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## Low-lying level structure of <sup>56</sup>Cu and its implications on the rp process

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22	The low-lying energy levels of proton-rich $^{56}$ Cu have been extracted using in-beam $\gamma$ -ray spec-
23	troscopy with the state-of-the-art $\gamma$ -ray tracking array GRETINA in conjunction with the S800
24	spectrograph at the National Superconducting Cyclotron Laboratory at Michigan State University.
25	Excited states in <sup>50</sup> Cu serve as resonances in the ${}^{55}Ni(p,\gamma){}^{50}Cu$ reaction, which is a part of the
26	rp-process in type I x-ray bursts. To resolve existing ambiguities in the reaction Q-value, a more
27	localized IMME mass fit is used resulting in $Q = 639 \pm 82$ keV. We derive the first experimentally-
28	constrained thermonuclear reaction rate for ${}^{50}Ni(p,\gamma)$ Cu. We find that, with this new rate, the
29	rp-process may bypass the $^{\circ\circ}$ Ni waiting point via the $^{\circ\circ}$ Ni(p, $\gamma$ ) reaction for typical x-ray burst con-
30	ditions with a branching of up to $\sim 40\%$ . We also identify additional nuclear physics uncertainties
31	that need to be addressed before drawing final conclusions about the rp-process reaction flow in the $\frac{56}{50}$ matrix
32	m region.

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### I. INTRODUCTION

Accreting neutron stars in binary systems undergo accreting neutron stars in binary systems undergo accreting neutron stars in binary systems undergo are episodes of explosive hydrogen and helium burning, obare served as Type-I x-ray bursts. The main observable of as these events, the x-ray burst light-curve, is shaped by the nuclear energy generation during the rapid protonto capture process (rp-process) [1, 2]. This process involves a series of proton captures and  $\beta$ -decays that proceed are the proton-drip line.

Reaction rates connected to the so-called waiting point
nuclei [3], where the reaction flow slows down significantly, have the most significant impact on the observed
light curve. The doubly-magic nucleus <sup>56</sup>Ni has been
identified as one of a few major waiting points in the

<sup>48</sup> rp-process. This is due to the combination of its long <sup>49</sup> stellar electron capture half-life of  $\sim 3$  hrs [4] (which for 50 the fully ionized ion differs from its terrestrial half-life <sup>51</sup> and depends on the stellar electron density), and its low <sup>52</sup> proton-capture Q-value (690 keV) [5]. The effective life-<sup>53</sup> time of <sup>56</sup>Ni under typical x-ray burst conditions, which 54 depends steeply on temperature, has been constrained <sup>55</sup> by experimental data related to the  ${}^{56}Ni(p,\gamma)$  [6, 7] and  ${}^{57}Cu(p,\gamma)$  [8] reaction rates. However, large uncertain-56 <sup>57</sup> ties exist in the nuclear physics of more neutron-deficient  $_{\rm 58}$  nuclei in the  $^{56}{\rm Ni}$  region. In particular, a sequence of <sup>59</sup> proton-capture reactions in the <sup>55</sup>Ni, <sup>56</sup>Cu, <sup>57</sup>Zn isotonic 60 chain may be strong enough for the rp-process to by-<sup>61</sup> pass <sup>56</sup>Ni (Fig. 1). In this case, <sup>56</sup>Ni would not be an <sup>62</sup> rp-process waiting point, reducing the sensitivity of burst <sup>63</sup> models to the <sup>56</sup>Ni(p, $\gamma$ ) rate. The <sup>57</sup>Cu(p, $\gamma$ ) reaction rate  $_{\rm 64}$  remains important because the bypass exits the N=27 <sup>65</sup> isotonic chain through  $\beta$ -decay of <sup>57</sup>Zn to <sup>57</sup>Cu. Con-<sup>66</sup> sequently, the reaction flow would proceed more rapidly 67 into the Ge-Se-Kr mass region and a lower amount of  $_{68}$  A = 56 material would be produced in the ashes.

<sup>69</sup> The <sup>55</sup>Ni(p, $\gamma$ ) reaction determines the branching at <sup>70</sup> <sup>55</sup>Ni into the <sup>56</sup>Ni bypass reaction sequence. Here, we

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FIG. 1. The nuclide chart in the region of the  ${}^{56}$ Ni waiting point. The conventional rp-process flow leading to <sup>56</sup>Ni is denoted by the solid line. The potential bypass, sequential proton-captures along the N = 27 isotonic chain, is denoted by the dashed line.

