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Phys. Rev. C **97**, 044906 — Published 16 April 2018

DOI: 10.1103/PhysRevC.97.044906

Magnetic field in expanding quark-gluon plasma

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(Dated: December 17, 2017)

Intense electromagnetic fields are created in the quark-gluon plasma by the external ultra-relativistic valence charges. The time-evolution and the strength of this field are strongly affected by the electrical conductivity of the plasma. Yet, it has recently been observed that the effect of the magnetic field on the plasma flow is small. We compute the effect of plasma flow on magnetic field and demonstrate that it is less than 10%. These observations indicate that the plasma hydrodynamics and the dynamics of electromagnetic field decouple. Thus, it is a very good approximation, on the one hand, to study QGP in the background electromagnetic field generated by external sources and, on the other hand, to investigate the dynamics of magnetic field in the background plasma. We also argue that the wake induced by the magnetic field in plasma is negligible.

I. INTRODUCTION

The main goal of the relativistic heavy-ion collisions program is to produce and study the quark-gluon plasma (QGP). Along with the plasma, the relativistic heavy-ion collisions produce intense electromagnetic fields that modify its properties. In order to infer the plasma properties from the experimental data one needs to quantify the effect of electromagnetic fields on the QGP dynamics. In principle, this can be accomplished by solving the relativistic magneto-hydrodynamic (MHD) equations. The electromagnetic field affects both the ideal plasma flow and the transport coefficients, while the electric currents in plasma affect the electromagnetic field. Since the QGP dynamics is determined mostly by the strong interactions, one may start by treating the electromagnetic interactions as a small perturbation. This approximation amounts to decoupling, to a certain extent, of the dynamics of the electromagnetic field and the plasma.

The MHD of ideal QGP in the background electromagnetic field was studied in [1–10]. It has been recently argued in [8] that the effect of the electromagnetic field on QGP is small for realistic fields justifying the decoupling approximation. Still, before making a final conclusion that the plasma flow is decoupled from the electromagnetic field, one needs to verify that the kinetic coefficients do not strongly depend on the field. In particular, significant enhancement of the viscous stress may invalidate the ideal fluidity assumption. Despite the recent progress in calculating the transport coefficients [11–26], their values at the temperatures of phenomenological interest are not yet certain.

Assuming perfect decoupling, i.e. that QGP does not affect the electromagnetic field at all, the electromagnetic field was computed in [27–33] using the hadron transport models. However, it was argued in [35, 36] that this approximation is adequate only at the earliest times after the plasma formation. At later times the plasma response plays the crucial role. Owing to its finite electrical conductivity it significantly enhances the electromagnetic field [34–38]. Thus far all calculations of the electromagnetic field assumed stationary plasma. The main goal of this paper is to compute the contribution of the plasma expansion to the magnetic field. We will argue that this contribution is on the order of a few per cent and thus can be safely neglected. Along the way, we will clarify a number of important points that were not sufficiently addressed in the previous publications.

The spacetime picture of a heavy-ion collision is shown in Fig. 1 and Fig. 2. In Fig. 1, which is nearly identical to the one found in the classical Bjorken's paper [39], we emphasize that the valence quarks, which are sources of the electromagnetic field, are external to the plasma. In fact, a small fraction of valence quarks can be found inside the QGP, which is known as the baryon stopping. However, the transfer of the valence quarks across the wide rapidity interval is strongly suppressed [51, 52]. Their contribution to the total field was estimated in [27] and turns out to be completely negligible at relativistic energies. In view of this observations we neglect the baryon stopping, assuming that all valence quarks travel along the straight lines. Furthermore, for our arguments in this paper it is sufficient to approximate the valence electric charges as classical point particles. In a more comprehensive treatment one has to replace the classical sources by the quantum distributions [56, 57].

In this paper we regard the QGP as a homogeneous plasma expanding according to the blast wave model [53–55] and having the electrical conductivity σ. We are going to neglect its mild time dependence [35] and treat it as a constant *. Recently, there has been a lively discussion of possible effects of the chiral anomaly [40–42] on the QGP dynamics in general and its electrodynamics in particular [43–50]. In this paper we adopt a conservative view and disregard these effects until they are firmly established.

The paper is organized as follows. In Sec. II we write down the basic equations that determine the electromagnetic fields in QGP. We derive the retarded Green's function of the electromagnetic field in the electrically conducting medium and show that it is a sum of two terms: the pulse and the wake. The wake field is usually neglected in calculations. We prove that this is a good

* Actually, even a mild time dependence of σ may be phenomenologically significant [38].

approximation. Indeed, at energies $\gamma = 100$ in a plasma with electrical conductivity $\sigma = 5.8$ MeV [12, 13], the wake term is small until $t \sim 100$ fm/c and thus can be neglected in phenomenological calculations. This is discussed in Sec. III in the stationary plasma limit. The main result of Sec. III is Eq. (17) which gives the analytical expression for the magnetic field of a point external charge in conducting medium. It agrees with the previous result derived by one of us [36], but has an advantage of being expressed in terms of the elementary functions. Expanding plasma is considered in Sec. IV were we treat the magnetic part of the Lorentz force perturbatively and derive the solution for the magnetic field. We summarize the results and discuss the prospects in Sec. V.

