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Isospin mixing from beta-delayed proton emission

N. A. Smirnova,[1] B. Blank,[1] B. A. Brown,[2] W. A. Richter,[3,4] N. Benouaret,[5] and Y. H. Lam[6]
[1] *CENBG, CNRS/IN2P3 and Université de Bordeaux, Chemin du Solarium, 33175 Gradignan cedex, France.*
[2] *Department of Physics and Astronomy, and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA.*
[3] *iThemba LABS, P.O. Box 722, Somerset West 7129, South-Africa.*
[4] *Department of Physics, University of the Western Cape, Private Bag X17, Bellville 7535, South-Africa.*
[5] *Faculté de Physique, USTHB, El-Alia 16111, Bab-Ezzouar-Alger, Algeria*
[6] *Key Laboratory of High Precision Nuclear Spectroscopy and Center for Nuclear Matter Science, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*
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We present a general scheme of a shell-model analysis of a β-delayed proton emission. We show that the experimental proton to γ-ray branching ratio for the isobaric analog state (IAS) populated in β decay of a precursor, supplemented by theoretical proton and γ-ray widths, can be used to extract spectroscopic factors for isospin-forbidden proton emission. In the case of a well-justified two-level mixing approximation and a relatively well known spectroscopic factor of the admixed state, the latter provides a new way to determine the amount of the isospin mixing in the IAS. This conjecture is illustrated by the theoretical analysis of ^{44}Cr and ^{48}Fe decay.

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Production of proton-rich nuclei in the last decades and advances in the development of experimental techniques made it possible to explore nuclear structure via exotic decay modes, such as direct or β-delayed emission of protons, di-protons or α-particles [1, 2]. In this context, β-delayed proton emission is of particular interest since it involves transitions proceeding via isospin-symmetry breaking and thus may provide a perfect testing ground for the determination of isospin mixing in nuclear states.

The isospin symmetry is an approximate symmetry of an atomic nucleus relying on the similarity between a proton and a neutron with respect to the strong interaction. However, due to the difference of proton and neutron masses and electromagnetic interactions, the isospin symmetry is broken. The precise theoretical estimation of this breaking is crucial for numerous applications, such as tests of the Standard Model of particle physics in nuclear decays [3]. At the same time, there are only very few experimental means to determine isospin mixing in nuclear states (see e.g. [4–8]). The most direct way stems from measurement of the Fermi-strength splitting (see Ref. [9] for review), since the Fermi matrix elements in the isospin-symmetry limit are model-independent. All other experimental measurements of isospin-forbidden decay rates and/or branching ratios, including electromagnetic $E1$ transitions or $E2/M1$ branching ratios in $N = Z$ nuclei, $E1$ mirror transitions, mixed Fermi–Gamow-Teller β-decay or isospin-forbidden proton emission require theoretical input in terms of the corresponding matrix elements of the electromagnetic, Gamow-Teller or particle transfer operator, respectively. Very few experimental cases could have been used up to now to bring substantial information on the isospin impurities of good precision. For example, in the case of $E1$ transitions, a precise theoretical estimation of the $E1$-matrix elements within a nuclear structure model is extremely difficult, so information of $E1$ transitions in self-conjugate nuclei remains unexploitable. Searches for new methods are thus crucial with the aim to put important constraints on theoretical modeling.

In this article we propose a novel approach to determine the amount of isospin impurity from β-delayed proton emission. For demonstration we consider even-even precursors from the pf shell with $T_z = (N - Z)/2 = -2$.