71 address uncertainties in this reaction rate experimentally, <sup>72</sup> and reanalyze theoretical predictions of the reaction Qvalue. We then use the new data to determine, in the 73 context of the remaining nuclear physics uncertainties, 74 the conditions under which the rp-process bypasses <sup>56</sup>Ni. 75 The  ${}^{55}\text{Ni}(p,\gamma){}^{56}\text{Cu}$  reaction proceeds through a few 76 isolated narrow resonances, and the astrophysical rate 77 78 can be approximated by

$$N_A \langle \sigma \upsilon \rangle \propto \sum_i (\omega \gamma)_i \exp(-E_i/kT)$$
 (1)

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<sup>79</sup> where  $E_i = E_i^x - Q$  is the resonance energy with reaction <sup>80</sup> Q-value Q and <sup>56</sup>Cu exitation energy  $E_i^x$ . The resonance <sup>81</sup> strength is given by

$$\omega\gamma = \frac{2J+1}{(2J_p+1)(2J_{55}N_i+1)} \frac{\Gamma_p\Gamma_\gamma}{\Gamma}.$$
 (2)

<sup>83</sup> the ground-sate spin of  ${}^{55}$ Ni,  $\Gamma_p$  the proton partial width,  ${}^{138}$   ${}^{56}$ Cu. GRETINA consists of 28 coaxial HPGe detector <sup>84</sup>  $\Gamma_{\gamma}$  the  $\gamma$  partial width and  $\Gamma = \Gamma_p + \Gamma_{\gamma}$ .

<sup>86</sup> nucleus exist in the literature. <sup>56</sup>Cu, as well as its well-<sup>141</sup> tum, angle, and position of each <sup>56</sup>Cu recoil at the target  $_{87}$  understood mirror nucleus  $^{56}$ Co, are part of the A = 56,  $_{142}$  based on observables at the S800 focal plane, combined  $_{**}T = 1$  isospin triplet. Based on this, the ground-state  $_{143}$  with the high position resolution for  $\gamma$ -ray detection in  $^{89}$  of  $^{56}$ Cu is assumed to be  $J^{\pi} = 4^+$  with a measured ter-  $^{144}$  GRETINA allow for accurate Doppler-shift corrections <sup>90</sup> restrial  $\beta$ -decay half-life of 93(3) ms [9]. To date, no <sup>145</sup> for  $\gamma$ -rays emitted in-flight. The recoil velocity  $\beta = v/c$ <sup>91</sup> low-lying excited states have been observed experimen-<sup>146</sup> used for the Doppler-shift correction was extracted using  $_{92}$  tally. In a recent  $\beta$ -delayed proton decay study of  $^{56}$ Zn,  $_{147}$  momentum information, and was determined for each in-<sup>93</sup> several higher-lying <sup>56</sup>Cu resonances above 1391 keV ex- <sup>148</sup> dividual event to correct for energy loss in the target. The <sup>94</sup> citation energy were observed [10]. Under astrophysical <sup>149</sup> <sup>56</sup>Cu recoils, after leaving the target, were identified us-96 97 <sup>99</sup> code ANTOINE have been used in the past [11]. How- <sup>154</sup> trajectory, a gas-filled ionization chamber that measured 100 ever, uncertainties in shell-model predictions of excita- 155 energy loss  $\Delta E$ , and a plastic scintillator that, along with <sup>101</sup> tion energies can amount up to 200 keV, leading to orders <sup>156</sup> the thin timing scintillator at the A1900 focal plane and <sup>102</sup> of magnitude uncertainty in the resonant-capture rate. <sup>157</sup> the scintillator at the S800 object position, were used

<sup>103</sup> Here we experimentally determine, for the first time, the excitation energies of low-lying states in  ${}^{56}$ Cu that serve as resonances in the  ${}^{55}\text{Ni}(p,\gamma){}^{56}\text{Cu}$  reaction 105

For a precise determination of the  ${}^{55}\text{Ni}(p,\gamma){}^{56}\text{Cu}$  rate, both the low-lying level scheme of  ${}^{56}$ Cu and the reaction Q-value need to be well-known since the resonance ener-108 gies enter the rate exponentially. While the mass of  ${}^{55}$ Ni <sup>110</sup> is experimentally well-known with an error of 0.75 keV [12], the mass of <sup>56</sup>Cu is not experimentally known. Con-111 flicting predictions for the <sup>56</sup>Cu mass exist in the litera-112 ture. The extrapolated  ${}^{56}$ Cu mass in the AME2003 compilation [13] results in a  ${}^{55}$ Ni proton-capture Q-value of 560(140) keV. A similar result of 600(100) keV is ob-115 tained with Coulomb shift calculations [14]. Using the 116 117 <sup>56</sup>Cu mass in the most recent AME2012 compilation,  $_{118}$  however, results in a Q-value of 190(200) keV [5]. We <sup>119</sup> obtain a new prediction for the reaction Q-value by us-<sup>120</sup> ing the *isobaric multiplet mass equation* (IMME).

