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Neutrino-nucleon scattering in supernova matter from the virial expansion

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We extend our virial approach to study the neutral-current neutrino response of nuclear matter at low densities. In the long-wavelength limit, the virial expansion makes model-independent predictions for neutrino-nucleon scattering rates and the density S_V and spin S_A responses. We find S_A is significantly reduced from one even at low densities. We provide a simple fit $S_A^f(n,T,Y_p)$ of the axial response as a function of density n, temperature T and proton fraction Y_p, which can be incorporated into supernova simulations in a straight forward manner. This fit reproduces our virial results at low densities and the Burrows and Sawyer random phase approximation (RPA) model calculations at high densities. Preliminary one-dimensional supernova simulations suggest that the virial reduction in the axial response may enhance neutrino heating rates in the gain region during the accretion phase of a core-collapse supernovae.

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I. INTRODUCTION

Neutrinos radiate 99% of the energy and play a crucial role in core-collapse supernovae [1–3]. The scattering of neutrinos and their transport of energy to the shock region are sensitive to the physics of low-density nucleonic matter, which is a complex problem due to bound nuclei and the strong correlations induced by nuclear forces. A recent three-dimensional supernova simulation was sensitive to modest changes in neutral-current neutrino-nucleon interactions and exploded when strange-quark contributions were included [4]. However, these strange-quark contributions were probbaly taken to be unrealistically large [5]. In this paper, we explore if similar reductions in neutral-current interactions can arise, not from strange-quark contributions but, from correlations in low-density nucleonic matter. The physics of neutrino-matter interactions is a broad and active field, where many interesting studies of neutrino-matter interactions have been performed recently [6–20].

For low densities and high temperatures, the virial expansion provides a model-independent approach. In previous works, we have presented the virial equation of state of low-density nuclear matter [21] and of pure neutron matter [22]. In particular, the virial expansion can be used to describe matter in thermal equilibrium around the neutrinosphere in supernovae. The temperature of the neutrinosphere is roughly $T \sim 4$ MeV from about 20 neutrinos detected in SN1987a [23, 24] and the mass density is $\rho \sim 10^{11} - 10^{12}\,\mathrm{g/cm^3}$. For pure neutron matter, the virial expansion in terms of the fugacity $z = e^{\mu/T}$ is valid for

$$\rho = \frac{2m}{\lambda^3}\, z + \mathcal{O}(z^2) \lesssim 4 \times 10^{11}\,(T/\mathrm{MeV})^{3/2}\,\mathrm{g/cm^3}\,, \quad (1)$$

where m is the nucleon mass and $\lambda = (2\pi/mT)^{1/2}$ denotes the thermal wavelength. A conservative validity range of the virial equation of state is given by $z < 1/2$, which gives the limiting density in Eq. (1). Therefore, the virial approach is applicable for the conditions of the neutrinosphere. Following our virial equation of state, we have generalized the approach to study spin-polarized neutron matter and the consistent neutrino response of neutron matter at low densities [25].

In this paper, we use the virial expansion to describe how neutrinos interact with low-density nuclear matter composed of protons and neutrons. We neglect alpha particles and other light nuclei [26, 27]. These will be included in later work. In Sec. II, we present our formalism. Our results for the axial response and preliminary one-dimensional supernova simulations are presented in Sec. III. Finally, we conclude in Sec. IV.

II. NEUTRINO RESPONSE

In this section, we use the virial expansion to describe how neutrinos interact with low-density nuclear matter. We focus on neutral-current neutrino interactions. We expect similar results for charged-current reactions, however we leave these to later work. We calculate the neutrino cross section per unit volume. The virial expansion provides model-independent results in the limit of low momentum transfer $q \to 0$.

The free cross section for neutrino-nucleon neutral-

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current scattering is

$$\frac{d\sigma_0}{d\Omega}_{\nu N} = \frac{G_F^2 E_\nu^2}{4\pi^2}\Big(C_{a,N}{}^2(3-\cos\theta) + C_{v,N}{}^2(1+\cos\theta)\Big)\,, \tag{2}$$

where G_F is the Fermi constant, E_ν the neutrino energy, and θ the scattering angle. The axial coupling up to strange-quark corrections is $|C_{a,N}| = |g_a|/2 = 0.63$ where g_a is the axial charge of the nucleon. The weak vector charge is $C_{v,n} = -1/2$ for scattering from a neutron n and $C_{v,p} = 1/2 - 2\sin^2\theta_W \approx 0$ for scattering from a proton p. Here θ_W is the weak mixing angle. The cross section in Eq. (2) neglects corrections of order E_ν/m from weak magnetism and other effects, for details see [28].

