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Pseudomagnetic effects for resonance neutrons

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Abstract

A general theory of pseudomagnetic effects on the propagation of polarized neutrons through a polarized target using multi-resonance approach is presented. Some applications related to proposed searches for time reversal invariance violation in neutron scattering are considered.

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I. INTRODUCTION

Neutron spin optics in polarized nuclear targets has become a very important topic because of recent proposals for searches for time reversal invariance violation (TRIV) in neutron-nucleus scattering (see, for example [1] and references therein). The proposed experiments require a understanding of neutron spin dynamics during propagation through polarized nuclear targets in the presence of multiple s-wave and p-wave resonances, since the neutron spin rotation due to strong spin-spin interactions can reduce the values of TRIV observables or, in some cases, mimic TRIV effects [2–10].

The phenomenon of neutron spin rotation, known as a pseudomagnetic effect, in the propagation of polarized slow neutrons through a polarized target was predicted in ref. [11]. The phenomenon is related to the fact that, due to strong spin-spin interactions, the value of the neutron wave index of refraction depends on the relative orientation of the neutron spin relative to the direction of nuclear polarization,

$$n_{\pm}^2 = 1 + \frac{4\pi}{k^2} \sum_i N_i f_{\pm}^i. \tag{1}$$

Here, N_i is the number of nuclei of type i per unit volume, k is the neutron wave number, and $f_{\pm}^i$ is the neutron elastic forward scattering amplitude on an type-i nucleus for the positive and negative projections of the neutron spin along the direction of the nuclear polarization. Taking into account that the second term in the above equation is much smaller than the unity, we can write the difference of the refractive indices with different neutron spin orientation as

$$\Delta n = n_+ - n_- = \frac{2\pi}{k^2} \sum_i N_i (f_+^i - f_-^i). \tag{2}$$

This difference in refraction indices leads to a rotation of the neutron spin around the direction of the nuclear polarization, through the angle of $\varphi = k\Delta n z$ after the neutrons have propagated through a target of thickness z. The corresponding frequency of the neutron spin rotation is given [11] as

$$\omega_{\mathrm{P}} = v_{\mathrm{n}} \frac{\mathrm{d}\varphi}{\mathrm{d}z} = \frac{2\pi\hbar}{M_{\mathrm{n}}} \sum_i N_i \mathrm{Re}\,(f_+^i - f_-^i), \tag{3}$$

where v_{n} is the neutron velocity, and M_{n} is the neutron mass. For very low energy neutrons, the scattering amplitudes do not depend on neutron energy, and as a consequence, the frequency $\omega_{\rm P}$ has a constant value, which depends only on the properties of the polarized target. Therefore, it was suggested in [11] to consider an effective pseudomagnetic field, which produces a precession of the neutron spin at the frequency $\omega_{\rm P}$,

$$B_{\rm P} = \frac{\hbar\omega_{\rm P}}{2\mu_{\rm n}}, \tag{4}$$

as a natural characteristic of the target. (Here $\mu_{\rm n}$ is the neutron magnetic moment). Numerically, $B_{\rm P}/(1{\rm T}) = 5.47\omega_{\rm P}/(1{\rm GHz})$. This phenomenon has been studied mostly for the case of very low energy (thermal) neutrons (see [12–15] and references therein). In paper [16], the pseudomagnetic spin precession was studied in the presence of a low energy s-wave neutron resonance. In that case, the parameters $B_{\rm P}$ and $\omega_{\rm P}$ show very strong energy dependencies in the vicinity of the resonance. In this paper we present a general formalism for the pseudomagnetic phenomena and apply it for the multi-resonance case, involving neutron resonances with different parities.

II. GENERAL FORMALISM FOR PSEUDOMAGNETIC [←MISSPELLED CORRECTED] FIELD

Let us consider the reaction matrix $\hat{T}$, which is related to the scattering matrix $\hat{\mathbb{S}}$ and the matrix $\hat{R}$ as

$$2\pi i\hat{T} = \hat{1} - \hat{\mathbb{S}} = \hat{R}. \tag{5}$$

Thus a reaction amplitude $\hat{f}$ can be written as $\hat{f} = -\pi(k_{\rm i}k_{\rm f})^{-1/2}\hat{T}$, where $k_{\rm i,f}$ are values of initial and final momentum, respectively. Then, to describe the scattering of polarized neutrons on a polarized target with the spin $\vec{I}$, we need to calculate the corresponding reaction matrix elements

$$\left\langle \vec{k}_{\rm f}\mu_{\rm f} \middle| T \middle| \vec{k}_{\rm i}\mu_{\rm i} \right\rangle, \tag{6}$$

where $\mu_{\rm i,f}$ is the projection of the neutron spin along the axis of quantization. For coherent elastic scattering at zero angle, the initial and final values of neutron momenta and spin projections are equal to each other, $\vec{k}_{\rm i} = \vec{k}_{\rm f} = \vec{k}$ and $\mu_{\rm i} = \mu_{\rm f} = \mu$.

