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Quasielastic neutrino scattering is an important aspect of the experimental program to study fundamental neutrino properties including neutrino masses, mixing angles, the mass hierarchy and CP-violating phase. Proper interpretation of the experiments requires reliable theoretical calculations of neutrino-nucleus scattering. In this paper we present calculations of response functions and cross sections by neutral-current scattering of neutrinos off ^{12}C. These calculations are based on realistic treatments of nuclear interactions and currents, the latter including the axial-, vector-, and vector-axial interference terms crucial for determining the difference between neutrino and anti-neutrino scattering and the CP-violating phase. We find that the strength and energy-dependence of two-nucleon processes induced by correlation effects and interaction currents are crucial in providing the most accurate description of neutrino-nucleus scattering in the quasielastic regime.

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Many accelerator experiments are running [1–4], or are being planned [5, 6], to investigate neutrino-nucleus interactions and/or to measure neutrino oscillation parameters including the neutrino mass hierarchy and the charge-conjugation parity (CP) violating phase that differentiates the oscillation probabilities of neutrinos (ν) and anti-neutrinos ($\overline{\nu}$). These experiments employ nuclear targets like ^{16}O (T2K) or ^{56}Fe and ^{208}Pb (MINERνA) or ^{40}Ar (DUNE), and use event generators (EGs) [7] to analyze the scattering data by modeling the nucleus and reaction mechanisms. The EGs also provide information on key features of the experiment (signal and background event-rate distributions, systematic errors, *etc.*) crucial for the interpretation of the data in terms of oscillation parameters. Even in an experiment involving both near and far detectors, the EGs and associated nuclear physics models are required to determine the neutrino energy in order to perform analyses where the ratio of length to energy (L/E) is a critical input.

A large and growing body of work highlights the sensitivities of experimental analyses to systematic uncertainties in the nuclear physics [8–10]. Phenomenological approaches exist (see Refs. [11, 12]) using electron scattering data as constraints, but they often incorporate significant approximations and are more directly tested in vector-current processes. For example, it is known that two-nucleon currents and correlations also play an important role in axial-current matrix elements [13], particularly in the axial-vector interference ones that determine the differences between ν and $\overline{\nu}$ cross sections.

In the present paper, we report on an *ab initio* quantum Monte Carlo (QMC) calculation, based on Green's function Monte (GFMC) methods [14], of ν-^{12}C inclusive scattering induced by neutral-current interactions. While limited in kinematical scope to the quasielastic region, it has nevertheless the advantage of relying on a first-principles description of nuclear dynamics. In such a description, the nucleons interact with each other via effective two- and three-body potentials—respectively, the Argonne v_{18} (AV18) [15] and Illinois-7 (IL7) [16] models—and with electroweak fields via effective currents, including one- and two-body terms [17]. The GFMC methods then allow us to fully account, without approximations, for the complex many-body, spin- and isospin-dependent correlations induced by these nuclear potentials and currents, and for interaction effects in the final nuclear states [18]. For moderate momentum transfers and quasielastic energy transfers, the results of these calculations will be useful for the analysis and interpretation of accelerator-neutrino experiments as well as provide benchmarks for testing predictions from EGs and/or approaches based on approximate schemes of nuclear dynamics.

A recent GFMC calculation of the ^{12}C longitudinal and transverse electromagnetic response functions [19] is in very satisfactory agreement with experimental data, obtained from Rosenbluth separation of inclusive (e, e') cross sections. This agreement validates the dynamical framework and, in particular, the model for the vector currents adopted here. An interesting outcome of this study is the realization that interaction effects in the final nuclear states and two-body terms in these currents substantially affect the distribution of strength in the response as a function of the energy transfer ω. Final-state interactions (and initial- and final-state correlations) shift strength away from the quasielastic region into the threshold and high ω regions in both the longitudinal and transverse response functions. Two-body current contributions, though, while negligible in the longitudinal response, significantly increase the transverse one in the quasielastic peak, thus off-setting the quenching.