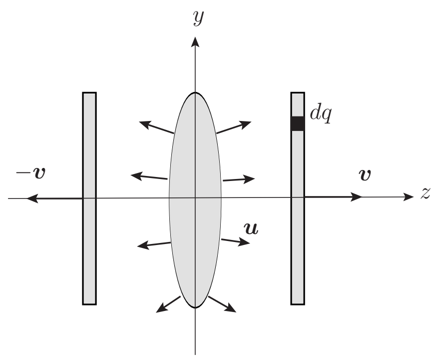

FIG. 1. The geometry of the heavy-ion collisions. Ion remnants move with velocity $\pm\boldsymbol{v}$. The plasma's velocity is $\boldsymbol{u}$. We emphasize that the valence electric charges dq are external to the plasma. The geometry in the xy plane is shown in Fig. 2.

II. MAXWELL EQUATIONS IN EXPANDING PLASMA

An electromagnetic field in flowing conducting medium satisfies the equations

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) + \boldsymbol{j}\,, \tag{1a}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho\,, \tag{1b}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0\,, \tag{1c}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,, \tag{1d}$$

where $\boldsymbol{u}$ is the fluid velocity, σ is electrical conductivity and $j^\mu = (\rho, \boldsymbol{j})$ is the external current created by the valence charges as shown in Fig. 1 . Replacing the fields with the potentials as usual

$$\boldsymbol{E} = -\boldsymbol{\nabla}\varphi - \partial_t \boldsymbol{A}\,, \qquad \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \tag{2}$$

and using the gauge condition

$$\partial_t \varphi + \boldsymbol{\nabla} \cdot \boldsymbol{A} + \sigma \varphi = 0 \tag{3}$$

we arrive at the equations

$$-\nabla^2 \varphi + \partial_t^2 \varphi + \sigma \partial_t \varphi = \rho \,, \tag{4a}$$

$$-\nabla^2 \boldsymbol{A} + \partial_t^2 \boldsymbol{A} + \sigma \partial_t \boldsymbol{A} - \sigma \boldsymbol{u} \times (\boldsymbol{\nabla} \times \boldsymbol{A}) = \boldsymbol{j} \,, \tag{4b}$$

We consider a point charge e moving in the positive z direction with constant velocity v:

$$\boldsymbol{j} = ev\hat{\boldsymbol{z}}\delta(\boldsymbol{b})\delta(z - vt) \,, \quad \rho = 0 \,. \tag{5}$$

In the experimentally interesting region of small z's (see Fig. 1), $|\boldsymbol{u}| \ll 1$. This allows us to treat the corresponding term in (4b) as a perturbation. Thus, writing $\boldsymbol{A} = \boldsymbol{A}^{(0)} + \boldsymbol{A}^{(1)}$ we obtain two equations

$$-\nabla^2 \boldsymbol{A}^{(0)} + \partial_t^2 \boldsymbol{A}^{(0)} + \sigma \partial_t \boldsymbol{A}^{(0)} = \boldsymbol{j} \,, \tag{6a}$$

$$-\nabla^2 \boldsymbol{A}^{(1)} + \partial_t^2 \boldsymbol{A}^{(1)} + \sigma \partial_t \boldsymbol{A}^{(1)} = \sigma \boldsymbol{u} \times \boldsymbol{B}^{(0)} \,. \tag{6b}$$

The first of these equations describes the field created by the external currents in the stationary plasma, whereas the second one takes expansion of plasma into account.

To find the particular solutions to these equations we introduce the retarded Green's function $G(\boldsymbol{r}, t|\boldsymbol{r}', t')$ that obeys the equation

$$-\nabla^2 G + \partial_t^2 G + \sigma \partial_t G = \delta(t - t')\delta(\boldsymbol{r} - \boldsymbol{r}') \,. \tag{7}$$

We note that the function $\mathcal{G}$ defined as

$$G(\boldsymbol{r}, t|\boldsymbol{r}', t') = e^{-\sigma t/2} \mathcal{G}(\boldsymbol{r}, t|\boldsymbol{r}', t') \tag{8}$$

is a Green's function of the Klein-Gordon equation with imaginary mass $m = i\sigma/2$

$$-\nabla^2 \mathcal{G} + \partial_t^2 \mathcal{G} + m^2 \mathcal{G} = e^{\sigma t'/2} \delta(t - t')\delta(\boldsymbol{r} - \boldsymbol{r}') \,. \tag{9}$$