The free cross section per unit volume for scattering from a mixture of neutrons and protons is then given by

$$\frac{1}{V}\frac{d\sigma_0}{d\Omega} = n_n \frac{d\sigma_0}{d\Omega}_{\nu n} + n_p \frac{d\sigma_0}{d\Omega}_{\nu p}\,, \tag{3}$$

$$= \frac{G_F^2 E_\nu^2}{16\pi^2}\Big(g_a^2(3-\cos\theta)(n_n+n_p) + (1+\cos\theta)n_n\Big)\,. \tag{4}$$

In the medium this cross section is modified by the density (vector) S_V and the spin (axial) S_A response. The response of the system to density fluctuations is described by S_V, while S_A describes the response of the system to spin fluctuations. The response functions are normalized to unity in the low-density limit $S_V, S_A \to 1$ as $n \to 0$. The cross section per unit volume in the medium is then given by

$$\frac{1}{V}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\Big(g_a^2(3-\cos\theta)(n_n+n_p)S_A + (1+\cos\theta)n_n S_V\Big)\,. \tag{5}$$

Note that $d\sigma/d\Omega$ reduces to the free cross section $d\sigma_0/d\Omega$ as $S_A, S_V \to 1$. In general both S_V and S_A depend on momentum transfer q. However, in the limit $q \to 0$ we can derive model independent virial results.

A. Virial equation of state

Next, we briefly review the virial equation of state for a system with neutrons and protons [21]. We will use this to calculate S_V and S_A. The pressure P is expanded to second order in the fugacities of neutrons z_n and protons z_p,

$$\frac{P}{T} = \frac{\ln Q}{V} = \frac{2}{\lambda^3}\Big[z_n + z_p + (z_n^2 + z_p^2)b_n + 2z_p z_n b_{pn}\Big]\,. \tag{6}$$

Here T is the temperature, V is the volume of the system, and Q is the grand-canonical partition function. The fugacities are related to the neutron μ_n and proton μ_p chemical potentials by $z_n = e^{\mu_n/T}$ and $z_p = e^{\mu_p/T}$. Finally the second virial coefficients b_n and b_{pn} are calculated from nucleon-nucleon elastic scattering phase shifts. These are tabulated in Ref. [21].

The neutron n_n and proton n_p densities follow from derivatives of $\ln Q$,

$$n_i = z_i \frac{\partial}{\partial z_i}\Big(\frac{\ln Q}{V}\Big)\Big|_{V,T}\,. \tag{7}$$

This gives

$$n_n = \frac{2}{\lambda^3}(z_n + 2z_n^2 b_n + 2z_p z_n b_{pn})\,, \tag{8}$$

$$n_p = \frac{2}{\lambda^3}(z_p + 2z_p^2 b_n + 2z_p z_n b_{pn})\,. \tag{9}$$

B. Vector response

The vector response S_V is equal to the static structure factor S_q, see for example, Refs. [25, 29]. For a single-component system

$$S_V(q=0) = \frac{T}{(\partial P/\partial n)_T}\,. \tag{10}$$

Using the virial equation of state this can be rewritten with $dP/dn = n/(Tz)(dz/dn)$ as

$$S_V = \frac{1}{n} z \frac{\partial}{\partial z} n\,. \tag{11}$$

Following Ref. [7], we generalize this result to a mixture of neutrons and protons:

$$S_V = \frac{C_v^{n\,2} S_{nn} + 2C_v^n C_v^p S_{np} + C_v^{p\,2} S_{pp}}{C_v^{n\,2} n_n + C_v^{p\,2} n_p}\,, \tag{12}$$

where

$$S_{nn} = z_n \frac{\partial}{\partial z_n} n_n = n_n + \frac{4}{\lambda^3} z_n^2 b_n\,, \tag{13}$$

$$S_{np} = z_p \frac{\partial}{\partial z_p} n_n = \frac{4}{\lambda^3} z_p z_n b_{pn}\,, \tag{14}$$

$$S_{pp} = z_p \frac{\partial}{\partial z_p} n_p = n_p + \frac{4}{\lambda^3} z_p^2 b_n\,. \tag{15}$$