#### EXPERIMENTAL DETERMINATION OF II. THE <sup>56</sup>CU LEVEL SCHEME

Excited states of <sup>56</sup>Cu were populated in inverse kine-123 124 matics in an experiment performed at the National Su-125 perconducting Cyclotron Laboratory (NSCL) at Michi- $_{126}$ gan State University [8]. A stable 160  ${\rm MeV}/u$   $^{58}{\rm Ni}$  pri- $_{127}$  mary beam impinged on a 752 mg/cm<sup>2</sup> <sup>9</sup>Be target placed 128 at the entrance of the A1900 fragment separator [15]. Af-<sup>129</sup> ter purification by the A1900 using the  $B\rho - \Delta E - B\rho$ <sup>130</sup> method, the produced <sup>56</sup>Ni secondary beam had a rate of  $\sim 10^5$  pps with a beam purity of  $\sim 75\%$ . The <sup>56</sup>Ni 131  $_{132}$  beam (E =  $\sim 75$  MeV/u) was then incident upon a 225  $_{133}~{\rm mg/cm^2~CD_2}$  target, producing  $^{56}{\rm Cu}$  through various re-<sup>134</sup> action channels. The CD<sub>2</sub> target was located in the cen-<sup>135</sup> ter of the  $\gamma$ -ray energy tracking array GRETINA [16], 136 which was used to measure energies of the prompt  $\gamma$ -rays <sup>82</sup> Here, J is the resonance spin,  $J_p$  the proton spin,  $J_{55Ni}$  is <sup>137</sup> emitted from the de-excitation of the excited states in <sup>139</sup> crystals, which are closely-packed to cover roughly  $1\pi$  in Only scarce experimental data for the odd-odd <sup>56</sup>Cu 140 solid angle. Kinematical reconstruction of the momenconditions, however, these resonances are too high in en- 150 ing detectors situated in the focal plane of the S800 specergy to be of relevance. In the absence of knowledge of 151 trograph [17] located downstream from GRETINA. The spectroscopic information, shell-model calculations using 152 S800 focal plane contained a set of two cathode readout the KB3 interaction in the pf-shell performed with the 153 drift counters that were used to determine the particle

<sup>158</sup> for time-of-flight (TOF) analysis. The measured time-<sup>159</sup> of-flight between the A1900 focal plane and S800 object <sup>160</sup> scintillators was used to uniquely identify the <sup>56</sup>Cu recoil <sup>161</sup> by  $\Delta E$ -TOF (Fig. 2).



FIG. 2. (Color online:)  $\Delta E$ -TOF particle identification for ions reaching the S800 focal plane. Color indicates the number of counts per bin. The Ni isotopic chain (dotted line) and the <sup>56</sup>Ni (leftmost ellipse), <sup>56</sup>Cu (rightmost ellipse), and <sup>57</sup>Cu (middle ellipse) isotopes are also marked (not actual analysis gates).

162 <sup>163</sup> using observed  $\gamma$ -ray transitions,  $\gamma - \gamma$  coincidences and <sup>201</sup> modes and similarities in energies, the two observed <sup>164</sup> guidance from the experimentally based level scheme of <sup>202</sup> states at  $E_x = 826(3)$  keV and  $E_x = 1037(3)$  keV <sup>165</sup> the mirror nucleus <sup>56</sup>Co. The Doppler-corrected spec- <sup>203</sup> are tentatively assigned as  $J^{\pi} = 4^+$  and  $J^{\pi} = 2^+$ , <sup>166</sup> trum of the  $\gamma$ -rays detected by GRETINA, in coincidence <sup>204</sup> respectively. <sup>167</sup> with the <sup>56</sup>Cu recoils in the S800, shows five  $\gamma$ -ray tran-<sup>205</sup> The observed line at  $E_{\gamma} = 572(1)$  keV is not seen in 168 sitions (Fig. 3). An additional line at  $E_{\gamma} = 1027$  keV 206 coincidence with the 166 keV line. The mirror <sup>56</sup>Co has <sup>169</sup> stems from contamination from a well-known  $\gamma$ -ray tran-<sup>207</sup> a  $J^{\pi} = 5^+$  state at  $E_x = 577$  keV that decays only <sup>170</sup> sition in <sup>57</sup>Cu which is located next to <sup>56</sup>Cu in the parti-<sup>208</sup> to the ground state. Based on the similar energies and <sup>171</sup> cle identification spectrum (Fig. 2). We confirmed that <sup>209</sup> similar decay modes, we tentatively assign the 572 keV <sup>172</sup> this  $\gamma$ -ray line disappears from the  $\gamma$  spectrum when the <sup>210</sup> transition to be the second excited  $J^{\pi} = 5^+$  state. <sup>173</sup> particle identification gate in Fig. 2 is tightened to only <sup>211</sup> The  $E_{\gamma} = 1224(4)$  keV line is not observed to be in  $_{174}$  include the most centrally located events in the  $^{56}$ Cu re-  $_{212}$  coincidence with any other  $\gamma$ -ray transition, and it is 175 coil region.