It is convenient to relate this matrix to the matrix $\hat{R}$ in the integral of motion representation of the S-matrix [17]

$$\left\langle S'l'\alpha' \middle| \mathbb{S}^J \middle| Sl\alpha \right\rangle \delta_{JJ'}\delta_{MM'}\delta(E'-E), \tag{7}$$

where J and M are the total spin and its projection, S is the channel spin, l is the orbital momentum, and α represents the other internal quantum numbers. Taking into account that the spin channel is a sum of the neutron spin $\vec{s}$ nucleus spin $\vec{I}$

$$\vec{S} = \vec{s} + \vec{I}, \tag{8}$$

and the total spin is

$$\vec{J} = \vec{S} + \vec{l}, \tag{9}$$

one can write T-matrix elements as

$$\begin{aligned} 2\pi i \left\langle \vec{k}\mu \middle| T \middle| \vec{k}\mu \right\rangle &= \sum_{JMlml'm'Sm_sS'm'_s} Y_{l'm'}(\theta,\phi) \left\langle s\mu I M_I | S'm'_s \right\rangle \left\langle S'm'_s l'm' | JM \right\rangle \\ &\quad \times \left\langle S'l'\alpha' \middle| R^J \middle| Sl\alpha \right\rangle \left\langle JM | Sm_s lm \right\rangle \left\langle Sm_s | s\mu I M_I \right\rangle Y^*_{lm}(\theta,\phi), \end{aligned} \tag{10}$$

where angles (θ, ϕ) describe the direction of the neutron momentum $\vec{k}$. For the simplicity of further formulae, let's choose the quantization axis along the vector $\vec{k}$. It should be noted that, for the case of s-wave neutrons, all the expressions do not depend on the choice of the quantization axis. The formulae for p-wave neutrons with an arbitrary choice of the quantization axis are presented in Appendix A. Then, the amplitude for neutron elastic scattering can be written as

$$\begin{aligned} f_\mu = \frac{i}{2k} \sum_{JlSS'M_I} & (2l+1) \left\langle s\mu I M_I | S'm'_s \right\rangle \left\langle S'm'_s l0 | JM \right\rangle \\ & \times \left\langle S'l \middle| R^J \middle| Sl \right\rangle \left\langle JM | Sm_s l0 \right\rangle \left\langle Sm_s | s\mu I M_I \right\rangle . \end{aligned} \tag{11}$$

It should be noted that, in the above expression for the amplitude, the sum over M_I must be taken carefully to be consistent with the polarization state of the nuclear target. For example, for the case of vector polarization, which we consider in details here, only the term with $M_I = I$ is presented in the sum.

The matrix elements in Eq.(11) for slow neutrons can be written in the Breit-Wigner resonance approximation with one s-resonance or p-resonance as

$$\begin{aligned} \left(F^J_{S'Sl}\right)_K &\equiv \left\langle S'_K l_K \middle| R^{J_K} \middle| S_K l_K \right\rangle \\ &= i \frac{\sqrt{\Gamma^{\mathrm{n}}_{l_K}(S'_K)}\sqrt{\Gamma^{\mathrm{n}}_{l_K}(S_K)}}{E - E_K + i\Gamma_K/2} e^{i(\delta_{l_K}(S'_K) + \delta_{l_K}(S_K))} - 2ie^{i\delta_{l_K}(S_K S'_K)} \sin\delta_{l_K}(S_K S'_K) \end{aligned} \tag{12}$$

where E_K, Γ_K, and $\Gamma^n_{l_K}$ are the energy, the total width, and the partial neutron width of the K-th nuclear compound resonance, E is the neutron energy, and δ_{l_K} is the potential scattering phase shift. For p-wave resonances we keep only the resonance term, because for low energy neutrons $\delta_l \sim (kR_0)^{2l+1}$ (where R_0 is nucleus radius), and, as a consequence, the contribution from p-wave potential scattering is negligible.

