Transport dynamics of parton interactions in pp collisions at energies available at the CERN Large Hadron Collider

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Transport Dynamics of Parton Interactions in pp Collisions at LHC Energies

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We investigate the transport dynamics of partons in proton-proton collisions at the Large Hadron Collider using a Boltzmann transport approach, the parton cascade model. The calculations include semi-hard pQCD interaction of partons populating the nucleons and provide a space-time description of the collision in terms of cascading partons undergoing scatterings and fragmentations. Parton production and number of collisions rise rapidly with increase in center of mass energy of the collision. For a given center of mass energy, the number of parton interactions is seen to rise stronger than linear with decreasing impact parameter before saturating for very central collisions. The strangeness enhancement factor γ_s for the semi-hard processes is found to rise rapidly and saturate towards the highest collision energies. Overall, our study indicates a significant amount of partonic interactions in proton-proton collisions, which supports the observation of fluid-like behavior for high multiplicity proton-proton collisions observed in the experiments.

I. INTRODUCTION

Relativistic heavy-ion collisions have been used with great success to probe the properties of hot and dense QCD matter, the quark-gluon-plasma (QGP) [1–7]. Proton-proton collisions at a given center of mass energy per nucleon have been thought to provide a base-line measurement without the creation of a QGP when extrapolated to the corresponding nucleus-nucleus case using simple geometric models [8]. However, recently this canonical picture has undergone a considerable change as several experimental "indications" of formation of a medium, e.g., flow and enhanced production of strangeness have also been seen in proton-proton collisions, albeit only when triggering on high multiplicity events [9].

One should note that on the theory side the notion of possible QGP formation in proton-proton collisions dates back several decades: hydrodynamics has been used for a long time while exploring pp collisions, with several theoretical studies assuming formation of QGP (see, e.g., Ref. [10–12]). It was suggested that the particle spectra for 1.8 TeV proton-antiproton collisions showed evidence of flow [13] which indicated formation of quark-gluon plasma in such collisions. One may recall though, that it was argued [14] that the increase in $\langle p_T^2 \rangle$ could be attributed to events simply having a large multiplicity, i.e. increased minijet activity, without the formation of a deconfined medium. Subsequently, additional data for the same system, along with measurements of HBT radii, were used [15] to claim evidence for a de-

confining phase transition in these collisions. However, only through the advent of recent high quality data have these calculations and analyses gained renewed traction. By now a large body of data for pp collisions at RHIC and ever-increasing LHC energies has been accumulated, which provides enough indications for flow and enhanced production of strangeness in events having large multiplicity [9], even though non-QGP based interpretations of the data have remained viable [16–18].

In the present work we aim at quantifying the amount of parton interactions and rescattering present in proton-proton collisions and whether the amount of interactions observed may lend credence to the notion of collectivity in these collision-systems and the application of hydrodynamic models. For this purpose we use a microscopic Boltzmann transport approach, the parton cascade model [19] as implemented in VNI/BMS [20] and extended recently to include heavy quark production [21, 22] to explore the emergence of semi-hard multi-partonic collisions and parton multiplications in pp collisions using pQCD matrix elements. This treatment [20] has several inherent advantages. First of all, all parton scatterings leading to $p_T \geq p_T^{\text{cut-off}}$ are treated within (lowest order) perturbative QCD, avoiding any arbitrariness, except for the dependence of the results on the momentum cut-off, introduced to avoid singular cross-sections for mass-less partons at lower momentum transfers. However, it is expected that spectra etc. for larger p_T should be reasonable. It should be noted, however, that the limitation to pQCD matrix elements with a momentum cut-off implies that our approach does not describe the dynamics of thermalized degrees of freedom. We thus will only be able to assess whether the conditions necessary for the formation of (equilibrated) QCD matter are met, but will not be able to describe the development and evolution of a QGP itself.

The tracking of the hard collision dynamics and all

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the partons involved in these interactions allows us to perform calculations at several levels of complexity: in a first step, we only allow the primary partons from the projectile nucleon to collide with primary partons from the target nucleon. Next we consider scattering among primary and secondary partons. This corresponds [20] most closely to minijet calculations [23]. Finally we perform calculations which account for fragmentation of final state partons following semi-hard scatterings. These radiative processes are included following the original PCM implementation [19] in the leading-logarithmic approximation (LLA).

We do not consider hadronization of either the partons which have undergone interaction or the un-interacted partons, and thus our findings relate only to the partons produced in the semi-hard processes.

We report our results for minimum bias collisions of protons at center of mass energies of 0.2, 2.76, 5.02, 7.00, and 14 TeV for the three implementations discussed above and study the evolution of parton production and multiple collisions with the increase in collision energy. Subsequently we explore the collision of protons at a center of mass energy of 7.00 TeV as a function of impact parameter and the $p_T^{\text{cut-off}}$ used for regularizing the infra-red divergences for pQCD cross-sections. Finally, we study some of these systematics as a function of number of quarks (charged particles).

II. FORMULATION

The Monte Carlo implementation of the parton cascade model has been discussed in [19, 20] for production of light quarks, gluons, and heavy quarks.

The $2 \rightarrow 2$ scatterings involving light quarks and gluons included in VNI/BMS are:

$$
\begin{aligned}
q_i q_j &\rightarrow q_i q_j, \; q_i \bar{q}_i \rightarrow q_j \bar{q}_j, \\
q_i \bar{q}_i &\rightarrow gg, \; q_i \bar{q}_i \rightarrow g\gamma, \\
q_i \bar{q}_i &\rightarrow \gamma\gamma, \; q_i g \rightarrow q_i g, \\
q_i g &\rightarrow q_i \gamma, \; gg \rightarrow q_i \bar{q}_i, \\
gg &\rightarrow gg.
\end{aligned}
\tag{1}
$$

The heavy quark production is included [21, 22] via,

$$
q\bar{q} \rightarrow Q\bar{Q}, \quad gg \rightarrow Q\bar{Q}
$$

processes, while their scatterings with light quarks or gluons are included via the

$$
qQ \rightarrow qQ, \quad gQ \rightarrow gQ,
\tag{2}
$$

processes, where g stands for gluons, q stands for light quarks, and Q stands for heavy quarks.

The $2 \rightarrow 3$ reactions are included via time-like branchings of the final-state partons (see Ref. [20]):

$$
\begin{aligned}
g^* &\rightarrow q_i \bar{q}_i, \; q_i^* \rightarrow q_i g, \\
g^* &\rightarrow gg, \; q_i^* \rightarrow q_i \gamma,
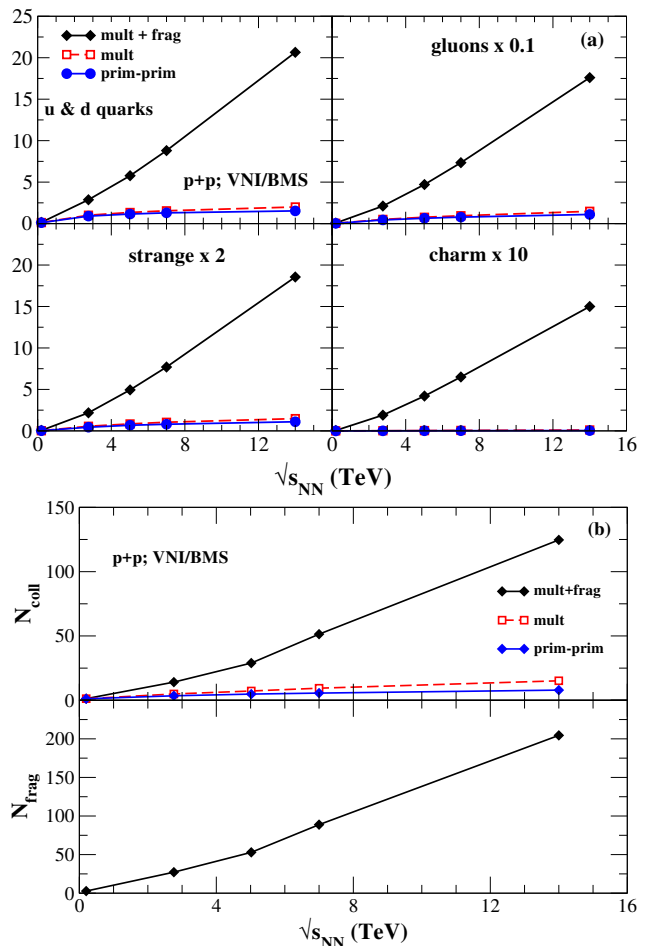\end{aligned}
\tag{3}
$$

