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Spectroscopic factors in ^{10}Be, ^{11}Be and ^{12}Be, extracted from (d,p), one neutron knockout, and (p,d) reactions are interpreted within the rotational model. Assuming that the ground state and first excited state of ^{11}Be can be associated with the $\frac{1}{2}[220]$ and $\frac{1}{2}[101]$ Nilsson levels, the strong coupling limit gives simple expressions that relate the amplitudes of these wavefunctions (in the spherical basis) with the measured cross-sections and derived spectroscopic factors. We obtain good agreement with both the measured magnetic moment of the ground state in ^{11}Be and the reaction data.

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I. INTRODUCTION

The lightest example of a so-called "Island of Inversion" is that at $N=8$, where the removal of $p_{3/2}$ protons from ^{14}C results in a quenching of the N=8 shell gap [1–4]. This is evident with the sudden drop of the $E(2^+)$ energy in ^{12}Be relative to the neighboring even-even isotopes, and the inversion of the ground state of ^{11}Be from the expected $1/2^-$ to the observed positive parity $1/2^+$ state.

The strong α clustering in ^{8}Be naturally suggests that deformation degrees of freedom will play an important role on the structure of the Be isotopes, a topic that has been extensively discussed in the literature (see [5] for a review). In fact, Bohr and Mottelson [6] proposed the effects of deformation to explain the inversion of the $1/2^+$ and the $1/2^-$ states.

In terms of a Shell Model picture, the underlying physics of such inversions is rather well understood, with the neutron-proton interaction playing an important role [2–4, 7]. Specifically for the case at N=8, the combined effects of the $V^{\pi\nu}_{p_{3/2},p_{1/2}}$ interaction, and the lowering of the $s_{1/2}$ orbit due to weak binding [8] erode the expected shell gap, and the quadrupole force takes over, driving the system to deform.

Given the discussions above, it is of interest to understand the structure of neutron-rich Be isotopes within the Nilsson model [9, 10], that captures the main effects of the quadruple force in a deformed mean field. Building on the arguments in Ref. [6], Hamamoto and Shimoura [11] presented a detailed interpretation of energy levels and available electromagnetic data on ^{11}Be and ^{12}Be in terms of single-particle motion in a deformed potential, using weakly bound one-particle wavefunctions calculated with a deformed Woods-Saxon (WS) potential instead of the standard harmonic-oscillator potential [12].

There are, of course, many theoretical approaches to describe the structure of neutron-rich ^{11}Be and ^{12}Be [13–20], which include ingredients the Nilsson model perhaps overlooks. Nevertheless, the deformed mean-field approach seems to capture the main physics ingredients [11]. In this work we analyze spectroscopic factors, obtained from studies of the ^{11}Be$(d,p)^{12}$Be [17, 21], ^{10}Be$(d,p)^{11}$Be [22, 23], ^{12}Be one neutron knockout $(-1n)$ [24, 25] and ^{11}Be$(p,d)^{10}$Be [16] reactions, in the Nilsson strong coupling limit. As we will show, the approach provides a satisfactory explanation of spectroscopic factors, in a simple and intuitive manner.

II. THE METHOD

In what follows, we use the formalism reviewed in Ref. [26], which we have recently applied to the N=20 Island of Inversion [27]. As in Refs. [6, 11] we associate the $1/2^+$ and the $1/2^-$ states in ^{11}Be to the Nilsson levels $\frac{1}{2}[220]$ and $\frac{1}{2}[101]$ respectively. In the spherical $|j,\ell\rangle$ basis these wavefunctions take the form:

$$|\tfrac{1}{2}[220]\rangle = C_{1/2,0}|s_{1/2}\rangle + C_{3/2,2}|d_{3/2}\rangle + C_{5/2,2}|d_{5/2}\rangle \quad (1)$$

$$|\tfrac{1}{2}[101]\rangle = C_{1/2,1}|p_{1/2}\rangle + C_{3/2,1}|p_{3/2}\rangle \quad (2)$$

where $C_{j,l}$ are the associated Nilsson wavefunction amplitudes.

For transfer reactions, such as (d,p), the spectroscopic factors $(S_{i,f})$ from an initial ground state $|I_iK_i\rangle$ to a final state $|I_fK_f\rangle$ can be written in terms of the Nilsson amplitudes [26]:

$$S_{i,f} = \frac{(2I_i+1)}{(2I_f+1)} g^2 \langle I_i j K_i \Delta K | I_f K_f\rangle^2 C^2_{j,\ell} \langle \phi_f|\phi_i\rangle^2 \quad (3)$$

where $g^2 = 2$ if $I_i = 0$ or $K_f = 0$ and $g^2 = 1$ otherwise, and $\langle\phi_f|\phi_i\rangle$ represents the core overlap between the initial and final states. A similar expression, without the spin factors, applies to the cases of 1n-KO and (p,d).