The energy dependence of the cross section is especially relevant for neutrino experiments, since it directly impacts the analysis of these experiments in terms of oscillation parameters and CP-violating phase. Earlier studies of integral properties of the response, either sum rules [13] or Laplace transforms of the response itself, so called Euclidean response functions [18, 20], have indicated that two-nucleon currents are important. However, these properties only provide indirect information on the strength distribution as a function of ω.

The differential cross section for ν and $\overline{\nu}$ inclusive scattering off a nucleus induced by neutral-weak currents can be expressed as [17]

$$\begin{aligned}\frac{d\sigma}{d\omega\, d\Omega} =& \frac{G_F^2}{2\pi^2} k' E' \cos^2\frac{\theta}{2}\Bigg[R_{00}(q,\omega) + \frac{\omega^2}{q^2} R_{zz}(q,\omega) \\ & - \frac{\omega}{q} R_{0z}(q,\omega) + \Big(\tan^2\frac{\theta}{2} + \frac{Q^2}{2\,q^2}\Big) R_{xx}(q,\omega) \\ & \mp \tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2} + \frac{Q^2}{q^2}}\, R_{xy}(q,\omega)\Bigg] \,, \end{aligned} \tag{1}$$

where $-$ $(+)$ refers to ν $(\overline{\nu})$, k' and E' are the momentum and energy of the outgoing neutrino, q and ω are the momentum and energy transfers with $Q^2 = q^2 - \omega^2$ being the four-momentum transfer, θ is the outgoing neutrino scattering angle relative to the incident neutrino beam direction, and $G_F = 1.1803 \times 10^{-5}$ GeV^{-2} as obtained from an analysis of super-allowed $0^+ \to 0^+$ β-decays [21].

The nuclear response functions are schematically given by

$$\begin{aligned} R_{\alpha\beta}(q,\omega) = & \sum_f \langle f | j^{NC}_\alpha(\mathbf{q},\omega)|0\rangle \langle f | j^{NC}_\beta(\mathbf{q},\omega)|0\rangle^* \\ & \times \delta(E_f - \omega - E_0) \,, \end{aligned} \tag{2}$$

where $|0\rangle$ and $|f\rangle$ represent the nuclear initial ground-state and final bound- or scattering-state of energies E_0 and E_f, and $j^{NC}_\alpha(\mathbf{q},\omega)$ denotes the appropriate components of the weak neutral current (NC). Explicit expressions for these currents and response functions are listed in Ref. [17]. By taking the momentum transfer $\mathbf{q}$ along z, the spin quantization axis, the subscripts 0 and z refer to, respectively, the charge ρ^{NC} and longitudinal component of the current $\mathbf{j}^{NC}$, and x and y to the transverse components of $\mathbf{j}^{NC}$.

The leading terms in the NC are those associated with individual nucleons. They are well known, see Ref. [17].