FIG. 1: (Color online) (a) Production of light quarks, strange quarks, charm quarks and gluons as a function of centre of mass energy for semi-hard partonic collisions in pp system for calculations involving scattering only between primary partons (filled circles), multiple scatterings *with-out* fragmentation of scattered partons (hollow squares) and multiple scatterings *with* fragmentations of scattered partons (filled diamonds). (b) Number of collisions and number of fragmentations as a function of centre of mass energy.

following the well tested procedure adopted in PYTHIA.

We add that the IR-singularities in these pQCD cross-sections are avoided in PCM by introducing a lower cut-off on the momentum transfer $p_T^{\text{cut-off}} \approx 2$ GeV. We also add that, as discussed in Ref. [22], the processes $gQ \rightarrow gQ$ and $qQ \rightarrow qQ$ have been explicitly excluded in these calculations when the heavy quark belongs to the sea, in order to account for the strong suppression of these interactions when NLO terms are included.

The $2 \rightarrow 3$ processes are included by inclusion of radiative processes for the final state partons in a leading logarithmic approximation. The collinear sigularities have been regularized by terminating the time-line branchings, once the virtuality of the parton drops to $Q_0^2 = m_i^2 + \mu_0^2$, where m_i is the current mass of the parton (zero for glu-
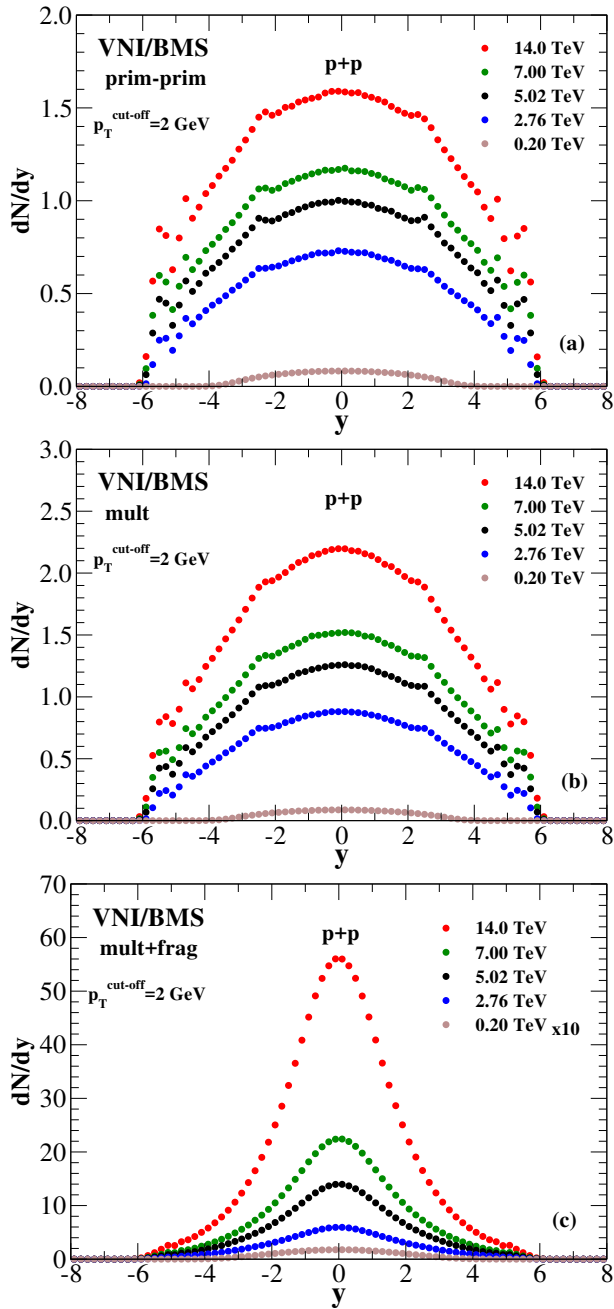
FIG. 2: (Color online) Rapidity density of partons produced in *pp* interactions due to semi-hard collisions among primary partons (a), multiple collisions *without* fragmentations of final state partons (b) and multiple collisions *with* fragmentations of final state partons (c) at different centre of mass energies.
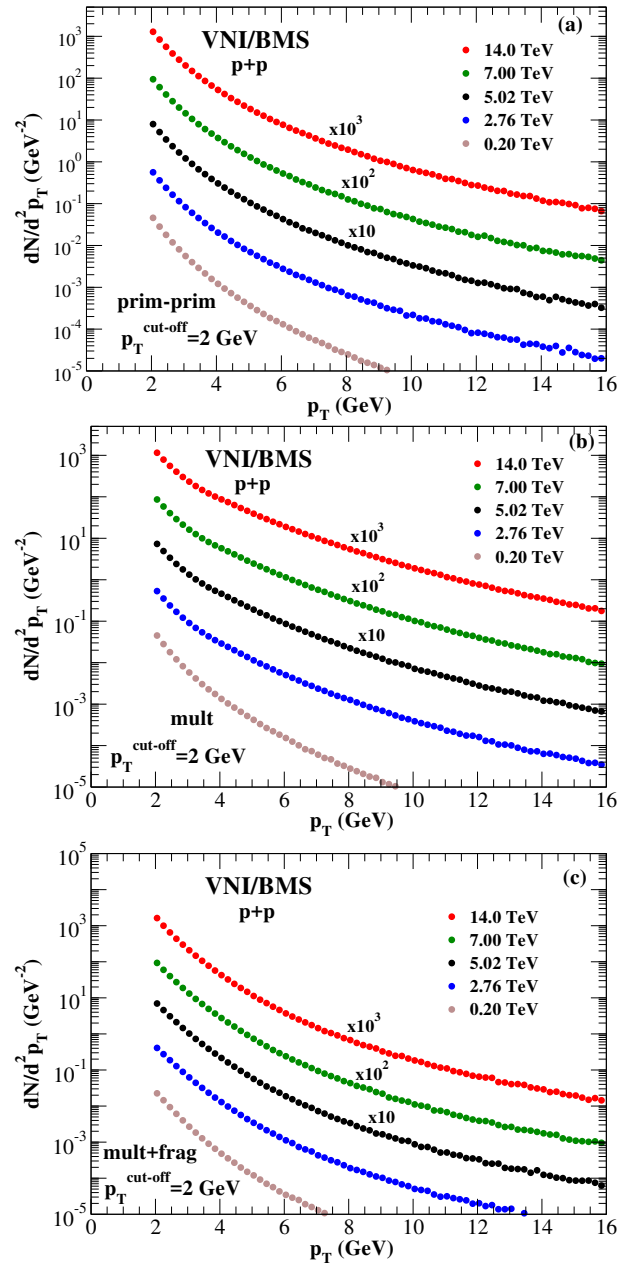