Schematically the process mentioned above can be understood as follows: a precursor, being in its 0^+, T=2 ground state, decays via positron emission. The main branch populates the IAS, 0^+ state at around 3–6 MeV excitation energy. In the isospin-symmetry limit, proton emission from that state is forbidden. If isospin symmetry holds, the IAS would decay via γ emission only. The observation of proton emission from the IAS provides evidence that isospin symmetry is broken. Obviously, the main effect of isospin mixing should be observed for the IAS, which may be surrounded by 0^+, T=1 states in the odd-odd daughter nucleus. The quantum numbers of the proton emitted from a 0^+ state are unambiguously fixed by angular momentum conservation. We are interested in the cases when the Q value allows for both proton and γ-ray emission to be observed. A measurement of the ratio of proton to electromagnetic decays of the IAS provides an important constraint on the proton width and allows the extraction of a spectroscopic factor for the isospin-forbidden proton emission. In the special case of two-level mixing, this ratio can be used to determine the value of isospin mixing in the IAS, provided that the spectroscopic factor of the admixed, unperturbed T=1 state is relatively well known.

The ratio of proton to γ decay intensities is directly proportional to the ratio of the proton width, Γ_p, to the

γ-decay width, Γ_γ, of a given state, i.e for the IAS,

$$\frac{I_p^{\mathrm{IAS}}}{I_\gamma^{\mathrm{IAS}}} = \frac{\Gamma_p^{\mathrm{IAS}}}{\Gamma_\gamma^{\mathrm{IAS}}} \, . \tag{1}$$

Up to now, the γ-ray intensity (I_γ^{IAS}) has been precisely measured only in the case of ^{56}Zn [10]. While not known in the other cases yet, we assign values deduced from the well known theoretical feeding of the IAS in super-allowed β decay (I_β^{IAS}) and from the experimental proton intensity (I_p^{IAS}):

$$I_\gamma^{\mathrm{IAS}} = I_\beta^{\mathrm{IAS}} - I_p^{\mathrm{IAS}} \, . \tag{2}$$

The total proton width can be expressed as a single-particle proton width multiplied by the corresponding spectroscopic factor [11], i.e. $\Gamma_p = \Gamma_{sp}\, S$. In the present work, the single-particle proton width has been calculated from the proton scattering cross-section in a Woods-Saxon potential with the potential depth adjusted to reproduce known proton energies. The electromagnetic widths can be obtained within the shell model. Thus, the spectroscopic factor for an isospin-forbidden proton emission from the IAS can be extracted via a simple relation:

$$S_{\mathrm{exp}}^{\mathrm{IAS}} = \frac{\Gamma_\gamma^{\mathrm{IAS}}}{\Gamma_{sp}^{\mathrm{IAS}}} \frac{I_p^{\mathrm{IAS}}}{I_\gamma^{\mathrm{IAS}}} \, . \tag{3}$$

In some cases, isospin mixing of the IAS can be modeled as due to the admixture of a close-lying state of the same spin and parity, but of a different isospin (0^+, T=1 states for the cases under consideration). Within a two-level mixing approximation we can express the IAS as $|\mathrm{IAS}\rangle = \sqrt{1-\alpha^2}|T=2\rangle + \alpha|T=1\rangle$. Then, the spectroscopic factor for proton emission from the IAS due to that mixing is

$$S^{\mathrm{IAS}} = \alpha^2 \, S^{\mathrm{T=1}} \, , \tag{4}$$

where $S^{\mathrm{T=1}}$ is the spectroscopic factor of the isospin-allowed emission from the admixed state. Hence, providing a theoretical value of $S^{\mathrm{T=1}}$, we can deduce the amount of the isospin-mixing in the IAS as

$$\alpha_{\mathrm{exp}}^2 = \frac{\Gamma_\gamma^{\mathrm{IAS}}}{\Gamma_{sp}^{\mathrm{IAS}} S^{\mathrm{T=1}}} \frac{I_p^{\mathrm{IAS}}}{I_\gamma^{\mathrm{IAS}}} \, . \tag{5}$$

For the lowest states with relatively large spectroscopic factors, the theoretical uncertainty on $S^{\mathrm{T=1}}$ is quite small, and Eq. (5) thus proposes a new way to determine isospin mixing from experimental data. Higher-lying states with relatively small spectroscopic factors (0.01 or less) can have large theoretical errors, and the values should be taken as upper limits. This may limit the applicability of Eq. (5).

