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Difference in proton radii of mirror nuclei as a possible surrogate for the neutron skin

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It has been recently suggested that differences in the charge radii of mirror nuclei are proportional to the neutron-skin thickness of neutron-rich nuclei and to the slope of the symmetry energy L [B.A. Brown Phys. Rev. Lett. 119, 122502 (2017)]. The determination of the neutron skin has important implications for nuclear physics and astrophysics. Although the use of electroweak probes provides a largely model-independent determination of the neutron skin, the experimental challenges are enormous. Thus, the possibility that differences in the charge radii of mirror nuclei may be used as a surrogate for the neutron skin is a welcome alternative. To test the validity of this assumption we perform calculations based on a set of relativistic energy density functionals that span a wide region of values of L . Our results confirm that the difference in charge radii of various neutron-deficient nickel isotopes and their corresponding mirror nuclei is indeed strongly correlated to both the neutron-skin thickness and L . Moreover, given that various neutron-star properties are also sensitive to L , a data-to-data relation emerges between the difference in charge radii of mirror nuclei and the radius of low-mass neutron stars.

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The neutron-rich skin of medium and heavy nuclei is a fundamental nuclear property that has gained prominence almost two decades ago because of its strong correlation to the equation of state (EOS) of neutron-rich matter, primarily to the slope of the symmetry energy L [1–4], a quantity that is closely related to the pressure of pure neutron matter at saturation density. In particular, the neutron-rich skin (R_{skin}) of heavy nuclei is highly sensitive to the *difference* between the symmetry energy at saturation density (as in the nuclear interior) and the symmetry energy at lower densities (as in the nuclear surface). As such, the thickness of the neutron skin emerges from a competition between the surface tension and the *slope* of the symmetry energy. This suggests that the neutron-skin thickness of heavy nuclei have a common dynamical origin: the the slope of the symmetry energy. Thus, besides the strong correlation between R_{skin} and L , a strong correlation also emerges between the neutron-skin thickness of different heavy nuclei; see Fig. 2 of Ref. [5].

Given that the weak charge of the neutron is significantly larger than the corresponding one for the proton, parity-violating electron scattering offers a clean probe of neutron densities that is free from strong-interaction uncertainties [6]. The pioneering Lead Radius Experiment (PREX) at the Jefferson Laboratory has provided the first model-independent evidence on the existence of a neutron-rich skin in ^{208}Pb [7, 8]. In the near future a follow-up experiment (PREX-II) is envisioned to reach the desired 0.06 fm sensitivity in the neutron radius of ^{208}Pb and a brand new experiment on ^{48}Ca (CREX) promises to bridge the gap between modern ab initio approaches and density functional theory [9]. Moreover, the study of neutron-rich matter with unusual features such as large neutron skins is one of the key science drivers for the Facility for Rare Isotope Beams (FRIB) [10, 11].

Besides being of fundamental importance in nuclear structure, the neutron-rich skin of medium to heavy nuclei plays a critical role in the determination of the equation of state (EOS) of neutron-rich matter. In turn, important dynamical signatures observed in the collision of heavy ions are encoded in the EOS [12–19]. Further, despite a difference in length scales of 18 orders of magnitude, the neutron-skin thickness of ^{208}Pb and the radius of a neutron star share a common dynamical origin [20–26]. Indeed, the only input that the structure of spherically symmetric neutron stars is sensitive to is the equation of state of neutron-rich matter. This fact alone has created a unique synergy between nuclear physics and astrophysics.

Although there is little doubt that parity violating electron scattering provides the cleanest probe of neutron densities, the experimental challenges associated with such experiments are enormous. This fact has motivated searches for complementary observables to the neutron skin that also display a strong sensitivity to the density dependence of the symmetry energy. Particularly valuable was the identification of the electric dipole polarizability (α_D) as a strong isovector indicator [27]. The electric dipole polarizability encodes the response of the nucleus to an externally applied electric field and is directly proportional to the *inverse* energy weighted sum of the isovector dipole response [28]. The isovector dipole resonance is commonly identified as an out-of-phase oscillation of protons against neutrons, with the symmetry energy acting as the restoring force. Since α_D was first identified as a strong isovector indicator, a flurry of activity ensued in both theoretical [5, 29–34] and experimental fronts [35–42].