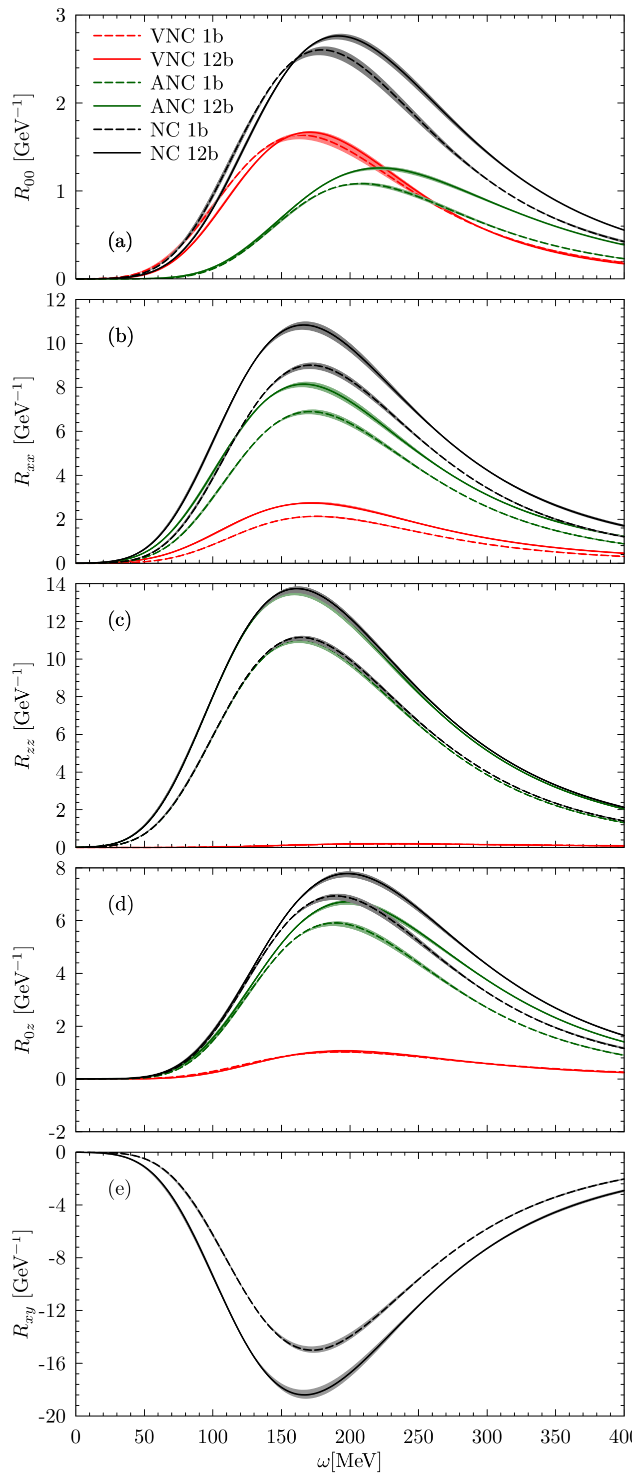

FIG. 1. (Color online) Neutral-current response functions in ^{12}C at momentum transfer $q = 570$ MeV/c, corresponding to the AV18/IL7 Hamiltonian and obtained with one-body only (dashed lines) and one- and two-body (solid lines) currents. The narrow bands indicate the uncertainty in the maximum-entropy inversion. The vector and axial contributions are shown separately in all cases but for R_{xy}. See text for further explanations.

Important two-body terms in the electromagnetic operators are obtained from the static part of the AV18 potential, which is assumed to be due to exchanges of effective pseudo-scalar ("π-like") and vector ("ρ-like") mesons. The effective propagators projected out of the AV18 static components are then used to construct the π-like and ρ-like two-body current and charge operators [17]. This procedure, while not unique (see Ref. [22] for an exhaustive discussion of this issue), leads to two-body currents, which are conserved (relative to this static part) and contain no free parameters, and whose short-range behavior is determined by that of the AV18—the latter is ultimately constrained by NN scattering data. Additional electromagnetic two-body currents taken into account [17] are purely transverse, and are associated with Δ excitation and $\rho\pi\gamma$ transition couplings.

Meson-exchange phenomenology based on effective π, ρ and $\rho\pi$ exchanges, including contributions from Δ excitation, is also used to construct the nuclear two-body axial currents. In particular, the poorly known $N \to \Delta$ transition axial coupling constant g_A^* has been determined by fitting the Gamow-Teller matrix element contributing to tritium β-decay [17].

The calculation of the response functions proceeds along similar lines to that of Ref. [19]. We compute the Laplace transforms of $R_{\alpha\beta}(q,\omega)$ with respect to ω which reduce to the following current-current correlators

$$E_{\alpha\beta}(q,\tau) = \langle 0| j_\alpha^{NC\,\dagger}(\mathbf{q},\omega_{\rm qe}) e^{-(H-E_0)\tau} j_\beta^{NC}(\mathbf{q},\omega_{\rm qe})|0\rangle - |F_{\alpha\beta}(q)|^2 {\rm e}^{-\tau\omega_{\rm el}} \ , \tag{3}$$

