

This is the accepted manuscript made available via CHORUS. The article has been published as:

# Reexamining the nuclear structure of $^{154}\text{Gd}$ in the dynamic pairing plus quadrupole model

J. B. Gupta and J. H. Hamilton

Phys. Rev. C **95**, 054303 — Published 5 May 2017

DOI: [10.1103/PhysRevC.95.054303](https://doi.org/10.1103/PhysRevC.95.054303)

# Reexamining the nuclear structure of $^{154}\text{Gd}$ in the dynamic pairing plus quadrupole model

J.B. Gupta<sup>+</sup>\* and J.H. Hamilton<sup>++</sup>

<sup>+</sup>Ramjas College, University of Delhi, Delhi-110007 India

<sup>++</sup>Physics Department, Vanderbilt University, Nashville, TN, U.S.A.

Received Aug. 15, 2016, revised Oct 26, Dec.23, 2016 and Feb.20. 2017

## Abstract

In a previous study of the collective multi-phonon bands in  $^{154}\text{Gd}$ , using the microscopic dynamic pairing plus quadrupole model, data for eight  $K$ -bands were analyzed. In the last four decades, its decay scheme is significantly revised and the nuclear theory has undergone a significant change. Special focus is on new weak intensity transitions in several bands and on the reassigned levels in its decay scheme. The present study represents a detailed revised analysis of the collective even parity bands below 2.1 MeV. Also, a discussion is given on the nature of the  $K^\pi = 0^+$  excited bands, validity of band mixing approach and of the assumption of shape coexistence of  $\beta$ -band with ground band. Comparison is made with the X(5) analytical symmetry and the algebraic interacting boson model predictions. Discussion of the  $2n$  transfer reactions is given. The validity of the multiphonon view of the  $K^\pi = 4^+$  and  $2_2^+$  bands is also studied.

PACS numbers 21.60.Ev, 21.60.Cs, 21.10.Re, 27.70+q

Key words:

[NUCLEAR STRUCTURE, Collective multiphonon bands,  $^{154}\text{Gd}$ . microscopic theory]

\*e-mail jbgupta2011@gmail.com

## I. INTRODUCTION

The nucleus  $^{154}\text{Gd}$  is softly deformed, and is one of the four  $N = 90$  isotones ( $Z=60-66$ ), which are considered to be good candidates for the analytically solvable critical point symmetry  $X(5)$  [1], lying at the edge of the spherical to deformed or  $U(5)$  to  $SU(3)$  transition path [2-5]. This nucleus serves as a good laboratory for the study of the multi-phonon collective bands in the framework of the unified collective model of Bohr-Mottelson [6], as well as for the algebraic Interacting Boson Model (IBM-1 and IBM-2) [7], and microscopic theories. Earlier in (1979), the band structures of  $^{154}\text{Gd}$  were studied in the microscopic theory of dynamic pairing plus quadrupole model [8], wherein the decay character of the eight even-parity collective bands of this nucleus were analyzed [9]. Much new data on this nucleus have become available in the last four decades and many new concepts in nuclear theory have evolved meanwhile. Thus a revisit of its structure in the microscopic view, vis-a-vis the new data and the current concepts, is warranted.

In the early seventies, there was much interest in the  $\beta$ -g,  $\gamma$ -g and  $\gamma$ - $\beta$  band mixing anomalies (to get consistent band mixing parameter  $Z_\gamma$  and  $Z_\beta$  for a band) in Sm and Gd isotopic chains and other deformed rare earths. The possibly larger  $MI$  admixtures in  $\beta$ -g transitions, which could partially resolve these anomalies for  $\beta$ -bands, were not found. A review of these anomalies may be seen in Ref. [10]. Sousa *et al.* [11] from  $^{154}\text{Tb}$  decay, and Gupta *et al.* [12] from  $^{154}\text{Eu}$  decay (using large volume Ge(Li) detectors), extended the study of the nucleus  $^{154}\text{Gd}$  to 25 even parity levels below 2.1 MeV, up to  $I^\pi \leq 6^+$ , spread over seven vibrational bands. The understanding of the structure of these multiphonon bands presented a challenge for the collective nuclear theories.

As stated above, the DPPQ model was employed to interpret the multiphonon structures in terms of full  $\beta$ - $\gamma$  vibration mixings [9]. In that work, besides the six bands, viz.  $K^\pi = 0_1^+$  ground band,  $K^\pi = 0_2^+$   $\beta$ -band,  $K^\pi = 2_1^+$   $\gamma$ -band,  $K^\pi = 0_3^+$   $\beta\beta$ -band,  $K^\pi = 2_2^+$   $\beta\gamma$ -band and  $K^\pi = 4_1^+$   $\gamma\gamma$ -band, predictions for two other higher excited bands ( $\gamma\gamma$   $K^\pi = 0_4^+$ ,  $3\beta = 0_5^+$ ) were made, for which scanty data were available or no confirmed spin parity were assigned. Static character of two other bands ( $\gamma + 2\beta$   $K^\pi = 0_6^+$ ,  $2\gamma + \beta$   $K^\pi = 4_2^+$ ) were also given. Due to lack of data, some predictions were just based on theory. In the present

study, we reexamine the DPPQ model predictions with the presently available data [13, 14]. In Sec. II, we review the present experimental information on new data and involving 2 nucleon transfer reactions. In Sec. III, a brief review of the DPPQ model is given to introduce the basic concepts and the terminology. The Interacting Boson Model (IBM) is also introduced there. The experimental data are compared with predictions from theory in Sec IV, where the results are analyzed. A summary is given in Sec. V. For convenience, we use the nomenclature of multiphonon  $\beta$ ,  $\gamma$  combinations, even if these need to be justified yet.

## II. NEW DATA ON THE LEVEL STRUCTURE OF $^{154}\text{Gd}$

In the last four decades, with the advances in experimental facilities, especially in the detector technology and computerized multi-channel analysis and storing of data, much additional information is available. Very low in energy and/or weak in intensity  $\gamma$ -rays have been identified and resolved in the decay scheme of  $^{154}\text{Gd}$ . The NDS 110 of 2009 [14] lists several new transitions in the decay scheme of  $^{154}\text{Gd}$ . Also several doubtful placements of transitions have been corrected.

Tonev *et al.* [2] determined the life times of some low spin levels in  $^{154}\text{Gd}$  and compared their results with the predictions of X(5) symmetry approximation method. Kulp *et al.* [15] used a high efficiency  $8\pi$  spectrometer to study the spectrum of  $^{154}\text{Gd}$ .

A major change concerns the assignment of the  $K^\pi = 0_3^+$  band head. From the  $^{154}\text{Eu}$  decay, Meyer [16] had assigned the 1292.7 keV level to  $0_3^+$  by erroneously assuming a 612 keV  $\gamma$ -ray ( $I_\gamma < 0.016$ ). The 612 keV  $\gamma$ -ray was not found in [12]. In the  $^{154}\text{Tb}$  decay, Sousa *et al.* [11] from the conversion electron data corresponding to a 615 keV  $E0$  transition to the 680.7 keV  $0_\beta$  state assigned a 1295.8 keV level and interpreted it as  $2\beta$  phonon band head [11]. A state at 1292.7 keV or 1295.8 keV was not observed in the (d, d') work [17] either, as noted earlier [12].

Kulp *et al.* [15] also excluded the 1292.7 keV and 1295.8 keV level assignments and instead assigned a new 1182 keV level to  $0_3^+$ . This also led them to question the interpretation of the  $0_3^+$  state as a good example of a  $2\beta$  phonon state [11, 16]. In view of the importance of this issue, it needs further consideration.

The  $2^+$  level at 1418.2 keV is invariably included in the  $K^\pi = 0_3^+$  band. A strong  $E2$  transition (602.7 keV) to the  $2_\beta$  level was confirmed in coincidence [12] and by  $E0$  decay to the 815.5 keV  $2_\beta$  level. In the singles spectrum too, a 1418.2 transition was seen by Gupta *et al.* [12].

The  $I^\pi = 4^+$  member of the  $0_3^+$  band assigned previously to a 1698.3 keV level in (d, d') work [17], is also ambiguous. Gupta *et al.* [12] did not find any of the reported de-exciting  $\gamma$ -rays [16] in the singles or coincidence work. Instead of the 1698.3 keV  $4^+$  level, a 1701 keV level was proposed to be the  $4^+$  member of this band from the study of decay of  $^{154}\text{Tb}$  in Ref. [18]. A detailed analysis for this assignment is given in [15, 18]. The study of the nature of the  $0_3^+$  band is of current interest, in the general context of the  $2_\beta$  phonon band.

Also the assignments of the members of the higher lying  $0_4^+$  band have ambiguities. A 2081 keV level assigned previously to  $(2^+, 3, 4^+)$  in [11], is now replaced by a 2080.2 keV level assigned to  $4^+$ , belonging to a new  $0_4^+$  band at 1573.9 keV and a 1716.1 keV  $2^+$  level [15,13,14]. Here we note that the high resolution spectroscopy has enabled one to distinguish the 2080.230 keV level from the 2080.791 keV  $I^\pi = 3^-$  level [13].

The 2277.1 keV ( $2^+$  or  $3^+$ ) level, taken as the  $I^\pi = 2^+$  member of a  $K^\pi = 0_5^+$  band in [9], is replaced by 1775.4 keV  $I^\pi = 2^+$  one, built on the band head  $0_5^+$  at 1650.34 keV [13].

The  $I=2, 3$  and  $4$  members of the band at 1531 keV level are firmly assigned and the predominant  $K=2$  character is confirmed from the decay characteristics (see [12]). Kulp *et al.* [15] questioned the multi-phonon nature of the  $K^\pi=2_2^+$   $\beta\gamma$  band at 1531 keV. They cited the results of our previous calculation [9] in the DPPQ model with inconclusive arguments and emphasized the need of a fresh analysis [15]. Further, Wu *et al.* [19] studied the issue of anharmonicity of the  $K^\pi=4^+$   $\gamma\gamma$  band in  $^{154}\text{Gd}$  and other nuclei of this region. The issue of anharmonicity of the multi-phonon bands needs further studies to reach to a consistent view of the problem

Thus these revisions of the level structures require a revisit of the predictions in the microscopic DPPQ model. For the sake of a full view of the nuclear structure of  $^{154}\text{Gd}$ , and of the ambiguities described above, we need to present the decay structure of all the

known even parity bands below 2.1 MeV in  $^{154}\text{Gd}$ , along with information on 2 nucleon transfer reactions. Partial level structure, as adopted here, is illustrated in Fig. 1a, data taken from Ref. [13, 14].

### III. THEORY

#### A. The microscopic DPPQ model

A detailed description of the Kumar-Baranger method of treating the pairing plus quadrupole model is given in Ref. [20, 21], which includes the improvement in the method of solution of the collective Hamiltonian  $H_{\text{coll}}$ , to the original work of Ref. [8] (and the earlier works of Kumar-Baranger). Basically, the DPPQ model is a microscopic theory in the sense that the parameters of the  $H_{\text{coll}}$  are derived from  $H_{\text{PPQ}}$ , and not obtained *ab initio* by a fit to the experimental data, as done in several other microscopic approaches.

The PPQ Hamiltonian is given by Eq. (1)

$$H_{\text{PPQ}} = H_{\text{sph}} + H_{\text{Q}} + H_{\text{P}}. \quad (1)$$

Here  $H_{\text{sph}} = \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$ , with  $\epsilon_{\alpha}$  spherical single particle energies of an isotropic harmonic oscillator, and  $c_{\alpha}^{\dagger}$ ,  $c_{\alpha}$ , the spherical harmonic oscillator operators, represent the average spherical field part. The term  $H_{\text{Q}}$  represents the quadrupole part of the Hamiltonian and  $H_{\text{P}}$  the monopole pairing part. The quadrupole term includes the quadrupole (particle-hole coupled to  $J=2$ ,  $T=0$ ) components of the residual interaction and the pairing term represents the monopole pairing (particle- particle, and hole-hole coupled to  $J=0$  and  $T=1$ ). Separable Elliot type quadrupole-quadrupole interaction is employed.