FIG. 3: (Color online) Rapidity integrated p_T spectra of partons produced in *pp* interactions due to semi-hard collisions among primary partons (a), multiple collisions *without* fragmentations of final state partons (b) and multiple collisions *with* fragmentations of final state partons (c) at different centre of mass energies.

ons, current mass for quarks) and μ_0 has been kept fixed as 1 GeV. We have included $g \to gg$, $q \to qg$, $g \to q\bar{q}$, and $q \to q\gamma$ branchings for which the relevant branching functions $P_{a\to bc}$ are taken from Altarelli and Parisi [24]. The initial state of the nucleons has been set up in terms of partons whose momentum distributions are described by the parton distribution functions initialized at the scale of $Q_{\text{ini}}^2 = 4$ GeV². We have used GRV-HO function for

our studies even though more modern functions are now available in the literature, primarily as we are more interested in the evolution of the multi-parton interactions when the centre of mass energy or the impact parameter or the lower momentum cut-off for parton scattering is altered. The partons are distributed around the centres
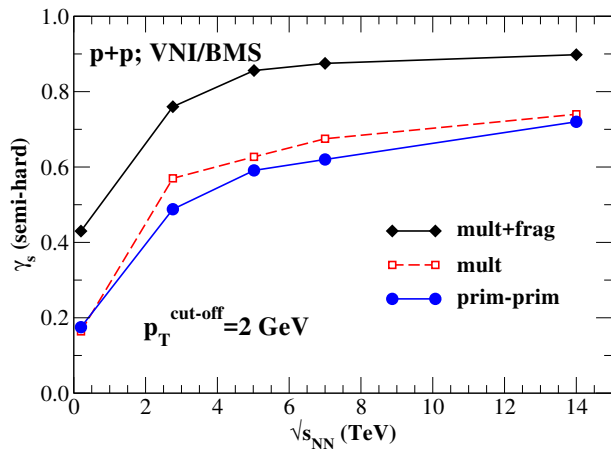
of nucleons according to the distribution,

$$h_N(\vec{r}) = \frac{1}{4\pi}\frac{\nu^3}{8\pi}\exp(-\nu r) \qquad (4)$$

with ν chosen to give the root mean square radius $R_N^{\mathrm{ms}} \equiv \sqrt{12/\nu} = 0.81$ fm.

Unless otherwise stated, we have kept $p_T^{\mathrm{cut\text{-}off}}$ fixed at 2 GeV. Most of our studies using PCM at RHIC energies used a more modest value of $p_T^{\mathrm{cut\text{-}off}} \approx 0.78$ GeV. For the corresponding results at $\sqrt{s} = 0.2$ TeV, the reader is referred to Ref. [20, 25–28]. Our calculations do not consider hadronization and subsequent interactions among the hadrons.

III.   ANALYSIS SETUP

As mentioned earlier, we perform three sets of calculations to investigate the essential features of the evolution of the partonic cascade in pp collisions. We define primary partons as partons which constitute the nucleon and which have not undergone any interaction. The secondary partons are those which are produced in collisions or fragmentation of scattered partons.

The first set of calculations look at the system which would be formed if only primary-primary collisions are included in the calculations. The second set of calculations look at the system when primary-primary, primary-secondary, and secondary-secondary collisions are permitted but fragmentation of final state partons is not permitted, thus effectively blocking parton multiplication. The final set of calculations describe the system when all possible multiple scatterings among partons are tracked and when the final state partons fragment, leading to a substantial increase in number of collisions and parton production from semi-hard processes.

We discuss our results in terms of partons produced in these semi-hard interactions and number of collisions as well as number of fragmentations (when applicable). We also give our results for relative abundance of strange quarks with respect to light quarks which are produced by semi-hard interactions considered here, defined by:

$$\gamma_s^{\mathrm{semi-hard}} = \frac{2(N_s + N_{\bar{s}})}{N_u + N_{\bar{u}} + N_d + N_{\bar{d}}}. \qquad (5)$$

We re-emphasize that the multi-parton interactions included in these calculations take place only if the momentum-transfer is larger than the $p_T^{\mathrm{cut\text{-}off}} \approx 2$ GeV. This necessarily provides that quite a large part of the initial state partons continue without interaction. These include valence/sea light quarks, sea strange quarks and gluons, which move with the momenta with which they were initialized. Many more partons would interact if $p_T^{\mathrm{cut\text{-}off}}$ is lowered or if a suitably screened interaction, e.g, by including Debye screening is considered[33]. The lack of interactions below the $p_T^{\mathrm{cut\text{-}off}}$ implies that we cannot directly study the possible formation of a thermalized medium, which would require abundant interactions

FIG. 4: (Color online) The ratio of strange quarks and light quarks produced in semi-hard processes for the three set of calculations discussed here.
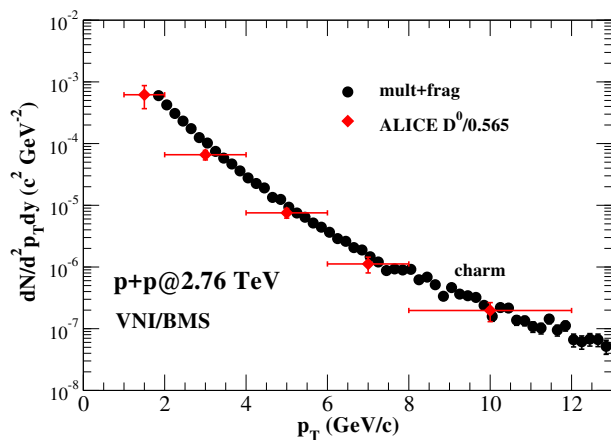

FIG. 5: (Color online) A comparison of our calculations with prompt charm production measured by ALICE experiment in pp collisions at 2.76 TeV [30].
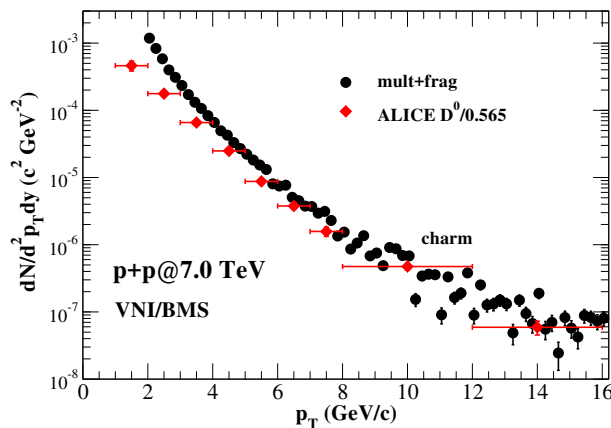