where H is the nuclear Hamiltonian and the elastic contributions proportional to the (elastic) form factors $F_{\alpha\beta}(q)$ have been removed ($\omega_{\rm el}$ is the energy of the recoiling ground state). The energy dependence of $\mathbf{j}^{NC}(q,\omega)$ comes in via the nucleon and nucleon-to-Δ transition weak neutral form factors, which are functions of Q^2. We freeze the ω dependence by fixing Q^2 at the value $Q^2_{\rm qe} = q^2 - \omega^2_{\rm qe}$ with the quasielastic energy transfer $\omega_{\rm qe}$ given by $\omega_{\rm qe} = \sqrt{q^2+m^2} - m$ (m is the nucleon mass). This is needed in order to exploit the completeness over the nuclear final states in evaluating the Laplace transforms of $R_{\alpha\beta}(q,\omega)$ [19]. Lastly, since terms in the states $j_\alpha^{NC}|0\rangle$ involve gradients of the ground-state wave function [17], we evolve, rather than the exact ground state $|0\rangle$, our best variational state $|0_T\rangle$ in order to reduce the computational cost. Comparison between sum-rule results obtained with either $|0\rangle$ or $|0_T\rangle$ indicates that this is an excellent approximation [23].

Evaluations of the various correlators are carried out in two steps. First, an unconstrained imaginary-time propagation of $|0_T\rangle$ is performed and stored. Next, the states $j_\beta^{NC}(\mathbf{q},\omega_{\rm qe})|0_T\rangle$ are evolved in imaginary-time following the path previously saved. During this evolution scalar products of $e^{-(H-E_0)\tau_i} j_\beta^{NC}(\mathbf{q},\omega_{\rm qe})|0_T\rangle$ with $j_\alpha^{NC}(\mathbf{q},\omega_{\rm qe})|0_T\rangle$ are computed on a grid of τ_i values, and from these scalar products estimates of $E_{\alpha\beta}(q,\tau_i)$ are obtained (a more extended discussion of the methods is in Refs. [18, 24]). The computer programs are written in FORTRAN and use MPI and OPENMP for parallelization. While Monte Carlo calculations are thought of as "embarrassingly parallel", the GFMC propagation involves killing and replication of configurations which in fact could lead to significant inefficiencies in a parallel environment—in the present case, the Mira supercomputer of the Argonne Leadership Computing Facility. Moreover, for a nucleus such as ^{12}C with its large number (about 4×10^6) of spin and isospin states, the calculation of a single Monte Carlo sample must be spread over many nodes. For these reasons, the Asynchronous Dynamic Load Balancing (ADLB) and Distributed MEMory (DMEM) libraries [25], which operate under MPI, were developed. As a consequence, parallelization efficiency close to 95% is achieved using 8192 nodes of Mira. Approximately one million node hours were used for the calculation reported here.

Maximum entropy techniques, developed specifically for the present problem in Ref. [19], are then utilized to perform the analytic continuation of the Euclidean response functions, corresponding to the "inversion" of the Laplace transforms. The resulting ^{12}C neutral weak response functions $R_{\alpha\beta}(q,\omega)$ are displayed in Fig. 1 for $q = 570$ MeV/c. In this connection, we note that the presence of low-lying excitations of the ^{12}C nucleus complicates the determination of $R_{\alpha\beta}(q,\omega)$ for ω values near threshold [19]. Resolving the corresponding peaks would require imaginary-time evolution to τ values of the order of $1/\Delta E$, where ΔE are the excitation energies of these states ($\Delta E =$ 4.44 MeV for the lowest 2^+ excited state in ^{12}C); due to the Fermion sign problem, this is not possible. Each of these peaks, however, is proportional to the square of weak neutral transition form factors between the ground and relevant excited state. Because of the rapid fall-off of these form factors with increasing momentum transfer this issue is not expected to be of any concern at the relatively high momentum transfer of interest here. Of course, these considerations remain valid for the elastic contributions alluded to earlier in Eq. (3).