A major contribution of Baranger and Kumar to the unification of the spherical vibration and deformed rotation-vibration theory [8], was the adoption of the Hartree-Bogoliubov transformation HBT [22] technique in treating the quadrupole and pairing interaction on equal footing, i.e. the use of the transformation of the spherical single particle operators  $c_{\alpha}^{\dagger}$ ,  $c_{\alpha}$  to deformed quasi-particle operators  $a_i^{\dagger}$ ,  $a_i$  (Eq. 2)

$$a_i^{\dagger} = \sum_{\alpha} (A_{\alpha}^{\dagger} c_{\alpha}^{\dagger} + B_{\alpha}^{\dagger} c_{\alpha}), \quad (2)$$

where  $A$ ,  $B$  are the wave functions, and for  $a_i$ , its complex conjugate (cc), a similar expression holds.

The  $Z=40$ ,  $N=70$  core is treated as inert core for this region. All the active nucleons over the two major shells of  $n=4$ , 5 for protons and  $n=5$ , 6 for neutrons are included. The quadrupole strength  $\chi$  and pairing strengths  $g_p$ ,  $g_n$  are fixed globally for the broad region. Then the deformed quasiparticle energies and wave functions are derived for a mesh of 92 points in the  $(\beta, \gamma)$  space ( $\beta=0-0.5$ , and  $\gamma=0^\circ-60^\circ$ ). These are used to derive the 7 parameters of the collective Hamiltonian:

$$H_{\text{coll}} = V(\beta, \gamma) + T_{\text{vib}}(\beta, \gamma) + T_{\text{rot}}(\beta, \gamma). \quad (3)$$

$$\text{where } T_{\text{vib}}(\beta, \gamma) = \frac{1}{2} B_{\beta\beta} (\partial\beta/\partial t)^2 + B_{\beta\gamma} \beta (\partial\beta/\partial t) (\partial\gamma/\partial t) + \frac{1}{2} B_{\gamma\gamma} (\beta \partial\gamma/\partial t)^2 \quad (4)$$

$$\text{and } T_{\text{rot}}(\beta, \gamma) = \frac{1}{2} \sum_k \theta_k(\beta, \gamma) (\hbar\omega_k)^2, k=1,2,3. \quad (5)$$

The scalar potential  $V(\beta, \gamma)$  of Eq. (3), and the other six kinetic terms (three mass parameters  $B_{\mu\nu}$  and three moments of inertia  $\theta_k$ ) are derived via the time dependent Hartree-Bogoliubov (TDHB) treatment of the pairing plus quadrupole (PPQ) Hamiltonian (Eq. 1), as functions of  $(\beta, \gamma)$ . The three moments of inertia  $\theta_k(\beta, \gamma)$  are given by

$$\theta_k(\beta, \gamma) = F_B \sum_{ij\tau} (u_i v_j - v_i u_j)^2 \langle i | J_k | j \rangle (E_i - E_j)^{-1}. \quad (6)$$

where  $F_B$  is the inert core renormalization factor and  $J_k$  are the single particle angular momentum operators. The three mass coefficients  $B_{\mu\nu}(\beta, \gamma)$  depend on the dynamic treatment of the quasi-particle amplitudes  $u_i$ ,  $v_i$ , quasi-particle energies  $E_i$ , and Fermi level  $\lambda_\tau$  (see Ref. [20] for the detailed expressions). It employs the cranking formulae.

Also the  $E2$ ,  $M1$  and  $E0$  operators are determined microscopically. The  $H_{\text{xoll}}$  are set up and the collective Hamiltonian is solved for each mesh point. Then a summation over the full  $(\beta, \gamma)$  space provides the dynamics of the Hamiltonian. The DPPQ model includes the dynamics of the 5-dimensional quadrupole motion and of the pair fluctuation.

A second unique feature of the dynamic PPQ method is that no fixed shape of the nucleus is assumed. Instead, the nucleus takes its own shape for each spin state. A third speciality is the full  $(\beta, \gamma)$  freedom of the mass coefficients  $B_{\mu\gamma}$  of the kinetic vibration and rotation terms in Eqs. (4, 5). However, slight variation in the quadrupole strength  $\chi = X \times A^{-1.4}$  of the quadrupole interaction in  $H_Q$  of Eq. (1) and the value of mass coefficient of the inert core normalization factor  $F_B$  for the kinetic energy terms of the Eqs. (4, 5) may

be adjusted by a few percent to approximately obtain the  $E(2_1^+)$  energy for the given nucleus.

### B. The interacting boson model

The interaction boson model IBM-1 treats the correlated valence nucleon pairs as proton bosons and neutron bosons equivalently [7]. In IBM-2, the proton bosons and neutron bosons are treated separately. The three limiting symmetries are labeled as U(5), SU(3) and O(6). In the MULT form with four terms,  $H_{\text{IBM-1}}$  can be written as:

$$H_{\text{IBM-1}} = \epsilon n_d + k Q \cdot Q + k' L \cdot L + k'' P \cdot P. \quad (7)$$

Here the first term represents the boson energy, the second term represents the quadrupole interaction of the  $L = 2$  d-bosons. The third term represents the O(3) angular momentum contribution and the fourth term represents the pairing operator interaction. The quadrupole operator is given by

$$Q = (d^+ s + s^+ d)^{(2)} + \chi (d^+ d)^{(2)}. \quad (8)$$

The boson energy operator  $n_d = [d^+ \cdot d]$ ,  $L = \sqrt{10} [d^+ \times d]^{(1)}$  and  $P = (1/2) ([d \cdot d] - [s \cdot s])$ . (Note for publishing: use tilde signs over d·d here for P term).

The coefficients of the four terms are varied to yield the least RMS deviation for the set of input energies (up to  $6^+$ ) in experiment. A computer Program *PHINT* [23] has been used for setting up and solving the IBM Hamiltonian. For  $^{154}\text{Gd}$  we set the parameters in Eq. (7) as  $\epsilon=350$  keV,  $2k=QQ= -26.7$  keV,  $2k'=ELL= 12.7$  keV,  $k''/2=PAIR= 1.9$  keV. For the transition operator  $T(E2)= e_B Q$ ,  $e_B=0.145$  eb and  $\chi= -0.20/0.145$  was adopted.

## IV. COMAPARISON OF DATA WITH THEORY

### A. The level structure of $^{154}\text{Gd}$

The partial level energy spectra of  $N=90$  isotones, depicting the near constancy of ( $2_g$ ,  $4_g$ ,  $0_\beta$  and  $2_\beta$ ) energies with varying  $Z$ , especially for (Nd, Sm, Gd and Dy) are illustrated in Fig. 2. These four isotones are considered to be the good examples of the X(5) analytical symmetry [2-5]. The energy of  $2_\gamma$  varies slightly with proton number  $Z$ . The energy difference  $E(2_\beta-0_\beta)$  is almost constant. Also it is almost equal to  $E(2_g)$  as



apparent in Fig. 3, where the  $N=90$  data lie on the diagonal for Ba-Dy ( $N = 88-96$ ). (The difference is largest for Nd with  $E(2_g) = 130$  keV). The continuous line is the diagonal. Thus the rotational moments of inertia  $\theta (=3/E_2)$  of the two bands are almost equal.

The nature of  $0_2^+$  states in deformed nuclei and of the rotational bands built on them, have been the topic of controversy in the last two decades. The proposed shape co-existence of the deformed ground band and vibrational  $\beta$ -band in  $^{152}\text{Sm}$  led to increased experimental and theoretical activity. Zamfir *et al.* [24] used the IBM prediction of weak  $B(E2, 2_\gamma \rightarrow 0_\beta)$  as an evidence of spherical vibrator shape of the  $\beta$ -band. The same views are adopted in a review of these issues in Ref. [25]. These views were supported from the large cross section amplitudes obtained in the  $^{150}\text{Sm}(t, p)$  reaction, [26], which led to the suggestion that the excited  $0^+$  states may be spherical vibrational states. Kumar [21] explained the large cross section for  $2n$  transfer for the excited state, without assuming a spherical shape in  $^{152}\text{Sm}$ . Jolie *et al.* [27] gave a different view on the shape coexistence proposition. Burke [28] analyzed the reasoning cited in [24], based mainly on the weak  $\gamma$ - $\beta$   $E2$  transition in  $^{152}\text{Sm}$ , and pointed out the inconsistency in assuming the shape co-existence. Later, Clark *et al.* [29], using band mixing and the results of DPPQM [21] for  $^{152}\text{Sm}$ , obtained results in agreement with experiment and opposed the proposition of shape co-existence in  $^{150}\text{Nd}$  and  $^{152}\text{Sm}$ . According to recent findings [30], it is explained that a large  $2n$  cross section of an excited  $0^+$  states [26] arises due to shape difference between the target and the product nucleus. According to DPPQM almost the same deformation is obtained for the three  $K$ -bands in  $^{152}\text{Sm}$  and  $^{154}\text{Gd}$  [31, 9].

Recently, Gupta and Hamilton [31] reviewed the issue of the nature of  $K^\pi = 0_2^+$  bands in deformed nuclei for the  $A=140-180$  region illustrating the universality of  $\beta$ -vibration. They illustrated the regularities in the systematics of the level energies and  $B(E2)$  values, and explained the experimental fact of  $B(E2, 0_g \rightarrow 2_\beta)$  being smaller than  $B(E2, 0_g \rightarrow 2_\gamma)$ . A prime cause of this difference is the lesser overlap of  $0_2^+$  wave function  $A_{200}$  with the ground state wave function  $A_{100}$ , leading to smaller intrinsic matrix elements, even if the quadrupole moments are equal. The above cited systematics includes the Figs. 2 and 3 here. Further justification based on  $B(E2)$  values are given below.

### B. $K$ -band structure of the states in $^{154}\text{Gd}$

The partial energy level spectrum of  $^{154}\text{Gd}$  up to  $I=6$  in the lowest seven even parity  $K$ -bands are illustrated above, in Fig. 1a. The DPPQ model spectrum is illustrated in (Fig. 1b and Table I). The spectrum details are given here in reference to their identification in the model calculation. As stated in [9], the energy scale for the calculated vibrational bands is rather expanded (Fig. 1b).

TABLE I. The energy spectrum of  $^{154}\text{Gd}$  is compared with the DPPQ model predictions. The level energies are in keV. Only even parity values are calculated.

Band		$0^+$	$2^+$	$3^+$	$4^+$	$5^+$	$6^+$
$K^\pi=0_1^+$	Exp	0	123		371		718
	DPPQ	0	126		313		585
$K^\pi=0_2^+$	Exp	680	815		1047		1366
	DPPQ	984	1180		1390		1682
$K^\pi=2_1^+$	Exp		996	1128	1264	1433	1607
	DPPQ		1505	1602	1776	1895	2178
$K^\pi=0_3^+$	Exp	1182	1418		1701		
	DPPQ	1842	2156		2487		
$K^\pi=2_2^+$	Exp		1531	1661	1789		
	DPPQ		2520	2688	3084		
$K^\pi=4_1^+$	Exp				1646	1770	1912
	DPPQ				2844	2981	3402
$K^\pi=0_4^+$	Exp	1574	1716		2080		
	DPPQ	2721	3042		3364		
$K^\pi=0_5^+$	Exp	1650	1775				
	DPPQ	2967	3315				

The vibrational energy scale may be affected by the pairing strength used here. Also the effect of the spherical single particle energies used is not known. No variation of the inertial factor for the rotation and vibrational terms exclusively is allowed, to maintain the inertia tensor symmetry relations. Recently Li *et al.* [32] studied the effect of the pairing strength in their RMF+BCS treatment of  $^{150}\text{Nd}$  using covariant density functional theory. As expected, the increasing pairing strength gave shallower PEC minima, and higher  $E(0_2^+)$ , lower  $R_{4/2}$  etc. In DPPQM, on account of the dynamic summation over the whole  $(\beta, \gamma)$  space, the effects may be more complex. So pairing strength is not varied.

The  $K=0$  and  $K=2$   $K$ -component admixtures predicted in DPPQM for  $I=2$  states of  $g$ -,  $\beta$ - and  $\gamma$ - bands exceed 99%, signifying pure  $K$  -bands. For  $I = 4$  states also, the  $K=0$  components are 99.8 %, and 96 % in ground and  $\beta$ -band respectively. The  $K=2$  component in  $4_3^+$  state exceeds 95%.with 3.9%  $K=0$  admixture in the  $\gamma$ -band. In the following, the numbers in the parenthesis for  $K$ -admixture, represent in percent: the  $K=0$ ,  $K=2$  (for  $I=2$ ), and  $K=4$  (for  $I=4$ ) and  $K=6$  (for  $I=6$ ), in that order.