Finally, we consider the final 0^+ states in ^{12}Be as superpositions of the neutron states in Eqs. (1,2) [11]:

$$|0^+_1\rangle = \alpha|\nu_1\bar{\nu}_1\rangle + \beta|\nu_2\bar{\nu}_2\rangle$$

$$|0_2^+\rangle = -\beta|\nu_1\bar{\nu}_1\rangle + \alpha|\nu_2\bar{\nu}_2\rangle \quad (4)$$

where $|\nu_1\rangle$ and $|\nu_2\rangle$ are the neutron states in Eq. 1 and Eq. 2 respectively and $\bar{\nu}$ indicates the time-reverse orbit. The $|2_1^+\rangle$ is associated with the 2^+ member of the rotational band built on the $|0_1^+\rangle$ state.

III. RESULTS

With the established framework for our calculations, we can derive specific formulae relating the Nilsson amplitudes $C_{j,l}$ to the experimental spectroscopic factors for the reactions considered here. The relations follow directly from Eqs. **??** and are given below for the four specific cases.

A. ^{11}Be$(d,p)^{12}$Be

For this first case we start from the ^{11}Be $1/2^+$ ground state, and consider transfer of a single neutron in (d,p) to populate the 0_1^+, 0_2^+, and 2_1^+ states. Following directly from Eqs.(1-4) the relevant spectroscopic factors are

$$S_{1/2^+,0_1^+} = 2C_{1/2,0}^2\alpha^2$$
$$S_{1/2^+,0_2^+} = 2C_{1/2,0}^2\beta^2$$

and

$$S_{1/2^+,2_1^+} = \frac{2}{5}(C_{3/2,2}^2 + C_{5/2,2}^2)\alpha^2.$$

In Ref. [18], a shell-model inspired solution was proposed to explain the spectroscopic factors data. In their analysis, the authors use simple mixed wavefunctions, which are naturally captured in the Nilsson model.

B. ^{10}Be$(d,p)^{11}$Be

In this case, since we start from the ^{10}Be 0^+ ground state, the angular momentum selection rules imposed by the Clebsch-Gordan coefficients in Eq. (2) the spectroscopic factors directly project out the amplitudes of the wavefunctions in the spectroscopic factors:

$$S_{0^+,1/2^+} = C_{1/2,0}^2$$

and

$$S_{0^+,1/2^-} = C_{1/2,1}^2.$$

It is worth noting that this case has been studied in the particle-vibration coupling and deformed-core-plus-neutron cluster models in Refs. [14, 15].

C. ^{12}Be$(-1n)^{11}$Be

The case of neutron knockout is essentially equivalent to the previous examples, but we now have the addition of the core overlaps in Eq. (4) as we consider the $K = 1/2^+$ and $K = 1/2^-$ final states, with spectroscopic factors given by:

$$S_{0_1^+,1/2^+} = 2C_{1/2,0}^2\alpha^2; \quad S_{0_1^+,5/2^+} = 2C_{5/2,2}^2\alpha^2$$

and

$$S_{0_1^+,1/2^-} = 2C_{1/2,1}^2\beta^2; \quad S_{0_1^+,3/2^-} = 2C_{3/2,1}^2\beta^2$$

D. ^{11}Be$(p,d)^{10}$Be

Finally, the spectroscopic factors for the (p,d) reaction populating states in ^{10}Be reduce to:

$$S_{1/2^+,0_1^+} = C_{1/2,0}^2$$

and

$$S_{1/2^+,2_1^+} = C_{5/2,2}^2.$$

In Ref. [16] the authors consider a particle-vibration coupling picture, suggesting an appreciable admixture of core excitation to explain their cross-section data.
In comparing with the experimental data (summarized in Table I) we have used the expressions above together with the condition of wavefunction normalization to empirically adjust the amplitudes of the Nilsson states in Eqs. (1) and (2). In addition we consider the measured magnetic moment (see Appendix) of the ground state in ^{11}Be, $\mu = -1.6813(5)\mu_N$ [28], as a constraint. There are in total 12 relations connecting the experimental data to four unknown amplitudes which we determine from a χ^2-minimization procedure. Given the possible systematic uncertainties in the determination of absolute spectroscopic factors, particularly from different experimental conditions and analysis, we have done a weighted fit of the relative spectroscopic factor values with respect to the ground state transition for each of the data sets, and of the absolute value of the ^{11}Be ground-state magnetic moment.

The following wavefunctions‡:

$$|\tfrac{1}{2}[220]\rangle = -0.72(3)|s_{1/2}\rangle - 0.09(2)|d_{3/2}\rangle + 0.69(2)|d_{5/2}\rangle$$

$$|\tfrac{1}{2}[101]\rangle = 0.68(4)|p_{1/2}\rangle + 0.73(3)|p_{3/2}\rangle$$

‡ Adopted signs follow the phases of a standard Nilsson calculation.