The corresponding retarded Green's function in the coordinate representation reads (see e.g. [58])

$$\begin{aligned} \mathcal{G}(\boldsymbol{r}, t|\boldsymbol{r}', t') =& \frac{1}{4\pi} e^{\frac{1}{2}\sigma t'} \left\{ \frac{\delta(t - t' - R)}{R} \right. \\ & \left. - \frac{m}{\sqrt{(t-t')^2 - R^2}} J_1\left(m\sqrt{(t-t')^2 - R^2}\right) \theta(t - t' - R) \right\} \theta(t - t') \,. \end{aligned} \tag{10}$$

Eqs. (8) and (10) furnish the retarded Green's function for the original Eq. (7):

$$G(\boldsymbol{r},t|\boldsymbol{r}',t') = G_a(\boldsymbol{r},t|\boldsymbol{r}',t') + G_b(\boldsymbol{r},t|\boldsymbol{r}',t') \tag{11a}$$

$$G_a(\boldsymbol{r},t|\boldsymbol{r}',t') = \frac{1}{4\pi}e^{-\frac{1}{2}\sigma(t-t')}\frac{\delta(t-t'-R)}{R}\theta(t-t') \tag{11b}$$

$$G_b(\boldsymbol{r},t|\boldsymbol{r}',t') = \frac{1}{4\pi}e^{-\frac{1}{2}\sigma(t-t')}\frac{\sigma/2}{\sqrt{(t-t')^2-R^2}}I_1\left(\frac{\sigma}{2}\sqrt{(t-t')^2-R^2}\right)\theta(t-t'-R)\theta(t-t')\,. \tag{11c}$$

We separated the Green's function into a sum of the two terms: the original pulse G_a and the wake G_b created by the currents induced in the plasma. The exponential factor $\exp[-\sigma(t-t')/2]$ indicates the decrease of the field strength due to the work done by the field on the electric currents in the plasma.

III. SOLUTION FOR THE STATIC PLASMA

The particular solution to (6a), namely the one induced by the external currents, is given by

$$\boldsymbol{A}^{(0)}(\boldsymbol{r},t) = \int G(\boldsymbol{r},t|\boldsymbol{r}',t')\boldsymbol{j}(\boldsymbol{r}',t')d^3r'dt'\,, \tag{12}$$

where the retarded Green's function is given by (11). Since the retarded Green's function breaks up into two physically meaningful terms we compute and analyze each term independently.

A. The pulse field

The argument of the delta function in G_a vanishes when $t-t' = |\boldsymbol{r}-vt'\hat{\boldsymbol{z}}|$. The corresponding retarded time t' satisfying $t>t'$ reads

$$t' = t_0 = \gamma^2\left(t - vz - \sqrt{(z-vt)^2+b^2/\gamma^2}\right)\,. \tag{13}$$

Writing

$$\delta(t-t'-R) = \frac{\delta(t'-t_0)(t-t_0)}{\sqrt{(z-vt)^2+b^2/\gamma^2}} \tag{14}$$

and denoting $\xi = vt - z$ we find

$$\boldsymbol{A}_a^{(0)}(\boldsymbol{r},t) = \frac{ev\hat{\boldsymbol{z}}}{4\pi}\frac{1}{\sqrt{\xi^2+b^2/\gamma^2}}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+b^2/\gamma^2}\right)\right\}\,. \tag{15}$$

It is readily seen that as $\sigma\to 0$ this term reproduces the vector potential of a charge uniformly moving in vacuum. The magnetic field corresponding to the vector potential (15) is given by

$$\boldsymbol{B}_a^{(0)} = -\frac{\partial A_{az}^{(0)}}{\partial b}\hat{\boldsymbol{\phi}} \tag{16}$$

$$= \frac{ev}{4\pi}\hat{\boldsymbol{\phi}}\left\{\frac{\sigma b/2}{\xi^2+b^2/\gamma^2} + \frac{b}{\gamma^2[\xi^2+b^2/\gamma^2]^{3/2}}\right\}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+b^2/\gamma^2}\right)\right\}\,. \tag{17}$$

The first term in the curly brackets dominates when $\sqrt{\xi^2+b^2/\gamma^2} \gg 1/\sigma\gamma^2 \sim 10^{-5}$ fm. Assuming that this is the case, (17) simplifies in the limit $b/\gamma \ll \xi$ yielding the "diffusion approximation"

$$\boldsymbol{B}_a^{(0)} \approx \frac{ev}{8\pi}\hat{\boldsymbol{\phi}}\frac{\sigma b}{\xi^2}e^{-\frac{\sigma\xi}{2(1+v)}}e^{-\frac{b^2\sigma}{4\xi}}\,, \quad \xi > 0\,. \tag{18}$$

Clearly, the second exponential factor in (18) can be dropped at later times $\xi \gg b^2\sigma/4 \sim 0.5$ fm.

The expression for the magnetic field was previously derived by one of us in [36] (see Eq. (7) there) and, unlike (17), is represented in a form of a one-dimensional integral. Both formulas reduce to (18) in the diffusion approximation.