The  $2_4^+$  state has (81.4+18.6)%  $K=0$  and  $K=2$  admixture associated with the 1418 keV  $2^+$  level, based on 1182 keV  $0_3^+$   $2\beta$ -band [13,14,15]. The  $4^+$  state has (70+22+8)%  $K=0$ , 2 and 4 admixture which we associate with the newly assigned 1701 keV  $4^+$  state. The  $2_5^+$  DPPQM state has (21+79)%  $K=0$ , 2 admixture, which we associate with  $K=2$   $\beta\gamma$  band based on the 1531 keV  $2^+$  state [13,14,15]. The sixth  $2^+$  DPPQM state with (95.7+4.3)%  $K=0$ , 2 admixture, is associated with the 1716.1 keV state, based on 1574.keV  $0_4^+$  state. Seventh  $2^+$  state in DPPQM has (44+56)%  $K=0$ , 2 admixture.

The fifth  $4_5^+$  state of DPPQM (at 2844 keV), with (22+13+65)%  $K=0$ , 2 and 4 admixture, is associated with  $K^\pi = 4^+ \gamma\gamma$  band head at 1645.8 keV. The sixth  $4^+$  state in DPPQM (at 3084 keV) with (9+63+28)%  $K=0$ , 2 and 4-component admixture corresponds to the 1789.2 keV  $4^+$  of the  $K=2$   $\beta\gamma$  band. The seventh  $4^+$  DPPQM state (at 3364 keV) with (94+4+2)%  $K=0$ , 2 and 4 admixture, is associated with 2080.2 keV state member of  $K^\pi=0_4^+$  band at 1574 keV. The  $K$ -components in these states are also useful in determining the degree of band mixing of the collective bands and their identification with respective  $K$ -bands (Figs.1a and 1b).

Second  $5^+$  of DPPQM (at 2981 keV) has (57+43)%  $K=2$  and 4 component admixture and  $5_3^+$  (at 3219 keV) has (40+60)%  $K=2$ , 4 admixture respectively. Fourth  $6^+$  in DPPQM (at 2893 keV) has (64+29+5+2)%  $K=0$ , 2, 4 and 6-component admixture. Fifth  $6^+$  (at 3402 keV) has (28+48+21 +3)%  $K=0$ , 2, 4 and 6-admixture, associated with the 1912 keV state. Such large admixtures present the band identity problem.

### C. The potential energy surface

In the DPPQ model [20], the potential energy function of the nucleus is given by Eq. (7)

$$V(\beta, \gamma) = \sum_i \tau_i v_i^2 \eta_i - \sum_\tau g_\tau^{-1} \Delta_\tau^2 + (1/2) \chi^{-1} \beta^2. \quad (7)$$

Here  $i$  represents all the deformed quasiparticle (dqp) states of the two oscillator shells,  $v_i^2$  are the occupation probability of a dqp state,  $\eta_i$  the dqp energy,  $g_\tau$  is the pairing strength ( $\tau = n, p$ ) and  $\Delta_\tau$  are the calculated pairing gaps. In the last term, the coefficient  $\chi = X \times A^{-1.4}$  (MeV) is the quadrupole force strength. Here we take  $X=70.0$  (a regional value).

The potential energy curve (PEC)  $V(\beta, \gamma=0^\circ)$  in  $^{154}\text{Gd}$  from the DPPQ model is depicted in Fig. 4. The  $V_{\min}$  lies on the prolate side at  $\beta=0.24$ ,  $\gamma=0^\circ$ , about 3.0 MeV below the spherical barrier at  $\beta=0$ . The horizontal line marked as ZPE denotes the zero point energy and lies  $\sim 1.8$  MeV above the  $V_{\min}$ . It signifies a spread from  $\beta=+0.1$  to  $+0.4$ , representing softness in the  $\beta$  degree of freedom of the transitional nucleus at  $N=90$ . The oblate minimum lies at  $\beta = -0.14$  and is only 0.34 MeV deep. The 0.680 MeV  $0_2^+$  and 0.815 MeV  $2_\beta$  and 0.996 MeV  $2_\gamma$  levels lie within the prolate part of the PES. The higher bands are mostly above the spherical barrier and oblate minimum, and hence exhibit considerable mixing of prolate, oblate and spherical shapes, resulting in large  $K$ -admixture.

The PES for  $^{154}\text{Gd}$  differs from the infinite square well potential  $u(\beta)$  assumed for the X(5) symmetry, with a separation of total  $V(\beta, \gamma) = u(\beta) + v(\gamma)$  in [1, 2]. In fact the same holds good for the other  $N=90$  isotones as well. The PEC minimum here is not wholly flat and is shifted towards the prolate side significantly. Also there is a contribution of the oblate side, corresponding to  $\gamma=60^\circ$ . In view of this, it is rather interesting to predict the properties of the  $N=90$  isotones from the assumptions for X(5) symmetry [2-5]. This difference is not accounted for in the confined beta soft (CBS) model of Pietralla and Gorbachev [33]. First, the CBS model applies to a deformed nucleus lying beyond X(5) towards the rigid deformed limit. Secondly, it has no  $\beta=0$  part and the oblate part, which in fact represent the  $\gamma=60^\circ$  side. The nucleus need not move across the  $\beta=0$  barrier, but can move rather directly along the  $\gamma$ -degree of freedom. The  $\beta$  and  $\gamma$  degrees of freedom in a nucleus do not represent independent variables, but are rather coupled. This is apparent on a  $(\beta, \gamma)$  space diagram for the potential  $V(\beta, \gamma)$  as well as for the wave functions of the nuclear collective states. An explicit explanation of this aspect of the PES would be interesting.

Using relativistic HFB with NL3 force, Fossion *et al.* [34] illustrated the  $V(\beta, \gamma)$  PES for  $N=90$  Nd, Sm and Gd isotones and noted the difference from the X(5) assumption of a flat square well potential  $u(\beta)$ . Li *et al.* [35] obtained a similar PES using relativistic Energy Density Functionals (EDF) for  $N=90$  isotones and noted the difference from X(5) symmetry. Jolie *et al.* [27] using 2-parameter  $H_{IBM}$  obtained the deformed minimum along with spherical maximum for  $^{152}\text{Sm}$ , as opposed to a double minimum in Ref. [24].

#### D. B(E2) values and other moments from DPPQM

##### 1. Ground, $\beta$ and $\gamma$ bands

Besides the energy eigenspectrum, the static and dynamic moments and B(E2) values from the model, test the validity of the model. Some absolute B(E2) values for the lower three bands are compared with the experimental data in Tables II-V. For the DPPQ model, the quadrupole strength factor  $X$  was taken 70.0 and the inert core re-normalization factor  $F_B = 2.4$  [9]. The IBM-1 Hamiltonian parameters are given in Sec. III(B).

TABLE II. Static moments and absolute B(E2) values ( $e^2b^2$ ) in  $^{154}\text{Gd}$ .

Quantity	Expt. [14,2]	DDM [37]	DPPQ	IBM-1	X(5) [2]
$Q(2_1^+) \text{ eb}$	-1.82 4	-1.41	-1.79	-1.75	
$Q(2_2^+) \text{ eb}$			-1.73	-1.24	
$Q(2_3^+) \text{ eb}$			+1.57	+1.35	
$\mu(2_1^+) \mu\text{n}$	+0.91 4	0.93	0.69		
$B(E2, 2_1-0_1)$	0.77 2	0.49 <sup>b</sup>	0.772	0.76	0.77 <sup>a</sup>
$B(E2, 4_1-2_1)$	1.20 3	0.73 <sup>b</sup>	1.163	1.09	1.22
$B(E2, 6_1-4_1)$	1.30 4		1.34	1.17	1.50

a) Normalized value. b) If normalized, it yields 0.77 and 1.15.

TABLE III. Absolute B(E2) values ( $e^2b^2$ ) in  $^{154}\text{Gd}$ , for excitation from the ground state.

$I_i \ I_f$	Expt. [2,14]	DPPQ	IBM-1
$0_g \ 2_g$	3.85 15	3.86	3.82
$2_\beta$	0.022 2	0.018	0.0025
$2_\gamma$	0.140 8	0.139	0.135
$2_{02}^+$		0.0065	0.0035
$2_{5=\beta\gamma}$		0.0002	

The static quadrupole moment  $Q(2_1^+)$  and magnetic moment  $\mu(2_1^+)$  (Table II) are in agreement with experiment, providing the first order test of collectivity predicted from theory. Here we note that in the DPPQ model calculation, the electric charge parameter  $e_n$  and  $e_p=(1+e_n)$  were kept constant for the whole broad region of rare earths. The same is true for the magnetic moment. The intraband  $B(E2)$  values in the ground band also test the validity of the model and indicate the quadrupole collectivity of the nucleus. The normalized value of  $X(5)$  symmetry for  $I=4$  also agrees with data. But the  $X(5)$  value for  $I^\pi=6^+$  is 15% higher [2] for  $^{154}\text{Gd}$ . Same is true for  $I=8$  [2]. Since the  $X(5)$  symmetry values do not have a fitting parameter, these deviations indicate its approximations [2] for  $^{154}\text{Gd}$ . Also note the almost equal quadrupole moments of the  $2_2=2_\beta$  and  $2_g$  states in DPPQM. The dynamic deformation model [36] (without any fitting parameters) also yields values [37] in approximate agreement with data. Our IBM-1 calculation also yields  $B(E2)$  values for the gsb, with deviations increasing with spin. The quadrupole moment of  $2_3^+$  state is positive corresponding to  $K=2$  band.

TABLE IV. Absolute  $B(E2)$  values ( $\times 100$ ) ( $e^2b^2$ ) from  $\beta$ -band in  $^{154}\text{Gd}$ .

$I_i$	$I_f$	$E_\gamma$ [13]	Expt. [9] <sup>a</sup>	Expt. [2]	DPPQ	$X(5)$ [2]	IBM-1
$0_\beta$	$2_g$	557.6	26 4	22 2	22	49 <sup>b</sup>	4 <sup>c</sup>
$2_\beta$	$0_g$	815.5	0.48 4	0.45 5	0.37	1.5	0.05 <sup>c</sup>
	$2_g$	692.4	4.0 4	3.3 2	3.3	6.8	1.3
	$4_g$	444.5	12 1	10 1	8.7	28	2.0
$2_\beta$	$0_\beta$	134.8	49 16 <sup>c</sup>	27.0 18 <sup>d</sup>	76	61	51
$4_\beta$	$2_g$	924.5	0.35 8	0.27 2	0.59	0.75	0
	$4_g$	676.6	3.8 6	2.5 3	3.40	4.4	2.0
	$6_g$	329.9	11.9 25	7.1 4	8.6	21	1.2
	$2_\beta$	232.1	122 35 <sup>c</sup>	93 7	120	93	75
$6_\beta$	$4_g$	994.9	0.27 10		0.08		0.02
	$6_g$	648.3	3.3 10		3.6		2.6
	$4_\beta$	318.3			149		85

a) Values deduced from  $B(E2, 0_g \rightarrow 2_\beta)$  and the  $B(E2)$  ratios, in [9].

b)  $X(5)$  values, normalized to  $B(E2, 2_1^+ \rightarrow 0_1^+)$  yield values larger than in experiment. A second alternative is to normalize to  $B(E2, 0_2^+ \rightarrow 2_1^+)$  as done in Ref. [3] for  $^{152}\text{Sm}$ .

c) In IBM-1,  $\beta$ -g transitions are weak.

d) Revised value of  $B(E2, 2_\beta \rightarrow 0_\beta)$  from life time data [2] seems to lie on the lower side, half of the previous value. It is also far from DPPQM and X(5) values. For  $^{152}\text{Sm}$  the value is  $>0.50 e^2b^2$  [24].  
e) Ref. [38].

Absolute  $B(E2)$  values for excitation from ground state (Table III), provide important information regarding the nature of the different excited  $2^+$  states. The weaker excitation strength of the  $K^\pi=0_2^+$   $2_\beta$  state compared to  $2_\gamma$  state is a typical feature of their identity. These differences are well given from DPPQM. As cited above, a prime cause of this difference is the lesser overlap of  $0_2^+$  wave function  $A_{200}$  with the ground state wave function  $A_{100}$ , leading to smaller intrinsic matrix elements [31]. In IBM-1 (last column) excitation of  $2_\beta$  and  $2_{03}^+$  are weaker (a usual feature of IBM).