In the present work we demonstrate the proposed method on ^{44}Cr and ^{48}Fe decays which satisfy the conditions described above. A detailed theoretical study of a number of pf shell emitters and the systematic extraction of spectroscopic factors for isospin-forbidden proton emission is performed elsewhere (Ref. [12] and work in preparation).

Theoretical calculations have been performed in the full pf shell using the NuShellX@MSU [13] shell-model code with specific charge-dependent Hamiltonians. The first one, cdGX1A, is based on the GXPF1A [14] interaction with the addition of Coulomb, strong charge-symmetry breaking and charge-independence breaking interactions from Ref. [15] and updated isovector single-particle energies from Ref. [16]. The other Hamiltonian, cdKB3G, was constructed on the basis of the KB3G interaction [17], with the addition of the Coulomb interaction and isovector single-particle energies scaled as $\sqrt{\hbar\omega(A)}$.

Experimentally established partial decay schemes of ^{44}Cr and ^{48}Fe are shown in Figs. 1 and 2, in comparison with theoretical calculations using the two charge-dependent interactions. There is a number of 1^+ states below and above the IAS, which are also described theoretically. In order not to overload the figures, we do not show them. Figures 3 and 4 demonstrate that theory predicts rather well the low-energy spectra of odd-odd ^{44}Sc and ^{48}V, which are the mirrors of the daughter nuclei, ^{44}V and ^{48}Mn, respectively. Let us remark that the experimental spectra of neutron-rich partners contain a few more states (of negative parity, but also of positive parity) which are not described by the pf shell model. Those states correspond to nucleon excitations from the sd to the pf shell (intruder states) and hence require calculations in the complete $sdpf$ space, which is not easily tractable. Thus, in the present study we explore the level schemes and decay properties of those nuclei in the pf shell-model space.

Theoretical β-decay half-lives of the precursors, excitation energies and electromagnetic decay widths of the IAS in the daughter nuclei are summarized in Table I. We used standard effective charges $e_p = 1.5e$, $e_n = 0.5e$ and optimized empirical g-factors as given in Ref. [14] for the cdGX1A interaction and in Ref. [17] for the cdKB3G interaction. We used experimental energies of γ-ray transitions from the IAS when available. We applied a quenching factor $q_F = 0.74$ to the Gamow-Teller (GT) operator to calculate the β-decay strength distribution in the Q window. The large uncertainty on the theoretical half-life of ^{44}Cr is due to the poorly known Q value. We note a generally good agreement between calculations and available experimental data (see also Ref. [20] for an earlier study of these and other pf shell $T_z = -2$ nuclei).

TABLE I. Theoretical half-lives of precursors, excitation energies and electromagnetic widths of the IAS in daughter nuclei.

Pre-	$T_{1/2}$ [ms]		E_{IAS} [MeV]		$\Gamma_\gamma^{\mathrm{IAS}}$ [eV]	
cursor	cdGX1A	cdKB3G	cdGX1A	cdKB3G	cdGX1A	cdKB3G
^{44}Cr	57(13)	56(14)	3.577	3.269	1.74	0.92
^{48}Fe	45.2(5)	47.4(6)	3.306	2.921	0.56	0.52