FIG. 3. Doppler-corrected  $\gamma$ -ray spectrum measured with GRETINA in coincidence with  ${}^{56}$ Cu ions in the S800 focal plane. A nearest neighbor addback algorithm has been applied. The asterisk indicates contamination from <sup>57</sup>Cu.

The left half of figure 5 shows the reconstructed 176 <sup>56</sup>Cu level scheme. The strongest observed line at 177  $_{178} E_{\gamma} = 166(1)$  keV is close in energy to the first excited <sup>179</sup> state at 158 keV  $(J^{\pi} = 3^+)$  in the mirror nucleus <sup>56</sup>Co. <sup>180</sup> Based on experimental information from the <sup>56</sup>Co mir-<sup>181</sup> ror nucleus, we expect the first excited state to be the 182 most intense transition as it is fed from several higher-183 lying states. This line is observed to be in coincidence <sup>184</sup> with two other  $\gamma$ -transitions, supporting its assignment  $_{185}$  as direct decay from the first excited  $3^+$  state (Fig. 186 4).

The transitions at  $E_{\gamma} = 660(3)$  keV and 187  $_{188} E_{\gamma} = 871(3)$  keV are observed to be in coincidence <sup>189</sup> with the  $E_{\gamma} = 166(1)$  keV transition as shown in Fig. 4, <sup>190</sup> but not with each other. Based on the prior assignment <sup>191</sup> of the 166 keV first excited state, two states are placed <sup>192</sup> at  $E_x = 826(3)$  keV and  $E_x = 1037(3)$  keV, respectively. <sup>193</sup> No ground state decays are observed for either of these  $_{194}$  states. There are three known states in the  $^{56}\mathrm{Co}$  mirror <sup>195</sup> at similar energies of  $E_x = 830,970$  and 1009 keV. Of <sup>196</sup> those, the 1009 keV state decays predominantly to the <sup>197</sup> ground state. Both the  $J^{\pi} = 4^+$  830 keV and  $J^{\pi} = 2^+$ <sup>198</sup> 970 keV states decay primarily to the first excited state <sup>199</sup> at 158 keV with only a 34% and 0.3% direct transition The low-lying level scheme of <sup>56</sup>Cu was constructed <sup>200</sup> to the ground state, respectively. Based on the decay

<sup>213</sup> therefore assigned to a level at that energy. The analog

<sup>214</sup> states in the mirror with the closest energies are 1009 keV  $_{215}$   $(J^{\pi} = 5^{+}_{2})$  and 1115 keV  $(J^{\pi} = 3^{+}_{2})$  which both decay <sup>216</sup> largely to the ground state. Other higher lying states in  ${}^{56}$ Co (the next one is at 1450 keV) decay predominantly 217 218 through cascades, which is not supported by our mea-<sup>219</sup> surement. We tentatively assign  $E_x = 1224(4)$  keV as 220 either the  $J^{\pi} = 3^+_2$  or the  $J^{\pi} = 5^+_2$  state. The observed 221 transitions, intensities and assignments are tabulated in <sup>222</sup> Table I. A comparison to the mirror nucleus is shown in 223 Fig. 5.



FIG. 4.  $\gamma - \gamma$  coincidences with  $E_{\gamma} = 871$  (3) keV (upper panel) and  $E_{\gamma} = 660$  (3) keV (lower panel).



FIG. 5. (Color online:) Proposed low-lying level scheme of <sup>56</sup>Cu (left) in comparison to its mirror nucleus <sup>56</sup>Co (right). Tentative spin and parity assignments are shown in parentheses. The observed  $\gamma$ -transitions are shown, with the corresponding transitions in the mirror shown with the same color.

#### III. MASS ESTIMATE OF <sup>56</sup>CU USING THE 224 ISOBARIC MULTIPLET MASS EQUATION 225

We use the isobaric mass multiplet equation (IMME) 226 to predict a new <sup>56</sup>Cu mass, which is needed to derive the 227 reaction Q-value and the resonance energies. The  ${}^{56}$ Cu  ${}^{242}$ 228 229 <sup>230</sup> triplet, and its mass excess can be calculated using

$$\Delta M = a + bT_z + cT_z^2. \tag{3}$$

TABLE I. Reconstructed level scheme of <sup>56</sup>Cu excitation levels with observed transition energies  $(E_{\gamma})$ , relative intensities  $(I_{\gamma})$  normalized to the  $E_{\gamma} = 166$  keV line, and tentative spinparity assignments (see text for details).

