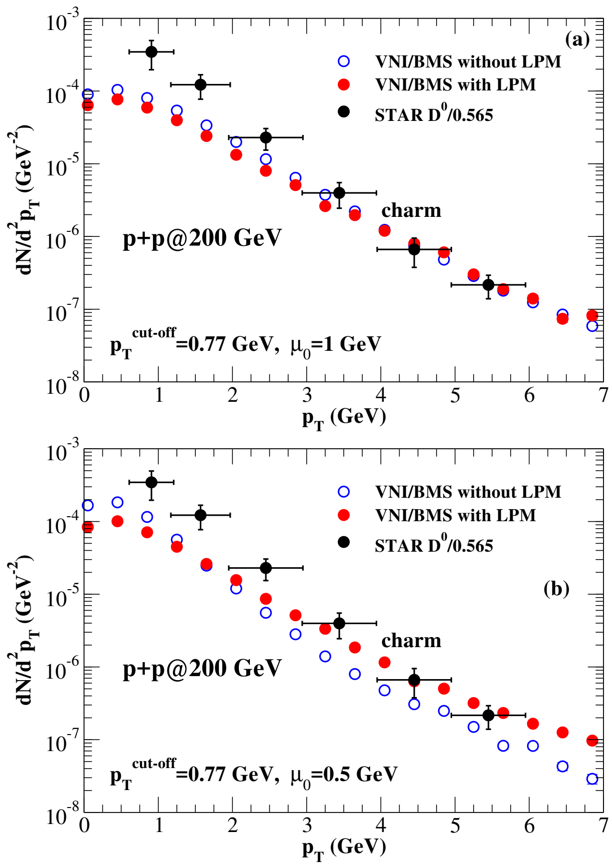

FIG. 4: (Color online) The transverse momentum spectra of charm quarks in pp collisions at 200 GeV, with $p_T^{\mathrm{cut-off}}$ =0.77 GeV and μ_0=1 GeV (a) and with $p_T^{\mathrm{cut-off}}$=0.77 GeV and μ_0=0.5 GeV (b) due to multiple collisions among partons and fragmentations off final state quarks, with and without inclusion of LPM effect.

evolution of the system. This leads to suppression of parton multiplication (see Refs. [17–19]). A similar procedure is adopted in the Boltzmann Approach to MultiParton Scattering, BAMPS, of the Frankfurt group [21]. This particular implementation of the LPM effect is quite common for semi-classical transport models, but by no means unique. An alternative method of implementing the LPM effect by Baier, Dokshitzer, Mueller, Peigne and Schiff (BDMPS) relies on recalculating the phase space for the emission of the radiated gluon [22–24] (see also Ref. [25]). Recently we have experimented with implementing the LPM effect in a scheme that is assured to reproduce the BDMPS limit of parton energy-loss [26, 27]. The energy loss suffered by charm quarks in an infinite medium (at a fixed temperature) was well described using this formalism [28]. However, this implementation, focussing on the evolution of the leading parton, is currently only feasible for infinite matter calculations in the PCM and further development is required to adapt the necessary algorithms to proton-proton or nucleus-nucleus calculations.

Our expectation is that the LPM effect will lead to a suppression of parton multiplication and thus to a reduction of primary-secondary or seconadary-secondary collisions, where primary partons make up the initial state of the two colliding protons and secondary partons are the partons emerging from scatterings and subsequent radiative interactions. It is expected that as the LPM effect reduces the number of multiple scatterings (which mainly produce charm quarks having low transverse momenta), we should expect a lowering of the production of charm quarks at smaller p_T. In addition, the suppression of radiation of gluons through the LPM effect should imply that charm quarks having large momenta radiate gluons less frequently. This should lead to a hardening of the transverse momentum spectra for charm quarks. Our analysis is set up to confirm/refute these expectations.

In order to clearly bring out the consequences of the LPM effect we proceed as follows: as a first step we study the production of charm quarks with multiple parton collisions and fragmentations without including the LPM effect. Next we give our results for calculations where the LPM effect is included. We investigate whether the LPM effect eliminates multiple parton scatterings by comparing the results from the above to a calculation with only primary-primary parton scatterings and fragmentations. The difference of the results of these calculations should clearly bring out the importance of multiple scatterings of partons in proton-proton collisions and indicate the possible emergence of an interacting medium created by semi-hard pQCD interactions.

Finally, in order to investigate the rapidity dependence of the LPM effect, we shall study the transverse momentum distribution of charm quarks at different rapidities, for which data have now become available.