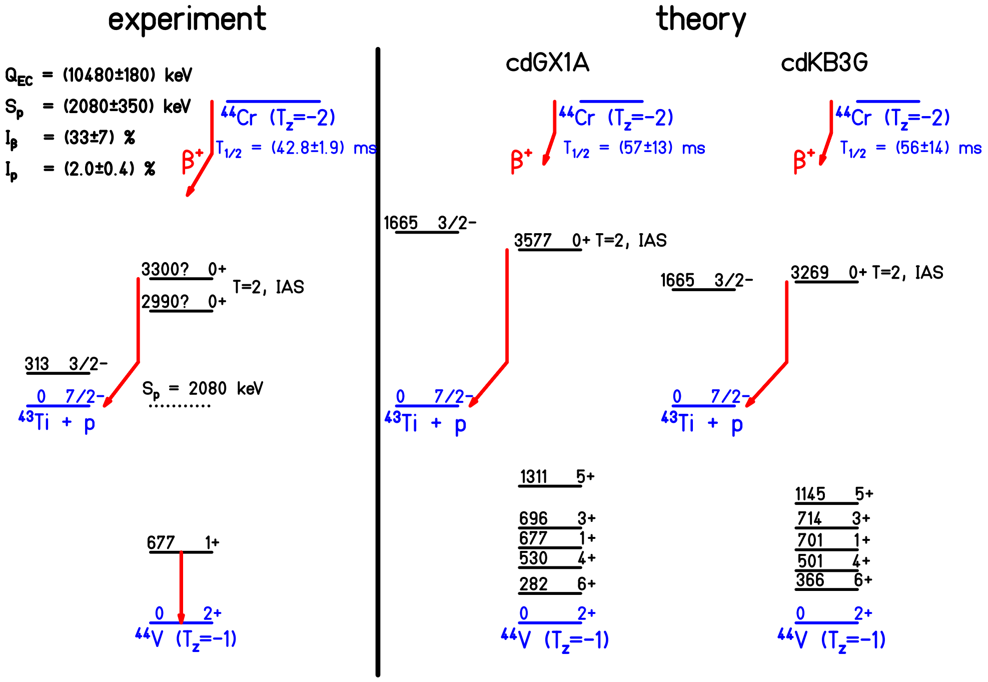

FIG. 1. (Color online) Experimental and theoretical partial decay schemes of ^{44}Cr. The intensity I_β is the theoretically calculated feeding of the IAS via a superallowed β decay, the proton intensity I_p is the experimentally observed proton-emission branching ratio tentatively assigned to be from the IAS. The Q value and the proton separation energy are from Refs. [18, 19]. The theoretical calculations have been performed with two different effective interactions, cdKB3G and cdGX1A.

^{44}Cr. The experimental work [21, 22] on β-delayed proton emission from ^{44}Cr reports on a few proton branches, with the one, Q_p=909(11) keV of 2.0(4)% branching ratio (average from three measurements), being tentatively assigned to the decay of the IAS.

The excitation energy of the IAS in ^{44}V is not firmly established yet due to a large uncertainty on the ground state mass excess. Based on the mass excess of ^{43}Ti and the proton energy, we could propose two tentative assignments for the IAS excitation energy, depending whether the proton is emitted to the ground (J^π=7/2$^-$) or to the first excited (J^π=3/2$^-$) state of ^{43}Ti (see Fig. 1 and discussion below).

Theory predicts the IAS to be the lowest 0^+ state in the spectrum of ^{44}V, well separated from the lowest energy 0^+, T=1 state (ΔE=1199 keV by cdGX1A and ΔE=427 keV by cdKB3G). The Coulomb mixing matrix element V between a neighboring state 0^+, T=1 at excitation energy E and the IAS can be estimated from the splitting of the Fermi strength due to isospin mixing. Let us denote $B(F)_{\rm IAS}$ as the main Fermi strength going to the IAS and $B(F)$ the Fermi branch populating the 0^+, T=1 state. If $R=\sqrt{B(F)/B(F)_{\rm IAS}}\ll 1$, then we can estimate the mixing matrix element as $V\approx\Delta E\times R$. In the case of ^{44}V, we get V=9 keV from cdGX1A and V=6 keV from cdKB3G. In first order perturbation theory, the magnitude of mixing is proportional to $(V/\Delta E)^2$. Due to a high uncertainty on the energies difference, ΔE, calculated in an odd-odd nucleus the theoretical amount of the isospin mixing is quite uncertain [23].