Now, following the definition in Eq.(3), one can obtain the frequency of neutron spin rotation due to the pseudomagnetic field for a nuclear target with a single element as

$$\omega_{\mathrm{P}} = \frac{2\pi N\hbar}{M_{\mathrm{n}}}\mathrm{Re}\,(f_{\frac{1}{2}} - f_{-\frac{1}{2}}). \tag{13}$$

One can see that for the case of s-wave neutron scattering on the vector polarized target, the difference of the amplitudes in Eq.(13) is

$$f_{\frac{1}{2}} - f_{-\frac{1}{2}} = \frac{i}{2k}\frac{2I}{2I+1}\left(F^{I+\frac{1}{2}}_{I+\frac{1}{2}\,I+\frac{1}{2}\,0} - F^{I-\frac{1}{2}}_{I-\frac{1}{2}\,I-\frac{1}{2}\,0}\right). \tag{14}$$

For the case of very slow neutrons one can neglect the resonance term contribution (the first term in Eq.(12)) to the R-matrices in the above equation. Then the R-matrix can be written in terms of the neutron scattering lengths $a_\pm$ for spin orientations parallel and antiparallel to the direction of nuclear polarization as

$$\left\langle \left(I \pm \frac{1}{2}\right) 0 \middle| R^{I\pm\frac{1}{2}} \middle| \left(I \pm \frac{1}{2}\right) 0 \right\rangle = -2ika_\pm, \tag{15}$$

which gives us the well known expression [11] for the pseudomagnetic frequency for thermal neutrons

$$\omega_{\mathrm{P}} = \frac{4\pi N\hbar}{M_{\mathrm{n}}}\frac{I}{(2I+1)}(a_+ - a_-). \tag{16}$$

For the low energy resonance region we need to take into account not only potential scattering, but also the contributions from each resonance. Thus, for example, with the presence of s-wave resonances with total spins $J = I \pm 1/2$, the pseudomagnetic frequency becomes neutron energy dependent and is given as

$$\omega^{\mathrm{s}}_{\mathrm{P}} = \frac{4\pi N\hbar}{M_{\mathrm{n}}}\frac{I}{(2I+1)}\left(a_+ - a_- - \sum_{K,l_K=0}\frac{\Gamma^{\mathrm{n}}_K}{2k}\frac{(E-E_K)}{(E-E_K)^2 + (\Gamma_K/2)^2}\beta_K\right), \tag{17}$$

$$\beta_K = \begin{cases} 1 & (J_K = I + \frac{1}{2}) \\ -1 & (J_K = I - \frac{1}{2}) \end{cases} \tag{18}$$

where the subscripts $\pm$ for resonance parameters corresponds to resonances with total spins $J = I \pm 1/2$, respectively. One can see that the pseudomagnetic frequency has a a sharp oscillation with the sign changing at the position of each s-wave resonance [16].

For the case of p-wave resonances the corresponding difference of amplitudes in Eq.(13) is

$$f_{\frac{1}{2}} - f_{-\frac{1}{2}} = \tag{19}$$

$$= \begin{cases} 0 & (J = I - \frac{3}{2}) \\ -\frac{3i}{k}\frac{I}{(2I+1)^2}\left((2I-1)F^J_{I-\frac{1}{2}\,I-\frac{1}{2}\,1} + 2\frac{\sqrt{2I-1}}{\sqrt{I+1}}F^J_{I-\frac{1}{2}\,I+\frac{1}{2}\,1} + \frac{1}{I+1}F^J_{I+\frac{1}{2}\,I+\frac{1}{2}\,1}\right) & (J = I - \frac{1}{2}) \\ -\frac{3i}{k}\frac{I}{(2I+1)^2}\left(2F^J_{I-\frac{1}{2}\,I-\frac{1}{2}\,1} - 2\frac{2I-1}{\sqrt{I(2I+3)}}F^J_{I-\frac{1}{2}\,I+\frac{1}{2}\,1} - \frac{(5+4I)(I+1)}{2I+3}F^J_{I+\frac{1}{2}\,I+\frac{1}{2}\,1}\right) & (J = I + \frac{1}{2}) \\ \frac{3i}{k}\frac{I}{(2I+3)(I+1)}F^J_{I+\frac{1}{2}\,I+\frac{1}{2}\,1} & (J = I + \frac{3}{2}) \end{cases}$$