Figure 1 shows that contributions from two-body terms in the NC significantly increase (in magnitude) the response functions obtained in impulse approximation (*i.e.*, with one-body currents) over the whole quasielastic region, except for R_{00} at low ω. This enhancement is mostly due to constructive interference between the one- and two-body current matrix elements, and is consistent with that expected on the basis of sum-rule analyses [13]. Counter to the electromagnetic case [19], we find that two-body terms in the weak neutral charge produce substantial excess strength in R_{00} and R_{0z} beyond the quasielastic peak. In the 00, $0z$, zz, and xx response functions the vector (VNC) and axial (ANC) components of the weak neutral current, $j_\alpha^{NC} = j_\alpha^{VNC} + j_\alpha^{ANC}$, do not interfere; in these cases, $R_{\alpha\beta} = R^{VNC}_{\alpha\beta} + R^{ANC}_{\alpha\beta}$ and the $R^{VNC}_{\alpha\beta}$ and $R^{ANC}_{\alpha\beta}$ are illustrated separately in Fig. 1. By contrast, the xy response function arises solely on account of this interference. The ANC contribution to $R_{\alpha\beta}$ is typically much larger than the VNC one (for example, $R^{ANC}_{xx} \simeq 3 \times R^{VNC}_{xx}$), except for the charge response R_{00}. Furthermore, in ^{12}C the 00 and xx VNC response functions are roughly proportional to the longitudinal and transverse electromagnetic response functions R_L and R_T, namely $R^{VNC}_{00/xx} \simeq R_{L/T}/4$. This is because the isoscalar and isovector pieces in j^{VNC} are related to the corresponding ones in the electromagnetic current j^{EM} by the factors, respectively, $-2\sin^2\theta_W$ and $(1 - 2\sin^2\theta_W)$ ($\sin^2\theta_W \simeq 0.23$), and the matrix elements of these pieces add up incoherently in the response of an isoscalar target such as ^{12}C.

The two-body terms in the ANC increase the one-body R^{ANC}_{xx} response by about 20% in the quasielastic region. This increase is much larger than the $\simeq$2–4% obtained in the case of Gamow-Teller rates between low-lying states near threshold, induced by the axial component of the weak charged current [26]. In those calculations a significant reduction of the relevant matrix elements arose from nuclear correlations, which are also included here. It would be very intriguing to study the interplay between, and evolution of, correlation effects and two-body current contributions as the momentum and energy transfers increase from the threshold regime of relevance in discrete transitions between low-lying states, to the intermediate regime ($\sim$ 50–100 MeV) of interest to neutrino scattering in astrophysical environments or neutrinoless double beta decay [27, 28], to the quasielastic regime being studied here.

In Fig. 2 we show the ν and $\overline{\nu}$ differential cross sections and the $\nu/\overline{\nu}$ ratios for a fixed value of the three-momentum transfer as function of the energy transfer for a number of scattering angles. In terms of these variables, the initial energy E of the neutrino is given by

$$E = \frac{\omega}{2}\left[1 + \sqrt{1 + \frac{Q^2}{\omega^2 \sin^2(\theta/2)}}\,\right] , \qquad (4)$$

and its final energy $E' = E - \omega$: for example, at $\theta = 15°$ the initial energy decreases from 2.2 GeV to 1.6 GeV as ω increases from threshold to 450 MeV; at $\theta = 120°$ the initial energy increases from roughly 0.3 GeV to slightly over 0.5 GeV as ω varies over the same range. Thus the present results computed at fixed $q = 570$ MeV/c as a function of ω span a broad kinematical range in terms of E and E'—the kinematical variables most relevant for the analysis of accelerator neutrino experiments.

Because of the cancellation in Eq. (1) between the dominant contributions proportional to the R_{xx} and R_{xy} response functions, the $\overline{\nu}$ cross section decreases rapidly relative to the ν cross section as the scattering angle changes from the forward to the backward hemisphere. For this same reason, two-body current contributions are smaller for the $\overline{\nu}$ than for the ν cross section, in fact becoming negligible for the $\overline{\nu}$ backward-angle cross section. As the angle changes from the forward to the backward hemi-