The proton single-particle width of the IAS in ^{44}V is $\Gamma^{\rm IAS}_{sp}(0f_{7/2})$=0.07(1) eV and $\Gamma^{\rm IAS}_{sp}(1p_{3/2})$=16.5(22) eV. The small spectroscopic factors for an isospin-forbidden proton emission (of the order of 10^{-4}) suggest that proton emission to the 7/2$^-$ ground state of ^{43}Ti should be strongly hindered. Exploring the possibility of a $p_{3/2}$ emission to the first excited state in ^{43}Ti, we get a proton width of the IAS $\Gamma^{\rm IAS}_p(1p_{3/2})=1.6(2)\times10^{-4}$ eV for cdGX1A or $\Gamma^{\rm IAS}_p(1p_{3/2})=4.1(6)\times10^{-3}$ eV for cdKB3G. This looks to be a more plausible scenario. With the average theoretical γ width of the IAS, $\Gamma^{\rm IAS}_\gamma=1.34(42)$ eV, we deduce from experimental data the spectroscopic factor for an isospin-forbidden proton emission $S^{\rm IAS}_{\rm exp}=5.3(24)\times10^{-3}$.

The shell-model spectroscopic factor for the lowest 0^+, T=1 state in ^{44}V is relatively large and is very similar for both interactions, on average $S^{\rm T=1}$=0.75(4). Although this state is little admixed it provides the major contribution to the IAS spectroscopic factor. Thus, the conditions for Eq. (5) are satisfied and, assuming that $I_p=2.0(4)\%$ belongs to the IAS, we can deduce the isospin-mixing from the data as $\alpha^2_{\rm exp}=0.7(3)\%$. We stress that this experimentally deduced value of the isospin mixing does not depend on a theoretical energy difference between the isospin-mixed states in an odd-odd nucleus.

The β-decay strength distribution indicates that al-

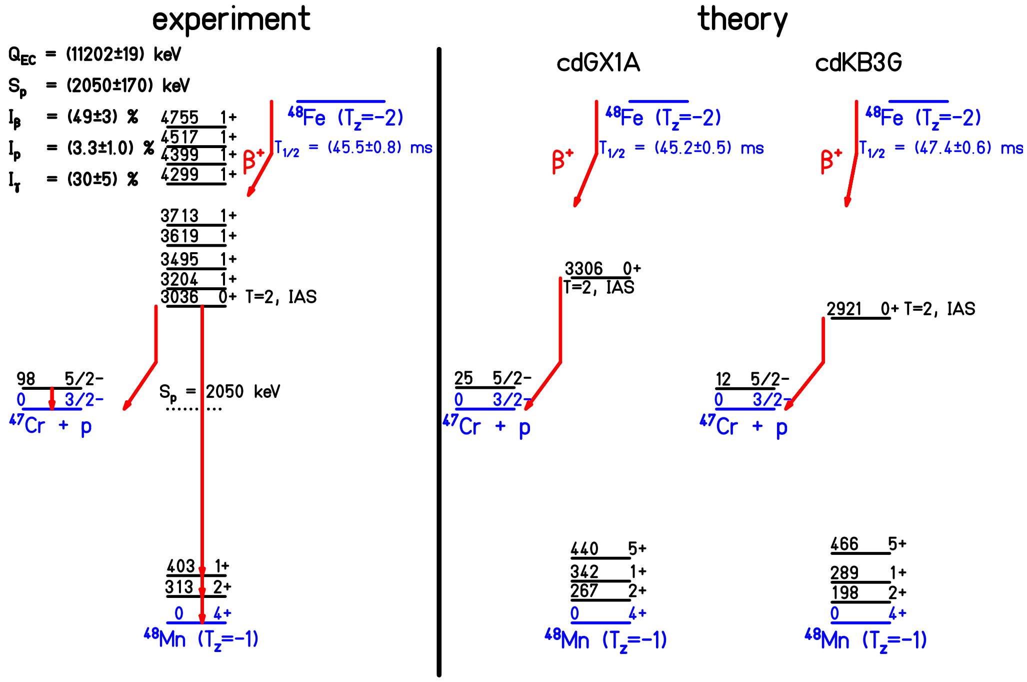

FIG. 2. (Color online) Experimental and theoretical partial decay schemes of ^{48}Fe. See caption to Fig. 1 for details.