FIG. 6: (Color online) A comparison of our calculations with prompt charm production measured by ALICE experiment in pp collisions at 7.00 TeV [31].
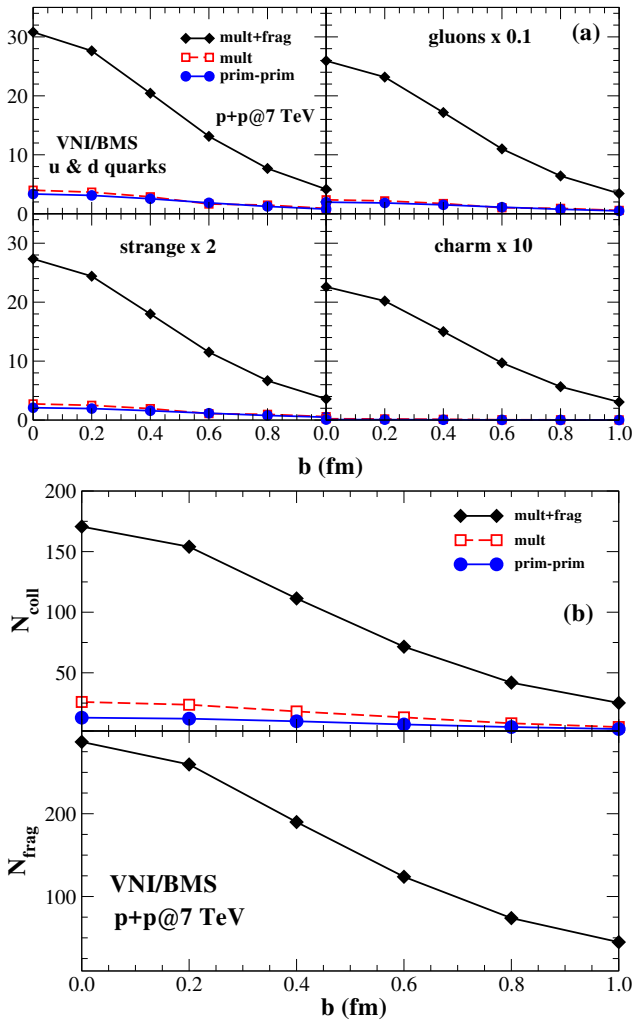
FIG. 7: (Color online) (a) Production of light quarks, strange quarks, charm quarks and gluons as a function of impact parameter for semi-hard partonic collisions in *pp* system for calculations involving scattering only between primary partons (filled circles), multiple scatterings *with-out* fragmentation of scattered partons (hollow squares) and multiple scatterings *with* fragmentation of scattered partons (filled diamonds). (b) Number of collisions and number of fragmentations as a function of impact parameter.

at scales below the $p_T^{\text{cut-off}}$. However, we can ascertain whether a sufficient number of interacting partons is deposited into the system that could potentially lead to the formation of a thermalized medium.

IV. EVOLUTION OF MULTI-PARTON INTERACTIONS WITH CENTRE OF MASS ENERGY

We first look at partons produced in minimum bias collisions of protons at varying center of mass energies (Fig. 1). We see only a marginal difference between the results for the calculations involving primary-
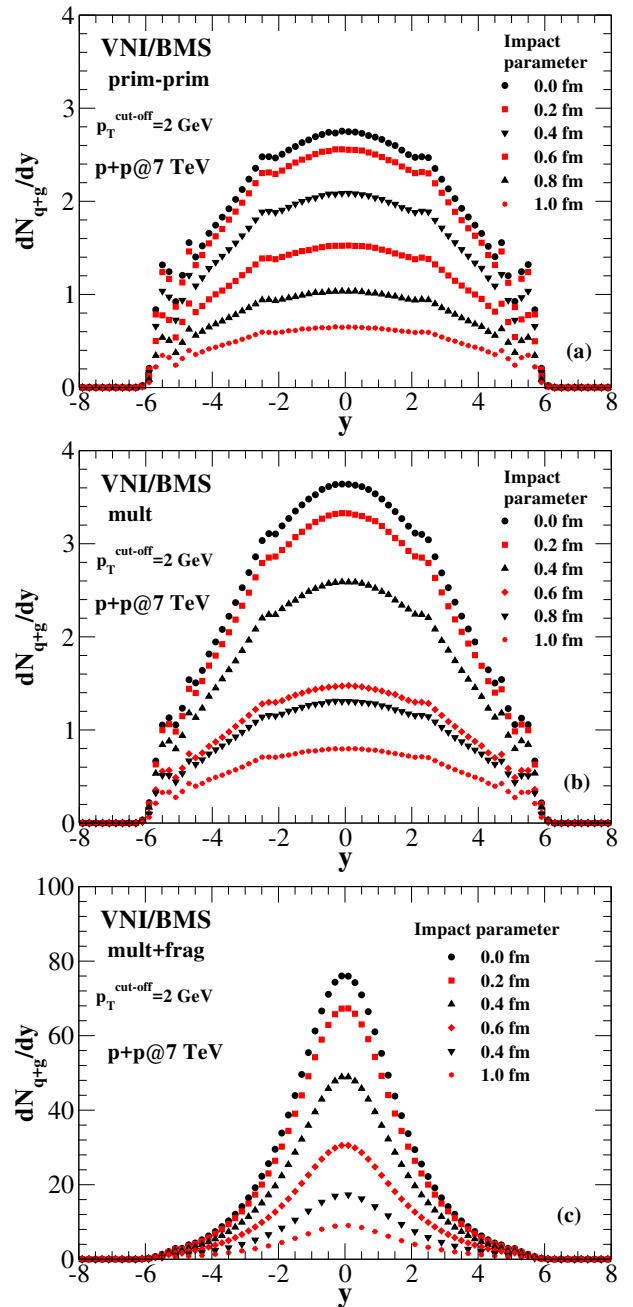
FIG. 8: (Color online) Rapidity density of partons produced in *pp* interactions at $\sqrt{s} = 7$ TeV due to semi-hard collisions among primary partons (a), multiple collisions *without* fragmentations of final state partons (b) and multiple collisions *with* fragmentations of final state partons (c) at different impact parameters.

primary collisions and multiple collisions (without fragmentation). The production of gluons, light quarks, and strange quarks is seen to rise monotonically along with the number of semi-hard collisions with the center of mass energy of the *pp* collision, but overall the number of partons involved in these interactions remains small, even at the highest beam energies.
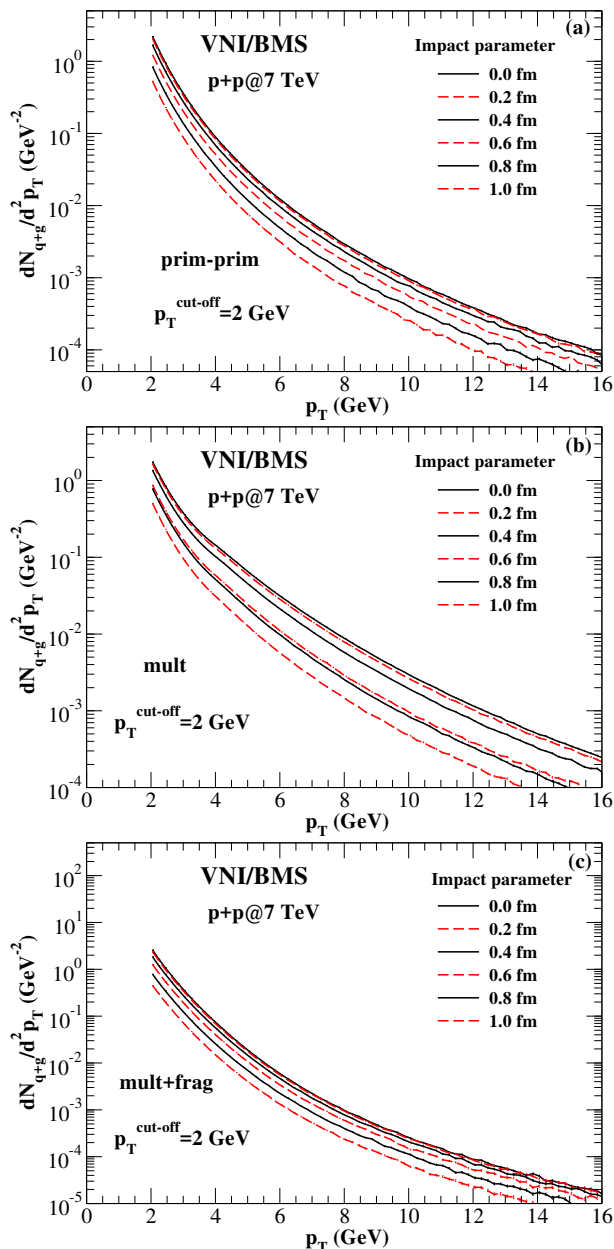
FIG. 9: (Color online) Rapidity integrated p_T spectra of partons produced in pp interactions at $\sqrt{s} = 7$ TeV due to semi-hard collisions among primary partons (a), multiple collisions *without* fragmentations of final state partons (b) and multiple collisions *with* fragmentations of final state partons (c) at different impact parameters.