B. The wake field

It has been tacitly assumed in [36] that the wake term is small. Using the Green's function (11c) we can compute this term explicitly:

$$\boldsymbol{A}_b^{(0)}(\boldsymbol{r},t) = \frac{e\hat{\boldsymbol{z}}}{4\pi}\frac{\sigma v}{2}\int_{-\infty}^{t_0}\frac{e^{-\sigma(t-t')/2}}{\sqrt{(t-t')^2-b^2-(z-vt')^2}}I_1\left(\frac{\sigma}{2}\sqrt{(t-t')^2-b^2-(z-vt')^2}\right)dt'\,. \tag{19}$$

It is useful to introduce a new integration variable λ such that

$$t' = \gamma^2\left(t - vz - \sqrt{(z-vt)^2+(b^2+\lambda^2)/\gamma^2}\right)\,. \tag{20}$$

It is straightforward to check that this implies

$$\lambda^2 = (t-t')^2 - b^2 - (z-vt')^2\,. \tag{21}$$

The vector potential (19) can now be represented as

$$\boldsymbol{A}_b^{(0)}(\boldsymbol{r},t) = \frac{e\hat{\boldsymbol{z}}}{4\pi}\frac{\sigma v}{2}\int_0^{\infty}\frac{d\lambda\, I_1\left(\frac{\sigma}{2}\lambda\right)}{\sqrt{\xi^2+(b^2+\lambda^2)/\gamma^2}}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+(b^2+\lambda^2)/\gamma^2}\right)\right\}\,. \tag{22}$$

The main contribution to this integral comes from the integration region $\sqrt{\gamma^2\xi^2+b^2} \ll \lambda \ll 2/\sigma\gamma$ where the integrand is approximately constant. At smaller λ's it vanishes as $\sim \lambda$, while at larger λ's it is exponentially suppressed. Thus, we can approximate the integral in (22) as

$$\begin{aligned}\boldsymbol{A}_b^{(0)}(\boldsymbol{r},t) &\approx \frac{e\hat{\boldsymbol{z}}}{4\pi}\frac{\sigma v}{2}\int_0^{\infty}\frac{d\lambda\,\frac{1}{2}\frac{\sigma}{2}\lambda}{\sqrt{\xi^2+(b^2+\lambda^2)/\gamma^2}}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+(b^2+\lambda^2)/\gamma^2}\right)\right\}\\ &= \frac{e\hat{\boldsymbol{z}}}{4\pi}\frac{\sigma v}{4}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+b^2/\gamma^2}\right)\right\}\,.\end{aligned} \tag{23}$$

Using (16) we derive the magnetic field

$$\boldsymbol{B}_b^{(0)}(\boldsymbol{r},t) = \frac{e\hat{\boldsymbol{\phi}}}{4\pi}\frac{\sigma^2 vb}{4}\frac{1}{\sqrt{\xi^2+b^2/\gamma^2}}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+b^2/\gamma^2}\right)\right\}\,. \tag{24}$$

Comparing (23) and (15) we conclude that the contribution of the wake to the retarded Greens function (11) is small in the phenomenologically relevant region $\sqrt{\xi^2+b^2/\gamma^2} \ll 4/\sigma \sim 10^2$ fm. However, it dominates in the opposite limit, i.e. at very late times.

C. Diffusion approximation

It is instructive to derive Eq. (18) directly from (7) as has been done in [38]. The diffusion approximation in (7) amounts to the assumption that $\partial_z^2 - \partial_t^2 \sim k_z^2/\gamma^2 \ll k_\perp^2, \sigma k_z$. In this case the retarded Green's function $G_{\mathcal{D}}(\boldsymbol{r}, t|\boldsymbol{r}', t')$ obeys the equation

$$-\nabla_\perp^2 G_{\mathcal{D}} + \sigma \partial_t G_{\mathcal{D}} = \delta(t-t')\delta(\boldsymbol{r}-\boldsymbol{r}')\,. \tag{25}$$

Its solution is

$$G_{\mathcal{D}}(\boldsymbol{r}, t|\boldsymbol{r}', t') = \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')+i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{p_\perp^2 - i\omega\sigma} = \frac{1}{4\pi t}\delta(z-z')\theta(t-t')e^{-\frac{\sigma(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp)^2}{4(t-t')}}\,. \tag{26}$$

Employing (5) and (12) one derives

$$\boldsymbol{A}^{(0)}(\boldsymbol{r}, t) = \frac{e\hat{\boldsymbol{z}}}{4\pi(t-z/v)} e^{-\frac{\sigma b^2}{4(t-z/v)}} \theta(t-z/v)\,, \tag{27}$$

which yields (18) for $\xi \ll 4/\sigma$.

