Kinematic sensitivity to the Fierz term of β-decay differential spectra

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The current most stringent constraints on exotic scalar or tensor couplings in neutron and nuclear β decay, involving left-handed neutrinos, are obtained from the Fierz interference term. The sensitivity to this term in a correlation coefficient is usually driven by an energy-averaged kinematic factor that increases monotonically toward smaller values of the β endpoint energies. We first point out here that this property does not hold for certain differential observables that are directly sensitive to the Fierz term, such as the β or the recoil energy spectrum. This observation is relevant for the selection of sensitive transitions in searches for exotic couplings through spectrum shape measurements. We then point out previous errors in the exploitation of measurements of the $\beta - \nu$ angular correlation coefficient and discuss their impact on the extraction of constraints on exotic couplings.

I. INTRODUCTION

Precision measurements in nuclear and neutron decays have played a crucial role in the development of the $V - A$ theory of the weak interaction, which is embedded in the framework of the standard electroweak model (SM). Today, these experiments constitute sensitive probes to "exotic" currents, such as right-handed vector currents or scalar and tensor currents, resulting from the exchange of new heavy bosons [1–3].

The description of weak decays using a model-independent Effective-Field-Theory approach [4] has recently made possible direct comparisons of sensitivity between searches for exotic interactions carried out at low energies and at the Large Hadron Collider (LHC) [5, 6]. Under the general assumption that the scale of new physics occurs at much higher energies than those accessible at the LHC, it appears that experiments at the LHC provide tight constraints on scalar and tensor couplings involving right-handed neutrinos [6], which are more stringent than those obtained from their quadratic contributions to β decays. On the other hand, for interactions involving left-handed neutrinos, experiments in nuclear and neutron decay can potentially be competitive with constraints reached or to be reached at the LHC provided they address observables that are linear in the exotic couplings with sufficient precision [5, 6]. This competition is also possible thanks to recent calculations of the corresponding hadronic form factors, which are now known with $\sim 10\%$ precision [5, 7, 8].

One of the most sensitive observables to these non-standard scalar and tensor interactions in nuclear β decay, which is linear in the couplings and that currently provides the tightest bounds on them [3, 9], is the Fierz interference term. This term enters many measured observables and affects in particular the shape of the β energy spectrum. The potential of the Fierz term to probe the presence of exotic couplings has motivated new measurements of β energy spectra with improved precision [10]. In nuclear β decay there is a variety of transitions that can be selected for such measurements but there appears to be some confusion as to which are the most sensitive ones in terms of their kinematic signature to the Fierz term. We first address this point below by discussing the sensitivity to the Fierz term of the total decay rate, of the β energy spectrum and of the recoil momentum spectrum. We then point out the improper use of some available experimental data of the $\beta - \nu$ angular correlation coefficient and its relation to the Fierz term, and discuss the implications of those errors on the extraction of constraints on exotic couplings.

II. TOTAL DECAY RATE

We restrict ourselves to allowed β decay transitions described by the statistical weight (phase space) of the form

$$P(W)dW = pWq^2dW , \qquad (1)$$

where p and W are respectively the momentum and total energy of the β particle and $q = W_0 - W$ is the momentum of the neutrino, with W_0 the maximal total energy of the β particle. We also consider two dynamic terms: the Fierz term, b, and the $\beta - \nu$ angular correlation coefficient, a. The decay rate function, averaged over the spin variables of the nucleus and the electron, is proportional to [11, 12]

$$N(W, \theta)dWd\Omega_\theta = P(W) \left[1 + b\frac{m}{W} + a\frac{p}{W} \cos\theta \right] dWd\Omega_\theta , \qquad (2)$$

where m is the mass of the β particle, θ is the angle between the momentum directions of the β particle and the neutrino, and $d\Omega_\theta$ is the differential solid angle around

θ. For simplicity, we do not include contributions due to recoil order terms [13] or to Coulomb and radiative corrections [14, 15], and neglect effects due to the neutrino mass.

The integration of Eq. (2) over the kinematic variables of the β particle and the neutrino, normalized by the integral over the phase space, gives

$$N_0 = 1 + b\langle\frac{m}{W}\rangle \,, \qquad (3)$$

where $\langle m/W\rangle$ denotes the average of m/W over the statistical weight given by Eq. (1). Figure 1 shows the variation of the factor $\langle m/W\rangle$ as a function of the endpoint energy, $E_0 = W_0 - m$, for values in the range 20 keV to 20 MeV. For reference, the values for neutron decay ($E_0 = 782$ keV) and for ^{6}He decay ($E_0 = 3.50$ MeV) are indicated with black points. It is obvious that $\langle m/W\rangle$ increases monotonically toward lower endpoint energies and tends asymptotically to 1 since the kinetic energy in the denominator becomes negligible relative to the electron mass.
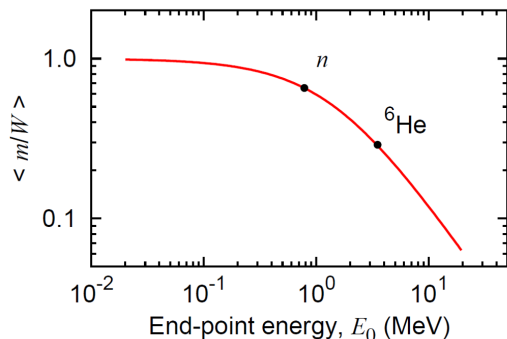