These results imply that semi-hard $2 \rightarrow 2$ interactions, without fragmentation of final state partons, are too few in pp collisions even at the highest energy considered to lead to a hot and dense interacting medium.

The corresponding results for the calculations with fragmentation of final state partons using the procedure indicated earlier, show a rapid multiplication of partons and a sharp rise in the number of collisions compared to the cases discussed above. This increase is clearly driven

by the number of fragmentations, We also see a sharp increase in production of strange and charm quarks due to the multiple interactions as gluons multiply and interact. The increase could also be partly due to the opening up of processes like $g^* \rightarrow Q\overline{Q}$ as the centre of mass energy increases. We note that already at $\sqrt{s}_{NN} = 200$ GeV more than 100 interacting partons are deposited into the system, which can participate in the partonic collisions having momentum transfers of more than 2 GeV. This number grows to about 500 at 2.76 TeV. These should be sufficient to lead towards a thermalized system of partons with signs of collectivity. This would become even more likely once softer collisions are accounted for. The calculations clearly indicate that parton multiplication following initial scattering among primary partons drives and is driven by substantially increased multiple scatterings. The system thus created has a large number of partons undergoing semi-hard multiple collisions and the multi-parton interactions rise rapidly with increase in the center of mass energy.

The corresponding rapidity density distributions of partons produced for the three sets of calculations are shown in Fig. 2. Once again we see that semi-hard collisions among partons *without* radiative processes do not lead to a substantial rise in the production of partons in pp collisions. The radiative processes following multiple collisions lead to an increase in parton production by a factor of 10–20 at LHC energies. We also note that the greatly increased multiple collisions can lead to a substantial production of strange and charm quarks as the centre of mass energy increases. One should note, however, that the average p_T of partons included in the analysis of the upper two frames is considerably higher due to the momentum cut-off of the scattering cross section than in the lowest frame, which includes fragmented partons that can carry a significantly lower p_T. Thus we add that the rapid increase in the parton production with the centre of mass energy seen in the calculations with multiple collisions and fragmentations is largely due to large production of partons (mostly gluons) having low transverse momenta from fragmentation and also multiple scatterings. In contrast, the calculations with only primary-primary and multiple collisions include only those partons which have transverse momenta larger than 2 GeV, leading to a slower growth in the multiplicity with collision energy. This growth may also get some what moderated if we include Landau Pomeranchuk Migdal suppression, neglected in these exploratory calculations. This is under investigation.

The transverse momentum spectra for these calculations (for $p_T \geq 2$ GeV) are shown in Fig. 3 and reveal a power-law behavior as expected. A closer scrutiny of these spectra also reveals a softening of the p_T distribution when fragmentation of the final state partons is included.

The ratio of the number of strange and light quarks produced in such collisions, γ_s (Eq.5), is often used as a measure of strangeness or chemical equilibration. The

results for γ_s as a function of center of mass energy for the three sets of the calculations for semi-hard processes considered here are shown in Fig. 4. We immediately see that fragmentations play an important role in increasing the value of γ_s which for the highest energy is seen to saturate at a value of about 0.9, suggesting that the fragmentations and enhanced multiple scatterings may push the system towards equilibration of strangeness even in pp collisions at higher energies.

Let us pause here to understand this large opening up of multi-partonic interactions in pp collisions as the energy of the collision increases. This has several origins. First of all, we recall that the center of mass energy in a collision of primary partons - $\widehat{s}$ is equal to $x_1 x_2 \sqrt{s}$, where x_i's stand for the fractions of nucleon momenta carried by the partons. The lower cut-off on the transverse momentum for the collision requires (see e.g., Ref. [29]) that $x_1 x_2 \sqrt{s} \geq 2 p_T^{\text{cut-off}}$. As $0 \leq x_i \leq 1$, the partons must have $x_i \geq 2 p_T^{\text{cut-off}}/\sqrt{s}$ to be able to participate in the semi-hard partonic collisions considered here. The structure functions for gluons and sea quarks increase with decreasing x and this will bring in many more partons which can participate in the semi-hard collisions as the center of mass energy increases. Secondly the parton-parton cross-sections rise as the available center of mass energy increases. And lastly with the increase in the center of mass energy, many more collisions will have large momentum transfers making it possible for the fragmentation processes to contribute to partons multiplications.

In Figs. 5 and 6 we give a comparison of our calculations with the prompt charm production measured by the ALICE experiment [30, 31] at 2.76 and 7.00 TeV respectively. The experimental values for D^0 have been divided by 0.565- the fraction for fragmentation of c quarks into D^0. We have limited the comparison to $p_T > 2$ GeV in view of the $p_T^{\text{cut-off}}$ used in our calculations. A fair agreement is seen, though we note a definite tendency of the calculations to give a larger production of charm as the p_T decreases, especially at the higher incident energy. This is under investigation.

V. EVOLUTION OF PARTONIC CASCADES AS A FUNCTION OF IMPACT PARAMETER (b) AND $p_T^{\text{cut-off}}$

Our calculations provide an opportunity to study the evolution of the partonic cascade as a function of impact parameter as the number of partons in the region of over-lap changes with the impact parameter. Thus for example in a collision of protons at the center of mass energy of 7 TeV (a case which we study in greater detail here), each proton is populated by about 270 partons, which include the up and down valence quarks, up, down and strange sea quarks, and gluons distributed according to the function given by Eq. 4 given earlier. This immediately provides for a larger possibility for multiple collisions for smaller impact parameters. We acknowl-
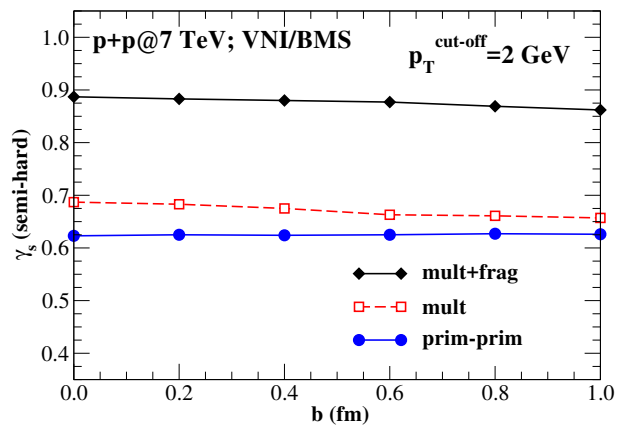

FIG. 10: (Color online) The ratio of strange quarks and light quarks produced in semi-hard processes for the three set of calculations discussed here as a function of impact parameter.

edge that the identification of the impact parameter in a pp collision is experimentally rather challenging, yet we proceed under the assumption that some measure of centrality can be identified that allows experimental data to be mapped to our systematic study as a function of the impact parameter (over which we have full control in our calculation). We add that authors of Ref. [32] have performed an early study of evolution of elliptic flow in pp collisions at 14 TeV as a function of impact parameter.