In the ongoing quest to determine the equation of state of neutron-rich matter, B.A. Brown has recently identified a physical observable that is closely related to the

neutron skin [43]. The argument is both simple and elegant: in the limit of exact charge symmetry, the neutron radius of a given nucleus is identical to the proton radius of its mirror nucleus. That is,

$$\begin{aligned} R_{\text{skin}}(Z, N) &\equiv R_n(Z, N) - R_p(Z, N) \\ &\stackrel{\text{c.s.}}{=} R_p(N, Z) - R_p(Z, N) \equiv R_{\text{mirr}}(Z, N). \end{aligned} \quad (1)$$

For example, in the case of ^{48}Ca [9]:

$$R_{\text{skin}}(^{48}\text{Ca}) \stackrel{\text{c.s.}}{=} R_p(^{48}\text{Ni}) - R_p(^{48}\text{Ca}) \equiv R_{\text{mirr}}^{48}. \quad (2)$$

While the basic idea is appealing, the ultimate test of its validity relies on its robustness against the all-important Coulomb corrections. Indeed, most of the work in Ref. [43] was devoted to show that the differences in the charge radii of mirror nuclei as predicted by a set of Skyrme functionals is proportional to the slope of the symmetry energy at saturation density—even in the presence of Coulomb corrections. In this work we show that those findings remain valid in the relativistic approach. Moreover, we also show how R_{mirr} , just as R_{skin} , is correlated to the radius of low-mass neutron stars, a stellar property that is highly sensitive to the density dependence of the symmetry energy.

In the relativistic mean field (RMF) approach pioneered by Serot and Walecka [44, 45], the basic fermionic constituents are protons and neutrons interacting via photon exchange as well as through the exchange of various mesons of distinct Lorentz and isospin character. Besides the conventional Yukawa couplings of the mesons to the relevant nuclear currents, the model is supplemented by several nonlinear meson coupling that are essential for its ultimate success [20, 46, 47]. Besides a progressive increase in the complexity of the model, sophisticated fitting protocols are now used for its calibration. Indeed, properties of finite nuclei, their monopole response, and even a few properties of neutron stars now provide critical inputs in the determination of the relativistic functional [26].

To explore some of the interesting correlations that emerged in Ref. [43], but now in the relativistic context, we employ a set of 14 energy density functionals that span a wide region of values of the slope parameter: $L \simeq 50\text{--}140$ MeV. In turn, this corresponds to a neutron-skin thickness in ^{208}Pb ranging from about $R_{\text{skin}}^{208} \simeq 0.15$ to 0.33 fm, well within the limits of the PREX measurement [7, 8]. Parameter sets for the models adopted in this contribution are: NL3 [48, 49], FSUGold [50], IU-FSU [51], TAMUC-FSU [52], FSUGold2 [26], and FSUGarnet [53]. Although most of these models have been accurately calibrated, a few of them were obtained by systematically varying their two isovector parameters, while leaving the isoscalar sector intact. This enables one to modify the poorly constrained density dependence of the symmetry energy without compromising the success of the model in the isoscalar sector. In essence, all these models reproduce nuclear observables near stability, yet may vary widely in their predictions for the properties of

exotic nuclei far from stability. We note that the only charge-symmetry breaking term included in this work is the Coulomb interaction. Sub-leading contributions such as isospin violations in the nuclear interaction or the neutron-proton mass difference have been ignored. Moreover, pairing correlations have also been omitted, as we expect that their impact on charge radii will be small. Indeed, although in a different context, we have seen that pairing correlations do not affect the radii of the tin isotopes, often regarded as the quintessential superfluid nuclei [54]. Nevertheless, including pairing correlations and additional isospin-breaking corrections in future calculations will help assess the robustness of the alleged correlation.