III. RESULTS

A. Multiple scatterings and consequences of LPM effect

An interacting medium would be characterized by partons undergoing multiple interactions. This is different from the case when we have several parton-parton interactions involving only primary partons from the projectile and the target, without any further interaction among the partons thus produced.

In Fig. 1 we show results for minimum bias collisions of protons at several incident beam energies and show the number of semi-hard partonic scatterings, number of fragmentations, and the number of charm quarks produced per collision.

The first set of calculations restricts the interactions to primary-primary collisions followed by fragmentations off the final state partons. These results will not be affected by assigning or not assigning a formation time (i.e, inclusion or non-inclusion of LPM effect) to the radiated gluons as, further scatterings are not considered. The second set of calculations allows for primary-primary, primary-

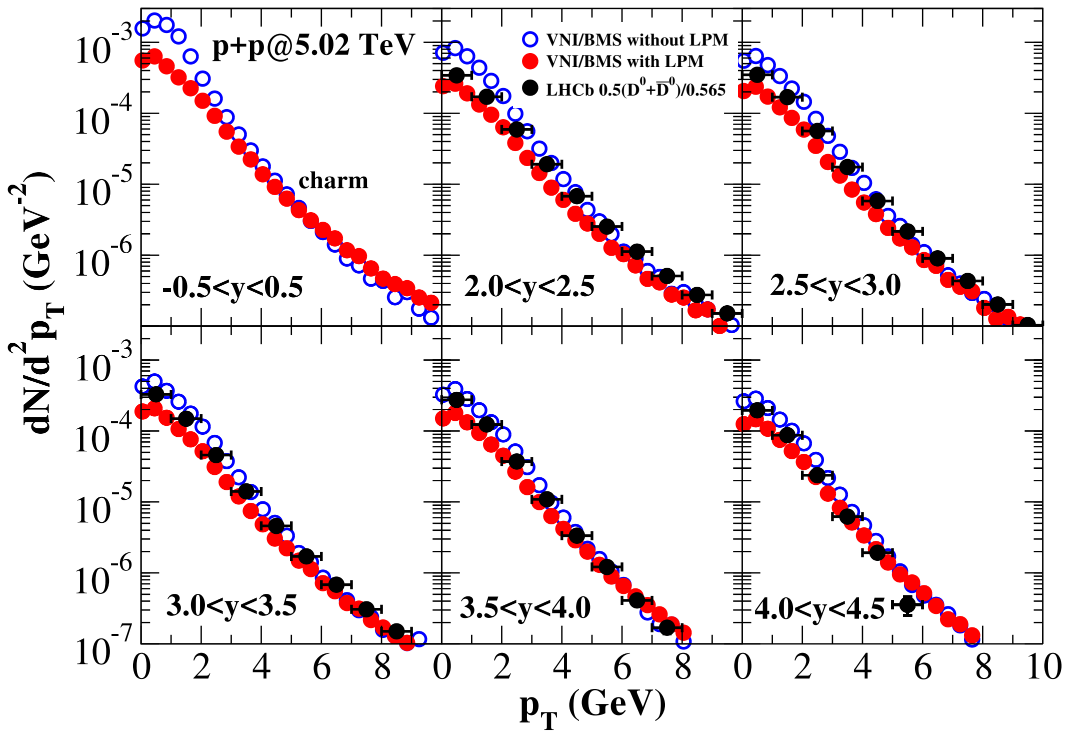

FIG. 5: (Color online) The transverse momentum spectra of charm quarks in pp collisions at 5.02 TeV due to multiple collisions among partons and fragmentations off final state quarks, with and without inclusion of LPM effect.

secondary, and secondary-secondary collisions along with fragmentations off the final state parton, but the LPM effect is not taken into account. The final set of calculations describe the system when all possible multiple scatterings and fragmentations off the final state partons are included and the LPM effect is accounted for, using the procedure discussed earlier.

We find that without the LPM effect, the number of collisions and fragmentations rise rapidly with increase in collision energy. The accounting of the LPM effect moderates this rise considerably. The reduction in the number of collisions is about 2% at 200 GeV and rises to almost 80% at 13.00 TeV, showing a strong dependence on the collision energy (for a fixed $p_T^{\text{cut-off}}$ of 2 GeV). The corresponding reduction in number of fragmentations is similar, being about 2% at 200 GeV and rising to about 70% at 13.00 TeV. The similarity of these numbers should not come as a surprise as in our approach scatterings are followed by fragmentations. The reduction in the production of charm quarks is smaller though, just about 1% at 200 GeV and about 60% at the top energy considered. We attribute the smaller reduction in the charm quark multiplicity compared to the reduction in overall scatterings and fragmentations to the large mass of the charm quark, which restricts the phase space for its production.