From their Coulomb excitation work, Tonev *et al.* [2] quoted  $B(E2, 0_g \rightarrow 2_\beta)$  equal to  $0.022(2) e^2b^2$ . In X(5) symmetry, the normalized value of  $0.075 e^2b^2$  relative to  $B(E2, 2_g \rightarrow 0_g)$  is rather higher by a factor of  $\sim 3$ . For the fourth  $2_4^+$  state, a factor of  $10^{-3}$  smaller strength and for the excitation of  $2_{\beta\gamma}$  a factor of  $10^{-4}$  weaker strength (Table III) are predicted in the DPPQ model calculation!

Some absolute  $B(E2)$  values for transitions from the  $K^\pi=0_2^+$   $\beta$ -band are cited in Table IV. The  $B(E2, 0_\beta \rightarrow 2_g) \sim 0.26(4) e^2b^2$  (average of two values in [9]), or  $\sim 52$  W.u is a good indicator of the collectivity of the  $\beta$ -band. This is reproduced in the DPPQ model. The X(5) value (normalized to  $B(E2, 2_1^+ \rightarrow 0_1^+)$  is  $0.49 e^2b^2$  as in Ref. [2]. The values of  $\beta$ -g  $B(E2)$ s from  $2_\beta$ , increasing with spins of the ground band, are also well given in theory (including X(5) and IBM-1). Also the strong intraband  $2_\beta \rightarrow 0_\beta$  transition (old value), only slightly smaller than  $B(E2, 2_1^+ \rightarrow 0_1^+)$  in ground band, is an indicator of good rotational features of the  $\beta$ -band. The same is true for  $B(E2, 4_\beta \rightarrow 2_\beta)$  and  $B(E2, 6_\beta \rightarrow 4_\beta)$ . The  $B(E2)$  values for transitions from  $4_\beta$  and  $6_\beta$  states also are well given in theory.

These results for  $^{154}\text{Gd}$  are in agreement with the conclusion by Clark *et al* [29] for  $^{152}\text{Sm}$  in regard to the shape coexistence and also on the nature of the  $K^\pi=0_2^+$  band in  $N=90$  isotones of (Nd-Gd). This also agrees with the observations on the nature of the  $K^\pi=0_2^+$  bands in deformed nuclei reviewed in Ref. [31], but differs from the views based on energy ratio  $R_{4/2}$  and (t, p) scattering cross section data and IBM-1 predictions in [24,

25]. The energy difference  $E(2_{\beta}-0_{\beta})$  is almost constant for  $N=90$  isotones, Also it is almost equal to  $E(2_g)$  as apparent in Fig. 3, where the  $N=90$  data lie on the diagonal for Ba-Dy ( $N = 88-96$ ). The values of  $\beta_{rms}$  for the states (up to  $I=6$ ) of the  $\beta$ -band are almost equal to the states in g-band (0.279 versus 0.262 for  $0_2^+$  state) (see Table II in [9] for higher states).

The DPPQ model  $B(E2)$  values for  $\gamma$ -g  $E2$  transitions from  $2_{\gamma}$  (Table V).are in good agreement with experimental values The weak intensity 315.6 keV  $2_{\gamma}-0_{\beta}$  transition of  $\sim 0.17$  W.u [24] (or  $0.0008$   $e^2b^2$ ) is also reproduced in DPPQM and IBM-1. This weak transition led to the anomalous concept of phase co-existence view in [24], but refuted in later works [27-29]. The  $B(E2)$  values for transitions from  $3_{\gamma}$  are similar in DPPQM, IBM and X(5), except for  $B(E2,3_{\gamma}-2_{\beta})$ .

TABLE V. Absolute  $B(E2)$  ( $e^2b^2$ )  $\times 100$  for  $\gamma$ -band in  $^{154}\text{Gd}$ .

$I_i - I_f$	$E_{\gamma}(\text{keV})$	Expt. [14]	DPPQ	IBM-1	X(5)
$2_{\gamma}-0_g$	996.3	2.86 22	2.78	2.7	2.86 <sup>b</sup>
$2_{\gamma}-2_g$	873.2	6.1 5	4.9	4.3	4.3
$2_{\gamma}-4_g$	625.3	0.86 7	0.44	0.65	0.22
$2_{\gamma}-0_{\beta}$	315.6	0.08 <sup>a</sup>	0.07	0.10	0.25
$3_{\gamma}-2_g$	1004.7		4.5	4.4	5.7
$3_{\gamma}-4_g$	756.8		3.1	3.1	2.4
$3_{\gamma}-2_{\gamma}$	131.5		114	99	143
$3_{\gamma}-2_{\beta}$	312.3		0.15	0.02	0.33

a) [24]. b) Normalized to  $B(E2, 2_{\gamma} \rightarrow 0_g)$  in experiment [39].

#### E. The inter band $B(E2)$ ratios

##### 1. $K^{\pi}=0_2^+$ $\beta$ -band and $K^{\pi}=2^+$ $\gamma$ -band

For proper identification of the energy levels and the de-exciting  $\gamma$ -rays, the transition energies  $E_{\gamma}$  and intensities  $I_{\gamma}$  are also listed in the following Tables. In Table-VI, we list the revised up to-date  $B(E2)$  ratios for transitions from the  $K^{\pi}=0_2^+$   $\beta$ -band. The new transitions, not available previously, are marked by an asterisk. The large ratio of intra-band to interband transition strengths are well supported by theory. The newly determined  $B(E2)$  ratios also agree reasonably well with DPPQ values. For  $B(E2, 6_{\beta} \rightarrow 4_g/6_g)$ , the value is revised from 0.080(30) in [40, 9] to 0.037 (4), in better agreement



with the DPPQ value. Most of the other DPPQ values are in fair agreement with experiment. Just as for  $N=90$  isotone  $^{152}\text{Sm}$ , the  $B(E2)$  ratios for  $^{154}\text{Gd}$  from X(5) symmetry also are in fair agreement with data. As stated above, the  $\beta$ -g transitions (especially  $2_\beta$ - $0_g$ ,  $4_\beta$ - $2_g$ ) are weak in IBM-1, which cause larger deviations from data.

TABLE VI.  $B(E2)$  ratios for transitions from  $K^\pi=0_2^+$   $\beta$ -band to ground band in  $^{154}\text{Gd}$ . Data taken from nndc [13, 14]. The new transitions, not available previously, are marked by an asterisk.

Transitions	$E_\gamma/E_{\gamma'}$ keV	$I_\gamma/I_{\gamma'}$	$B(E2)$ ratio	DPPQ	IBM-1	X(5)[2]
$2_\beta$ $0_g/2_g$	815.5/692.4	0.2876	0.127 1	0.11	0.04	0.22
815.5 $4_g/2_g$	444.5/692.4	0.3078	2.82 5	2.61	1.60	4.0
$0_g/4_g$	815.5/444.5	0.934	0.045 2	0.043	0.025	0.056
$0_\beta/0_g$	134.8*/815.5	0.0156	126 15*	205	1025	40
$4_\beta$ $2_g/4_g$	924.6/676.6	0.388	0.081 2	0.18	0	0.16
1047 $6_g/4_g$	329.9/676.6	0.056	2.03 20	2.54	0.62	4.7
$2_g/6_g$	924.6/329.9	6.93	0.040 5	0.070	0	0.036
$2_g/2_\beta$	924.6/232.1*	28.5	0.003 1*	0.005	0	0.008
$6_\beta$ $4_g/6_g$	994.9/648.3	0.312	0.037 4	0.024	0.008	
1366 $4_g/4_\beta$	994.9/318.3*	0.774	0.0026 3*	0.0005	0.0005	

Clark *et al.* [29], using the DPPQM results from Kumar's calculation [21] for  $^{152}\text{Sm}$ , illustrated the consistent set of  $\beta$ -g  $B(E2)$  ratios through band mixing. Similar good agreement with experiment is apparent (Table VI) here for  $^{154}\text{Gd}$ , without resorting to shape co-existence assumption. In fact, in DPPQ model, there is complete freedom of shape dynamically. However, as stated above, no sharp difference in shape between ground and  $\beta$ -band is predicted.

TABLE VII.  $B(E2)$  ratios for transitions from  $K^\pi=2_1^+$   $\gamma$ -band in  $^{154}\text{Gd}$ .

Transitions	$E_\gamma/E_{\gamma'}$ (keV)	$I_\gamma/I_{\gamma'}$	$B(E2)$ ratios				
$I_i I_f/I_f'$	[13]	[13]	Expt. [9] <sup>a</sup>	Expt.[13]	DPPQ	IBM-1	X(5)
996.3 keV							
$2_\gamma$ $4_g/2_g$	625.3/873.2	2.61/100	0.144 5	0.139 2	0.090	0.15	0.052
$0_g/2_g$	996.3/873.2	86.8/100	0.46 1	0.45 1	0.55	0.63	0.66
$0_g/4_g$	996.3/625.3	86.8/2.61	3.3 1	3.3 1	6.1	4.17	1.28
<b><math>2_\beta/2_g</math></b>	180.7/873.2	0.043/100	1.03 23	1.13 16	1.47	4.1	0.03
$0_\beta/0_g$	315.6/996.3	0.073/86.8	0.14 1	0.26 1	0.03 <sup>b</sup>	0.037 <sup>b</sup>	0.09
$0_\beta/2_\beta$	315.6/180.7	0.073/0.043	0.063 14	0.104 16	0.010 <sup>b</sup>	0.006 <sup>b</sup>	1.85

1127.8keV							
$3_\gamma \ 2_g/4_g$	1004.7/756.8	100/25.11	1.06 4	0.97 2	1.41	1.42	2.38
$2_\gamma/2_g$	131.5/1004.7	0.073/100	17 1	19 1	26	23	
$4_\beta/2_\beta$	80.4/312.3	0.016/0.101	182 91	140 70	34	694	0.063
$2_\beta/2_g$	312.3/1004.7	0.101/100	0.289 13	0.35 1	0.04 <sup>c</sup>	0.01 <sup>c</sup>	0.06
$4_\beta/4_g$	80.4/756.8	0.016/25.11	50 25	47 24	1.6 <sup>c</sup>	4.2 <sup>c</sup>	0.01
1263.8keV							
$4_\gamma \ 6_g/4_g$	546.1/892.8	1.68/100	0.27 4	0.20 2	0.37	0.38	0.09
$2_g/4_g$	1140.7/892.8	45.6/100	0.14 1	0.13 1	0.32	0.25	0.31
$2_\beta/2_g$	448.5/1140.7	0.49/45.6	1.15	1.14 17	0.50	0.75	
$2_\gamma/2_g$	267.5/1140.7	1.4/45.6	43.3	43 9	29	33	
1432.6keV							
$5_\gamma \ 4_g/6_g$	1061.4/714.9	100/25.5	0.74 15	0.54 2	0.77	0.78	1.67
$4_\gamma/4_g$	168.8/1061.4	5/100	491	<491	19	18	
1606.6keV							
$6_\gamma \ 4_g/6_g$	1235.1/888.7	43/100	0.08 2	0.08 2	0.14	0.17	0.25

- a) Values adopted in [9] from the weighted average of  $I_\gamma$  in [16] and [41].  
b) Weak  $E2$  transition  $2_\gamma \rightarrow 0_\beta$  in DPPQM agrees with experiment (Table V), but the  $B(E2)$  to  $0_g$  and  $2_\beta$  are relatively stronger. The same applies to IBM-1  
c) Again (odd spin)  $\gamma$ - $\beta$  band mixing is small in DPPQM and much smaller in IBM-1. Also  $X(5)$  values [39] differ by large factors.

The revised values of  $B(E2)$  ratios for transitions from  $\gamma$ -band are listed in Table VII along with the old values in [9]. Most values are almost the same, but a few newly measured values, e.g. for  $2_\gamma - 0_\beta/0_g$  and  $2_\gamma - 0_\beta/2_\beta$  differ by almost factors of two. For transitions from the  $2_\gamma$  state, most values are well predicted in DPPQM and IBM-1. The  $X(5)$  values for  $\gamma$ -g  $B(E2)$  ratios are almost equal to the rotor values. The increased  $B(E2, 2_\gamma - 0_\beta/0_g)$  [13], nearly double of the older value, is not supported in theory. The  $3_\gamma$  to  $4_\beta$  transition of 80 keV is weak in DPPQM. Other  $B(E2)$  ratios are given reasonably well in theory. For transitions from the higher states of  $4_\gamma$ ,  $5_\gamma$  and  $6_\gamma$  our theory values are in fair agreement with most data.