We note that our calculation contains a large number of uninteracted partons which will subsequently hadronize. It is quite likely that the hadrons arising from these uninteracted partons will have lower transverse momenta while those resulting from the partons which have undergone semi-hard collisions will have larger transverse momenta.

We give results of our calculations for impact parameter, b, equal to 0.0, 0.2, 0.4, 0.6, 0.8, and 1.0 fm. The $p_T^{\text{cut-off}}$ for these calculations has been fixed at 2 GeV.

Looking at the total number of light quarks, strange quarks, charm quarks, gluons and number of collisions and fragmentations (when applicable) (see Fig 7), we see a very clear increase in the number of collisions as the impact parameter decreases. These variations are large enough to provide a distinctive classification of events with large semi-hard partonic collisions for the more realistic calculation of partonic collisions along with radiative processes.

How are the rapidity density and transverse momentum distributions of partons produced in these semi-hard processes affected by variation in impact parameter? We give these results for the rapidity densities for the three set of calculations in Fig.8.

We see once again that multiple collisions along with parton fragmentations lead to a large production of partons due to semi-hard processes. This production is seen to rise with decrease in impact parameter which leads to a larger overlap of partonic clouds. The production is seen to be largest at central rapidities. (The structures seen
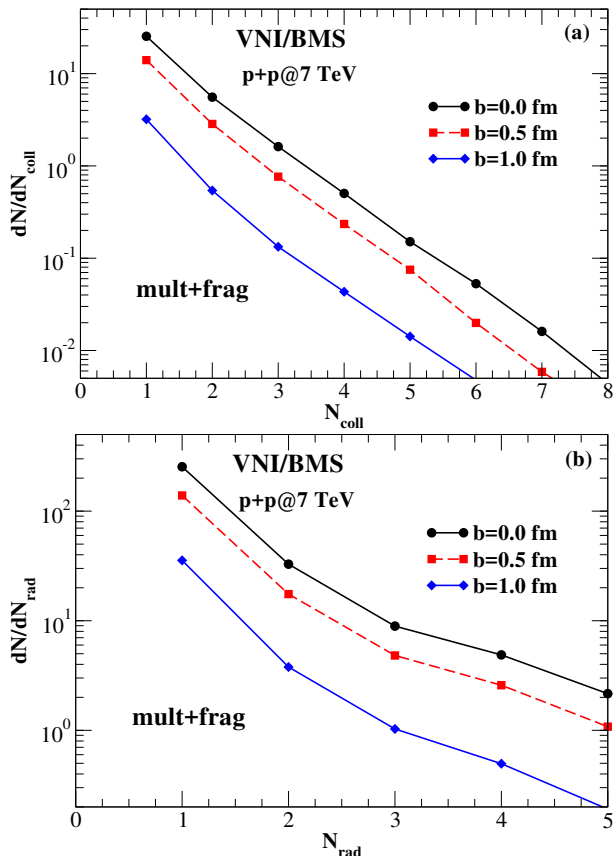
FIG. 11: (Color online) The frequency distribution of number of collisions (a) and number of radiations (b) suffered by partons for different impact parameters at $\sqrt{s}$ of 7 TeV for pp collisions. The multiple collisions among the partons and radiative processes are included.
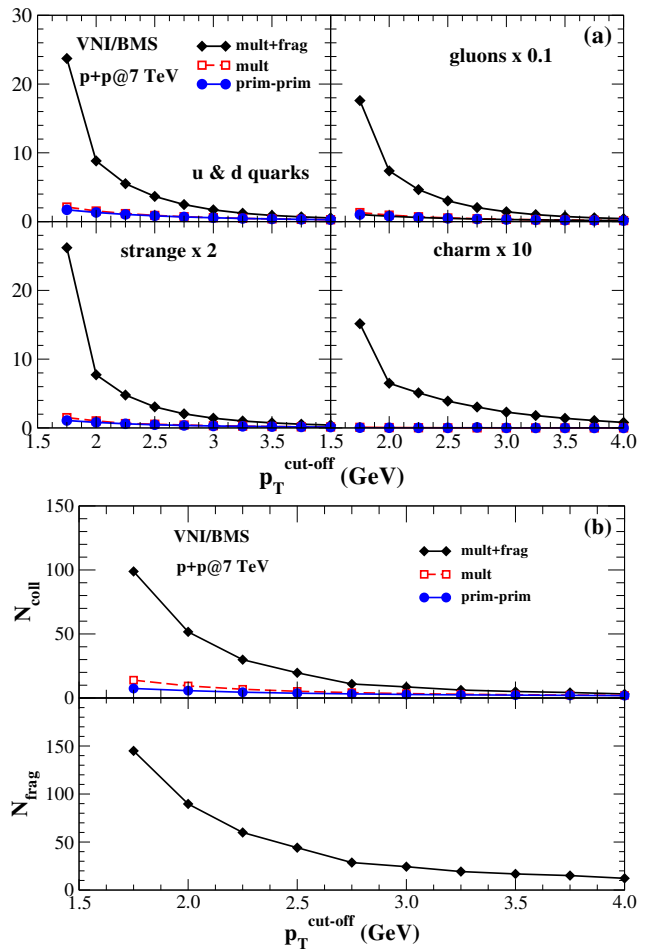

FIG. 12: (Color online) (a) Effect of changing the lower $p_T^{\text{cut-off}}$ on production of light quarks, strange quarks, charm quarks and gluons as a function of $p_T^{\text{cut-off}}$ for semi-hard partonic collisions in pp collisions at $\sqrt{s} = 7$ TeV for calculations involving scattering only between primary partons (filled circles), multiple scatterings *with-out* fragmentation of scattered partons (hollow squares) and multiple scatterings *with* fragmentation of scattered partons (filled diamonds). (b) Number of collisions and number of fragmentations as a function of $p_T^{\text{cut-off}}$ set of calculations.

in the dN/dy distributions for the calculations invoking only primary-primary or multiple collisions without fragmentations arise due to cut-offs on p_T.)

Fig.9 shows the respective transverse momentum spectra: we find that the distributions are quite similar for different impact parameters but differ in magnitude. This may indicate that in high energy pp interactions even though the number of partonic collisions increases as we reduce the impact parameter, the number of semi-hard collisions suffered by individual partons may not increase very substantially - as that could alter the shape of these momentum distributions. We shall come back to this point again.

Later we shall study the behaviour of γ_s as a function of multiplicity of charged particles (Fig. 17). We note that γ_s rises rapidly and saturates with rise in multiplicity, in contrast to the behaviour seen here as a function of the impact parameter. The difference between these two systematics is mostly due to event-by-event fluctuations in the spatial distribution of partons that allow for a large variability in the number of collisions and fragmentations for a given impact parameter. The number of collisions in turn drives the multiplicity and thus the

rise in γ_s. Events with small multiplicity (which will be dominated by but not limited to events with larger impact parameter) have a moderate value for γ_s, while the large multiplicity events, which will have larger number of multiple collisions (with $p_T \geq 2$ GeV) and fragmentations will show a larger value for γ_s. We find that γ_s is around 0.78 for average multiplicity and rises to 0.9 for large multiplicity.