$E_x$ (keV)	$E_{\gamma} \; (\mathrm{keV})$	$I_{\gamma}$ (%)	$J_i^{\pi} \to J_f^{\pi}$
166(1)	166(1)	100	$(3_1^+) \to \text{g.s.}$
572(1)	572(1)	122(8)	$(5_1^+) \rightarrow \text{g.s.}$
826(3)	660(3)	28(8)	$(4_2^+) \to (3_1^+)$
1037(3)	871(3)	50(8)	$(2_1^+) \to (3_1^+)$
1224(4)	1224(4)	19(10)	$(3_2^+, 5_2^+) \to \text{g.s.}$

The *a* coefficient for integer triplets is the mass excess of the isobaric analogue state (IAS) of the  $T_z = 0$  member of the triplet, in this case the  $J^{\pi} = 4^+$  state in <sup>56</sup>Ni, and can be calculated from the reported IAS excitation energy of 6432 keV [18]. The IMME b and c coefficients for the A =56 triplet have not been published, but can be estimated using fits to coefficients of triplets in the vicinity of A =56. Global fit functions of IMME parameters have been discussed in [19], where the authors treat the nucleus as a homogeneous charged sphere, and coefficients a, b and care reported for the A = 4n subgroups. Here, we fit only to coefficients for a local region with A=32, 36, 40 and 48. As per the homogeneous charged sphere approximation of [20], the b and c coefficients can be parametrized in the following manner:

$$b = C_b^1 - C_b^2 \times (A - 1)/A^{-1/3} \tag{4}$$

$$c = C_c^1 + C_c^2 \times A^{-1/3}$$
(5)

 $_{\rm 231}$  where  $C_b^1,C_b^2,C_c^1,C_c^2$  are fit parameters. The fits ob- $_{232}$  tained for b and c in the local vicinity are then used for the A = 56, T = 1 subgroup. The resulting fit extrapolated to A = 56 results in c = 110(95) keV and b =-8680(109) keV. Along with the result for the *a* coeffi-235 cient from [18] of 6431.9 (7) keV, this provides a mass 236 excess prediction for  ${}^{56}$ Cu of -38685(82) keV and, thus, 237  $_{238}$  a Q-value of 639  $\pm$  82 keV. The error is taken from the 239 largest deviation between a measured mass and the pre-<sup>240</sup> dicted value from the fit function in the local region of 241 interest.

TABLE II. Summary of predictions for the Q-value of  ${}^{55}\mathrm{Ni}(p,\gamma).$ 

$560 \pm 140$ Mass extrapolationAME2003 [13] $190 \pm 200$ Mass extrapolationAME2012 [5] $600 \pm 100$ Coulomb Shift / Shell ModelBrown et al. [14] $639 \pm 82$ IMMEThis work	Q-value (keV)	Method	Reference
	$560 \pm 140$	Mass extrapolation	AME2003 [13]
	$190 \pm 200$	Mass extrapolation	AME2012 [5]
	$600 \pm 100$	Coulomb Shift / Shell Model	Brown et al. [14]
	$639 \pm 82$	IMME	This work

As seen in Table II, the more precise estimate from this ground state  $(J^{\pi} = 4^+)$  is part of the A = 56,  $T = 1^{243}$  work agrees within errors with the Coulomb-shift calcula-<sup>244</sup> tion from [14], favoring a higher Q-value compared to the <sup>245</sup> lower extrapolated value reported in the AME2012 com-B) 246 pilation. A recent IMME-based estimate using the T=2

<sup>247</sup> quintet [21] reported a Q-value of 651(88) keV. Moreover, <sup>248</sup> requiring reasonable Coulomb shifts for higher-lying mir-<sup>249</sup> ror states, as extracted experimentally in [10] between <sup>56</sup>Cu and <sup>56</sup>Co, also favors a higher Q-value. 250

#### THERMONUCLEAR REACTION RATE IV. 251

With our measurement and our predicted <sup>56</sup>Cu mass, 252 we have determined the resonance energies of the 253  ${}^{55}\text{Ni}(p,\gamma){}^{56}\text{Cu}$  reaction. In order to determine the as-254 <sup>255</sup> trophysical reaction rate, proton- and  $\gamma$ -widths ( $\Gamma_p$  and  $_{256}$   $\Gamma_{\gamma}$  respectively) were calculated for each state using a shell-model with the GXPF1A interaction [22] (Table 257 III). These calculations allowed up to 3-particle 3-hole 258 excitations in the pf-shell. 259