most half of the β strength goes to the lowest 1^+ state of ^{44}V (57(2)% from cdGX1A and about 48(2)% from cdKB3G), about 22-28% populates the IAS and the rest is distributed among a few 1^+ states (one of them is below, while the others are above the IAS). The proton widths of those 1^+ states are typically much larger than their respective electromagnetic widths, resulting thus in 11(1)% (cdGX1A) or 17(1)% (cdKB3G) as an upper limit of the total proton branching ratio ($E_p > 1$ MeV). This corresponds well to the experimental value of 10(2)% (an average from Refs. [21, 22] with the emission from the IAS being subtracted).

^{48}Fe. The latest experimental work [19] with γ-ray - proton coincidences assigns a 1018(10) keV proton group with $I_p = 4.8(3)\%$ to be emitted from the IAS in ^{48}Mn. This assignment is in agreement with that done in the earlier literature (summarized in [21], however with a much lower intensity, $I_p = 1.7(3)\%$). For our theoretical consideration, we adopt that a 1013(9) keV proton group of $I_p = 3.3(10)\%$ is emitted from the IAS (average values from three measurements). A few more proton groups seen are supposed to correspond to the proton emission from the GT-fed 1^+ states. In addition, three gamma lines of 90, 313 and 98 keV have been observed in [19]. A 2634-keV γ-ray line with $I_\gamma = 30(5)\%$, registered in [21], most likely corresponds to the decay of the IAS.

In general, the partial decay scheme of ^{48}Fe is well supported by the shell-model calculations. Both charge-dependent interactions predict the IAS to be the lowest 0^+ state in the spectrum of ^{48}Mn, separated by 245 keV with V=17 keV (cdGX1A) or by 295 keV with V=15 keV (cdKB3G) from the next excited 0^+, T=1 state.

The theoretical proton width of the IAS is 14.6(16) eV, while its average electromagnetic width from the two interactions for a measured energy of E_γ=2.634 MeV is $\Gamma_\gamma^{\mathrm{IAS}}$=0.54(2) eV. Since no full proton-γ spectroscopy has been performed in Ref. [21], we estimate the intensity of the γ decay from the IAS based on the calculated β feeding 49(3)% [19] and the measured proton intensity (Eq. (2)). As shown in Eq. (3), we can deduce from the experiment the spectroscopic factor $S_{\mathrm{exp}}^{\mathrm{IAS}} = 2.7(9) \times 10^{-3}$. This value is close to the predictions of the cdGX1A interaction (see Table II).

To determine the spectroscopic factor of the admixed 0^+, T=1 state, we notice that both interactions predict a summed single-particle strength of 0.676 (cdGX1A) and 0.673 (cdKB3G) which is split among three T=1 states as shown in Table III. With the cdKB3G interaction, the spectroscopic strength is accumulated mainly in the 0_3^+ state, while the cdGX1A interaction predicts that it is split about 1/3 in the 0_2^+ state and 2/3 in the 0_3^+ state. This distribution correlates with the corresponding energy differences between the states. The Coulomb mixing matrix elements between the states 0_i^+, T=1 and the IAS are also shown in Table III, and it is seen that there is a good agreement between the two interactions. The difference in the spectroscopic strength distribution is due to the difference in energy splitting between the 0_2^+ and 0_3^+ states, which as we remarked already is uncertain in an odd-odd nucleus. While there is no experimental data on those excitation energies and keeping in mind that the