## 2. $K^\pi = 0_3^+$ band

The  $0_3^+$  at 1182.1 keV and  $4^+$  at 1701 keV states are the new assignments in this band, and some weak  $I_\gamma$  for the transitions from 1418 keV  $2^+$  are revised, (see footnote (b) of Table VIII). Accordingly, several  $B(E2)$  ratios in Table VIII are different from the

previous values referred to in [9] (columns 4 and 5). For the newly assigned 1182.1 keV  $0_3^+$  state, the  $B(E2, 0_3^+ \rightarrow 2_\beta/2_g)$  value of  $\sim 40$  (in experiment) favors it as collective structure, (much larger in theory because of the much weaker  $E2$  to  $2_g$ ) ( $< 3 \times 10^{-4} e^2 b^2$ ). The absolute  $B(E2, 0_3^+ \rightarrow 2_\beta) = 0.52 e^2 b^2$  in theory, equivalent to 104 W.u, indicates good collectivity. It is larger than  $B(E2, 0_\beta \rightarrow 2_g) = 0.22(2) e^2 b^2$ . In  $N=90$  isotone  $^{152}\text{Sm}$ ,  $B(E2, 0_3^+ \rightarrow 2_\beta/2_g)$  is 43 in experiment and  $(0.8/0.0064) \sim 126$  in DPPQM, larger by a factor of  $\sim 3$ .

For the 1418 keV  $2^+$  state which is well established, preferential decay to  $\beta$ - and  $\gamma$ -band members are also supported in theory, though because of the very weak transition to  $2_g$  ( $4 \times 10^{-5} e^2 b^2$ ) in theory, the five  $B(E2)$  ratios involving it, are larger in theory. The  $B(E2, 2_{03}^+ \rightarrow 2_\beta) = 0.1 e^2 b^2$  or 20 W.u in DPPQM indicates a collective  $E2$  transition. Most of the other  $B(E2)$  ratios are well given in theory. The 1418 keV  $2_{03}^+$  to  $0_3^+$  125 keV transition, replaced by 236.1 keV intra-band transition, yields  $B(E2)$  ratio as in theory, indicating large intraband  $E2$  transition. The absolute  $B(E2)$  value for this newly assigned intraband transition is predicted to be  $0.63 e^2 b^2$ , a rotational model value, almost equal to  $B(E2, 2_g \rightarrow 0_g) = 0.77 e^2 b^2$  in experiment.

For the 1701 keV  $4^+$ , the weak transition to  $4_g$  ( $\geq 10^{-7} e^2 b^2$ ) and  $2_\beta$  ( $\sim 10^{-5}$ ) yield large (small)  $B(E2)$  ratio in theory. For other transitions, the theory values reproduce the basic pattern of the experimental data. The predicted 80%  $K=0$  admixture and the stronger  $E2$  transitions to  $4_\beta$ , as also given in theory, indicate large band mixing with  $\beta$ -band. On account of the 1182 keV  $0_3^+$  state, yielding energy ratio  $E_{03}/E_{0\beta}$  less than 2.0, Kulp *et al.* [15,18] questioned its  $2\beta$  vibrational character. However, from the general decay pattern of the  $0_3^+$  and other higher spin members, anharmonic vibration can give a lower energy ratio.

TABLE VIII.  $B(E2)$  ratios for transitions from  $K^\pi = 0_3^+$  ( $2\beta$ )-band at 1182.1 keV  $0_3^+$  in  $^{154}\text{Gd}$ . The DPPQM values for energies are 1842, 2156 and 2487 keV respectively.

Transitions	$E_\gamma/E_{\gamma'}(\text{keV})$	$I_\gamma/I_{\gamma'}$	$B(E2)\text{ratios}$	Expt.	DPPQ
$I_i I_f/I_{f'}$	[13]	[13]	Expt [9,16]	[13]	
1182.1 $0_3$					
$0_3^+ 2_\beta/2_g$	366.6/1059.0	21/100		42 18	1910 <sup>a</sup>
1418.1 keV					
$2_{03} 2_\beta/2_\gamma$	602.7/421.9	47.7/2.2	$>4.8$	3.6 28	4.4
$2_\gamma/2_g$	421.9/1295.1	2.2/18.9	$>96$	32 24	560

$2_{\beta}/2_g$	602.7/1295.1	47.7/18.9	450	116 11	2470
$4_g/2_g$	1047.2/1295.1	100/18.9	>38	15 1	59
$0_g/2_g$	1417.9/1295.1	13.6/18.9	>1.4	0.5 1	33
$0_{\beta}/2_{\beta}$	737.5/602.7	3.7/47.7	0.06 1	0.03 1	0.01
$4_{\beta}/2_{\beta}$	370.6/602.7	6.88/47.7	2.0 7	1.6 1	2.7
$0_{\beta}/0_g$	737.5/1417.9	3.7/13.6	26 5	7.1 14	0.60
$4_{\beta}/4_g$	370.6./1047.2	6.88/100	20 5	12 1	111
$0_{03}/0_{\beta}$	236.1/737.5	2.8/3.7	b)	225 50	810
$0_{03}/2_g$	236.1/1295.1	2.8/18.9	b)	736 160	1.6+E4
$2_{\gamma}/3_{\gamma}$	421.9/290.4	2.2/2.8	c)	0.12 10	0.26
1701.4keV					
$4_{03} 4_{\beta}/4_g$	653.7/1330.3 <sup>d</sup>	100/75		47 <sup>d</sup>	2 E+05
$6_g/4_g$	983.7/1330.3	92/75		5.6	1.0 E+04
$2_{\beta}/2_{\gamma}$	885.8/705.1	8.9/2.2		1.3	0.01
$2_{\gamma}/3_{\gamma}$	705.1/573.5	2.2/12		0.07	0.26
$2_{\gamma}/2_g$	705.1/1578.2	2.2/9.2		13	5.2
$2_{\beta\beta}/2_{\gamma}$	283.0/705.1	17/2.2		742	144

- a) The  $B(E2, 0_3^+ \rightarrow 2_g)$  value is of the order of  $10^{-4}$  smaller in theory, unlike the 100 to 21 relative  $I_{\gamma}$  in experiment [13,14], for the newly assigned 1182 keV  $0_3^+$  state.
- b) 1418 keV  $2_{03}^+$  to  $0_3^+$  125 keV transition is now replaced by 236.1 keV transition.
- c) The 290.4 keV transition to  $3_{\gamma}$  is new.
- d) No intensity errors are listed [13] for  $I_{\gamma}$  from 1701.4 keV state.

The nature of second excited  $0^+$  state of deformed nuclei of this region, also has been under discussion for a long time [42, 18, 30]. Shahabuddin *et al.* (1980) [42] proposed the 1182 keV level of this band in their (t, p) reaction work. On the basis of large 2n transfer amplitude, which results from the  $\nu[505]\uparrow$  Nilsson intruder orbital, following [42], Kulp *et al.* [18] called this state as a “pairing isomer”. A pairing isomer implies pairing vibration, decoupled from the superfluid ground state. Earlier, Ragnarsson and Broglia [43] noted the larger  $\sigma(p, t)$  and small  $\sigma(t, p)$  in actinides, and interpreted it as pairing isomer on account of the increased pairing gap  $\Delta$  than for the ground state. They explained that the pairing isomers are formed because of the inhomogeneity of oblate and prolate levels in the vicinity of the Fermi surface. However, in  $^{154}\text{Gd}$ , the situation is different.

Meyer *et al.* [44] have measured the 2n transfer cross section for the (p, t) reaction on  $^{152,154}\text{Gd}$ . For g.s. to g.s. 2n cross section  $\sigma$ , the values are 0.913(4) for  $^{152}\text{Gd}$  and

2.37(1) for  $^{154}\text{Gd}$ . This supports the interpretation as above [30], in terms of the difference in deformation of the target and the product nucleus. Further, for the 680 keV  $0_2^+$  in  $^{154}\text{Gd}$ , 20% of the ground state  $\sigma$  value is indicated and for the 1182 keV  $0_3^+$ , only a 0.005 fraction is indicated [44].

Using IBM-1 and coherent state formalism, Fossion *et al.* [45], explained the peaks in (t, p) 2n transfer cross sections in Gd and other nuclei, and correlated them with the shape phase transition on the U(5) to SU(3) path. They obtained enhanced  $\sigma$  for the excited  $0_2^+$  state in  $^{152}\text{Gd}(t, p)^{154}\text{Gd}$  reaction using the  $s^+$  operator of IBM. Clark *et al.* [30] reviewed the role of the 2n transfer reaction strength for the ground state and excited  $0^+$  states in deformed nuclei, vis-a-vis the shape co-existence view and the shape difference of the target and product nucleus. They refuted the assumption of spherical excited state in the product nucleus, based on the earlier interpretations, in [42] and [18]. By comparing with the IBM predictions, they concluded that large  $\sigma$ s (2n transfer) for (t, p) and (p, t) reactions arise from the shape difference. This agrees with the results of Ref. [46].

In Ref. [46], the occupation probabilities of protons and neutrons in relevant Nilsson orbits for  $N=88, 90$  isotopes of Gd from DPPQM, were presented (see Tables VII-X and Figs. 5, 6 and 7 in [46]). A sharp rise of the occupation of  $vi_{13/2}$  and  $vh_{9/2}$  down sloping orbitals is indicated, which explains the sharp shape transition at  $N=88-90$ . and may give rise to the large 2n transfer amplitude, g.s. to g.s., and to the excited states.

Kumar [21] has pointed out for  $^{152}\text{Sm}$ , that the fluctuations in pairing vibration, giving rise to large  $\sigma(t, p)$ , should be weak. Also, he noted that there are no two minima in PES corresponding to the excited  $0^+$  states. However, as illustrated in Fig.4 for the PES, the multiphonon bands lie above the spherical barrier, hence some reduction in  $\beta_{\min}$  and  $\beta_{\text{rms}}$  for these states is possible. Thus, the pairing vibration may contribute simultaneously, to the quadrupole vibration, supported by the B(E2) data (absolute and relative) for the  $0_3^+$  and higher spin members and the collectivity of the band.

In this context, it is noted that in the present treatment of DPPQM, we have not calculated explicitly the state wise 2n transfer cross section for (p,t) and (t,p) stripping and pick up reactions, though we have calculated the occupation probabilities of the individual Nilsson orbits as referred to in the paragraph above. As per Kumar's observation, one needs to estimate the fluctuations in the pairing field and the quadrupole field for

individual excited  $0^+$  states. Kumar [21, (Table 15)] also estimated the summed up occupation of Nilsson orbits and obtained increased  $\sigma(t, p)$  for excited state in  $^{152}\text{Sm}$ . The difference between  $\sigma(p, t)$  and  $\sigma(t, p)$  can also be on account of the use of  $v$  and  $u$  probability amplitudes of pairing in the two cases. However, if the change in deformation for excited state is small, this will be difficult to estimate.

### 3. $K^\pi=4^+$ band at 1645.8 keV

As stated above, the fifth  $4^+$  of DPPQM has (22+13+65)%  $K=0, 2$  and  $4$ -component admixture, which we associate with the 1646 keV state. In the first 4 rows of Table IX, the  $B(E2)$  values for transitions from  $I=4_{\gamma\gamma}$  at 1646 keV to the 996 keV  $\gamma$ -band, relative to 680 keV  $\beta$ -band and the  $g$ -band are listed, which display the preferential decay to the  $\gamma$ -band, rather than to  $(\Delta K=4)$   $\beta$ -band. The  $(\Delta K=4)$  transitions to  $g$ -band are weaker by two orders of the magnitude (with large error margin). These features are supported in DPPQM. Also, the other  $B(E2)$  ratios involving two levels of the same band in lower six rows agree with experiment (with one exception).

TABLE IX.  $B(E2)$  ratios for transitions from  $K^\pi=4_1^+$  ( $\gamma\gamma$ )  $I=4$  (1645.8 keV)-band in  $^{154}\text{Gd}$ .