The behavior of the strangeness enhancement parameter can be understood by noting that the frequency of number of collisions and number of radiations suffered by the parton indicate the possibility of creating an interacting medium of partons that would allow for a measure of chemical equilibration as well. Thus, for the most central collisions in our most realistic calculation (the scenario
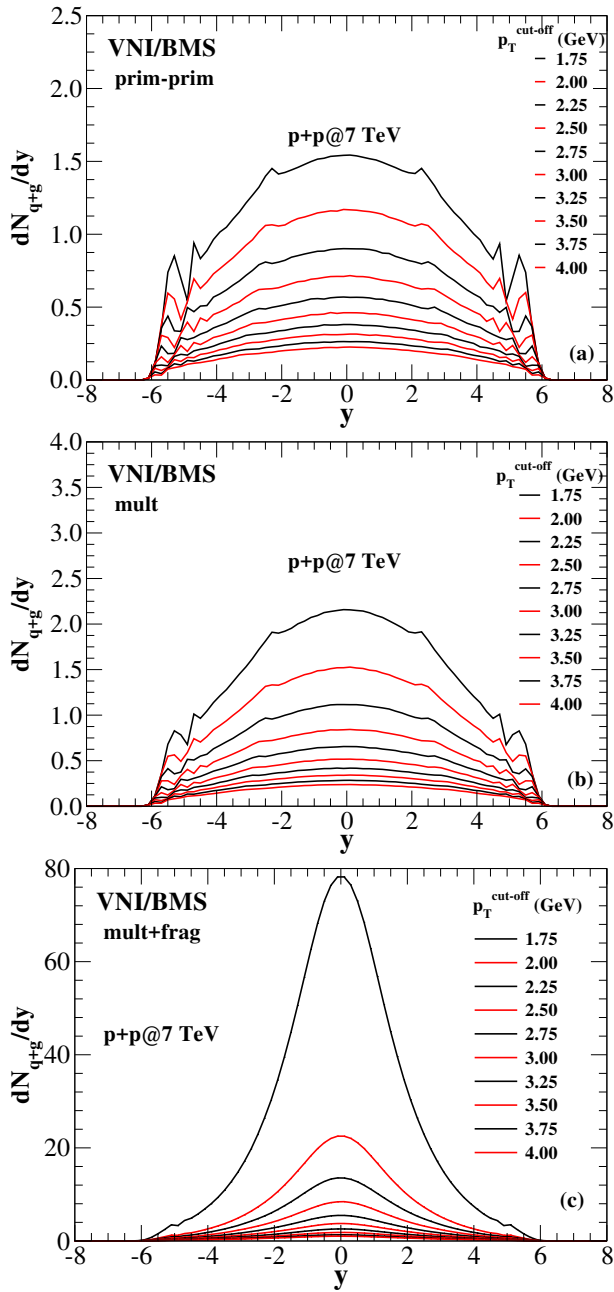
FIG. 13: (Color online) Rapidity density of partons produced in pp interactions at $\sqrt{s} = 7$ TeV due to semi-hard collisions among primary partons (a), multiple collisions *without* fragmentations of final state partons (b) and multiple collisions *with* fragmentation of final state partons (c) at different cut-offs for transverse momentum in semi-hard collisions.

FIG. 14: (Color online)Rapidity integrated p_T spectra of partons produced in pp interactions at $\sqrt{s} = 7$ TeV due to semi-hard collisions among primary partons (a), multiple collisions *without* fragmentations of final state partons (b) and multiple collisions *with* fragmentation of final state partons (c) at different cut-offs for transverse momentum in semi-hard collisions.

that includes primary and secondary scattering as well as fragmentation) we find on average in excess of 170 parton-parton scatterings and more than 300 fragmentations, even in the absence of low momentum interactions below the $p_T^{\text{cut-off}}$ (see Fig. 11). This figure shows the frequency of number of collisions and number of radiations suffered by individual partons at three impact parameter: a substantial number of partons interact multiple
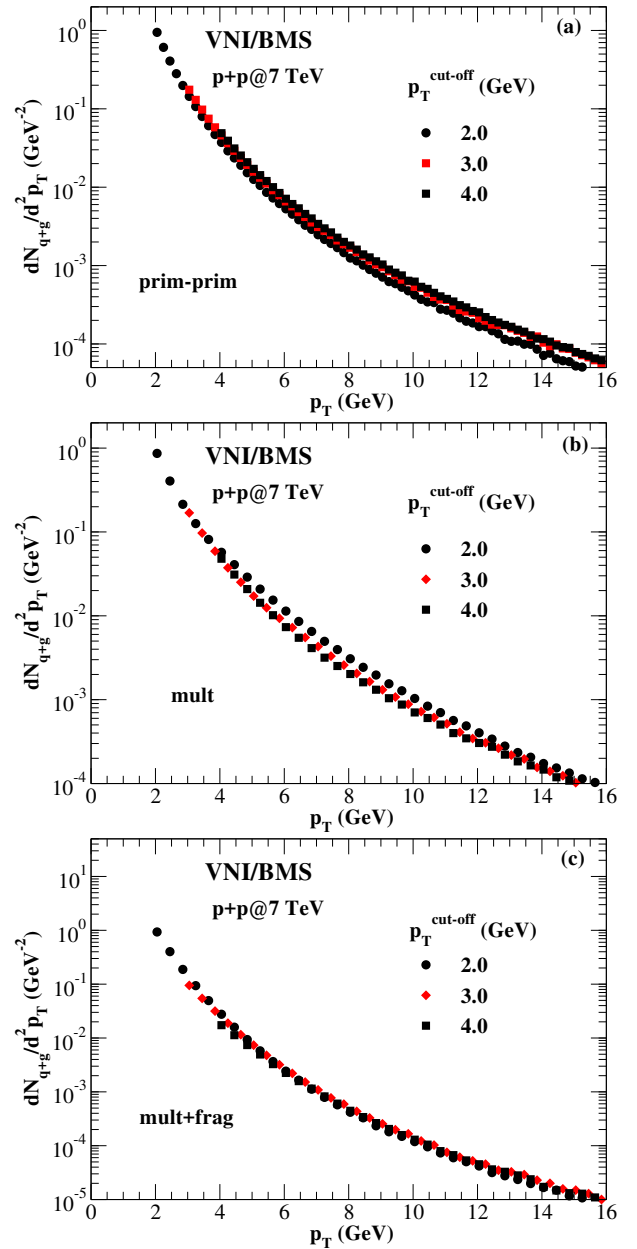
times. The number of collisions as well as the parton rescattering that we observe indicate the formation of an interacting medium and a system that could potentially thermalize, as is hinted by experimental observations. Stronger statements regarding thermalization are hampered by the presence of the $p_T^{\text{cut-off}}$ that is used to regularize the interaction cross sections in our model.

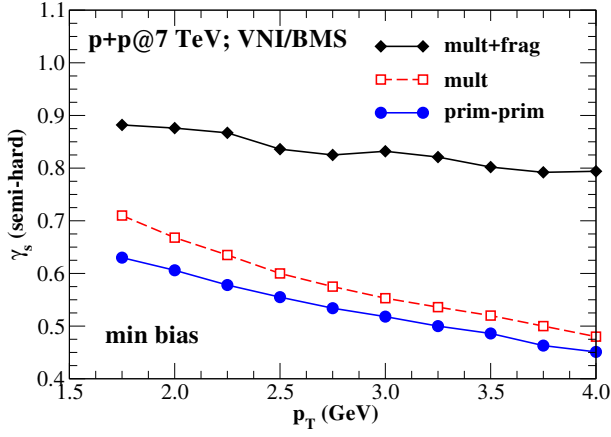In Fig. 12, we vary the value of $p_T^{\text{cut-off}}$ (for a minimum

FIG. 15: (Color online) The ratio of strange quarks and light quarks produced in semi-hard processes for the three set of calculations discussed here as a function of $p_T^{\text{cut-off}}$ in pp collisions at $\sqrt{s} = 7$ TeV.

bias sample of p+p collisions) to investigate these quantities within a reasonable range of cut-off values. The observed trends (strongly rising collision and fragmentation numbers with reduced values of the cut-off) are certainly favorable for the formation of a thermalized medium.