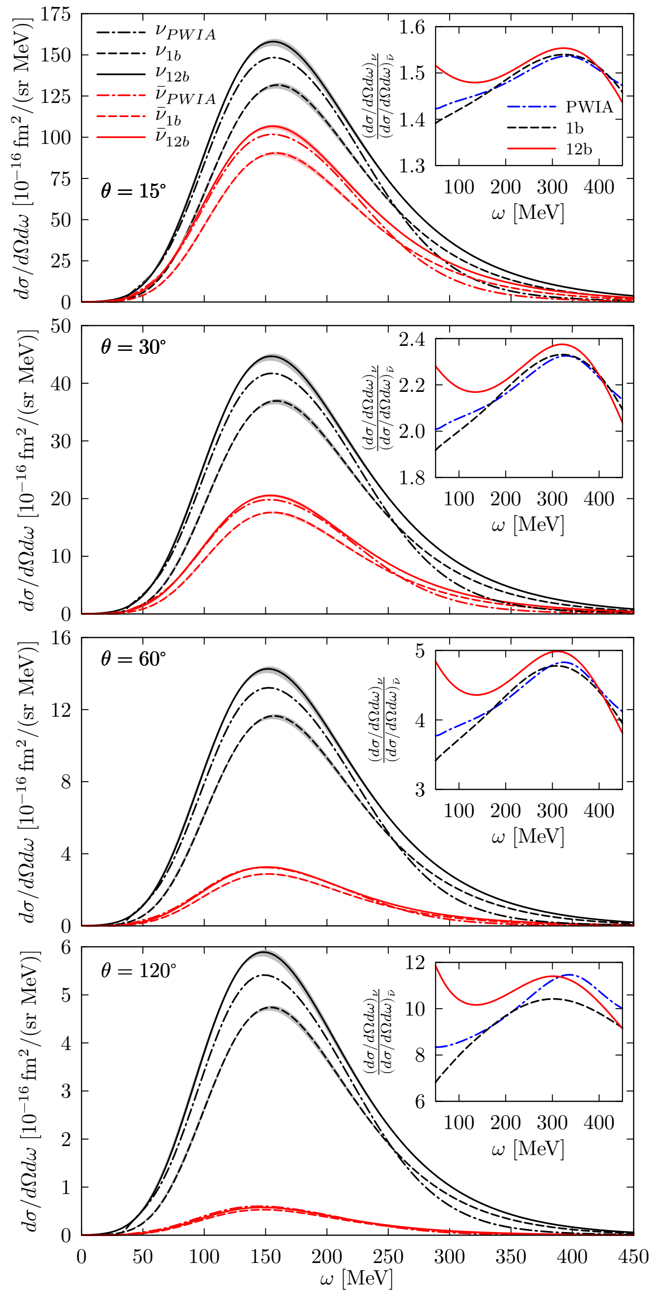

FIG. 2. (Color online) Weak neutral ν (black curves) and $\overline{\nu}$ (red curves) differential cross sections in ^{12}C at $q = 570$ MeV/c, obtained with one-body only and one- and two-body terms in the NC. The final neutrino angle is indicated in each panel. The insets show ratios of the ν to $\overline{\nu}$ (central-value) cross sections. Also shown are the PWIA results.

sphere, the ν cross section drops by almost an order of magnitude, and in the limit $\theta = 180^\circ$ is just proportional to $R_{xx}(q,\omega) - R_{xy}(q,\omega)$.

For comparison, we also show results obtained in the plane-wave impulse approximation (PWIA), in which only one-body currents are retained $j_\alpha^{NC} = \sum_i j_\alpha^{NC}(i)$. This comparison highlights how correlations and final state interactions quench the quasielastic peak, redistributing strength to the threshold and high-energy transfer regions.

In summary, we find substantial two-nucleon contributions to the neutral-current scattering of neutrinos and anti-neutrinos from ^{12}C. These contributions are significant over the entire quasielastic region, and are very important in each of the vector, axial, and axial-vector interference response functions. They significantly impact the magnitude of the cross sections, their energy dependence, and particularly the ratio of neutrino to anti-neutrino cross sections. It will be important to compare different, more approximate treatments of ν-A scattering to these calculations, and also to extend the present calculations over a wider range of energy and momentum transfers.

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