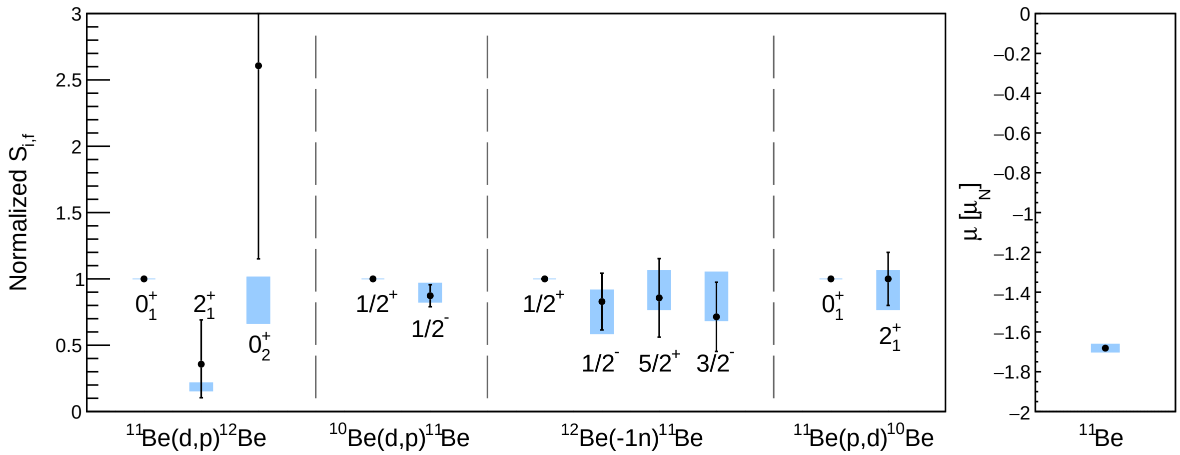

FIG. 1: Relative experimental spectroscopic factors and magnetic moment (data points) compared to the strong coupling limit results obtained in our analysis (blue boxes), which encompass the 1σ confidence level in our fit.

TABLE I: Summary of experimental relative spectroscopic factors in ^{10,11,12}Be compared to the Nilsson calculations using amplitudes empirically adjusted from a weighted fit to the data.

Initial State	Final State	Energy [MeV]	ℓ	Experimental $S_{i,f}$ [17]	[23]	[24, 25]	[16][a]	Theoretical $S_{i,f}$ Present work	[17][b]	[18]	[15]	[20]	[24, 25][c]	[14]	[13]
^{11}Be	^{12}Be														
$\frac{1}{2}^+$	0_1^+	0.00	0	1				1	1	1					
	2_1^+	2.11	2	0.36(29)				0.19(3)	1.41 [1.41]	0.9					
	0_2^+	2.24	0	2.61(134)				0.84(17)	1.84 [1.55]	0.47					
^{10}Be	^{11}Be														
0^+	$\frac{1}{2}^+$	0.00	0		1			1			1	1			
	$\frac{1}{2}^-$	0.32	1		0.87(8)			0.90(7)			1.24	0.97			
^{12}Be	^{11}Be														
0^+	$\frac{1}{2}^+$	0.00	0			1		1					1		
	$\frac{1}{2}^-$	0.32	1			0.82(22)		0.75(17)					0.84 [0.69]		
	$\frac{5}{2}^+$	1.78	2			0.86(29)		0.92(15)					0.80 [0.8]		
	$\frac{3}{2}^-$	2.69	1			0.71(26)		0.87(18)							
^{11}Be	^{10}Be														
$\frac{1}{2}^+$	0_1^+	0.00	0				1	1			1	1		1	1
	2_1^+	3.4	2				1.0(2)	0.92(15)			0.26	0.24		0.25	0.73

[a]Here we consider the values obtained from their SE analysis. See text for further discussion.
[b]The values correspond to two WBP interaction calculations
[c]Values are for WBT2 and [WBT2'] interactions

and $\alpha = 0.74(4)$ and $\beta = 0.68(4)$ are obtained. The resulting spectroscopic factors are summarized in Table I and, with the magnetic moment, in Fig. 1, showing good agreement with the experimental data. The wavefunctions as well as α and β are fairly consistent with those used in Ref. [11], α=β=0.707.

A brief discussion is in order, regarding the experimental result of Ref. [16] quoted in Table I. We have adopted the average value corresponding to their SE (Single-particle form factor) analysis. We note, however, that their alternate analysis using VIB (Vibrational) form factor yields a relative spectroscopic factor for the 2_1^+ state in the ^{11}Be(p, d)^{10}Be of 0.26(5). If we use this value instead, our fit finds a reduced amplitude of the $d_{5/2}$ component of the $\frac{1}{2}[220]$ wavefunction, but we still obtain good overall agreement with the experimental data.