In Fig. 1(a) we display data-to-data relations between the neutron-skin thickness of ^{48}Ca and the difference in proton radii between three neutron-deficient nickel isotopes and their corresponding mirror nuclei; the same information is shown in Fig. 1(b) but now for ^{208}Pb . For example, the blue circles denote the difference in proton radii in the $A=50$ sector: $R_{\text{mirr}}^{50} \equiv R_p(^{50}\text{Ni}) - R_p(^{50}\text{Ti})$. Besides computing proton radii along the $A=50, 52, 54$ isobars, we tried to calculate the proton radius of ^{48}Ni but were unable to bind the $f^{7/2}$ protons—especially in RMF models having a soft symmetry energy. As indicated by the correlation coefficients displayed in parentheses in Fig. 1, there is a strong correlation between R_{skin} and R_{mirr} for both ^{48}Ca and ^{208}Pb , at least for the representative set of models used in this work. Also shown are the linear regression slopes obtained from the statistical analysis. Clearly, the larger the value of the slope the more accurate the determination of the charge radius of the *unstable* neutron-deficient nickel isotope needs to be. Indeed, if one is interested in the determination of the neutron skin of ^{48}Ca to a precision of 0.02 fm [9], then one must measure the charge radius of ^{54}Ni to better than 0.004 fm; note that the charge radius of its *stable* mirror nucleus ^{54}Fe is already known to 0.002 fm [55]. On the other hand, for the $A=50$ case the charge radius of ^{50}Ni needs to be determined to “only” 0.012 fm. However, in this case the experimental challenge is formidable, as ^{58}Ni is the most neutron-deficient isotope with a well measured charge radius [55]. Yet, we are confident that with the commissioning of new and more intense radioactive beam facilities, the experimental community will continue to rise up to the challenge. Note that while the regression slopes almost double for ^{208}Pb , the aim of the PREX-II experiment is to determine the neutron radius of ^{208}Pb to 0.06 fm. Nevertheless, we caution against a possible model dependence of our results, as the correlation between the neutron skin of ^{48}Ca and L —or equivalently the correlation between R_{skin}^{48} and R_{skin}^{208} —does not appear to be as strong as suggested here; see for example Fig. 2(b) of Ref. [29].

Having established the existence of a strong correlation between R_{skin} and R_{mirr} , we now proceed to explore the sensitivity of the latter to the slope of the symmetry energy. Recall that the slope of the symmetry energy is

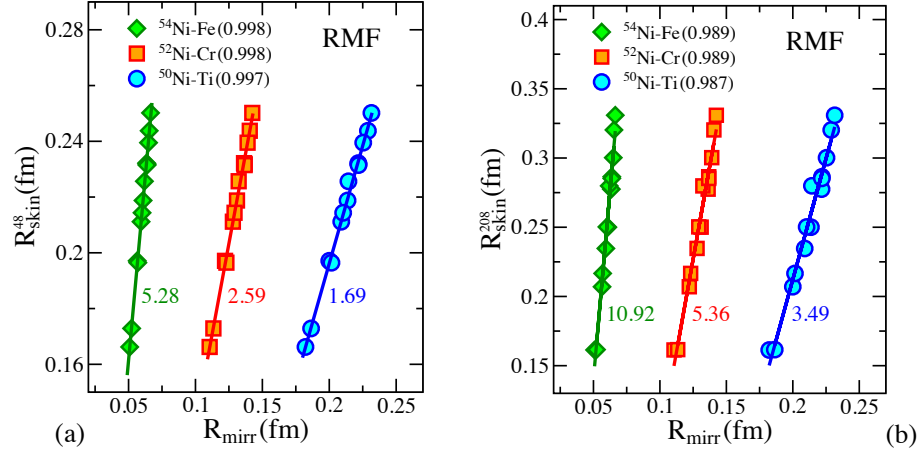


FIG. 1: (Color online) (a) Data-to-data relations between the neutron-skin thickness of ^{48}Ca and the difference in proton radii between a few neutron-deficient nickel isotopes and their corresponding mirror nuclei along the $A=50, 52, 54$ isobars. Numbers in parentheses represent the correlation coefficients and the numbers next to the lines the linear regression slopes. (b) Same but now for the neutron-skin thickness of ^{208}Pb .