Using Eqs. (13,14,15), we have for S_V

$$S_V = 1 + \frac{4}{\lambda^3}\,\frac{C_v^{n\,2} z_n^2 b_n + 2C_v^n C_v^p z_n z_p b_{pn} + C_v^{p\,2} z_p^2 b_n}{C_v^{n\,2} n_n + C_v^{p\,2} n_p}\,. \tag{16}$$

In the limit $C_v^p \approx 0$ this reduces to the neutron-matter result [25]

$$S_V = 1 + \frac{4}{\lambda^3}\,\frac{z_n^2 b_n}{n_n}\,. \tag{17}$$

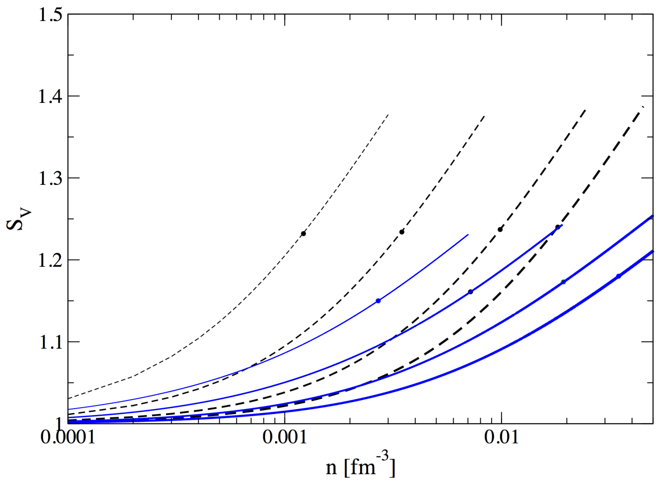

FIG. 1. (Color online) Vector response S_V versus density n for proton fractions of $Y_p = 0$ dashed and 0.3 solid lines. The curves are for temperatures of (left to right with increasing thickness) 2.5, 5, 10, and 15 MeV. The solid circles show where $z_n = 0.5$. The virial expansion is most valid to the left of these points.

Here the impact of protons is to somewhat modify the neutron fugacity z_n because of the b_{pn} term in the neutron density, Eq. (8). The virial coefficient $b_n \approx 0.32$ is small and positive. As a result the vector response is slightly enhanced (larger than one) as shown in Fig. 1. Attractive nucleon-nucleon interactions increase the probability to find nucleons close together. These density fluctuations increase the (local) weak charge and produce a vector response $S_V > 1$.

C. Axial response

To calculate the axial response S_A we generalize our virial equation of state to describe spin-polarized nuclear matter. Let z_p^+, z_n^+ be the fugacities for spin up p and n, and z_p^-, z_n^- be the spin down fugacities. Generalizing the results of Ref. [25], we have for the density of spin-up neutrons n_n^+

$$n_n^+ = \frac{1}{\lambda^3}\left[z_n^+ + 2b_+ {z_n^+}^2 + 2z_n^+(b_- z_n^- + b_{pn}^+ z_p^+ + b_{pn}^- z_p^-)\right] . \tag{18}$$

We discuss the spin virial coefficients b_+, b_-, b_{pn}^+ and b_{pn}^- in Sec. II D. Likewise the density of spin-down neutrons n_n^- is

$$n_n^- = \frac{1}{\lambda^3}\left[z_n^- + 2b_+ {z_n^-}^2 + 2z_n^-(b_- z_n^+ + b_{pn}^+ z_p^- + b_{pn}^- z_p^+)\right] . \tag{19}$$

Similarly, the density of spin-up protons n_p^+ is

$$n_p^+ = \frac{1}{\lambda^3}\left[z_p^+ + 2b_+ {z_p^+}^2 + 2z_p^+(b_- z_p^- + b_{pn}^+ z_n^+ + b_{pn}^- z_n^-)\right] , \tag{20}$$

while the density of spin-down protons n_p^- is

$$n_p^- = \frac{1}{\lambda^3}\left[z_p^- + 2b_+ {z_p^-}^2 + 2z_p^-(b_- z_p^+ + b_{pn}^+ z_n^- + b_{pn}^- z_n^+)\right] . \tag{21}$$