There is continuing interest in this region of the nuclear chart, and with the availability of radioactive beams of ^{12}Be and ^{13}B as well as new instrumentation, further experimental work will be carried out. With this in mind, we take the Nilsson approach a little further, and predict estimates for spectroscopic factors for the reactions ^{12}Be$(d,p)^{13}$Be and ^{13}B$(d,^3$He$)^{12}$Be which are likely to be studied in the near future. There is some discrepancy in the literature about the low-lying level

TABLE II: Predicted spectroscopic factors in the Nilsson scheme for the reactions ^{12}Be$(d,p)^{13}$Be and ^{13}B$(d,^{3}He)^{12}$Be.

Initial State	Final State	Energy [MeV]	ℓ	Calculated $S_{i,f}$
^{12}Be	^{13}Be			
0_1^+	$\frac{1}{2}^+$	0.00	0	0.52
	$\frac{5}{2}^+$	~1.8	2	0.47
	$\frac{1}{2}^-$	0+x	1	0.46
^{13}B	^{12}Be			
$\frac{3}{2}^-$	0_1^+	0.00	1	0.5
	2_1^+	2.11	1	0.5
	0_2^+	2.24	1	0

assignments of ^{13}Be [29], but in any scenario the $\frac{1}{2}[220]$ and $\frac{1}{2}[101]$ Nilsson levels play a center role (as in ^{11}Be). The calculations are straightforward and the results are summarized in Table II.

It is also of interest to consider proton spectroscopic factors within the Nilsson scheme for $Z = 5$ where the proton is expected to occupy the $\frac{3}{2}[101]$ level, an assignment supported by the ground state spin $3/2^-$ and measured magnetic moment in ^{13}B, $\mu = 3.1778(5)\mu_N$ [30], for which we calculate $\mu \approx 3.2\mu_N$. Since the level parentage is attributed only to the $p_{3/2}$ orbit, the spectroscopic factors depend only on the Clebsch-Gordan coefficients, our predictions for the reaction ^{13}B$(d,^{3}He)^{12}$Be are included in Table II.

IV. CONCLUSION

We have analyzed spectroscopic factors in ^{11}Be and ^{12}Be, obtained from (d,p), $(-1n)$, and (p,d) reactions, in the Nilsson strong coupling limit. Using the formalism developed for studies of single-nucleon transfer reactions in deformed nuclei we derived, for the cases considered, simple formulae for spectroscopic factors in terms of the amplitudes of the deformed wavefunctions. These amplitudes were empirically adjusted to reproduce the experimental data, including the magnetic moment of the ^{11}Be ground state. We have also used these wavefunctions to make some predictions for reactions such as ^{12}B$(d,p)^{13}$Be and ^{13}B$(d,^{3}He)^{12}$Be, that will likely be studied in the near future.

V. APPENDIX

We present here the formulae used to calculate the magnetic moment (see Ref. [6]). For a $K = 1/2$ band the magnetic moment of the $I = 1/2$ state is given by:

$$\mu = \frac{1}{2}g_R + \frac{g_K - g_R}{6}(1-2b)$$

where $g_R \approx Z/A$ and g_K are the collective and single-particle gyromagnetic factors respectively, and b is the magnetic decoupling parameter.

The gyromagnetic factor g_K depends on the C_{jl} amplitudes through the following relation:

$$g_K = g_s(C^2_{1/2,0} + \frac{1}{5}(C^2_{5/2,2} - C^2_{3/2,2}) - 2\sqrt{\frac{24}{25}}C_{5/2,2}C_{3/2,2})$$

and the magnetic decoupling parameter b is related to the decoupling parameter a:

$$b = \frac{g_R a - (g_s + g_K)/2}{(g_K - g_R)}$$

with a:

$$a = C^2_{1/2,0} - 2C^2_{3/2,2} + 3C^2_{5/2,2}$$

Using the wavefunctions derived, the calculated gyromagnetic factor g_K, decoupling and magnetic-decoupling parameters for the ground state of ^{11}Be are: $g_K = -2.79$, $a = 1.93$ and $b = -1.27$ respectively. We note that, associating the $5/2^+$ state at 1.78 MeV with the second member of the rotational band, its energy is given by:

$$E_{rot} = A(\frac{5}{2}(\frac{5}{2}+1) - a(\frac{5}{2}+1))$$

With the rotational constant $A = 0.35$ MeV, determined from the 2^+ in ^{12}Be, we estimate $a = 1.85$, in excellent agreement with the value calculated from the magnetic moment.

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