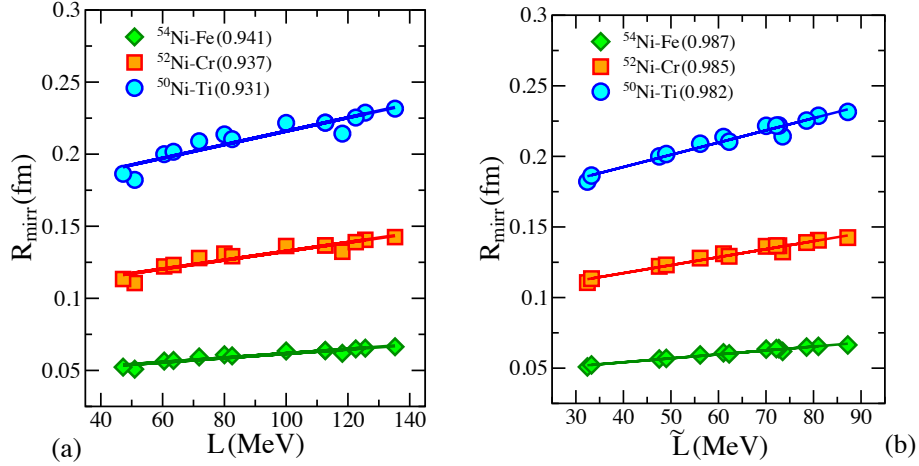


FIG. 2: (Color online) (a) Difference in proton radii along the $A=50, 52, 54$ isobars as a function of the slope of the symmetry energy at saturation density. Numbers in parentheses represent the correlation coefficients. (b) Same but now as a function of the slope of the symmetry energy at the lower density of 0.10 fm^{-3} .

defined as

$$L = \left(3\rho \frac{\partial \mathcal{S}}{\partial \rho} \right)_{\rho=\rho_0}, \quad (3)$$

where $\mathcal{S}(\rho)$ is the symmetry energy, namely, the energy cost of turning neutrons into protons (or vice versa) in symmetric nuclear matter.

In Fig. 2(a) we plot the difference in the proton radii of mirror nuclei as a function of L . The observed correlation is as strong as the one between the neutron-skin thickness of ^{48}Ca and L (figure not shown). While this suggests an efficient tool to constrain a fundamental parameter of the equation of state, the robustness of this result should be tested against a possible model dependency. Clearly, it would be ideal to extend this approach to the heavy-mass region where the surface to volume

ratio is more favorable, as in the case of ^{208}Pb whose neutron skin has been firmly established as a proxy for L . Unfortunately, exploiting the isovector character of mirror nuclei is limited to a fairly narrow region of the nuclear chart.

Shown in Fig. 2(b) is a similar plot but now against the slope of the symmetry energy at the slightly lower density of $\tilde{\rho}_0 = 0.10 \text{ fm}^{-3}$, or about 2/3 of the density at saturation. Note that \tilde{L} is defined exactly as in Eq. (3) but now evaluated at $\rho = \tilde{\rho}_0$. In all three cases the correlation becomes tighter. That a lower density than saturation represents a better choice to determine the symmetry energy has been emphasized repeatedly; see for example Refs. [1, 2, 19–21, 56–59]. Indeed, given that so far the isovector sector is largely informed by the binding energy of stable neutron-rich nuclei, the symmetry energy is bet-

ter constrained at a density that results from an average of the nuclear interior and the nuclear surface.

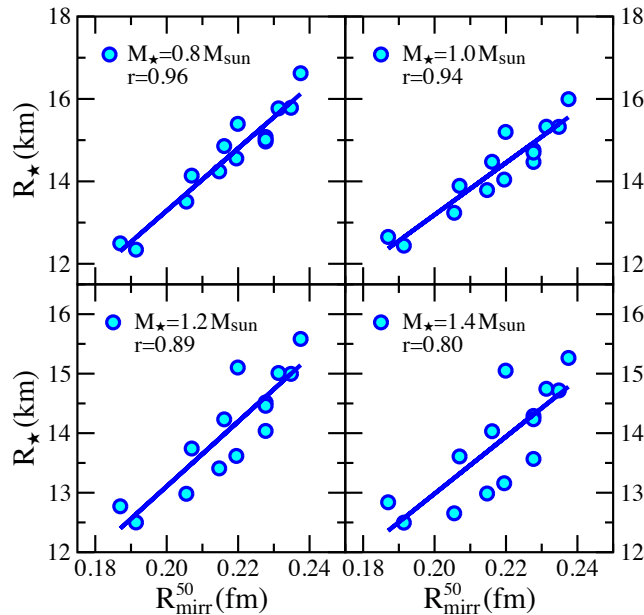


FIG. 3: (Color online) Stellar radii for neutron stars with masses of $M_\star = 0.8, 1.0, 1.2, 1.4 M_\odot$ as a function of the difference in proton radii between ^{50}Ni and ^{50}Ti . Here r is the correlation coefficient deduced from a linear regression.