which leads to the pseudomagnetic frequency from p-resonances

$$\omega^{\mathrm{p}}_{\mathrm{P}} = \frac{6\pi N\hbar}{M_{\mathrm{n}}k}\frac{I}{(2I+1)}\sum_{K,\,l_K=1}\gamma_K\frac{E-E_K}{(E-E_K)^2+(\Gamma_K/2)^2} \tag{20}$$

$$\gamma_K = \begin{cases} 0 & (J_K = I - \frac{3}{2}) \\ \frac{1}{2I+1}\big((2I-1)\Gamma^{\mathrm{n}}_K(I-\frac{1}{2}) \\ \quad +2\sqrt{\frac{2I-1}{I+1}}\sqrt{\Gamma^{\mathrm{n}}_K(I+\frac{1}{2})}\sqrt{\Gamma^{\mathrm{n}}_K(I-\frac{1}{2})} \\ \quad +2\Gamma^{\mathrm{n}}_K(I+\frac{1}{2})\big) & (J_K = I - \frac{1}{2}) \\ \frac{1}{2I+1}\big(2\Gamma^{\mathrm{n}}_K(I-\frac{1}{2}) \\ \quad -2\sqrt{\frac{2I-1}{I(2I+3)}}\sqrt{\Gamma^{\mathrm{n}}_K(I+\frac{1}{2})}\sqrt{\Gamma^{\mathrm{n}}_K(I-\frac{1}{2})} \\ \quad -\frac{5+4I(I+1)}{2I+3}\Gamma^{\mathrm{n}}_K(I+\frac{1}{2})\Big) & (J_K = I + \frac{1}{2}) \\ -\frac{2I+1}{(2I+3)(I+1)}\Gamma^{\mathrm{n}}_K(I+\frac{1}{2}) & (J_K = I + \frac{3}{2}) \end{cases} \tag{21}$$

It should be noted that the signs of the amplitudes of the neutron decay widths $\sqrt{\Gamma^{\mathrm{n}}_K(I \pm \frac{1}{2})}$ must be obtained from experiments. This expression looks complicated; however, since p-wave resonances are very weak in low energy region, usually only the closest resonance contribution needs to be taken into account. Therefore, at most three terms in the above expression will actually contribute to p-wave dependent part of the pseudomagnetic frequency. Moreover, for the case of TRIV searches only resonances with $J = I - 1/2$ and $J = I + 1/2$ are of interest, since only these resonances can be mixed with s-resonances (which have spins $J = I - 1/2$ and $J = I + 1/2$) by weak and TRIV interactions. One can see also that in contrast to s-wave resonances this pseudomagnetic frequency depends, in general, not on total neutron widths, but on the partial neutron widths for different spin channels. (For the relation of the spin channel formalism with the spin-orbital scheme formalism see Appendix B.)

Up to now we considered the case of pure vector polarized mono isotopic target. Based on the coherent nature of the pseudomagnetic effect, it is easy to generalized all the above expressions for the case of a composite target with an arbitrary polarization. Thus, for composite (multi isotope) target, the total pseudomagnetic frequency is a linear sum of frequencies from all isotopes presented in the target. The case of arbitrary polarization of each isotope is accounted by a summation of differences of amplitudes of Eq.(11) taken with the corresponding weights $w(M_I)$ for each spin projection quantum number M_I, which is the weight in the density operator used for the description of the general polarization in terms of the density polarization matrix. Therefore, the resulting pseudomagnetic frequency ω_P^* can be written as [11]

$$\omega_{\mathrm{P}}^* = \omega_{\mathrm{P}} \frac{1}{I} \sum_{M_I} w(M_I) M_I. \tag{22}$$

It should be noted that, in Eqs.(19) and (21), there are no contributions from the resonance with a total spin $J = I - 3/2$, but there is a contribution with $J = I + 3/2$. This asymmetry simply reflects the fact that we consider the case with a pure vector polarization of the target, which corresponds to $M_I = I$. For the case of mixed target polarization with a fractional population of the target nuclear level of $M_I = -I$, the resonance with a spin $J = I - 3/2$ can also lead to pseudomagnetic precession due to corresponding difference of amplitudes

$$f_{\frac{1}{2}} - f_{-\frac{1}{2}} = \frac{3i}{k} \frac{I}{(2I+3)(I+1)} F^J_{I-\frac{1}{2}\,I-\frac{1}{2}\,1}. \tag{23}$$

However, as it was mentioned above, these resonances cannot lead to TRIV effects.