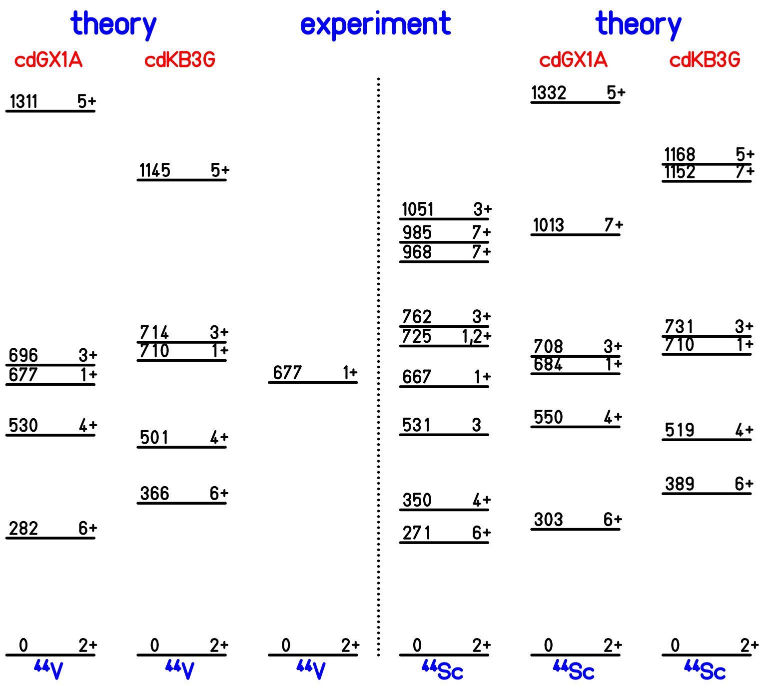

FIG. 3. (Color online) Experimental (in the center) and theoretical (on the right and on the left) level schemes of mirror nuclei ^{44}V and ^{44}Sc. The experimental data are from Refs. [18, 19]. The theoretical calculations have been performed with two different effective interactions, cdGX1A and cdKB3G.

TABLE II. Theoretical single-particle proton widths ($\Gamma_{sp}^{\rm IAS}$) and shell-model spectroscopic factors ($S^{\rm IAS}$), the experimentally deduced values of the spectroscopic factors ($S_{\rm exp}^{\rm IAS}$) and isospin-mixing of the IAS ($\alpha^2_{\rm exp}$). The spins of the final nuclei and the measured proton decay energies from Refs. [19, 21] are indicated for reference.

Pre-cursor	J_f^π	Q_p [keV]	$\Gamma_{sp}^{\rm IAS}$ [eV]	$S^{\rm IAS}$ cdGX1A	$S^{\rm IAS}$ cdKB3G	$S_{\rm exp}^{\rm IAS}$	$\alpha^2_{\rm exp}$ (%)
^{44}Cr	$3/2^-$	909(11)	16.5(22)	1.0×10^{-4}	2.5×10^{-4}	$5.3(24) \times 10^{-3}$	0.7(3)
^{48}Fe	$3/2^-$	1018(10)	14.6(16)	1.5×10^{-3}	1.0×10^{-4}	$2.7(9) \times 10^{-3}$	1.4(5)

cdGX1A spectroscopic factor for the IAS is closer to the experimentally deduced value, we adopt for further analysis the cdGX1A value of the spectroscopic factor of the admixed state, $S^{\rm T=1}$=0.19. Eq. (5) is thus applicable, and we can deduce the isospin-mixing from the data as $\alpha^2_{\rm exp}$=1.4(5)%.

TABLE III. Shell-model excitation energies, interaction mixing matrix elements and spectroscopic factors of the lowest 0^+ states in ^{48}Mn with respect to proton emission to the $3/2^-$ ground state of ^{47}Cr.