IV. SOLUTION FOR THE EXPANDING PLASMA

A. Contribution of the plasma flow

Now we turn to Eq. (6b) that takes the plasma flow into account. Suppose that a point source is moving along the trajectory $z = vt$, $x = \tilde{x}$, $y = \tilde{y}$, where $\tilde{x}$ and $\tilde{y}$ are constants, see Fig. 2. Denote by $\tilde{\boldsymbol{r}}$ a vector with components $\tilde{x}, \tilde{y}, z$ and let $\tilde{\boldsymbol{b}}$ be its transverse part. The magnetic field created by this charge in the stationary plasma is then given by (17) and (24) with the replacement $b \to |\boldsymbol{b} - \tilde{\boldsymbol{b}}|$; denote it as $\boldsymbol{B}^{(0)}(\boldsymbol{r} - \tilde{\boldsymbol{r}}, t)$. The solution to (6b) can be written right away using the Green's function as

$$\boldsymbol{A}^{(1)}(\boldsymbol{r}, t|\tilde{\boldsymbol{r}}) = \sigma \int G_a(\boldsymbol{r}, t|\boldsymbol{r}', t')\boldsymbol{u}(\boldsymbol{r}', t') \times \boldsymbol{B}_a^{(0)}(\boldsymbol{r}' - \tilde{\boldsymbol{r}}, t') d^3r' dt'\,. \tag{28}$$

The contribution of the wake is neglected as per the results of the previous section.

The longitudinal expansion of QGP is usually described by the Bjorken model [39] in which the flow velocity in the lab frame is given by

$$\boldsymbol{u}(\boldsymbol{r}, t) = \frac{\boldsymbol{z}}{t}\,. \tag{29}$$

Since the plasma velocity is non-vanishing only in the forward light-cone, i.e. $\boldsymbol{u}^2 \leq 1$, the integral in (28) is restricted to the region $|z'| \leq t'$. Using $t' = t - R$ this implies that the integral over z'

runs between the following limits:

$$-\frac{t^2 - z^2 - (\boldsymbol{b} - \boldsymbol{b}')^2}{2(t+z)} \le z' \le \frac{t^2 - z^2 - (\boldsymbol{b} - \boldsymbol{b}')^2}{2(t-z)} \,. \tag{30}$$

In fact, the applicability of the Bjorken model is restricted to the central plateau region in the inclusive particle spectrum at a given energy. If $2Y$ is the extent of the plateau in rapidity, then $|\boldsymbol{u}| \le \tanh Y$. For a conservative estimate of the flow correction we set Y to infinity, which yields (30).

A more sophisticated blast wave model [53–55] takes the transverse flow into account

$$\boldsymbol{u}(\boldsymbol{r}, t) = \frac{u_o}{R_o} \boldsymbol{b}\, \theta(R_o - b) + \frac{\boldsymbol{z}}{t} \,, \tag{31}$$

where u_o and R_o are parameters fitted to the experimental data. We use $R_o = 7.5$ fm, $u_o = 0.55$ from [59]. This time, restriction to the forward light-cone $\boldsymbol{u}^2(\boldsymbol{r}', t') \le 1$ reads

$$\left(\frac{u_o b'}{R_o}\right)^2 + \left(\frac{z'}{t-R}\right)^2 \le 1 \,. \tag{32}$$

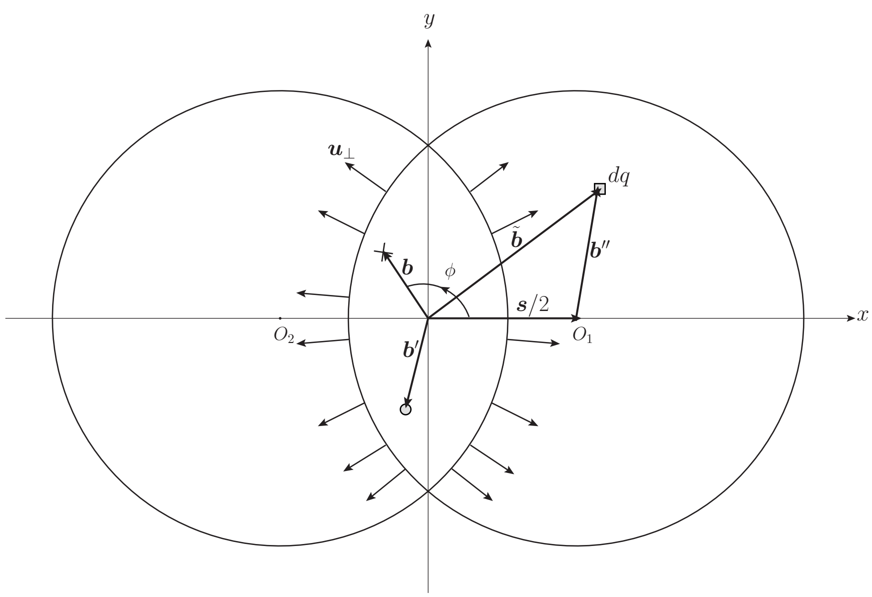

FIG. 2. The geometry of the heavy-ion collisions in the transverse plane. The two heavy-ion remnants (big circles) move in opposite directions along the z-axis, see Fig. 1. The element of charge dq is located at the same z as an ion remnant (i.e. it is not inside the plasma). Its projection on the transverse plane is depicted by the square. The small circle indicates the element of plasma moving with velocity $\boldsymbol{u}$. The observation point is denoted by the + symbol. The impact parameter $\boldsymbol{s}$ points from one nuclear center to another one.