III. PSEUDOMAGNETIC EFFECTS IN LANTHANUM ALUMINATE

Let us consider the application of the present formalism to the pseudomagnetic effect in lanthanum aluminate crystals. Since a very large parity violating effect was observed on ^{139}La in the vicinity of the 0.734 eV resonance [18–21], this isotope looks like a promising target for a search of for TRIV effects in nuclei [1]. ^{139}La nuclei can be polarized in lanthanum aluminate crystals with currently experimentally achieved value of ^{139}La polarization [22, 23] of 47.5%.

Since we do not know partial neutron widths for the p-wave resonance, we describe the ratio $x_s = \sqrt{\Gamma_{\rm p}^{\rm n}(I-\frac{1}{2})}/\sqrt{\Gamma_{\rm p}^{\rm n}(I-\frac{1}{2})+\Gamma_{\rm p}^{\rm n}(I+\frac{1}{2})}$ (see Appendix B) using a parameter α, such that $x_s = \sin\alpha$. Fig. 1 shows the pseudomagnetic field in the 100% polarized lanthanum target as a function of neutron energy in the vicinity of p-wave resonance for $\alpha = 0$, $\alpha = \pi/4$, $\alpha = -\pi/4$, and $\alpha = \pi/2$.

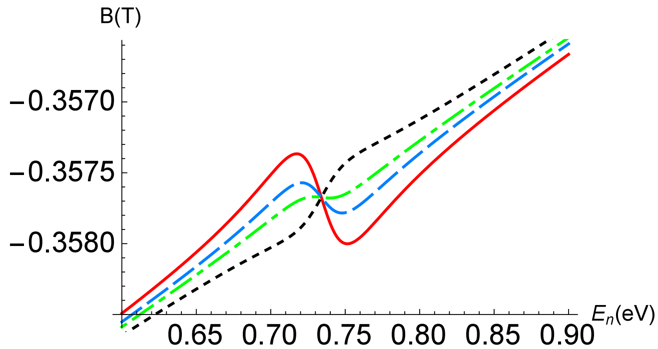

FIG. 1. (Color online) Pseudomagnetic field in the fully polarized lanthanum target for $\alpha = 0$ (solid line), $\alpha = \pi/4$ (dashed line), $\alpha = -\pi/4$ (dashed-dotted line), and $\alpha = \pi/2$ (dotted line).

Assuming that initial neutrons are polarized perpendicular to the quantization axis z and along the axis x, we can calculate the neutron polarization P_x (an expectation value of the spin projection operator) as a function of the propagation distance L in the target [11, 16]

$$P_x(L) = \frac{\cos\left(\frac{\omega_{\rm P} L}{v_{\rm n}}\right)}{\cosh\left(\frac{\omega' L}{v_{\rm n}}\right)}, \tag{24}$$

where $v_{\rm n}$ is neutron velocity and

$$\omega' = \frac{2\pi N\hbar}{M_{\rm n}} {\rm Im}\,(f_{\frac{1}{2}} - f_{-\frac{1}{2}}) \tag{25}$$

is the imaginary part of the pseudomagnetic frequency, which is related to neutron absorbtion in the target. For the case of a La target with the parameter $\alpha = 0$ and for neutron energy of 0.734 eV, the polarization as a function of L is shown in Fig. 2. One can see that the value of the neutron polarization is gradually decreasing due to neutron absorbtion. Fig. 3 shows the polarization as a function of the neutron energy at $L = 0$ cm (solid line), $L = 2$

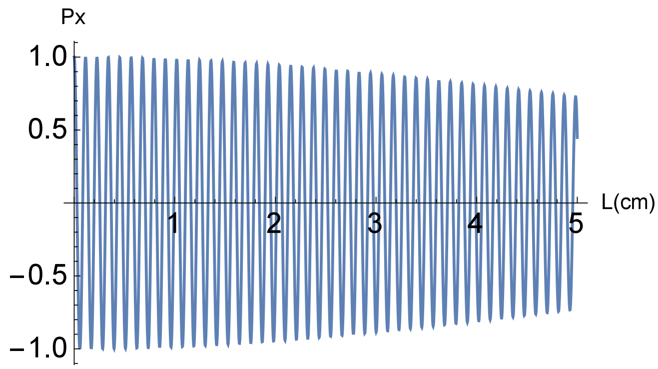

FIG. 2. (Color online) Polarization of 0.734 eV neutrons in the fully polarized lanthanum target as a function of the propagation distance L.