State	cdGX1A E [MeV]	cdGX1A V [keV]	cdGX1A S	cdKB3G E [MeV]	cdKB3G V [keV]	cdKB3G S
0_1^+ (IAS)	3.039	-	0.0015	2.921	-	0.0001
0_2^+	3.284	17	0.1897	3.216	15	0.0100
0_3^+	3.417	9	0.3672	3.785	8	0.6628
0_4^+	3.900	12	0.0545	4.655	11	0.0029

Both interactions predict the existence of at least three 1^+ states above the IAS, carrying each a few percent of the GT strength, and characterized by small electromagnetic widths. The electromagnetic widths of those states are calculated to be much smaller than their proton widths, so these three states are thus other possible candidates for the observed proton emission. Calculations of the proton widths of all those 1^+ states confirm that all of them should indeed be doublets corresponding to the $p_{3/2}$ proton emission from 1^+ parent states to the $3/2^-$ ground state and the $5/2^-$ state at 98 keV excitation energy in ^{47}Cr (see discussion in Ref. [19]).

Both interactions predict that about 13(1)% of the β-decay strength populates 1^+ states above the IAS via a GT component. This estimate, considered as an upper limit for the proton emission probability, agrees with the experimental value for the total proton branching ratio of 11.9(17)%, without taking into account the IAS [19].

In conclusion, we have proposed a new way to extract a tiny amount of isospin mixing using the experimental β-delayed proton to γ emission branching ratios. It requires a simple shell-model input for the proton and γ widths which may be obtained from well established

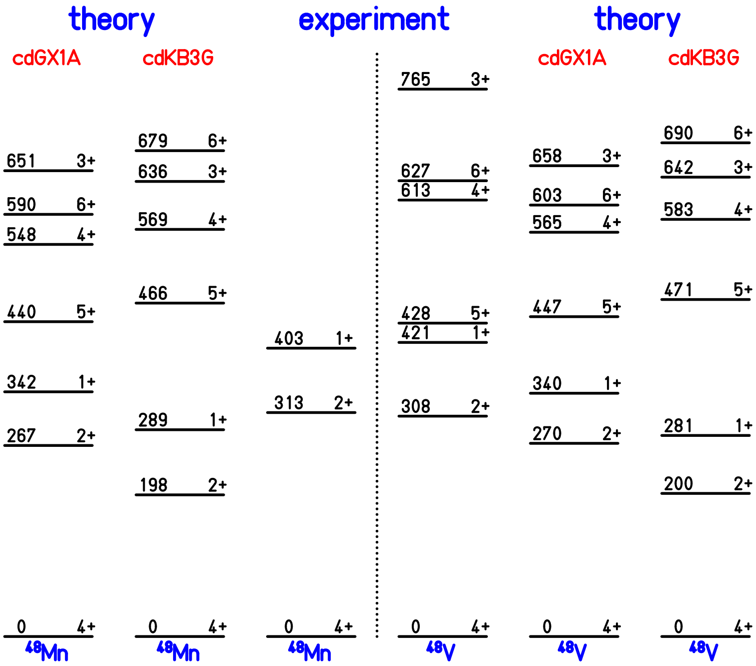

FIG. 4. (Color online) Experimental (in the center) and theoretical (on the right and on the left) level schemes of mirror nuclei ^{48}Mn and ^{48}V. See caption to Fig. 3 for details.

isospin-conserving calculations. The necessary conditions are (i) a two-level mixing approximation and (ii) a large value for the spectroscopic factor of the admixed state as provided by the shell model. This is typically the case when it is the lowest $J^\pi, T = T_z$ state which is admixed to a $J^\pi, T = T_z + 1$ state. The application is demonstrated on ^{44}Cr and ^{48}Fe. Preliminary considerations indicate that the case of ^{24}Si, an *sd*-shell precursor, could also be explored in future.

Dedicated high-resolution experiments with full proton - γ-ray spectroscopy are required to unambiguously identify the IAS and to measure precisely its proton/γ branching ratio in order to test predictions of the charge-dependent shell-model Hamiltonians presented here. The understanding of these processes will shed light on the mechanisms behind the isospin mixing and will be important to constraint theoretical models.

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