The perturbative approach developed in (6) assumes that the term proportional to the plasma velocity can be treated as a perturbation. This is justified only if each component of $\boldsymbol{u}$ is small.

Inspection of (31) indicates that for the phenomenologically relevant values of t, z and b this condition holds. We will establish the accuracy of this approximation a posteriori by a direct numerical calculation later in this section.

B. Initial conditions

Thus far we assumed that a particle moves in plasma all the way from $t = -\infty$. In fact, a physical scenario more relevant for relativistic heavy-ion collisions is that the valence charges move in vacuum until a certain time τ when the plasma emerges. We neglect the finite thermalization time. Let the initial conditions be

$$\boldsymbol{A}(\boldsymbol{r},\tau) = \boldsymbol{\mathcal{A}}(\boldsymbol{r})\,, \quad \frac{\partial \boldsymbol{A}(\boldsymbol{r},\tau)}{\partial t} = \boldsymbol{\mathcal{V}}(\boldsymbol{r})\,, \tag{33}$$

where $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{V}}$ are determined by the field that existed before the plasma emergence at $t = \tau$ [38]. Then, the solution to (6a) can be written as

$$\begin{aligned}
\boldsymbol{A}^{(0)}(\boldsymbol{r},t) = &\int_{\tau}^{t+} dt' \int d^3r' \boldsymbol{j}(\boldsymbol{r}',t') G(\boldsymbol{r},t|\boldsymbol{r}',t') && \text{(34a)}\\
&+ \int d^3r' \left\{\sigma \boldsymbol{\mathcal{A}}(\boldsymbol{r}') + \boldsymbol{\mathcal{V}}(\boldsymbol{r}')\right\} G(\boldsymbol{r},t|\boldsymbol{r}',\tau) && \text{(34b)}\\
&- \int d^3r' \boldsymbol{\mathcal{A}}(\boldsymbol{r}') \frac{\partial}{\partial t'} G(\boldsymbol{r},t|\boldsymbol{r}',\tau)\,. && \text{(34c)}
\end{aligned}$$

The initial conditions (34b) and (34c) are satisfied at the leading order. Since they are independent of the plasma flow, we are not going to be concerned with them anymore in this paper. Thus, the solution to (6b) takes form

$$\boldsymbol{A}^{(1)}(\boldsymbol{r},t|\tilde{\boldsymbol{r}}) = \sigma \int_{\tau}^{t+} dt' \int d^3r' G(\boldsymbol{r},t|\boldsymbol{r}',t')\, \boldsymbol{u}(\boldsymbol{r}',t') \times \boldsymbol{B}^{(0)}(\boldsymbol{r}' - \tilde{\boldsymbol{r}},t')\,. \tag{35}$$

The initial time is chosen to be $\tau = 0.2$ fm/c in accordance with the phenomenological models of relativistic heavy-ion collisions [54, 55].

C. Magnetic field of a nucleus

The total field created by a nucleus is

$$\boldsymbol{A}_{\text{nucl}}(\boldsymbol{r},t) = \int \rho(\boldsymbol{r}'')\boldsymbol{A}^{(0)}(\boldsymbol{b} - \tilde{\boldsymbol{b}}, z - \tilde{z}, t) d^3r'' + \int \rho(\boldsymbol{r}'')\boldsymbol{A}^{(1)}(\boldsymbol{r},t|\tilde{\boldsymbol{b}},\tilde{z}) d^3r''\,, \tag{36}$$

where we slightly modified the notation by replacing $\tilde{\boldsymbol{r}}$ with $\tilde{\boldsymbol{b}}, \tilde{z}$ in the vector potential argument. In the laboratory frame, the proton distribution in the nucleus in the z-direction is very narrow with average coordinate $\tilde{z} = vt$ depending on the direction of motion. Assuming that the nuclear density ρ is constant throughout the nucleus of radius R_A and using Fig. 2 one can compute the vector potential as

$$\boldsymbol{A}^{(0)}_{\rm nucl}(\boldsymbol{r},t) = 2\int \rho\sqrt{R_A^2-(b'')^2}\boldsymbol{A}^{(0)}(\boldsymbol{b}-\boldsymbol{b}''-\boldsymbol{s}/2, z-vt, t)d^2b'' \tag{37}$$

$$\boldsymbol{A}^{(1)}_{\rm nucl}(\boldsymbol{r},t) = 2\int \rho\sqrt{R_A^2-(b'')^2}\boldsymbol{A}^{(1)}(\boldsymbol{r},t|\ \boldsymbol{s}/2+\boldsymbol{b}'', vt)d^2b''\,. \tag{38}$$

The nuclear density is normalized as $\rho(4\pi/3)R_A^3 = Z$, where Ze is the nucleus electric charge. The contribution of another heavy-ion can be calculated by simply replacing $\boldsymbol{v} \to -\boldsymbol{v}$. In the figures below we show only the single nucleus contribution.