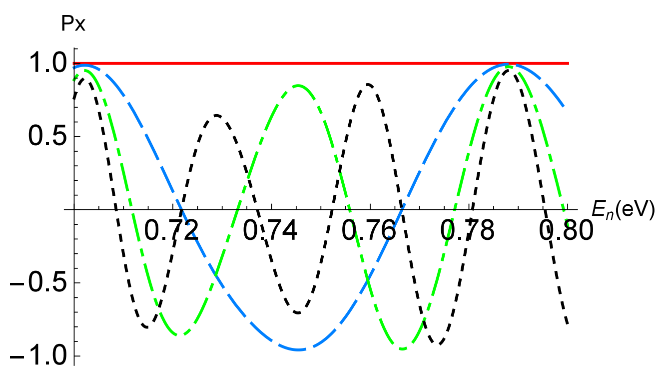

FIG. 3. (Color online) Neutron polarization in the fully polarized lanthanum target as a function of neutron energy at $L = 0$ cm (solid line), $L = 2$ cm (dashed line), $L = 4$ cm (dashed-dotted line), and $L = 6$ cm (dotted line).

cm (dashed line), $L = 4$ cm (dashed-dotted line), and $L = 6$ cm (dotted line), which clearly demonstrate the energy dependance of the pseudomagnetic effect.

For the case of $LaAlO_3$ we need also to include the pseudomagnetic field from polarized Al. Then, assuming a pure vector 100% polarizations for both ^{139}La and ^{27}Al nuclei, the calculated pseudomagnetic fields in the $LaAlO_3$ target for $\alpha = 0$, $\alpha = \pi/4$, $\alpha = -\pi/4$, and $\alpha = \pi/2$ are shown in Fig. 4.

From the above pictures we can see that pseudomagnetic fields of La and Al in the vicinity of the La p-wave resonance are oriented in opposite directions. This demonstrates that, in principle, one can essentially reduce the pseudomagnetic field in a compound by choosing an appropriate combination of the elements with opposite directions of pseudomagnetic fields.

Here we discuss the case of $LaAl_xX_{1-x}O_3$ where Al is partially replaced by the element X.

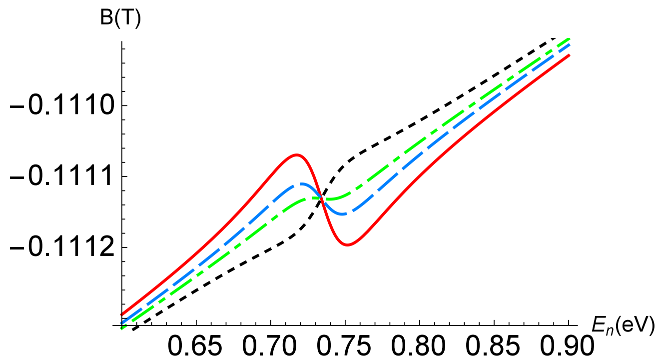

FIG. 4. (Color online) Pseudomagnetic field in fully polarized $LaAlO_3$ target for $\alpha = 0$ (solid line), $\alpha = \pi/4$ (dashed line), $\alpha = -\pi/4$ (dashed-dotted line), and $\alpha = \pi/2$ (dotted line).

TABLE I. The replacement fraction x for the cancelation of the pseudomagnetism in $LaAl_xX_{1-x}O_3$ at the thermal neutron energy.

Element	I	Abundance	$(a_+ - a_-)$[fm]	x
^{139}La	7/2	0.9991	6.9	-
^{27}Al	5/2	1	0.52	-
^{45}Sc	7/2	1	-12.08	0.59
^{59}Co	7/2	1	-12.79	0.56

Table I shows the replacement fraction x for X=^{45}Sc and X=^{59}Co to cancel the pseudomagnetic field of ^{139}La. It should be noted that the cancelation is calculated only at the thermal neutron energy neglecting all resonance contributions. In general, it depends strongly on the neutron energy. As we can see from this table, for example, the La and Al pseudomagnetic fields are parallel at the thermal neutron energy but have an opposite directions in the resonance region (see Figs. (4) and (1)). It should be also noted that the additional absorption with the replacement of Al should be carefully considered in the design of the experiment.