Reaction-rate uncertainties were calculated with a 260 Monte-Carlo approach, similar to that of [23], to prop-261 erly account for the uncertainties in the excitation ener-262 <sup>263</sup> gies. Resonance energies and the reaction Q-value were allowed to vary assuming a Gaussian distribution within 264 the uncertainties given in Table III. The uncertainty in 265 the spin assignment for the 1224 keV state was also taken 266 into account, but this represented only a small percent-267 age of the uncertainty. The sampled resonance energy 268 and corresponding rescaled proton-widths are used as input to Eq. 1, producing a sample of rates. At a given 293 temperature, the 50<sup>th</sup>, 16<sup>th</sup> and 84<sup>th</sup> percentiles of the 271 distribution of rate values provides the median, and 1- $\sigma$ 272 uncertainty, respectively. The results are shown in Fig. 6. To assess the reaction rate uncertainty prior to our 274 275 measurement, we used the shell-model calculation and  $_{\rm 276}$  assumed a 200 keV uncertainty for the resonance ener-277 gies. The resulting rate uncertainty (the light blue band <sup>278</sup> in Fig. 6) ranges from 4 orders of magnitude at 0.1 GK to <sup>279</sup> about an order of magnitude at 2.0 GK. This is reduced at low temperatures to less than two orders of magni-280 tude by our measurement (the gray band in Fig. 6). The 281 282 additional uncertainty from the calculated proton and  $\gamma$ partial widths is estimated to be significantly smaller, 283 about of a factor of 2 [8]. Thus, the dominant remaining 284 source of uncertainty is the  $\sim 80$  keV error in the  ${}^{56}Cu$ mass, with smaller contributions from the uncertainties 286 of the experimentally-unmeasured proton and  $\gamma$  partial 287 288 widths.

Table V gives the corresponding REACLIB rate fit co-280 290 efficients, using the parametrization given in Eqn. 6, for <sup>291</sup> our updated <sup>55</sup>Ni(p, $\gamma$ ) reaction rate.

$$N_A < \sigma \upsilon > = \sum_i \exp(a_{0i} + a_{1i}T_9^{-1} + a_{2i}T_9^{-1/3} + a_{3i}T_9^{1/3} + a_{4i}T_9 + a_{5i}T_9^{5/3} + a_{6i}\ln T_9)$$
(6)



FIG. 6. (Color online:) Rate predictions showing the reduction of rate uncertainty by this work, assuming Q =639 (82) keV. We only consider uncertainties from resonance energy errors. The light band (blue) shows the  $1-\sigma$  uncertainty in the shell model rate, whereas the dark band (grey) shows the 1- $\sigma$  uncertainty in the experimentally-constrained rate. A clear reduction of the rate uncertainty in the temperature region of interest can be seen, especially at lower temperatures.

#### V. CONSEQUENCES ON THE RP-PROCESS FLOW AROUND <sup>56</sup>NI

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The astrophysical conditions that would lead the rp-<sup>295</sup> process flow to bypass the <sup>56</sup>Ni waiting point were inves-<sup>296</sup> tigated using a limited reaction network that includes the <sup>297</sup> nuclides in Fig. 1. The network was seeded with <sup>55</sup>Ni, <sup>298</sup> where the rp-process enters the A = 56 region. <sup>56</sup>Ni was treated as a sink in the network calculation, with only 300 flow into this nuclide being allowed. In this case, the ra-<sup>301</sup> tio of the abundance of all other nuclei (<sup>57</sup>Ni, <sup>57,58</sup>Cu,  $_{302}$  and  $^{58}$ Zn) to the total abundance in the N = 28 and  $_{303} N = 29$  chains is a measure of the fraction of the rp-<sup>304</sup> process reaction flow that bypasses <sup>56</sup>Ni, as it measures  $_{305}$  the amount of material trapped in neither  ${}^{55}$ Co nor  ${}^{56}$ Ni. The reaction network was run at constant temperature 306 and proton density for 1 s, approximately 5 half-lives of 307 <sup>55</sup>Ni. A constant proton density was ensured by keeping 308 <sup>309</sup> the mass density constant, and by using a large proton- $_{310}$  to-seed ratio of  $\sim 400$  such that the change in the proton 311 abundance due to the comsumption of protons is negli-312 gible.

Even with the constraint on the  ${}^{55}Ni(p,\gamma)$  rate from 313 314 this work, there remain additional uncertainties that af- $_{315}$  fect the rp-process flow. The proton-capture rate on  $_{^{316}}$   $^{56}\mathrm{Cu}$  determines the branching at  $^{56}\mathrm{Cu},$  where  $\beta$  decay <sup>317</sup> leads back to <sup>56</sup>Ni, and also determines the total proton-<sup>318</sup> capture flow at <sup>55</sup>Ni in the case of  $(p, \gamma) - (\gamma, p)$  equilib-<sup>319</sup> rium between <sup>55</sup>Ni and <sup>56</sup>Cu. In addition, the mass of <sub>320</sub> <sup>57</sup>Zn has not been measured and its uncertainty affects <sub>321</sub> the  ${}^{57}$ Zn( $\gamma$ ,p) rate, which hampers the flow bypassing