It follows from (37) that the magnetic field created by a single nucleus in a stationary plasma is

$$\boldsymbol{B}^{(0)}_{\rm nucl}(\boldsymbol{r},t) = 2\int \rho\sqrt{R_A^2-(b'')^2}\boldsymbol{B}^{(0)}_a(\boldsymbol{b}-\boldsymbol{b}''-\boldsymbol{s}/2, z-vt, t)d^2b''\,, \tag{39}$$

where only the pulse contribution (17) is taken into account, whereas the wake contribution (24) is neglected. Since $\boldsymbol{A}^{(0)}_{\rm nucl}$ is directed along the z-axis, the corresponding magnetic field $\boldsymbol{B}^{(0)}_{\rm nucl}$ is circularly polarized in the $\hat{\boldsymbol{\phi}}$ direction with respect to the nuclear center O_1 (or O_2). It is related to the radial $\hat{\boldsymbol{b}}$ and the polar $\hat{\boldsymbol{\varphi}}$ unit vectors of the cylindrical coordinate system defined with respect to the "lab" reference frame shown in Fig. 2 as

$$\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{b}}\sin(\phi-\zeta) + \hat{\boldsymbol{\varphi}}\cos(\phi-\zeta)\,, \tag{40}$$

where ζ given by

$$\cot\zeta = \frac{b\cos\phi - s/2}{b\sin\phi} \tag{41}$$

is the angle between the vector pointing from O_1 to the observation point and the x-axis. The correction (38) due to the plasma expansion can be written down using (39) as

$$\boldsymbol{A}^{(1)}_{\rm nucl}(\boldsymbol{r},t) = \sigma\int_\tau^{t+} dt' \int d^3r' G_a(\boldsymbol{r},t|\boldsymbol{r}',t')\boldsymbol{u}(\boldsymbol{r}',t')\times\boldsymbol{B}^{(0)}_{\rm nucl}(\boldsymbol{r}',t')\,. \tag{42}$$

In view of (31), this equation indicates that the longitudinal expansion of plasma induces the transverse $\hat{\boldsymbol{\varphi}}$ and $\boldsymbol{b}$ components of the vector potential, while the transverse expansion induces a small z-correction to the vector potential. Moreover, according to (40), $A^{(1)}_\varphi/A^{(1)}_b = -\tan(\phi-\zeta)$.

In the left panel of Fig. 3 we show the time-dependence of the vector potential in the stationary plasma $\boldsymbol{A}^{(0)}$ at a representative point indicated in the caption. This calculation agrees with the previous results [36]. It is seen that the magnetic field appears at $t = \tau = 0.2$ fm/c because we assumed that QGP emerges at that time. It is important to mention that in this calculation we do

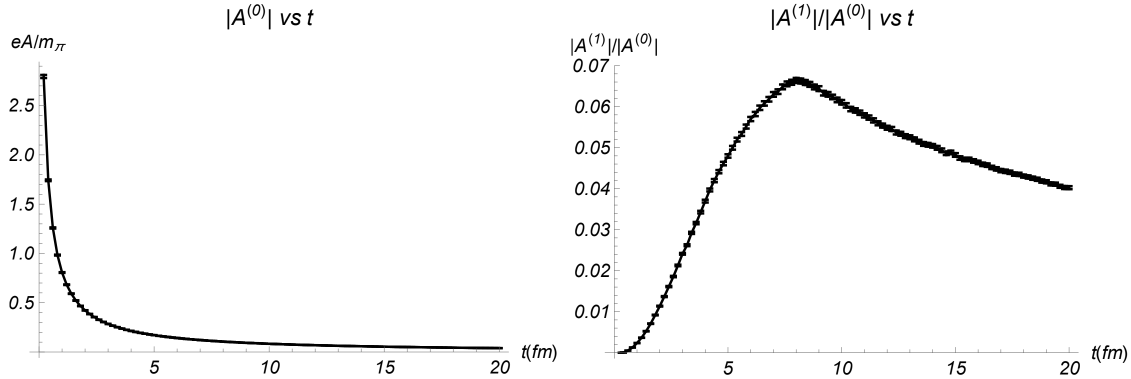

FIG. 3. The vector potential $\boldsymbol{A} = \boldsymbol{A}^{(0)} + \boldsymbol{A}^{(1)}$ created at a representative point $z = 0$, $b = 1$ fm, $\phi = \pi/6$ (see Fig. 2) in QGP by a remnant of the gold ion moving with the boost-factor $\gamma = 100$ ($\sqrt{s} = 0.2$ TeV) and impact parameter $|\boldsymbol{s}| = 3$ fm. Left panel: vector potential $\boldsymbol{A}^{(0)}$ in the non-expanding plasma. Right panel: the relative contribution of the plasma expansion. The plasma emerges at $\tau = 0.2$ fm/c.

not consider the contributions from the fields that existed at $t < \tau$. They are given by Eqs. (34b) and (34c) and are not affected by the plasma flow, even though they give a significant contribution to $\boldsymbol{A}^{(0)}$ as shown in [38].