IV. CONCLUSIONS

The presented study of the pseudomagnetic spin rotation for the propagation of polarized neutrons through polarized targets shows the importance of a multi resonance description of the effects. The general theoretical framework considered in this paper can be used for the analysis of pseudomagnetic effects in any experimental setup (see, for example [1–10] and references therein) for a search for TRIV in neutron scattering. We show that the effective pseudomagnetic field has a noticeable energy dependence in the vicinity of a p-wave resonance and it is rather sensitive to target structure, to the polarization pattern of different nuclei in the target, and to the values of partial neutron widths. Therefore, by changing the composition materials of the target and by applying an external magnetic field, it is possible to reduce the effect of the pseudomagnetic field in the given interval of the neutron energy for a particular target. The partial neutron widths have been measured only by using angular distribution measurements in neutron radiative capture. The sensitivity of the pseudomagnetic field to the values of the partial neutron widths gives a new method to measure them in the neutron transmission through polarized targets.

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Appendix A: A general form for the difference of p-wave amplitudes

The general formula for the difference of p-wave amplitudes amplitudes for a pure vector polarized target can be obtained fron Eq.(10). Then, choosing the direction of the target polarization along the axis z and the momentum direction along the vector $\vec{n}(\theta,\phi)$ we obtain

$$f_{\frac{1}{2}} - f_{-\frac{1}{2}} = \tag{A1}$$

$$= \begin{cases} A_{--}^{-3/2} F^J_{I-\frac{1}{2}\, I-\frac{1}{2}\, 1} & (J = I - \frac{3}{2}) \\ A_{--}^{-1/2} F^J_{I-\frac{1}{2}\, I-\frac{1}{2}\, 1} + A_{-+}^{-1/2} F^J_{I-\frac{1}{2}\, I+\frac{1}{2}\, 1} + A_{++}^{-1/2} F^J_{I+\frac{1}{2}\, I+\frac{1}{2}\, 1} & (J = I - \frac{1}{2}) \\ A_{--}^{1/2} F^J_{I-\frac{1}{2}\, I-\frac{1}{2}\, 1} + A_{-+}^{1/2} F^J_{I-\frac{1}{2}\, I+\frac{1}{2}\, 1} + A_{++}^{1/2} F^J_{I+\frac{1}{2}\, I+\frac{1}{2}\, 1} & (J = I + \frac{1}{2}) \\ A_{++}^{3/2} F^J_{I+\frac{1}{2}\, I+\frac{1}{2}\, 1} & (J = I + \frac{3}{2}) \end{cases}$$

Where the spin-angular coefficients are given by

$$A_{--}^{-3/2} = -\frac{3i}{2k}\frac{I-1}{2I+1}\sin^2\theta \tag{A2}$$

, for $I > 1$,

$$A_{--}^{-1/2} = -\frac{3i}{k}\frac{I\left(\sin^2\theta + \sqrt{2I-1}\sin 2\theta\cos\phi + (2I-1)\cos^2\theta\right)}{(2I+1)^2}, \tag{A3}$$

$$A_{-+}^{-1/2} = \frac{3i}{k}\frac{I\left((2I-3)\sin\theta\cos\theta\cos\phi + \sqrt{2I-1}\,(1-3\cos^2\theta)\right)}{\sqrt{I+1}(2I+1)^2}, \tag{A4}$$

$$A_{++}^{-1/2} = \frac{3i}{2k}\frac{I}{(I+1)(2I+1)^2}\left(\sqrt{2I-1}\sin 2\theta\cos\phi + (I+1)(2I-1)\sin^2\theta - 2\cos 2\theta\right), \tag{A5}$$

$$\begin{aligned} A_{--}^{1/2} = -\frac{3i}{2k}\frac{1}{(2I+1)^2}\Big[&4I\cos^2\theta + 2I\sqrt{2I+1}\sin 2\theta\cos\phi \\ &+ \left(2I^2+I+1-2\sqrt{2I+1}\sqrt{I}\cos 2\phi\right)\sin^2\theta + 2\sqrt{I}\sin 2\theta\cos\phi\Big], \end{aligned} \tag{A6}$$