We define axial or spin fugacities $z_n^a = (z_n^+/z_n^-)^{1/2}$ and $z_p^a = (z_p^+/z_p^-)^{1/2}$ and following Ref. [7] write the axial response as

$$S_A = \frac{S_{pp}^A + S_{nn}^A - 2S_{np}^A}{n_n + n_p} , \tag{22}$$

with

$$S_{nn}^A = z_n^a \frac{\partial}{\partial z_n^a}(n_n^+ - n_n^-)\Big|_{z_n^a=1} = n_n + \frac{4}{\lambda^3}(b_+ - b_-)z_n^2 , \tag{23}$$

$$S_{np}^A = z_p^a \frac{\partial}{\partial z_p^a}(n_n^+ - n_n^-)\Big|_{z_p^a=1} = \frac{4}{\lambda^3} z_n z_p (b_{pn}^+ - b_{pn}^-) , \tag{24}$$

$$S_{pp}^A = z_p^a \frac{\partial}{\partial z_p^a}(n_p^+ - n_p^-)\Big|_{z_p^a=1} = n_p + \frac{4}{\lambda^3}(b_+ - b_-)z_p^2 . \tag{25}$$

Note that the minus sign for the S_{np}^A term in Eq. (22) is because the axial charge of a neutron is opposite to that of a proton. To clean up the notation, we define the axial virial coefficients

$$b_a = b_+ - b_- , \tag{26}$$

$$b_{pn}^a = b_{pn}^+ - b_{pn}^- . \tag{27}$$

The final result for S_A can then be written as

$$S_A = 1 + \frac{4}{\lambda^3}\,\frac{(z_p^2 + z_n^2)b_a - 2z_p z_n b_{pn}^a}{n_n + n_p} . \tag{28}$$

To lowest order in the density one has $z_p \approx \lambda^3 n_p/2$ and $z_n \approx \lambda^3 n_n/2$ so that

$$S_A \approx 1 + \lambda^3\,\frac{(n_n^2 + n_p^2)b_a - 2n_n n_p b_{pn}^a}{n_p + n_n} . \tag{29}$$

Note that we use the full Eq. (28) for results in the next section. Because the spin virial coefficient $b_a \approx -0.6$ (see below), the axial response is reduced $S_A < 1$. This is because two neutrons or two protons are likely to be correlated in a $^1S_0$ state because of the Pauli principle and this spin zero state reduces the spin response.

We define the total response $S_{\rm tot}$ as the ratio of the in-medium transport cross section to the free one,

$$S_{\rm tot} = \frac{\int d\Omega\, \frac{d\sigma}{d\Omega}(1-\cos\theta)}{\int d\Omega\, \frac{d\sigma_0}{d\Omega}(1-\cos\theta)} . \tag{30}$$

From Eqs. (4) and (5), we thus have

$$S_{\rm tot} = \frac{5g_a^2 S_A + (1-Y_p)S_V}{5g_a^2 + 1 - Y_p} , \tag{31}$$

where Y_p is the proton fraction. The total response depends on both S_V and S_A. However, in general S_A is most important because of the large factor $5g_a^2$. We present results for S_A in Sec. III. However first we discuss the spin virial coefficients.

D. Spin virial coefficients

The virial coefficient $b_a = b_+ - b_-$ is discussed in Ref. [25] and describes spin interactions between two protons or two neutrons. We now discuss $b_{pn}^a = b_{pn}^+ - b_{pn}^-$ that involves interactions between protons and neutrons. The virial coefficient b_{pn}^+ describes interactions between a p and a n with like spin projections, while b_{pn}^- describes interactions between nucleons with unlike spins. We therefore have

$$b_{pn}^a = \frac{1}{2^{1/2}}(e^{E_d/T}-1) + \frac{2^{1/2}}{\pi T}\int_0^\infty dE e^{-E/2T}[\delta_{pn}^+(E) - \delta_{pn}^-(E)]\,, \quad (32)$$

with

$$\delta_{pn}^+(E) = \frac{1}{2}\delta_{^3S_1} + \frac{1}{6}\delta_{^3P_0} + \frac{1}{2}\delta_{^3P_1} + \frac{5}{6}\delta_{^3P_2} + \frac{1}{2}\delta_{^3D_1} + \frac{5}{6}\delta_{^3D_2} + \frac{7}{6}\delta_{^3D_3} + \ldots\,. \quad (33)$$