As discussed earlier, a comparison between results including the LPM effect with those for only primary-primary collisions and fragmentations reveal the extent of multiple scatterings. We note that collisions involving primary and secondary partons account for about 2% of the total number of collisions when LPM is accounted for at 200 GeV and increase to about 45% at the top energy considered. The number of fragmentations also rises similarly.

These results suggest that the semi-hard partonic interactions in pp collisions at LHC energies produce a dense medium, where partons undergo multiple interactions, even when the LPM suppression of fragmentations off final state partons is accounted for. These (additional) multiple collisions are sufficiently large to leave an imprint even in minimum bias events which are dominated by collisions involving larger impact parameters where the produced medium may not be very dense.

Evidence of the increasing importance of the LPM effect in more central collisions (which are likely to have a larger multiplicity) is seen from Fig. 2 where the corresponding results are plotted for zero impact parameter. We see that the number of collisions, fragmentations, and charm quarks for all the cases rise significantly and so also the effect of LPM supression.

B. Transverse momentum distribution of charm quarks

Next we discuss our results for the p_T distribution of charm quarks. Given the nature of charm quark fragmentation into D mesons, the p_T spectra can be used as

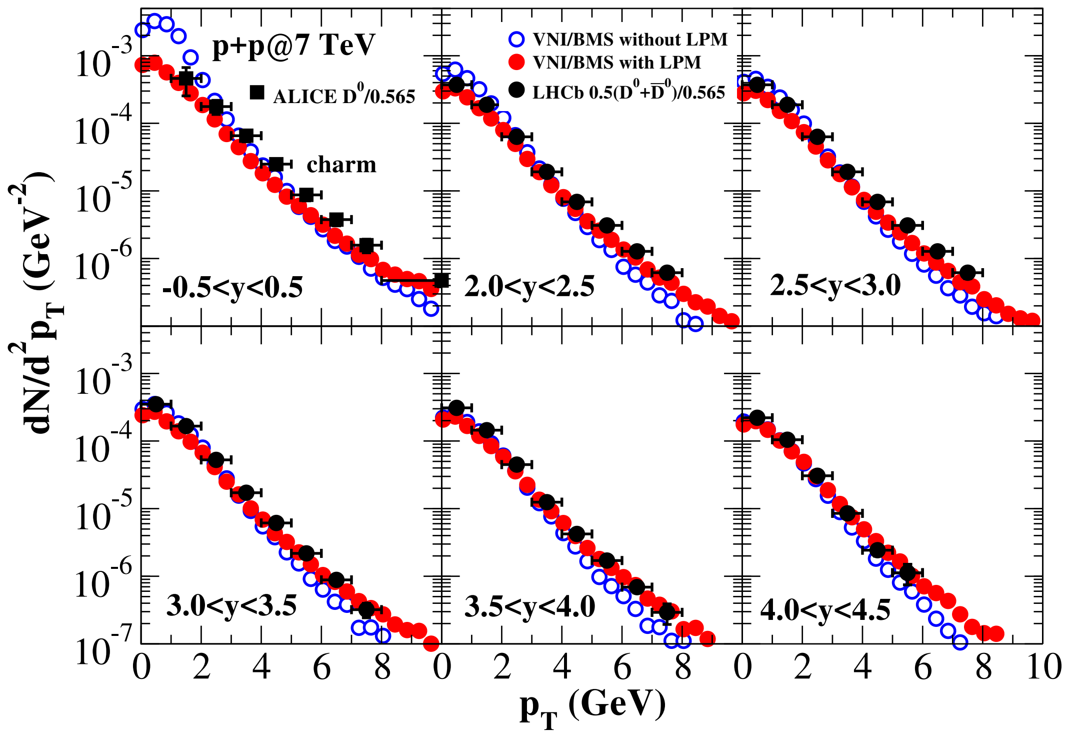

FIG. 6: (Color online)The transverse momentum spectra of charm quarks in pp collisions at 7.00 TeV due to mutiple collisions among partons and fragmentaions off final state quarks, with and without inclusion of LPM effect.

a proxy for the p_T distribution of prompt D^0 mesons, by accounting for the fraction (0.565) for which the charm quark fragments into a D^0 meson.