Transitions $I_i \rightarrow I_f$	$E_\gamma/E_{\gamma'} \text{ (keV)}$ [13]	$I_\gamma/I_{\gamma'}$ [13]	Expt. [9,11]	Expt. [13]	DPPQ
1646 keV					
$4_{K=4} \rightarrow 2_\gamma/2_\beta$	649.6/830.5	100/7.1	57 8	48 5	28
$2_g/2_\gamma$	1522.8/649.6	1.0/100	0.004 1	0.0014 3	0.021
$4_\gamma/4_\beta$	382.0/598.2	10.8/12.0	6.6 10	8.5 16	22
$4_g/4_\gamma$	1275.7/382.0	2/10.8	0.0013	0.0004 4 <sup>a</sup>	0.026
$4_{K=4} \rightarrow 3_\gamma/2_\gamma$	518.0/649.6	57.0/100	1.72 17	1.77 9	1.29
$4_\gamma/2_\gamma$	382.0/649.6	10.8/100	1.33 22	1.54 6	1.02
$2_\beta/4_\beta$	830.5/598.2	7.1/12.0	0.10 3	0.11 2	0.67
$3_\gamma/4_\gamma$	518.0/382.0	57.0/10.8	1.30 22	1.15 8	1.26
$2_g/4_g$	1522.8/1275.7	1.0/2	0.43 15	0.2 2 <sup>a</sup>	0.8
$6_g/4_g$	928.2/1275.7	3.4/2	4.9 16	8 8 <sup>a</sup>	0.6

a) For 1275.7 keV,  $I_\gamma=2(2)$  listed in [13,14], indicates 100 % error.

For  $I=5$  state at 1770 keV (Table X), the first row displays a factor of 250 stronger decay for the intra-band transition [13]. The revised  $B(E2)$  ratio for  $5_2 \rightarrow 4_\gamma/4_\beta$  of 21(9) again indicates a stronger decay to  $\gamma$ -band. The transition to  $4_g$  is further weakened by a factor of 60 compared to  $4_\beta$ . In Table X, two sets of values ( $5_2$  and  $5_3$ ) of DPPQM are listed. The  $5_2^+$  state in DPPQM has (56+44)%  $K=2, 4$  components and  $5_3^+$  has (40+60)%  $K=2, 4$  admixture, indicating good mixing with  $K=2$   $\gamma$ -vibration band. The values from  $5_2^+$  seem to be closer to data, as in [9], than those from  $5_3^+$  state (last column), though it's  $K=4$  component admixture is smaller than  $K=2$ .

For the  $6^+$  at 1911.5 keV, strong intra band transition strength, and preferential decays to  $\gamma$ -band compared to  $\beta$ -band and  $g$ -band are indicated. The fifth  $6^+$  DPPQ model values are compared with data. The  $B(E2)$  ratios from theory agree with data in some cases and are far in other cases. The absolute intra band  $B(E2)$  value from  $6^+$  [19] is reproduced approximately.

TABLE X.  $B(E2)$  ratios for transitions from  $K^\pi=4_1^+$  ( $\gamma\gamma$ )  $I=5, 6$  spins in  $^{154}\text{Gd}$ .

$I_i$	$I_f/I_i'$	$E_\gamma/E_{\gamma'}(\text{keV})$	$I_\gamma/I_{\gamma'}$	$B(E2)\text{ratio}$	$B(E2)$	DPPQ	DPPQ
		[13]	[13]	Expt. [9,11]	Expt.[13]	$5_2^+$	$5_3^+$
1770.2 keV							
$5^+$	$4_\gamma/4_{K=4}$	506.4/124.4	93/21	0.0012	0.004 1	0.06	0.20
	$4_\gamma/4_\beta$	506.4/722.6	93/26	9.1 40	21 9	1.6	5.2
	$4_g/4_\beta$	1400/722.6	11/26		0.015 9	0.001	0.002
	$4_\gamma/3_\gamma$	506.4/642.4	93/100	2.8 5	3.1 6	2.1	1.1
	$5_\gamma/3_\gamma$	337.4/642.4	35/100	7 2	9 3	3.0	0.7
1911.5 keV						$6^+$	
	$B(E2, 6_4-4_4)$	0.24 =49W.u			0.24 <sup>a</sup>	0.63	
	$B(E2 6_4-5_\gamma)$	0.015=3.1W.u			0.015 <sup>a</sup>	0.023	
$6^+$	$4_{K=4}/4_\gamma$	265.8/647.6	54/71		65 13	498	
	$5_\gamma/5_{K=4}$	479.2/141.3	52/100		0.0012 2	0.057	
	$4_{K=4}/5_{K=4}$	265.8/141.3	54/100		0.023 3	1.3	
	$4_\beta/4_\gamma$	545.7/647.6	8/71		0.26 11	0.30	
	$4_g/4_\gamma$	1541.2/647.6	5.7/71		0.001 1	0.26	
	$6_g/6_\gamma$	1193.3/304.8	41.6/19.5		0.0023 4	0.037	
	$6_\gamma/5_\gamma$	304.8/479.2	19.5/52	3.5 8	3.6 5	1.6	

a) Estimated by Wu *et al.* [19].

The nature of  $K^\pi=4^+$  in deformed nuclei has been in discussion for a long time. The support for the 2056 keV  $K^\pi=4^+$  state in  $^{168}\text{Er}$  as a two-phonon  $\gamma\gamma$  excitation, is based on the  $B(E2)$  ratio  $R(4^+)=B(E2, 4_{\gamma\gamma}\rightarrow 2_\gamma)/B(E2, 2_\gamma\rightarrow 0_g) \geq 1.1$  derived from life time data by Borner *et al.* [47], besides the anharmonicity estimates from various theoretical models. From the Coulomb excitation (CE) work [48],  $B(E2, 4_{\gamma\gamma}\rightarrow 2_\gamma)=0.060(15) e^2b^2$  and the ratio  $R(4^+)$  is 1.9(4). In DPPQM, Gupta *et al.* [49] predicted  $R(4^+)=2.5$  in agreement with CE data. Similarly, measured  $R(4^+)=(0.5-3.9)$  for  $^{164}\text{Dy}$  [50] for 2173.1 keV  $4^+$  supports the  $4_{\gamma\gamma}$  interpretation. For  $^{154}\text{Gd}$ , no life time or Coulomb excitation data are available for the  $K^\pi=4^+$  state at 1646 keV.

The absolute  $B(E2, 4_{\gamma\gamma}\rightarrow 2_\gamma)$  in DPPQM of  $0.047 e^2b^2$  or  $\sim 9$  W.u compared to  $B(E2, 2_\gamma\rightarrow 0_g)=0.029 e^2b^2$  or  $\sim 6$  W.u yielding  $R(4^+)=1.6$ , and  $B(E2, 4_\gamma\rightarrow 2_g)=0.015$  or 3 W.u indicates good collectivity. Wu *et al.* estimated  $B(E2, 6_4 \rightarrow 4_4)=0.24 e^2b^2$  or 49 W.u (DPPQM value exceeds this, Table X), which supports the collective structure of the  $K^\pi=4^+$  band. If one assumes a contribution of 2 quasi-particle components, the  $B(E2)$  value will be less than one W.u. [19]. The energy levels (upto  $I^\pi=9^+$ ) of this band display a rotational pattern as for the ground band [19,13]. The above stated (Tables IX and X) preferential decays of all levels of the  $K^\pi=4^+$  band to  $K^\pi=2^+$   $\gamma$ -band, support the view of calling it a  $2\gamma$  band.

In an alternative approach, Burke [51] on the basis of  $E4$  strengths, and single-nucleon-transfer results, suggested the  $K^\pi=4^+$  band as predominantly hexadecapole vibrations. Ronnigen *et al* [52] measured the  $E4$  component in the ground band of  $^{154-160}\text{Gd}$  isotopes, yielding a small fraction of the quadrupole strength. For the excited bands, this admixture of  $E4$  component should be smaller. Thus a small  $E4$  admixture in the  $K^\pi=4^+$  is possible, along with the two phonon components.

Wu *et al.* [19] studied the problem of large anharmonicity in the energy of  $K=4$  band versus  $K=2$  band, the ratio  $R = E(4_{\gamma\gamma})/E(2_\gamma)$  being = 1.65 in  $^{154}\text{Gd}$ , lower than 2.0 expected for a harmonic  $2\gamma$ - phonon band. The problem of energy anharmonicity in the 16 nuclei in Gd-Hf studied in [19] is rather complex and it varies with  $N$  and  $Z$ . Wu *et al.* [19] suggested that a greater  $\gamma - \beta$  separation leads to a higher value of the ratio  $R$ . On the basis of  $\Delta E(\beta-\gamma)$ , the ratio  $R$  falls in two groups. For  $E(2_\beta)>E(2_\gamma)$ ,  $R$  is greater than 2.0 and



for low  $2_\beta$ ,  $R < 2.0$ . In Gd isotopes, at  $N=90$  in  $^{154}\text{Gd}$ ,  $2_\beta$  is below  $2_\gamma$ . In  $^{156}\text{Gd}$ ,  $\Delta E(\beta-\gamma)$  is reduced and in  $^{158}\text{Gd}$  there is a crossover. The ratio  $R$  reduces from 1.6 to 1.34 in  $^{156}\text{Gd}$ , and then increases to 1.52 in  $^{158}\text{Gd}$ .

In Dy isotopes, the ratio  $R$  varies from 2.0 to 1.75, 1.73 and 2.85 for  $N=92$  to  $N=98$ . However, at  $N=92$ ,  $2_\beta$  just crosses over  $2_\gamma$  and there after,  $2_\beta$  moves up and  $2_\gamma$  moves down continuously. Thus, its behavior is different from that of Gd isotopes. In  $^{160-168}\text{Er}$  isotopes, the ratio  $R$  increases monotonically from 1.89 through 2.98, 2.52 2.60 at  $N=94$  to 2.65 at  $N=100$ . and  $\Delta E(\beta-\gamma)$  increases from  $N=92$  to 98, but the increase in  $R$  at  $N=100$  is not related to the decrease in  $\Delta E(\beta-\gamma)$ . Thus the relation of the anharmonicity  $R$  to  $\Delta E(\beta-\gamma)$  seems to be partially valid. Three body interaction in IBM have been used to obtain the anharmonicity in the ratio  $R$  for  $^{166}\text{Er}$  in Ref. [53], with partial success. A major difference of the Gd isotopes from the Dy and Er isotopes is in the value of  $R$ , being less than two, while most multiphonon theories [47] predicted  $R > 2$ .

Variations of  $R$  and  $\Delta E(\beta-\gamma)$  with  $N$  and  $Z$  are related to more microscopic factors at play. In the microscopic approach of dynamic PPQ model, a fair reproduction of the basic decay pattern is reproduced. Also in DPPQM, the ratio  $R$  for  $^{154}\text{Gd}$  is close to the experimental value. This should justify the  $K=4$  band to be labeled as two  $\gamma$ -phonon  $\gamma\gamma$ -band along with strong mixing with the  $\gamma$ -band and  $\beta$ -band, even if a small  $E4$  component is present.

#### 4. $K^\pi=2^+$ $\beta\gamma$ band at 1531 keV

The revised B(E2) ratios (column 5) for  $E2$  transitions from  $K=2$   $\beta\gamma$  band are almost the same as used earlier in [9] (column 4) in Table XI. The well established 1531 keV  $2_{\beta\gamma}$  level in the decay scheme of  $^{154}\text{Gd}$  is predicted to have a (21+79)%  $K=0, 2$  - component admixture. Kulp *et al.* [15] assigned the 349.2 keV  $\gamma$ -ray to populate the newly assigned 1182.2 keV  $0_3^+$  level from the  $2_{\beta\gamma}$  state. The B(E2) ratio relative to  $0_\beta^+$  state agrees with the DPPQ model value (Table XI). Stronger decays to  $0_\beta$ ,  $2_\beta$  and  $2_\gamma$  than to g-band are also given in DPPQM. The other B(E2) ratios involving two levels of the same band also are reasonably well predicted (except for  $4_g$ ) in our calculation. DPPQM values

of  $B(E2, 2_5^+ \rightarrow 4_\gamma) = 0.10 e^2 b^2$  and  $B(E2, 2_5^+ \rightarrow 0_3^+) = 0.05 e^2 b^2$  indicate coupling with the  $\gamma$ - and  $0_3$  bands.

TABLE XI.  $B(E2)$  ratios for transitions from  $K^\pi=2^+$  ( $\beta\gamma$ ),  $I^\pi=2^+$  1531.3 keV.  $2_5^+$ -state in  $^{154}\text{Gd}$ .