The effect of the variation of the cut-off on the rapidity density distribution and rapidity integrated transverse momentum spectra is shown in figures 13 and 14 respectively. We again see a rapid rise in the parton production as the $p_T^{\text{cut-off}}$ is reduced leading to more collisions and fragmentations. We have shown only a selected set of results for the parton spectra to avoid severe over-crowding. The results for other values of the cut-off parameter lie between appropriate curves given here. We see a near identity of p_T spectra beyond the largest $p_T^{\text{cut-off}}$, as expected, i.e. while the low p_T results of our calculation are significantly affected by the cut-off, the high momentum results remain stable.

Finally the dependence of the strangeness enhancement factor γ_s on the transverse momentum cut-off is shown in Fig. 15. We see a mild rise in γ_s as the $p_T^{\text{cut-off}}$ is decreased for all the calculations reported here as these lead to increased number of partonic collisions.

VI. EVOLUTION OF STRANGENESS FOR EVENTS WITH LARGE MULTIPLICITY

In the previous section we saw that production of partons rises with decrease in impact parameter. While the determination of impact parameter for pp collisions may be non-trivial, one can easily compare the results for minimum bias events to those with high multiplicity.

In Fig. 16 we show the increase in production of strangeness and charm in collisions with large multiplicity, for the calculations allowing multiple scattering among partons and the fragmentation of final state par-
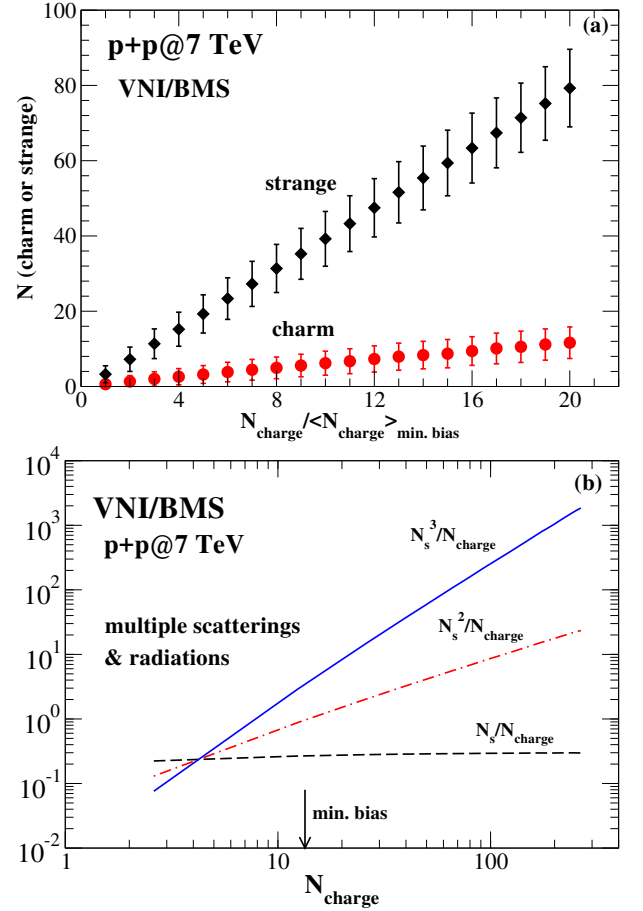

FIG. 16: (Color online) (a) Number of strange and charm quarks produced in semi-hard interactions vs. average number of charge particles produced. (b) The variation of number of strange quarks, its square and cube.
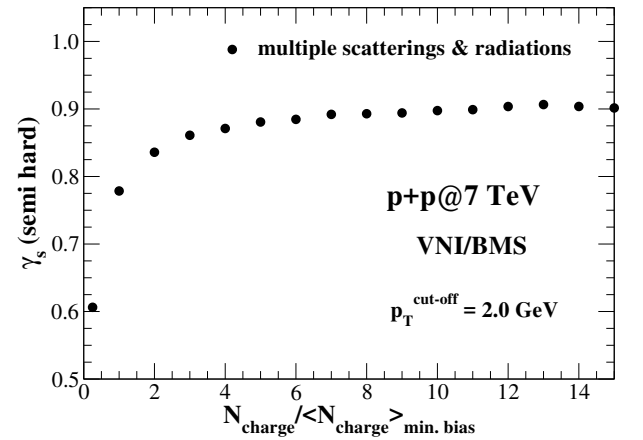

FIG. 17: (Color online) The ratio of strange quarks and light quarks produced in semi-hard processes as a function of charged particles (quarks) produced.

tons. A rapid increase in strangeness and charm multiplicity is seen as the number of partons rises.

The lower frame of Fig. 16 shows the variation of the ratio of strange quarks vs. the number of quarks (charged particles) produced against the multiplicity of the charged particles produced in semi-hard interactions. We find it to be essentially constant, basically as the $p_T^{\text{cut-off}}$ of 2 GeV chosen in these studies is much larger than the mass of strange quark taken in the calculations. The behavior of the square and the cube of the number of strange quarks is also shown. Taken together, these results indicate an increasing production of hadrons with one, two, and three strange quarks as we look at events with larger multiplicity, as indeed observed in recent experiments at LHC.

It is not straight forward to compare our qualitative findings with the ALICE experimental data. Our calculations include only the partons which are produced in semi-hard collisions having $p_T^{\text{cut-off}} \geq 2$ GeV and the associated branchings with the cut-off on virtuality discussed above. Any comparison to data would require a selection criterion for final state hadrons that emerge from hadronization of only these high momentum partons, which is non-trivial, given that hadrons from parton fragmentation reside at lower p_T, whereas hadrons emerging from recombination reside at larger p_T than the hadronizing parton(s).

The total number of light quarks and gluons produced in the collisions are definitely much larger than the results obtained in our calculations. Thus any comparison to actual data would involve addition of a large unknown (to us) number of charged/strange quarks to the numerator as well as the denominator while defining the ratio plotted in Fig. 16 (b).

Thus, in all fairness our results are a qualitative indication that the production of hadrons in high multiplicity collisions can have a large component of single and multi-strange hadrons, and that it is definitely so for the partons which are produced in semi-hard pQCD interactions.

In Fig. 17 we have plotted our result for strangeness enhancement factor as a function of charged particles. We see that as the number of quarks (charged particles) produced rises in the event, the strangeness tends to equilibrate, as indeed indicated in results from the ALICE experiment.

VII. SUMMARY AND CONCLUSIONS

We have studied the formation of a partonic medium produced in pp collisions using parton cascade model. The production of light quarks, strange quarks, charm quarks, and gluons in calculations permitting multiple collisions and fragmentation of scattered partons is seen to rise rapidly with the energy due to semi-hard interactions treated using pQCD. The multi-parton interactions (collisions and radiations) are seen to rise with decrease in impact parameter and decrease in the lower cut-off on the transverse momenta at a given centre of mass energy. The strangeness enhancement factor for the semi-hard processes at higher energies is seen to be close to unity and only marginally dependent on the impact parameter. A detailed analysis of number of collisions and radiations suffered by partons reveals that while many partons undergo only one collision and one fragmentation per event, the chances of the same parton undergoing multiple collisions or several radiations is still substantial. This indicates the emergence of an interacting medium. The strangeness and charm production is also seen to rise as a function of charged particles as indicated in recent experiments. The strangeness is seen to equilibrate in high multiplicity events and as the centre of mass energy increases. Overall we deem the amount of partons produced and the multiple interactions among these partons favorable for the formation of an interaction medium that may give rise to collective effects.

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