Exper	iment		Shell	Model		C	$^2S$		
$E_x$ (keV)	$E_r \; (\text{keV})$		$E_x$ (keV)	$E_r \; (\text{keV})$	$J^{\pi}$	l = 1	l = 3	$\Gamma_{\gamma} (eV)$	$\Gamma_p \ (eV)$
166(1)			146		$(3_1^+)$	0.84	$9.1 \times 10^{-3}$	$8.4 \times 10^{-5}$	
572(1)			483		$(5_1^+)$	0.70	0.16	$1.1 \times 10^{-3}$	
826(3)	187(82)		1066	427	$(4^+_2)$	0.12	0.69	$4.5 \times 10^{-4}$	$1.2 \times 10^{-16}$
1037(3)	398(82)		1023	384	$(2_1^+)$	0.64	0.16	$1.2 \times 10^{-2}$	$1.7 \times 10^{-7}$
1994(4)	585(82)	ſ	1146	507	$(5^+_2)$	0.15	0.71	$2.0 \times 10^{-3}$	$4.3 \times 10^{-6}$
1224(4)	363(62)	Ì	1474	835	$(3^+_2)$	0.10	0.68	$1.9 \times 10^{-3}$	$3.9 \times 10^{-5}$
			1582	943	$0_{1}^{+}$		$3.8 \times 10^{-2}$	$1.6 \times 10^{-6}$	$1.5 \times 10^{-4}$
			1913	1274	$2^{+}_{2}$	0.15	0.57	$1.4 \times 10^{-2}$	3.7
			2036	1397	$1_{1}^{+}$		$1.3 \times 10^{-2}$	$4.5 \times 10^{-4}$	$8.1 \times 10^{-3}$
			2066	1427	$3^{+}_{3}$	0.59	$9.1 \times 10^{-2}$	$2.2 \times 10^{-2}$	48
			2226	1587	$2^{+}_{3}$	$1.7 \times 10^{-3}$	$3.9 \times 10^{-2}$	$3.8 \times 10^{-3}$	0.53
			2272	1633	$4_{3}^{+}$	0.63	0.13	$5.3 \times 10^{-2}$	210
			2350	1711	$7_{1}^{+}$		$9.9 \times 10^{-3}$	$1.2 \times 10^{-4}$	$5.8 \times 10^{-2}$
			2393	1754	$6_{1}^{+}$		0.72	$3.1 \times 10^{-2}$	5.5
			2419	1780	$1_{2}^{+}$		$1.0 \times 10^{-2}$	$9.8 \times 10^{-3}$	$8.8 \times 10^{-2}$
			2483	1844	$3_{4}^{+}$	$5.5 \times 10^{-2}$	$1.3 \times 10^{-2}$	$8.9 \times 10^{-3}$	59
			2505	1866	$1_{3}^{+}$		$7.3 \times 10^{-3}$	$2.0 \times 10^{-2}$	0.11
			2543	1904	$2_{4}^{+}$	$1.6 \times 10^{-2}$	$7.5 \times 10^{-3}$	$9.2 \times 10^{-3}$	23
			2630	1991	$3_{5}^{+}$	$8.8 \times 10^{-3}$	$3.6 \times 10^{-3}$	$9.6 \times 10^{-3}$	19
			2723	2084	$4_{4}^{+}$	$2.3 \times 10^{-2}$	$2.0 \times 10^{-2}$	$9.9 \times 10^{-3}$	75
			2762	2123	$6^{+}_{2}$		$5.0 \times 10^{-2}$	$1.5 \times 10^{-2}$	2.6
			2914	2275	$5^{+}_{3}$	$1.1 \times 10^{-2}$	$1.0 \times 10^{-2}$	$1.7 \times 10^{-2}$	77

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TABLE IV. The recommended reaction rate  $N_A \langle \sigma v \rangle$  as a function of temperature T (GK) from this work, together with  $1-\sigma$  uncertainties (higher and lower).

$T_9$	$N_A\langle a$	$\sigma v \rangle (\text{cm}^3/\text{s/mole})$	)
	Recommended	Lower	Upper
0.1	1.497e-19	8.583e-20	2.422e-19
0.2	8.661e-12	8.121e-12	1.082e-11
0.3	1.666e-08	1.183e-08	2.084e-08
0.4	1.244e-06	7.796e-07	1.746e-06
0.5	1.859e-05	1.237e-05	3.606e-05
0.6	1.156e-04	8.254e-05	2.911e-04
0.7	4.677 e-04	3.197e-04	1.357 e-03
0.8	1.529e-03	8.877e-04	4.423e-03
0.9	3.880e-03	2.009e-03	1.149e-02
1.0	8.951e-03	4.287e-03	2.551e-02
1.5	1.583e-01	8.221e-02	3.294e-01
2.0	9.620e-01	6.177 e-01	$1.655e{+}00$