We finish this contribution by exploring a possible connection between $R_{\text{mirr}}^{50} \equiv R_p(^{50}\text{Ni}) - R_p(^{50}\text{Ti})$ and the radius of a neutron star, a stellar property that is known to be particularly sensitive to the density dependence of the symmetry energy [60]. Note that an intriguing correlation exists that involves objects that differ in size by 18 orders of magnitude: the smaller the neutron-skin thickness of ^{208}Pb the smaller the size of the neutron star [21]. That is, whether pushing against surface tension in ^{208}Pb or against gravity in a “low-mass” neutron star [22], it is the pressure of neutron-rich matter around saturation density that determines both the thickness of the neutron skin and the radius of a neutron star. Given the strong correlation between R_{skin}^{208} and R_{mirr}^{50} displayed in Fig. 1b, we find natural to explore a possible connection between the latter and the stellar radius. Thus, we display in Fig. 3 neutron-star radii as a function of R_{mirr}^{50} for neutron stars with masses of $M_\star = 0.8, 1.0, 1.2, 1.4 M_\odot$. We observe a strong correlation—with a correlation coefficient of $r = 0.96$ —between R_{mirr}^{50} and the radius of a $M_\star = 0.8 M_\odot$ neutron star. The correlation is strong because for such a relatively light neutron star, the central density remains below twice nuclear-matter saturation density. Indeed, in the case of the FSUGold2 functional, the central density remains below 1.5 times saturation density. This intriguing fact provides a fundamental link between the laboratory and the cosmos. However, the correlation weakens with increasing stellar mass because the radius becomes sensitive to the pressure at densities

significantly higher than those probed in the laboratory. For example, in the case of a $M_\star = 1.4 M_\odot$ the correlation weakens to $r = 0.8$ because now the density in the stellar core exceeds three times nuclear matter saturation density. As in the case of the neutron-skin thickness of ^{208}Pb , we conclude that it may be possible to infer some fundamental properties of low-mass neutron stars from the structure of atomic nuclei.

In summary, inspired by the simple and elegant idea presented in Ref. [43] that suggests that differences in the charge radii of mirror nuclei are correlated to both the neutron-skin thickness of neutron-rich nuclei and the slope of the symmetry energy, we have investigated the validity of these correlations in the relativistic framework. Using a set of accurately calibrated relativistic energy density functionals that span a wide range of values for the slope of the symmetry energy L , we have confirmed the results of Ref. [43]. Moreover, we have extended our results to the neutron-star domain and reported a strong correlation between the difference in the proton radii between ^{50}Ni and ^{50}Ti , and the radii of low-mass neutron stars. Thus, at least within the context of the RMF models employed in this work, we have established that the difference in charge radii may serve as a credible surrogate for the neutron-skin of neutron-rich nuclei. Moreover, we concluded that accurate measurements of the charge radii of neutron deficient nickel isotopes may have important implications for the structure of low-mass neutron stars.

Shortly after the submission of this paper, the LIGO-Virgo collaboration announced the historical first detection of gravitational waves from a binary neutron star merger [61]. Based on the extraction of the tidal deformability, it was concluded that the radius of a neutron star can not be overly large. This, in turn, provides strong evidence that the pressure of neutron-rich matter at intermediate densities can not be too stiff. Thus, the deep connection between R_{skin}^{208} and stellar radii now offers the unique possibility of discerning a possible phase transition in the interior of neutron stars. Indeed, if the follow-up PREX-II experiment confirms that R_{skin}^{208} is large, this would suggest a softening of the symmetry energy with increasing density, likely indicative of a phase transition [62].

The realization that the neutron-skin thickness of neutron-rich nuclei could have such a dramatic impact in areas far beyond the nuclear-structure domain has created a flurry of activity that continues until today. We trust that the ideas introduced in Ref. [43] and expanded in this presentation will also stimulate considerable experimental and theoretical activity. Theoretically, both ab initio models and energy density functionals of increasing sophistication are in an excellent position to predict with quantifiable uncertainties the charge distribution of neutron-deficient nuclei. Experimentally, enormous technical advances have resulted in pioneering measurements of the charge radii of unstable neutron-rich isotopes at such facilities as ISOLDE-CERN [63] and

soon at RIKEN-SCRIT [64]. We are confident that these techniques may also be used to measure the charge radius of the neutron-deficient isotopes discussed in this work. Moreover, such remarkable level of activity will only increase with the commissioning of new radioactive beam facilities throughout the world. As we enter a golden era in nuclear structure that will see a paradigm shift in fundamental core concepts, we are confident that “unprecedented access to a vast new array of nuclei will result in scientific breakthroughs and major advances in our un-

derstanding of nuclei and their role in the cosmos” [11].

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