$$\begin{aligned} A_{-+}^{1/2} = -\frac{3i}{2k}\frac{1}{(2I+1)^2\sqrt{2I+3}}\Big[&-4\sqrt{I}(2I-1)\cos^2\theta \\ &+ 2(2I-1)\left(\sqrt{2I+1}\cos 2\phi + \sqrt{I}\right)\sin^2\theta \\ &+ \left\{\sqrt{I}\sqrt{2I+1}(2I-3) - (6I-1)\right\}\sin 2\theta\cos\phi\Big], \end{aligned} \tag{A7}$$

$$\begin{aligned} A_{++}^{1/2} = \frac{3i}{k}\frac{1}{(2I+1)^2(2I+3)}\Big[&I(4I^2+4I+5)\cos^2\theta \\ &+ \left(\sqrt{2I+1}(2I^2+I+1) - (2I-1)\sqrt{I}\right)\sin 2\theta\cos\phi \\ &+ \left(I(2I-1) - 2\sqrt{I}\sqrt{2I+1}\cos 2\phi\right)\sin^2\theta\Big], \end{aligned} \tag{A8}$$

$$\begin{aligned} A_{++}^{3/2} = \frac{3i}{2k}\frac{1}{(I+1)(2I+3)}\Big[&2I\cos^2\theta \\ &+ \left\{\frac{I\,(2I^2+5I+5)}{2I+1} + \left(\sqrt{\frac{3(I+1)}{(2I+1)}} - \sqrt{I+1}\sqrt{2I+3}\right)\cos 2\phi\right\}\sin^2\theta \\ &+ \left(2\sqrt{I+1} - \sqrt{2I+3}(I+1) - \sqrt{\frac{3}{2I+1}}\right)\sin 2\theta\cos\phi\Big]. \end{aligned} \tag{A9}$$

Appendix B: Relations between different spin-coupling schemes

The relation between two spin coupling schemes $\vec{J} = (\vec{I} + \vec{s}) + \vec{l}$ and $\vec{J} = (\vec{s} + \vec{l}) + \vec{I}$ is given by

$$\left\langle \left(\left(l, \frac{1}{2} \right) j, I \right) J \middle| \left(l, \left(\frac{1}{2}, I \right) S \right) J \right\rangle = (-1)^{l+I+j+S} \sqrt{(2j+1)(2S+1)} \begin{Bmatrix} l & \frac{1}{2} & j \\ I & J & S \end{Bmatrix}, \quad \text{(B1)}$$

where $\vec{S} = \vec{I} + \vec{s}$ and $\vec{j} = \vec{s} + \vec{l}$.

Now, defining

$$\begin{aligned} x &= \left| j = \frac{1}{2} \right\rangle \\ y &= \left| j = \frac{3}{2} \right\rangle \\ x_s &= \left| S = I - \frac{1}{2} \right\rangle \\ y_s &= \left| S = I + \frac{1}{2} \right\rangle, \end{aligned} \quad \text{(B2)}$$

one can write for $l = 1$

$$\begin{aligned} x_s &= (-1)^{2I+1} \sqrt{4I} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ I & J & I - \frac{1}{2} \end{Bmatrix} x + (-1)^{2I} \sqrt{8I} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ I & J & I - \frac{1}{2} \end{Bmatrix} y \\ y_s &= (-1)^{2I} \sqrt{4(I+1)} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ I & J & I + \frac{1}{2} \end{Bmatrix} x + (-1)^{2I+1} \sqrt{8(I+1)} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ I & J & I + \frac{1}{2} \end{Bmatrix} y. \end{aligned} \quad \text{(B3)}$$

Appendix C: Spin-operator representation

Sometimes for the description of neutron propagation through a polarized target, it is convenient to use a spin operator [24, 25]

$$\hat{f} = a + b(\vec{s} \cdot \vec{I}) \quad \text{(C1)}$$

whose eigenvalues for $J = I \pm 1/2$ are scattering amplitudes $f_{\pm 1/2}$. In that case, one can calculate the coefficients a and b as

$$\begin{aligned} a &= \frac{1}{2I+1} \left[(I+1) f_{\frac{1}{2}} + I f_{-\frac{1}{2}} \right] \\ b &= \frac{2}{2I+1} (f_{\frac{1}{2}} - f_{-\frac{1}{2}}), \end{aligned} \quad \text{(C2)}$$

with the amplitudes $f_{\pm\frac{1}{2}}$ as given in Eq.(11).

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