We have already seen that the LPM effect has only a very small effect at the lowest incident beam energy considered here, namely 200 GeV. This is again confirmed by Fig. 3 (upper panel), where the momentum distribution of the charm quarks with and without the LPM effect are shown. These are essentially identical. (The deviation of experimental data [29] from the theoretical calculations is mainly due to the value of $p_T^{\text{cut-off}}$ of 2 GeV used at all the energies.

We believe that a more appropriate value for this particular case could be about 0.7 GeV used earlier [15]). Thus we digress here to explore this in a little more detail. A reduction in $p_T^{\text{cut}-\text{off}}$ affects our results in a complex manner. For example, reducing $p_T^{\text{cut}-\text{off}}$ increases two body scatterings, due to increased cross-sections. However for a given $\sqrt{\hat{s}}$, the cross-sections for $q\overline{q} \to c\overline{c}$ and gg $\to c\overline{c}$ do not change due to the threshold of $2M_c$ which is larger than $p_T^{\text{cut}-\text{off}}$ used here. However increase in parton multiplication still raises charm production.

In Fig. 4 we show our results at 200 GeV where we reduce $p_T^{\text{cut}-\text{off}}$ to 0.77 GeV, keeping μ_0=1 GeV (upper panel). We see a a much improved description of charm production now. The effect of including or not including LPM effects remains marginal. How do our results depend on the value used for μ_0? In the same figure (lower panel) we show our results with a smaller value of μ_0 for $p_T^{\text{cut}-\text{off}}$=0.77 GeV. The first thing we notice is only a modest sensitivity of the results on μ_0 when LPM effect is accounted for. However the results without the inclusion of LPM effect, show a larger production of charm quarks at smaller p_T and a smaller production at larger p_T. We do note however that these values for $p_T^{\text{cut}-\text{off}}$ and μ_0 are rather uncomfortably small, for pQCD results to be taken literally.

The LPM effect starts to become relevant in the theoretical results for the p_T distribution of charm quarks at 2.76 TeV (see Fig. 3, lower panel), where a larger production of charm quarks is seen at lower momenta. We note however, that the results above p_T equal to 2 GeV, where we can trust our results, can not distinguish between the calculations with and with-out the LPM effect at this beam energy. We also add that the agreement of the calculation with the experimental data [30] is likely to improve with a slight decrease in $p_T^{\text{cut-off}}$ as it will increase the number of partonic collisions and the accompanying fragmentations.

LHCb has measured charm production at several forward rapidities at 5.02 [31], 7.00 [32], and 13.00 TeV [33]. The results at central rapidity for the same at 7.00 TeV beam energy from ALICE [34] are also available.

We see a good description of p_T spectra of charm quarks at all rapidities at 5.02 TeV (Fig. 5). An enhanced production of charm quarks is seen at lower momenta, when the LPM effect is neglected and the enhancement decreases with increase in rapidity. It remains to be seen