Here E_d is the deuteron binding energy and the factor in front of each phase shift is $(2J+1)/[2(2S+1)]$ where the factor of 1/2 is from the average over isospin 1 and 0 states. In our calculation, we have neglected states with $L > 2$. Similarly, we have

$$\delta_{pn}^-(E) = \frac{1}{4}\delta_{^1S_0} + \frac{1}{4}\delta_{^3S_1} + \frac{3}{4}\delta_{^1P_1} + \frac{1}{12}\delta_{^3P_0} + \frac{1}{4}\delta_{^3P_1} + \frac{5}{12}\delta_{^3P_2} + \frac{5}{4}\delta_{^1D_2} + \frac{1}{4}\delta_{^3D_1} + \frac{5}{12}\delta_{^3D_2} + \frac{7}{12}\delta_{^3D_3} + \ldots\,. \quad (34)$$

Now the factor for each phase shift is $(2J+1)/[4(2S+1)]$, where the 1/4 is from an average over both isospin 1 and 0 and spin 1 and 0 states. We calculate the spin virial coefficients based on the Nijmegen partial-wave analysis of nucleon-nucleon scattering [30]. Our results for b_a and b_{pn}^a are collected in Table I. The other virial coefficients b_n and b_{pn} needed to calculate the neutrino responses have all ready been tabulated in Ref. [21].

In contrast to b_a, b_{pn}^a is positive because a proton and a neutron can be correlated into the spin one $^3\mathrm{S}_1$ state (deuteron like), enhancing the spin response. However the axial charge of a proton is opposite to that of a neutron. This leads to a minus sign in Eq. (28) for the b_{pn}^a term. As a result, both b_a and b_{pn}^a reduce the total axial response and lead to $S_A < 1$.

T (MeV)	b_a	b_{pn}^a
1	-0.638	6.18
2	-0.653	1.74
3	-0.651	1.05
4	-0.648	0.785
5	-0.643	0.640
6	-0.637	0.561
7	-0.631	0.504
8	-0.625	0.463
9	-0.620	0.432
10	-0.615	0.408
12	-0.605	0.374
14	-0.597	0.352
16	-0.589	0.336
18	-0.583	0.324
20	-0.577	0.315

TABLE I. Spin virial coefficients b_a and b_{pn}^a based on nucleon-nucleon phase shifts.

E. Combining correlations, strange-quark, weak magnetism, and recoil corrections

We end this formalism section by describing the combination of correlation, strange-quark, weak magnetism, and recoil corrections. To a good approximation all of these effects can be combined in a straight forward way that avoids double counting. We write for the neutrino cross section per unit volume, see Eq. (5),

$$\frac{1}{V}\frac{d\sigma}{d\Omega} \approx \frac{G_F^2 E_\nu^2}{16\pi^2}\Big(\big[(g_a+g_a^s)^2 n_n + (g_a-g_a^s)^2 n_p\big] (3-\cos\theta)S_A + (1+\cos\theta)n_n S_V\Big)R(E_\nu/m)\,. \quad (35)$$

Effects from nucleon-nucleon correlations in the medium are described by the vector S_V and axial S_A response functions, see Eqs. (17) and (28), respectively. The vector response is slightly greater than one and the axial response is significantly less than one. As a result correlations reduce the cross section for both neutrino-proton and neutrino-neutron scattering.