$I_i \ I_f/I_f'$	$E_\gamma/E_{\gamma'} \text{ (keV)}$	$I_\gamma/I_{\gamma'} \text{ [13]}$	Expt. [9,53]	Expt. [13]	DPPQ
$2_5(\beta\gamma)1531$					
$0_3/0_\beta$	349.2/850.6	2.96/100		2.54 10	2.0
$0_\beta/0_g$	850.6/1531.3	100.0/2.64	713 51	716 30	550
$2_\beta/2_g$	715.5/1408.2	76.9/10.2	246 36	223 15	7.1
$2_\gamma/2_g$	535.1/1408.2	7.0/10.2		87 8	6.6
$0_g/2_g$	1531.3/1408.2	2.64/10.2	0.17 4	0.17 2	0.01
$4_g/2_g$	1160.5/1408.2	19.0/10.2	4.86 73	4.90 30	0.1
$0_\beta/2_\beta$	850.6/715.8	100.0/76.9	0.55 2	0.55 2	1.0
$4_\beta/2_\beta$	483.7/715.8	3.86/76.9	0.20 5	0.36 2	1.6
$3_\gamma/4_\gamma$	403.5/267.4	9.2/1.58	0.25	0.74 7	0.95
$3(\beta\gamma) 1661$					
$2_\beta/2_g$	845.4/1537.8	100/10.12	219 11	197 8	368
$4_\beta/4_g$	613.3/1289.9	16.4/3.70	333 19	182 9	140
$4_\gamma/4_g$	397.1/1289.9	4.86/3.70	943 53	572 32	365
$2_\gamma/2_g$	664.7/1537.8	4.60/10.12	38.4	30 2	50
$2_\beta/2_\gamma$	845.4/664.7	100/4.60	5.7 3	6.5 4	7.3
$4_\beta/4_\gamma$	613.3/397.1	16.4/4.86	0.35 2	0.38 2	0.38
$4_g/2_g$	1289.9/1537.8	3.70/10.12	0.55 13	0.88 3	1.1
$4_\beta/2_\beta$	613.3/845.4	16.4/100	0.84 4	0.82 4	0.7
$4_\gamma/2_\gamma$	397.1/664.7	4.86/4.60	13.6 8	13.8 7	13
$2_{\beta\gamma}/2_\gamma$	129.6/664.7	0.28/4.60		216 36	188
$2_{\beta\beta}/2_\gamma$	242.9/664.7	0.72/4.60		26 3	12
$4(\beta\gamma)1789$					
$4_\beta/4_g$	740.9/1417.9	19/100		5 1	83
$2_g/4_g$	1665.8/1417.9	38/100	0.26 5	0.17 2	0.31
$6_g/4_g$	1071.2/1417.9	4.6/100	<5.7	0.19 4	0.8

The predominant decay from  $3_{\beta\gamma}$  (1661 keV) state to members of  $\beta$ -, and  $\gamma$ -band are indicated in theory as well. Other  $B(E2)$  ratios also are given reasonably well in theory. Same trend is apparent for  $4_{\beta\gamma}$  as well. The tentative assignment of 1789 keV  $4_{\beta\gamma}$  was made from the 1417 keV  $4_{\beta\gamma} - 4_g$  firmly assigned in coincidence [12], and is supported by the 1071.2 keV  $\gamma$ -ray to  $6_g$ , and other  $E2$  transitions. The  $4_6^+$  in DPPQM with (9+63+28)%  $K=0, 2$  and 4-component admixture in DPPQM is associated with this state,

which gives reasonable B(E2) ratios (Table XI). Thus we get an over all reasonable agreement from theory of the basic pattern of decay from the members of the  $2_{\beta\gamma}$  band at 1531 keV in  $^{154}\text{Gd}$ .

Kulp *et al.* [15] cited DPPQ results in [9] for this band, but expressed the need of fresh analysis. The transition quadrupole moments obtained from the relative B(E2) values divided by Clebsch Gordan coefficients for  $E_\gamma$  from the levels in  $K=2_2$  band showing increase with final spin, indicated band mixing with lower bands [15]. As stated above,  $2_{\beta\gamma}$  has a large (21+79)%  $K=0$ , 2-component admixture, and the B(E2) ratios also indicate good mixing with  $\beta$  and  $\gamma$ -bands. The PEC also indicates the mixture of prolate, oblate and spherical shapes (Fig. 4). The  $H_{\text{coll}}$  includes the full  $(\beta, \gamma)$  dependence in the kinetic terms  $T_{\text{rot}}(\beta, \gamma)$  and  $T_{\text{vib}}(\beta, \gamma)$  terms, the latter involving the  $B_{\beta\beta}$ ,  $B_{\gamma\gamma}$  and  $B_{\beta\gamma}$  mass coefficients. Thus it well qualifies to be the  $K=2$   $\beta\gamma$  vibrational band.

### 5. $K^\pi=0_4^+$ ( $2\gamma$ )-band

TABLE XII. B(E2) ratios for transitions from 1574.0 keV,  $K^\pi=0_4^+$  ( $2\gamma$ )-band in  $^{154}\text{Gd}$ .

Transitions	$E_\gamma/E_\gamma'$ (keV)	$I_\gamma/I_\gamma'$	B(E2)ratios	DPPQ
$I_i I_f/I_f'$	[13]	[13]	Expt.[13]	
1573.97= $0_4^+$				
$0_4 \rightarrow 2_\gamma/2_\beta$	577.7/758.5	85.3/100	3.3 2	308
$2_g/2_\beta$	1451.7/758.5	27/100	0.011 3	3.4
1716.1 keV				
$2_{04} \rightarrow 2_g/4_g$	1593.4/1345.0	75/100	0.32 21	0.29
$0_g/2_g$	1715.7/1593.4	60/75	0.6 4	0.001
$2_\gamma/2_g$	719.8/1593.4	45/75	32 18	200
$3_\gamma/2_\gamma$	588.3/719.8	73/45	4.5 9	5.0
2080.2 keV				
$4_{04} \rightarrow 3_{\beta\gamma}/2_{04}$	419.3/364.3	9.5M1/3.2		1.2
$2_\gamma/3_\gamma$	1084.3/952.4	75/100	0.39 8	0.02
$4_{\gamma\gamma}/4_g$	434.4/1709.7	6.9/47	139 62	38
$4_g/6_g$	1709.7/1363.1	47/71	0.21 5	
$2_{04}/2_\gamma$	364.3/1084.3	3.2/75	10 4	5380

Sousa *et al.* [11] had assigned 2081.0 keV level to  $(2, 3, 4)^+$  on the basis of energy fits, and 2230 keV level to  $(2, 3, 4)^+$  on the basis of coincidence relations. But, in the

NDS, 110 (2009) [14] and in [15], these assignments are replaced by 1573.97 keV ( $0^+$ ), 1716.1 keV ( $2^+$ ) and 2080.2 keV ( $4^+$ ). Accordingly, the calculated B(E2) ratios from these levels of the  $K^\pi=0_4^+$  band (which may be called  $2\gamma K=0_4^+$  band) are listed in Table XII and compared with DPPQM results. The state  $2_6^+$  with 96%  $K=0$  component is associated with 1716 keV  $2^+$ . The  $4_7^+$  of DPPQM is associated with 2080.2 keV  $4_{K=04}$  state.

The 1574 keV  $0_4^+$  level decays preferentially to  $2_\gamma$  versus  $2_\beta$  and weakly to  $2_g$ . Thus the B(E2) ratio supports the  $K^\pi=0^+$   $2\gamma$  assignment. The B(E2) ratios from other states of the band also support its collective nature. The intra band transition of  $4_{04}$  to  $2_{04}$  of 364.3 keV supports the 2080 keV  $4^+$  assignment. The 434 keV transition to  $K=4$  1646 keV state yields large B(E2) ratio. Thus the assignments in this band seem reasonable. The DPPQ model calculation with sixth  $2^+$  (95%  $K=0$  component), and seventh  $4^+$  yields reasonable B(E2) ratios corresponding to data (Table XII), with some ratios far off too.

The  $2\gamma$  and  $3\beta$  respective assignment to  $0_4^+$  and  $0_5^+$  bands, is supported by the pattern of B(E2) data (Tables XII and XIII), in comparison to a possible suggestion of regarding the  $0_4^+$  or  $0_5^+$  states as  $2\beta$  multiphonon state [15], The smaller collectivity in  $0_4^+$  and  $0_5^+$  bands compared to  $0_3^+$  band, as reflected in the intraband transitions ( $2^+ \rightarrow 0^+$ )  $\sim 0.2 e^2 b^2$  versus  $0.5 e^2 b^2$  in DPPQM do not support the suggestion put forth by Kulp *et al.* [15].

## 6. The $K^\pi=0_5^+$ -band

TABLE XIII. B(E2) ratios for transitions from 1650.34 keV,  $K^\pi=0_5^+$ -band in  $^{154}\text{Gd}$ . No previous data was available in (1979) Paper [9].

$I_i \ I_f/I_f'$	$E_\gamma/E_{\gamma'} \text{ keV}$ [13]	$I_\gamma/I_{\gamma'}$ [13]	B(E2).ratios Expt. [13]	DPPQ
1650.3= $0_5^+$				
$0_5^+ \ 2_2/2_1$	834.8/1650.3	17/100	5.5 19	0.1
1775.4 keV				
$2^+ \ 0_2/0_1$	1094.9/1775.7	21/46	5.1 13	34
$0_1/2_1$	1775.7/1652.4	46/100	0 32 6	0.10
$4_1/2_1$	1404.6/1652.4	40/100	0.90 19	0.07
$2_2/4_2$	960.1/727.8	9/15	0.15 6	0.40
$3_1/4_2$	647.7/727.8	3/15	0 36 18	4.5

The  $0_5^+$  state at 1650.34 keV decays predominantly to  $2_\beta$  level at 815.5 keV compared to  $2_g$  level, with B(E2) ratio  $\sim 5.5$  (Table XIII). The 1775.4 keV  $2^+$  level decays to g-band ( $I=0,2,4$ ),  $0_\beta$  ( $I=0,2,4$ ) band and to  $3_\gamma$  level. The DPPQM values for  $2_7^+$  are listed in Table XIII. The preferential decay of  $2_{05}^+$  to  $0_2^+$  is given in DPPQM, though with larger ratio. *Also, the decays to the g-,  $\beta$ - and  $\gamma$ -band from the  $2_{05}^+$  state, are of the order of  $0.01 e^2b^2$  and yield varied agreement with data.* The stronger decay of 1650 keV  $0_5^+$  to  $2_2$  is also not reproduced in DPPQ model. The intra band B(E2,  $2_{05} \rightarrow 0_5$ ) is only  $0.24 e^2b^2$  (same for B(E2,  $2_{04} \rightarrow 0_4$ ), compared to B(E2,  $2_{04} \rightarrow 0_3$ ) of  $0.63 e^2b^2$  in DPPQM.

## V. DISCUSSION AND SUMMARY

After referring briefly to the new works related to  $^{154}\text{Gd}$  in the last four decades, in Sec. II. we have reviewed the new data on its level scheme. Major changes have been reported on the levels of the excited  $K^\pi=0^+$  bands. Even if there is agreement on the nature of the  $K^\pi=0_2^+$  band in  $^{154}\text{Gd}$  (after differing views for long), there remains an issue of different interpretations on the higher  $K^\pi=0^+$  bands. New data on  $K^\pi=0_{3,4,5}^+$  bands and other higher bands in  $^{154}\text{Gd}$  are reviewed. In Sec.III, we have reviewed the salient features of the 4 decades old dynamic PPQ model. While DPPQM has been successful in explaining the general features of shape transitional nuclei, the problem of energy scale of vibrational bands remains, as pointed out above. In a further development of the microscopic theory, Kumar (and coworkers) developed the dynamic deformation model (DDM) [55] using the Strutinsky shell correction method, replacing the use of Nilsson single particle orbits, which overcomes some of the problems of DPPQM. But use of parameters free process, produces its own limitations. Also, in DPPQM one uses the Bohr Mottelson unified collective model for predicting the nuclear structure, unlike some other theories, based on the collective aspects of the shell model itself.

In Sec.IV, first we have compared the data on *absolute B(E2) values* with DPPQ model, which validates the model and the implied collectivity of the lower three bands. The data on B(E2) ratios, intra band and interband, are compared with IBM and X(5) symmetry in the Tables. Agreement is better for stronger  $E2$  transitions, and deviations are larger for weak  $E2$  transitions. In many cases IBM and X(5) predictions also are good.

The PES of  $^{154}\text{Gd}$  is given, which illustrates the prolate, oblate and spherical shape mixings in the higher bands.