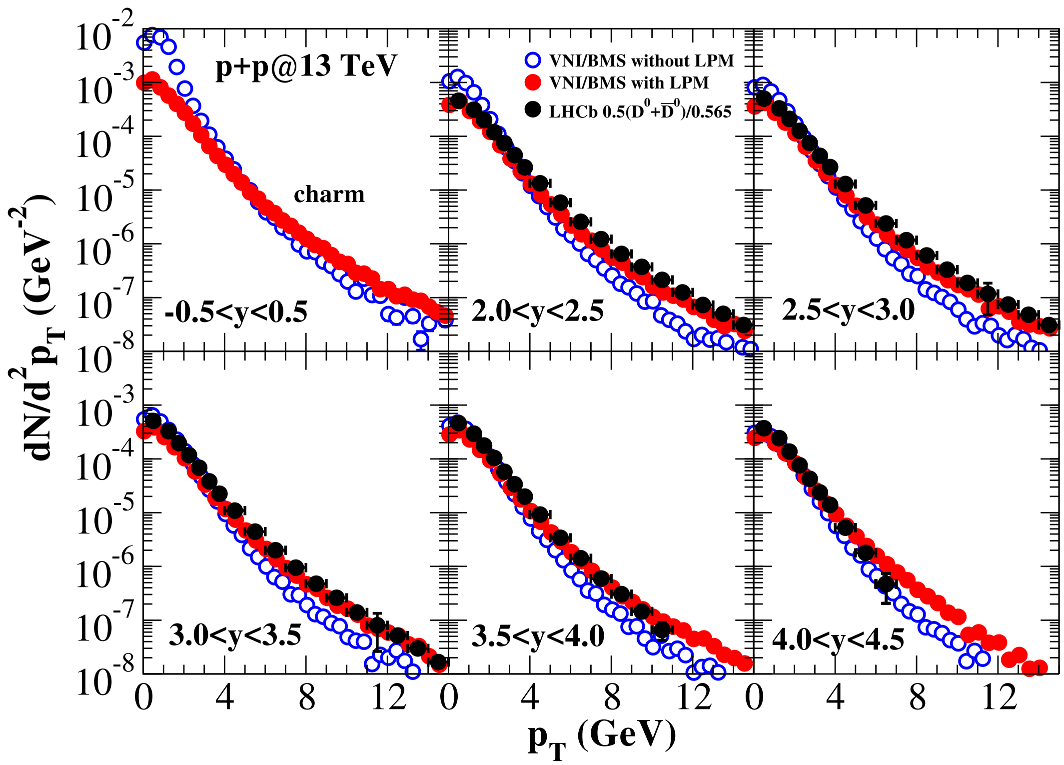

FIG. 7: (Color online) The transverse momentum spectra of charm quarks in pp collisions at 13.00 TeV due to mutiple collisions among partons and fragmentaions off final state quarks, with and without inclusion of LPM effect.

if the data likely to be available at 5.02 TeV (and 13.00 TeV, see later) at central rapidity are in agreement with these results.

The results for 7.00 TeV (Fig. 6) are of particular relevance, since experimental data also exist at central rapidity. Our calculations show a large suppression of charm production at lower p_T when the LPM effect is included and closely reproduce the transverse momentum spectra at all rapidities. We also see a hint of the hardening of the p_T spectra for large values of p_T, which is also reproduced by our calculations, even though the effect is not large. As indicated earlier, this happens as the LPM effect also suppresses the radiation of gluons by charm quarks traversing the medium at large energy/momenta.

The hardening of the transverse momentum spectra and suppression of charm quarks having low p_T (the suppression decreasing with increase in rapidity) is seen more clearly at 13.00 TeV (Fig. 7). The experimental results at all rapidities are adequately explained when the LPM effect is accounted for. It will be interesting to see if the substantial suppression predicted at central rapidity is supported by data.

IV. SUMMARY AND CONCLUSIONS

We have studied the impact of the Landau Pomeranchuk Midgal effect on the dynamics of parton transport in proton-proton collisions at LHC energies. In particular, we have focused on the production of charm quarks, since these are only produced in hard pQCD interactions for which the parton cascade model utilized in our study is uniquely suited. We find that the inclusion of the Landau Pomeranchuk Midgal effect, which suppresses the radiation of gluons off scattered partons leads to a reduction in the number of scatterings, number of fragmentations and number of charm quarks which are produced. Even after this suppression, however, these quantities remain larger than the corresponding numbers for calculations where only primary-primary collisions among partons is included along with fragmentation off final state partons.

The results indicate the formation of an interacting medium, which is dense enough for the LPM suppression of radiation to set in and yet permits multiple scatterings among partons. The LPM effect plays an important role in moderating the production of charm quarks having low transverse momenta. It also leads to a hardening of their transverse momentum spectra at larger p_T. The impact of the LPM effect is found to rise with increasing collision energy and to decrease with increase in rapidity.

Before closing, we add that the charm production in pp collisions has been studied in detail using Fixed Order Next to Leading Log (FONLL) calculations [35]. The data at higher LHC energies are generally found to be slightly above the upper limit given by these calculations (see Refs. [31–34]). Realizing that our calculations with only primary-primary collisions and fragmentations tend to roughly account for the higher order corrections in a Leading Log Approximation, these studies then suggest additional contributions from multiple scatterings. The precise extent of this contribution and its dependence on some of the parameters, e.g., current mass of the charm quark, $p_T^{\text{cut-off}}$ and μ_0 remain to be investigated. We do believe, however, that the additional contributions arising due to the multiple scatterings and suject to LPM effect will be there, unless of-course $p_T^{\text{cut-off}}$ and μ_0 are taken too large and too few interactions take place and too few radiations occur.

In brief, our results provide an indication of emergence of a dense and interacting medium of partons in *pp* collisions at LHC energies due to semi-hard pQCD interactions, even when LPM suppression of radiation of gluons from scattered partons is accounted for.

Acknowledgments

DKS gratefully acknowledges the support by the Department of Atomic Energy. This research was supported in part by the ExtreMe Matter Institute EMMI at the GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt, Germany. DKS also acknowledges valuable discussions with Jurgen Schukraft and Dirk Rischke. SAB acknowledges support by US Department of Energy grant DE-FG02-05ER41367.

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