Strange quark contributions to the nucleon spin are described by the parameter g_a^s (note $g_a = 1.26$). Melson *et al.* [4] consider $g_a^s = -0.2$ and this value reduces neutrino-neutron and increases neutrino-proton scattering cross sections. For neutron-rich conditions this leads to a net reduction in the neutrino scattering opacity. Therefore both correlation effects and strange quarks, if present (with $g_a^s < 0$), reduce the opacity, and the two effects add. Note that in Ref. [4] strange-quark contributions were probbaly taken to be unrealistically large [5]. On the one side, strange-quark contributions to the vector current have been measured by several parity-violating electron scattering experiments to be small [31], but direct experimental limits on strange-quark contributions to the nucleon spin are relatively poor and are based on an old Brookhaven neutrino scattering experiment [32]. Therefore, it would be very useful to have a better labo-

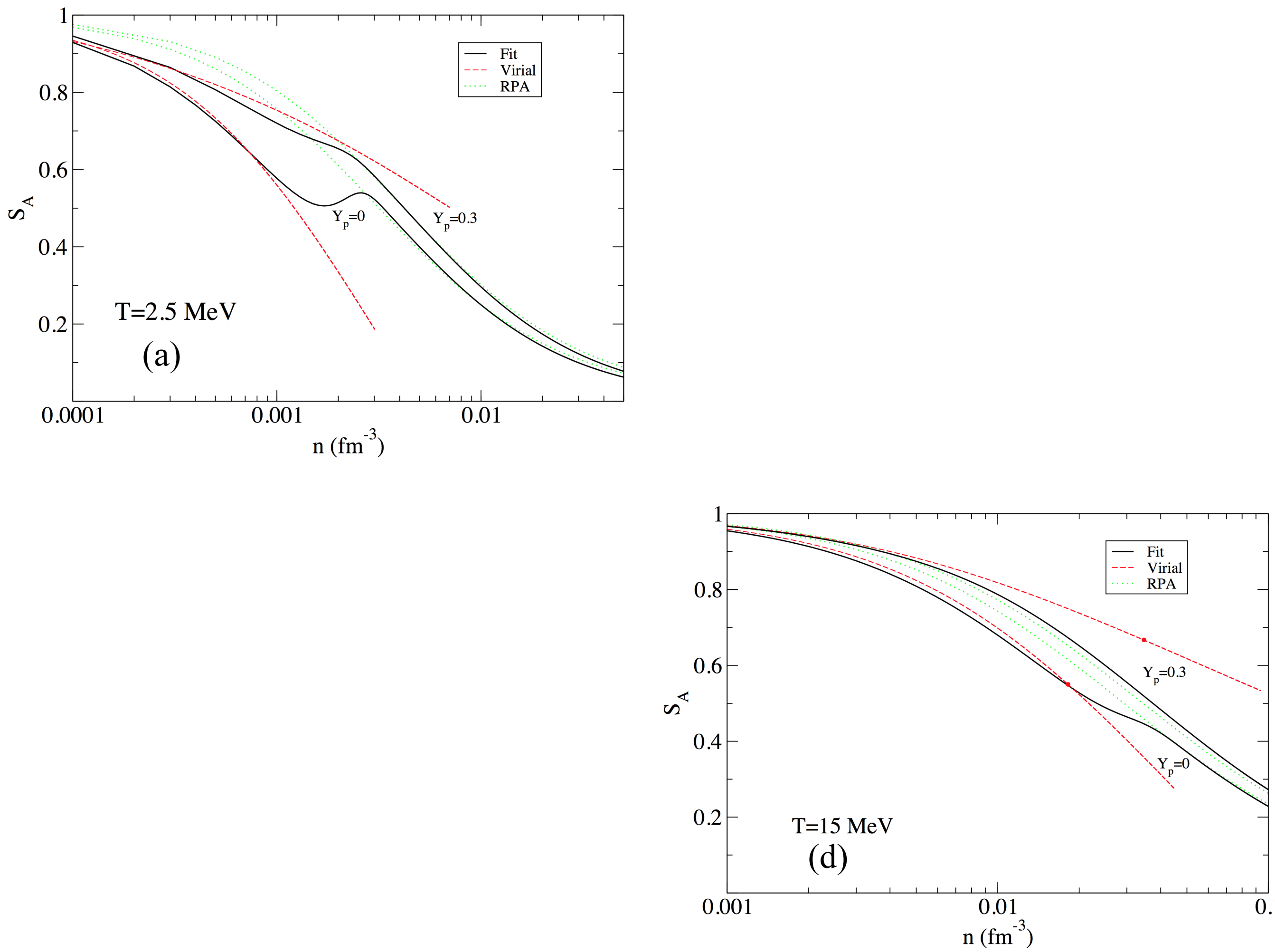

FIG. 2. (Color online) Axial response S_A versus density n for temperatures of $T = 2.5$ MeV (a), 5 MeV (b), 10 MeV (c), and 15 MeV (d). The red dashed lines are our virial expansion results, Eq. (28), for the indicated proton fractions. The solid red dots indicate where $z_n = 0.5$. The virial expansion is most valid to the left of these points. The green dotted lines show the original Burrows and Sawyer RPA results [7]. Finally the solid black lines show the interpolating fit S_A^f, Eq. (36).