At respective places we have reviewed the different views on the nature of the excited bands, in relation to their vibrational and multiphonon interpretations. The low energy of  $\beta$ -vibration in  $^{154}\text{Gd}$  and other  $N=90$  isotones allows the formation of multiphonon bands at low energy. The differing views in [24-31] on the shape coexistence of  $\beta$ -band and ground band are discussed in section IV D. The views as prevalent in the literature for higher lying bands have been reviewed. The very existence of regular collective bands in  $^{154}\text{Gd}$  favors the collective multiphonon view of these bands, in spite of the anharmonicities which arise on account of the band mixings. Note the order of the different  $K$ -bands related to the low  $0_\beta$  and higher  $2_\gamma$  states. In deformed isotopes of Gd and Dy, problems of reproducing even the  $\beta$ -band versus  $\gamma$ -band (in  $2^+$  and higher spin states) in proper order arise (e.g. see [56] for  $^{158}\text{Dy}$ ).

The  $2\beta$  phonon versus pairing vibration view is reviewed in some detail in Sec.IVE(2). The  $2n$  transfer reactions indicate an admixture of pairing and quadrupole vibrations may be present. The problem of higher excited  $0^+$  states has a long history and is currently of great interest [44]. The  $E(0_3^+)$  versus  $E(0_2^+)$  ratio also varies widely with  $N$  and  $Z$ , and the anharmonicity alone does not explain this variation. Then one resorts to other alternatives, like single particle admixture or pairing vibration etc. In fact the same problem occurs for  $K^\pi=4^+$  bands also, as discussed in the above in Sec.IVE(3), where, besides Wu *et al.* [19] analysis for  $^{154}\text{Gd}$ , we have cited the  $K^\pi=4^+$  bands of  $^{168}\text{Er}$  and  $^{164}\text{Dy}$  and the alternative view of  $E4$  admixture, in Burke's work [51]. The DPPQM results (Tables IX, X) support the collective aspects of the  $K^\pi=4^+$  band. It is possible for a small  $E4$  component to be present in the  $2_\gamma$  band.

There is a view that  $2n$  transfer reactions provide more microscopic information on the spectral features. This information also varies with  $N$  and  $Z$ . It is true that one cannot rely on the level energy and  $B(E2)$  ratio data alone. But the information on collectivity may not be ignored either. The absolute  $B(E2)$  values obtained in multiple Coulomb excitation and life time data, where possible, can provide additional support on collectivity.  $E0$  transition strength data can provide additional information. The view of pairing isomer needs to be supported by a possible change of deformation (shape

coexistence) for those states in the overall spectrum. We have illustrated the different views as in Refs. [18, 21, 30, 42-45]. As noted by Kumar in [21] one needs to compare the fluctuations in the pairing field and the quadrupole collective field in order to justify either view. The recent works on 2n transfer using  $s^+$  and  $s$  operators of IBM in [30, 45] have merit and further progress in this direction will be useful.

Also,  $2\gamma$   $K=0,2,4$  combinations of the three bands have different anharmonicities, as expected. This also remains an open problem for further experimental and theoretical developments. The life time data for these states can help to determine the nature of these states. In theory as well, estimate of  $E4$  component, in addition to  $E2$  component (e.g. in g-boson IBM) can help in determining the predominant  $2\gamma$  component in the  $K^\pi=4^+$  band.

The microscopic collective model approach in dynamic PPQ treatment accounts not only the shape degrees of freedom but also the variation in the kinetic coefficients. This allows such a large number of data on the eight bands to be compared with experiment, even if the agreement is not close always. The spectrum of  $^{154}\text{Gd}$  with eight collective bands, analyzed previously in (1979), and updated now after four decades, demonstrates the validity of the DPPQ model and of its predictive power on collective states. Kumar *et al.* have pointed out earlier [9] that in Dynamic deformation theory of KB, each state has full freedom on its shape and kinematics. Also two and four quasiparticle excitations are already included in the treatment of the kinetic coefficients of the rotational,  $\beta$ - and  $\gamma$ -vibration terms. However, the DPPQ model while it gives extensive agreement with data, is giving only a partial reproduction of all the experimental data, so a more complete understanding of all the experimental data is needed. It would be helpful to have measurements of more lifetimes of excited states and  $E0$  transitions to more fully understand the structure of  $^{154}\text{Gd}$ .

### ACKNOWLEDGEMENT

J. B. Gupta appreciates the post retirement association with Ramjas College, University of Delhi. The work at Vanderbilt University is supported by the U.S. Department of Energy under Grant No.DE-FG05-88ER40407.

### REFERENCES

- [1] F. Iachello, Phys. Rev. Lett. **87**, 052502 (2001).
- [2] D. Tonev, A. Dewald, T. Klug, P. Petkov, J. Jolie, A. Fitzler, O. Möller, S. Heinze, P. von Brentano, and R.F. Casten, Phys. Rev. **C69**, 034334 (2004).
- [3] R.F. Casten and N.V. Zamfir, Phys. Rev. Lett. **87**, 052503 (2001).
- [4] R. Krucken, Phys. Rev. Lett. **88**, 232501 (2002).
- [5] M. A. Caprio *et al.*, Phys. Rev. **C66**, 054310 (2004).
- [6] A. Bohr and B.R. Mottelson, *The Nuclear Structure* Vol. II (W.A. Benjamin, 1975).
- [7] F. Iachello and A. Arima, *The Interacting Boson Model*  
(Cambridge University Press, Cambridge, 1987).
- [8] K. Kumar and M. Baranger, Nucl. Phys. **A110**, 529 (1968).
- [9] K. Kumar, J.B. Gupta and J.H. Hamilton, Aust. J. Phys. **32**, 307 (1979).
- [10] J.H. Hamilton, Bull. Acad. Sci. USSR Phys. Ser. **36**, 17 (1972).
- [11] D.C. Sousa, L.L. Riedinger, E.G. Funk and J.W. Mihelich, Nucl. Phys. **A238**, 365 (1975).
- [12] J.B. Gupta, S.L. Gupta, J.H. Hamilton and A.V. Ramayya, Z. Phys. **A282**, 179 (1977).
- [13] Brookhaven National Laboratory, Chart of nuclides of National Nuclear Data Center, <http://www.nndc.bnl.gov/ENSDF>.
- [14] C.W. Reich, Nuclear Data Sheets **110**, 2257 (2009).
- [15] W.D. Kulp, J.L. Wood, K.S. Krane, J. Loats, P. Schmelzenbach, C.J. Stapels, R.-M. Larimer, E.B. Norman, Phys. Rev. **C69**, 064309 (2004).
- [16] R.A. Meyer, Phys. Rev. **170**, 1089 (1968).
- [17] R. Bloch, B. Elbek and P.O. Tjom, Nucl. Phys. **A91**, 576 (1967).
- [18] W.D. Kulp, J.L. Wood, K.S. Krane, J. Loats, P. Schmelzenbach, C.J. Stapels, R.-M. Larimer, E.B. Norman, Phys. Rev. Lett. **91**, 102501 (2003).
- [19] X. Wu, A. Aprahamian, S.M. Fischer, W. Reviol, G. Liu and J.X. Saladin, Phys. Rev. **C49**, 1837 (1994).
- [20] K. Kumar, in *The Electromagnetic Interaction in Nuclear Physics*, (ed W.D. Hamilton N. Holland Ch 3) (1975).
- [21] K. Kumar, Nucl. Phys. **A231**, 189 (1974).



- [22] N.N. Bogoliubov, Usp. Fiz. Nauk. **67**, 549 (1959).
- [23] O. Scholten, *Computer program package PHINT*. KVI Internal Report (1979).
- [24] N.V. Zamfir, R.F. Casten, M.A. Caprio, C.W. Beausang, R. Krucken, J.R. Novak, J.R. Cooper, G. Cata Daniel and C.J. Barton, Phys. Rev. **C60**, 054312 (1988).
- [25] P. E. Garrett, J. Phys. **G27**, R1 (2001).
- [26] P. Debenham and N.M. Hintz, Nucl. Phys. **A195**, 385 (1972)..
- [27] Jan Jolie, Pavel Cejnar and Jan Dobes, Phys. Rev. **C60**, 061303 (1999).
- [28] D.G. Burke, Phys. Rev. **C66**, 024312 (2002).
- [29] R.M. Clark *et al.* Phys. Rev. **C67**, 041302R (2003).
- [30] R.M. Clark, R. F. Casten, L. Bettermann, and R. Winkler, Phys. Rev. **C80**, 011303R (2009).
- [31] J.B. Gupta and J.H. Hamilton, Eur Phys. J. **A51**, 151 (2015).
- [32] Z.P. Li, J. Xiang, J.M. Yao and H. Chen, Int. J. Mod. Phys. **E20**, 494 (2011).
- [33] N. Pietralla and O.M. Gorbachenko, Phys. Rev. **C70**, 011304R (2004).
- [34] R. Fossion, D. Bonatsos and G.A. Lalazissis, Phys. Rev. **C73**, 044310 (2006).
- [35] Z.P. Li, T. Niksic, D. Vretenar, J. Meng, G.A. Lalazissis and P. Ring, Phys. Rev. **C79**, 054301 (2009).
- [36] K. Kumar, *Nuclear model and search for unity in nuclear physics* (Universitetsforlaget, Bergen, 1984).
- [37] K. Kumar (pvt. Communication to JBG).
- [38] P. Van Isacker, P.O. Lipas, K. Helimaki, I. Koivistoinen and D.D. Warner, Nucl. Phys. **A476**, 301 (1988).
- [39] R. Bijker, R.F. Casten, N.V. Zamfir and E.A. McCutchan, Phys. Rev. **C68**, 064310 (2003).
- [40] N. Rud, H.L. Nielsen and K. Wilsky, Nucl. Phys. **A167**, 401 (1971).
- [41] L.L. Riedinger, J.H. Hamilton and N.R. Johnson, Phys. Rev. **C2**, 2358 (1970).
- [42] M.A.M. Shahabuddin, D.G. Burke, I. Nowikow and J.C. Waddington, Nucl. Phys. **A340**, 109 (1980).
- [43] I. Ragnarsson and R.A. Broglia, Nucl. Phys. **263**(2), 315 (1976).
- [44] D.A. Meyer *et al.*, Phys. Rev. **C74**, 044309 (2006).

- [45] R. Fossion, C.E. Alanso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. **C76**, 014316 (2007).
- [46] J.B. Gupta, Phys. Rev. **C87**, 064318 (2013).
- [47] H.G. Borner, J. Jolie, S.J. Robinson, B. Krusche, R. Piepenbring, R. F. Casten, A. Aprahamian, and J. P. Draayer, Phys. Rev. Lett. **66**(6), 601 (1991).
- [48] T. Härtlein, M. Heinebrodt, D. Schwalm and C. Fahlander, Eur. J. Phys.A2, 253 (1998).
- [49] J.B. Gupta, J.H. Hamilton and A.V. Ramayya, Phys. Rev. **C63**, 044308 (2001).
- [50] F. Corminboeuf, J. Jolie, H. Lehmann, K. Fohl, F. Hoyler, H.G. Borner, C. Doll, and P.E. Garrett, Phys. Rev. **C56**, R1201 (1997).
- [51] D.G. Burke, Phys. Rev. Lett. **73**(14), 1899 (1994).
- [52] R.M. Ronningen *et al.* Phys. Rev. **C16**(6), 2208 (1977).
- [53] J. E. García-Ramos, C. E. Alonso, J. M. Arias and P. Van Isacker, Phys. Rev. **C61**, 047305 (2000).
- [54] D.R. Zolnowski, E.G. Funk and J.W. Mihelich, Nucl. Phys. **A177**, 513 (1971).
- [55] K. Kumar, *Nuclear models and search for unity in nuclear physics*, Universitetforlaget (Bergen, 1984).
- [56] J.B. Gupta, Int. J. Mod. Phys. **E25**(10) 16500070 (2016).

### Figures captions

FIG. 1a. The partial energy level spectrum in  $^{154}\text{Gd}$  [13].

FIG. 1b. The partial energy level spectrum in  $^{154}\text{Gd}$  from DPPQ model.

FIG. 2.  $N = 90$  energy levels. The  $K^\pi = 0_2^+$  band are well visible.  $2_3 = 2_\gamma$  lie above  $2_2 = 2_\beta$ .

FIG. 3. The energy difference  $E(2_\beta - 0_\beta)$  versus  $E(2_g)$  for Ba-Dy ( $N = 88-96$ ). The continuous line is the diagonal.

FIG. 4. The potential energy curve  $V(\beta, \gamma=0^\circ)$  in  $^{154}\text{Gd}$  from DPPQM.

ZPE denotes the zero point energy  $\sim 1.8$  MeV above the  $V_{\min}$ .