In the right panel of Fig. 3 we show the time-dependence of the ratio $A^{(1)}/A^{(0)}$ at a representative point inside QGP, which illustrates the relative significance of the plasma expansion in the magnetic field calculations. The main observation is that the relative contribution of the plasma expansion is below 10%. With this accuracy, the plasma expansion effect on the magnetic field can be safely neglected.

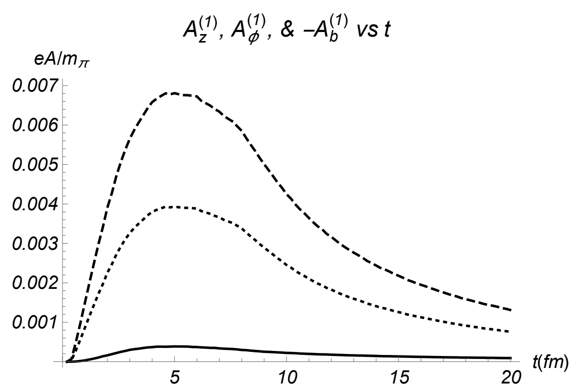

FIG. 4. Dotted line: $A_z^{(1)}$, dashed line: $A_\phi^{(1)}$, solid line $-A_b^{(1)}$ components of the correction $\boldsymbol{A}^{(1)}$ to the vector potential (in units of m_π/e) due to the plasma expansion. The geometric and kinematic parameters are the same as in Fig. 3. The cylindrical coordinates are defined with respect to the z-axis of Fig. 2, which is the lab frame for heavy-ion collisions.

Fig. 4 shows the components of the correction to the vector potential due to the plasma expansion. The vector potential in the stationary plasma always points in the direction of the external charge motion ($\pm\hat{\boldsymbol{z}}$-directions) generating the total magnetic field as a superposition of the circularly polarized fields of the individual charges. In contrast, flow of plasma generates additional components of the vector potential in the transverse plane.

The vector potentials shown in Fig. 3 and Fig. 4 is produced by a relativistic heavy-ion in a single event. We assumed that the electric charge distribution in the rest frame is uniform across the nucleus. Using a more accurate Woods-Saxon distribution gives a tiny correction. Many transport models treat heavy ion as a collection of electric charges of finite radius that are randomly distributed according to a given average charge distribution. This produces large event-by-event fluctuations of charge positions, which in turn induces large event-by-event fluctuations of electromagnetic field [31]. However, it was shown in [60] that the quantum treatment of the nuclear electric charge distribution yields fluctuations which are roughly an order of magnitude smaller than the flow contribution. In view of this observation we neglected the event-by-event fluctuations in this paper.

V. SUMMARY

We computed the effect of the QGP expansion on the magnetic field created inside the plasma by external valence charges of the heavy-ion remnants. Our main assumption is that the plasma flow is not affected by the magnetic field and is given by the phenomenological blast-wave model. We treated the effect of plasma flow as a perturbation of the magnetic field in a stationary plasma. The result shown in Fig. 3 indicates that the contribution of the plasma flow to the magnetic field is less than 10%. Our main conclusion is that there is no urgent need to solve the comprehensive MHD equations in order to describe the QGP dynamics at present energies, unless one wishes to reach precision of about 10%. It is a very good approximation, on the one hand, to study QGP in the background electromagnetic field generated by external sources and, on the other hand, to investigate the dynamics of magnetic field in the background plasma.

Since in this paper we focused on the contribution of plasma flow to the magnetic field of external charges, we disregarded the magnetic field created by the fields that existed before the plasma emergence. However, in phenomenological applications they certainly have to be taken into account as argued in [38]. Incidentally, we observed that the diffusion approximation used in [38] to analyze the initial conditions is quite reasonable.

In our previous calculations of magnetic field we always tacitly neglected the wake produced by the currents induced in plasma. In Sec. III we derived the analytic expressions for the pulse and wake fields, given by (17) and (24) respectively, and argued that the wake field is indeed negligible in the phenomenologically relevant regime due to the smallness of the electrical conductivity as compared to the inverse QGP lifetime.

Our paper paves the road to a comprehensive computation of electromagnetic field with quantum sources, whose importance was demonstrated in [56, 57]. The fact that the flow of plasma and the wake effects are but small corrections is enormous simplification of the MHD equations. Computing such a field with the appropriate initial conditions is the subject of our forthcoming paper.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-87ER40371.

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