ratory limit on g_a^s from a modern neutrino-nucleon scattering experiment.

Finally recoil and weak magnetism corrections can be approximately described by a factor $R(E_\nu/m)$. This is discussed in Ref. [28] and reduces antineutrino-nucleon scattering cross sections, while having only a modest effect on neutrino-nucleon cross sections.

III. RESULTS FOR THE AXIAL RESPONSE

In this section, we focus on results for the axial response S_A, and not on the vector response S_V, for two reasons. First, the axial response is more important for neutrino-transport cross sections because of a large $5g_a^2$ factor, see Eq. (35). Second, we have not included alpha particles or other light nuclei. Preliminary calculations suggest that spin zero alpha particles can significantly enhance S_V, but do not strongly impact S_A. Therefore we postpone a full discussion of S_V to later work, where we will explicitly include alpha particles and other light nuclei. For the present, a reasonable approximation is to simply set $S_V = 1$ in Eq. (31).

In Fig. 2 we show S_A for temperatures of $T = 2.5$ to 15 MeV. Our virial results (red dashed lines) are valid at low densities. To evaluate S_A for higher densities, where $z_n > 0.5$, one presently needs to employ a model-dependent calculation. We consider the random phase approximation (RPA) calculations of Burrows and Sawyer [7], because they are simple, well known, and have been employed in supernova simulations. We caution that these calculations may have a number of limitations. First they predict that the vector response is less than one $S_V < 1$ while Fig. 1 shows $S_V > 1$. Second the calculations use a Landau parameter for the effective interaction that is appropriate for symmetric nuclear matter. A Landau parameter appropriate for pure neutron matter could lead to a smaller S_A. See also the discussion

Fit parameter	Value
A_0	920
B_0	3.05
C_0	6140
D_0	1.5×10^{13}

TABLE II. Fit parameters for S_A^f fitting function, see Eqs. (36–39). These assume n in fm^{-3} and T in MeV.

in Ref. [25]. In future work we will revisit the behavior of S_A at high densities, but for now we consider the Burrows and Sawyer results. The green dotted lines in Fig. 2 show the Burrows and Sawyer RPA calculation [7]. Note that the RPA depends very weakly on the momentum transfer q and we use $q = 3T$.

Finally, we fit the virial results for S_A at low densities and the RPA results at high densities with an interpolating function $S_A^f(n, T, Y_p)$ that is a simple function of density n, temperature T, and proton fraction Y_p:

$$S_A^f(n, T, Y_p) = \frac{1}{1 + A(1 + Be^{-C})}\,, \tag{36}$$

where the functions A, B, and C are given by

$$A(n, T, Y_p) = A_0 \frac{n(1 - Y_p + Y_p^2)}{T^{1.22}}\,, \tag{37}$$

$$B(T) = \frac{B_0}{T^{0.75}}\,, \tag{38}$$

$$C(n, T, Y_p) = C_0 \frac{nY_p(1 - Y_p)}{T^{0.5}} + D_0 \frac{n^4}{T^6}\,. \tag{39}$$

The fit parameters A_0, B_0, C_0, and D_0 are collected in Table II for n in fm^{-3} and T in MeV. This fit is most accurate for $5 < T < 10$ MeV, $Y_p \leq 0.3$, and $n < 0.05$ fm^{-3}, but yields reasonable values outside this range. We make an implementation of this fitting function available in NuLib [33] at `http://www.nulib.org`. Note that if one sets $B_0 = 0$ in Eq. (38), Eq. (36) will approximately reproduce the original Burrows and Sawyer RPA results at low density. We see that S_A is significantly reduced from 1 even at relatively low densities. This is especially the case at low Y_p.