 $_{322}$   $^{56}\rm{Ni}$  at high temperatures. Finally, the uncertain 78  $\pm$  $_{323}$  17 %  $\beta$ -delayed proton branch of  $^{57}$ Zn [24] directs the <sup>324</sup> reaction flow back to <sup>56</sup>Ni and needs to be better con-325 strained. To explore the effect of these uncertainties, we  $_{326}$  considered two scenarios of maximal and minimal favor- $_{350}$  This work presents the first experimentally- $_{327}$  ability for the bypass. In the case of the maximal (min- $_{351}$  constrained  $^{55}Ni(p,\gamma)^{56}Cu$  thermonuclear reaction  $_{328}$  imal) favorability: (1) the  ${}^{56}$ Cu(p, $\gamma$ ) rate was increased  $_{352}$  rate, utilizing 5 newly identified excited states in  ${}^{56}$ Cu, 329 (decreased) by a factor of 100, the expected uncertainty 353 a new theoretically-constrained reaction Q-value, and  $_{330}$  of a shell-model rate; (2) the  $^{55}$ Ni(p, $\gamma$ ) rate was increased  $_{354}$  a new shell-model calculation of  $\gamma$ - and proton-widths.

<sup>331</sup> (decreased) by the uncertainty reported in this work; (3) <sup>332</sup> the  $\beta$ -delayed proton-emission rate of <sup>57</sup>Zn was decreased <sup>333</sup> (increased) by the uncertainty reported by [24].

<sup>334</sup> Fig. 7 shows the resulting fraction of the reaction flow <sup>335</sup> that bypasses <sup>56</sup>Ni as a function of temperature and pro-336 ton density for the two scenarios. In the scenario with <sup>337</sup> the most favorable nuclear physics assumptions, <sup>56</sup>Ni is 338 significantly bypassed for temperatures in the range of about 0.4 - 1.2 GK and proton densities above  $10^4$  g/cm<sup>3</sup>. 340 These are within the range of typical X-ray burst con-<sup>341</sup> ditions, with peak temperatures of 1-2 GK and proton  $_{342}$  densities up to  $10^6$  g/cm<sup>3</sup>. On the other hand, for 343 the most unfavorable scenario proton densities in ex- $_{344}$  cess of  $10^6$  g/cm<sup>3</sup> are required for the reaction flow to <sup>345</sup> bypass <sup>56</sup>Ni. Therefore, in the favorable scenario, <sup>56</sup>Ni <sup>346</sup> would be partially bypassed by the rp-process in all X-<sup>347</sup> ray bursts, while in the unfavorable scenario the full rp-<sup>348</sup> process would always pass through <sup>56</sup>Ni.

#### CONCLUSION VI.

TABLE V. REACLIB fit coefficients for our recommended  $^{55}\mathrm{Ni}(\mathrm{p},\gamma)$  reaction rate.

$E_x$	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$
1224	1.052854	-6.805068	7.127737E-01	-1.049583	5.849955E-02	-3.234916E-03	-9.787774E-01
Other	-5.223069E+01	-9.902812	1.336866E + 02	-7.623392E+01	-8.335959E-01	2.019964E-01	$6.914259E{+}01$
1038	-5.177171	-4.627019	-7.755680E-02	8.817104 E-02	-4.086783E-03	1.981643E-04	-1.549327
826	-2.601956E + 01	-2.170262	4.521332E-04	-6.347735E-04	3.674535E-05	-2.248394E-06	-1.499681



FIG. 7. (Color online:) Phase space diagram showing the region where the bypass may be effective, demonstrating the impact of the remaining nuclear physics uncertainties. The color and contours indicate the strength of the bypass. The most unfavorable (left) and most favorable (right) conditions are chosen to demonstrate the full range of the uncertainties.

<sup>355</sup> Below a temperature of 0.5 GK, the experimental data  $_{356}$  reduce the rate uncertainty from a factor of  $10^5$  to  $10^2$ 357 at 0.1 GK and by almost an order of magnitude at  $0.5~\mathrm{GK}$  . The dominant remaining uncertainty is the 358  $_{\rm 359}$  reaction Q-value due to the unknown mass of  $^{56}{\rm Cu}.$  $_{360}$  For temperatures above 0.5 GK, the reaction rate is 361 dominated by higher-lying resonances that have not been determined experimentally. With the new data, 362 <sup>363</sup> and using a detailed network analysis, we find that within remaining uncertainties the rp-process can bypass 364 the <sup>56</sup>Ni waiting point for typical x-ray burst conditions 365 with a bypass branch as high as  $\sim 40\%$ . We also identify 366 additional nuclear physics uncertainties in the  ${}^{56}Cu(p,\gamma)$ 367 reaction rate, the  ${}^{57}$ Zn mass, and the  ${}^{57}$ Zn  $\beta$ -delayed 368 proton emission branch that need to be addressed. 369 370

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