We briefly explore the effect of the reduced axial response arising from the virial expansion on the heating rate obtained in simulations of the accretion phase of a core-collapse supernovae. To compare with Ref. [4], we use the Lattimer and Swesty [34] equation of state (with $K_0 = 220$ MeV) and the $20\,M_\odot$ progenitor star from Ref.[35]. We use the one-dimensional code GR1D [33], available at `http://www.GR1Dcode.org`. For the sake of comparison with Ref. [4], we perform simulations with and without strange-quark contributions. We show the heating rate realized in the gain region in Fig. 3. The solid lines denote simulations using no strange quark corrections, while the dashed lines denote simulations using $g_a^s = -0.2$. The black lines are for simulations that use free particle rates, while the orange lines denote simulations using the virial corrected rates. Similar to Ref. [4],

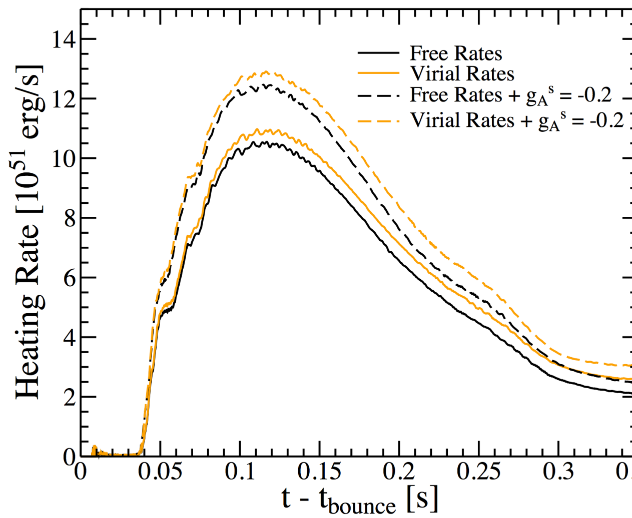

FIG. 3. (Color online) Heating rate in the gain region for one-dimensional simulations of the accretion phase of core-collapse supernovae for various assumptions on the neutral-current neutrino-nucleon scattering cross section. The solid lines represent simulations that ignore the contribution to the rates from strange quarks, whereas dashed lines denote simulations that include strange-quark contributions, assuming $g_a^s = -0.2$ for comparison with Ref. [4]. The black lines show the heating rate for simulations that assume $S_A = 1$, while orange lines show the larger heating rates with S_A^f from Eq. (36).

the assumed strange-quark contribution raises the heating rate by $\sim 20\%$ around 100 ms after bounce. We find the virial rates increase the heating by $\sim 5\%$ around 100 ms after bounce and higher at later times. We expect the reduction in S_A to play an even larger role at later times when the neutrinos decouple from regions with higher matter densities.

Our preliminary two-dimensional SN simulations, with a reduced axial response S_A^f, Eq. 36, explode 100-150 ms earlier than simulations with $S_A = 1$, for 15, 20, and 25 $M_\odot$ stars. For a 12 $M_\odot$ star, S_A^f leads to an explosion at late times while a simulation with $S_A = 1$ fails to explode. Further details of these simulations will be provided in a later publication. Very recent results by Burrows et al. [36], based on an earlier version of this paper, find similar but perhaps somewhat larger effects of the reduced axial response S_A^f.

IV. SUMMARY AND CONCLUSIONS

Supernova simulations may be sensitive to the neutral-current interactions of mu and tau neutrinos at low densities near the neutrinosphere. In this paper, we have calculated the axial or spin response S_A of nuclear matter in a virial expansion that is model-independent at low densities and high temperatures. We find S_A to be significantly reduced. Our results can be incorporated into supernova simulations by multiplying both the neutrino-proton and neutrino-neutron neutral current scattering rates by $S_{\rm tot}$ given by Eq. (31) with $S_V = 1$ and S_A given by our fit function S_A^f from Eqs. (36), (37), (38), and (39). Preliminary one-dimensional supernova simulations suggest that the reduction in the axial response may enhance the neutrino heating rates in the gain region during the accretion phase of a core-collapse supernova. In future work, we will extend the calculation to the vector response S_V in a virial expansion and study the impact of light nuclei.

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