FIG. 1: (Color on-line) Variation of the sensitivity factor $\langle m/W\rangle$ as a function of the endpoint energy E_0.

This property has been exploited to extract very stringent constraints on scalar couplings from the contribution of the Fierz term to the $\mathcal{F}t$-values in super-allowed pure Fermi transitions [16]. Nuclei with the lowest endpoint energies, such as ^{10}C and ^{14}O, have the largest sensitivity to the Fierz term, whereas the b-contamination to the $\mathcal{F}t$-values of transitions with larger endpoints, such as ^{26m}Al, is smaller.

It is clear from Fig. 1 that, from a purely statistical standpoint, the uncertainty on the Fierz term extracted from a measurement of the rate in Eq. (3) would decrease monotonically toward lower energies. For a sample with 10^8 events, the smallest statistical uncertainty would be $\Delta b = 10^{-4}$.

III. DIFFERENTIAL DISTRIBUTIONS

The monotonic increase of sensitivity to b in Eq. (3) does not imply, however, that this property also holds

when the Fierz term is extracted from the measurement of a differential distribution such as the β energy spectrum or the recoil momentum spectrum. This is so, simply because in differential distributions one measures the effect on the shape of the distribution and not on the number of events. To illustrate this quantitatively we have performed simple Monte-Carlo studies where the statistical uncertainty on the Fierz term is determined from fits of differential spectra.

A. The β energy spectrum

We consider first the distribution in electron energy, resulting from the integration of Eq. (2) over the directions of the neutrino,

$$N_e(W)dW \;\propto\; P(W) \cdot \left(1 + b\frac{m}{W}\right)dW \qquad (4)$$
$$= [P(W) + b\,g(W)]\,dW \,. \qquad (5)$$

We have generated β-energy spectra following the shape of the phase space $P(W)$ in Eq. (1), for different values of the endpoint energy, E_0. Each spectrum contained 10^8 events. The generated spectra were then fitted between 5% and 95% of their kinetic energy range, with a function given by Eq. (4). The fits had two free parameters: the overall normalization and the Fierz term b.
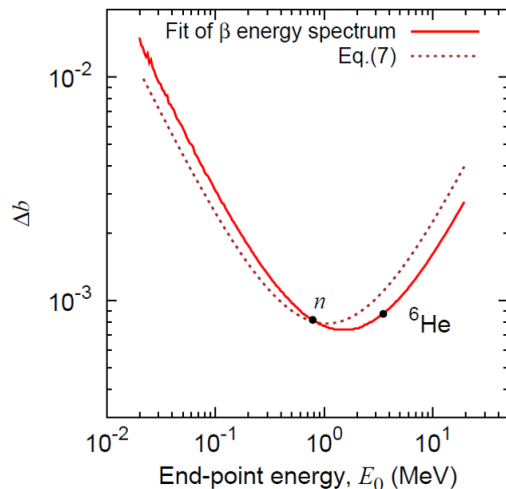

FIG. 2: (Color on-line) The solid red line shows the 1σ statistical uncertainties obtained from fits of simulated β energy spectra as a function of the endpoint energy E_0. The dashed brown line shows the result obtained with the approximation given by Eq. (7).

The red solid curve on Fig. 2 shows the 1σ statistical uncertainty on the Fierz term obtained from these fits as a function of the endpoint energy, E_0. For endpoint energies larger than about 1-2 MeV, the statistical uncertainty increases roughly linearly with the endpoint energy, due to the $1/W$ factor. For endpoint energies smaller than 1-2 MeV the statistical uncertainty

does not decrease monotonically but, instead, it also increases and equally fast on the log-log scale. The origin of this loss of sensitivity toward smaller endpoint energies is rather simple. The sensitivity to b in the fits is driven by the differences in shape between $g(W)$ and $P(W)$ in Eq. (5). As the average kinetic energy becomes smaller, these two functions become identical and the fitting function becomes proportional to $(1+b)$ with a loss of the specific kinematic signature to b. In other words, although the effect of the b term in the overall normalization, *c.f.* Eq. (3), is maximal for very low endpoints, its effect on the shape becomes very weak simply because the factor m/W becomes almost energy independent, namely $m/W \approx 1$. This is illustrated in Fig. 3.
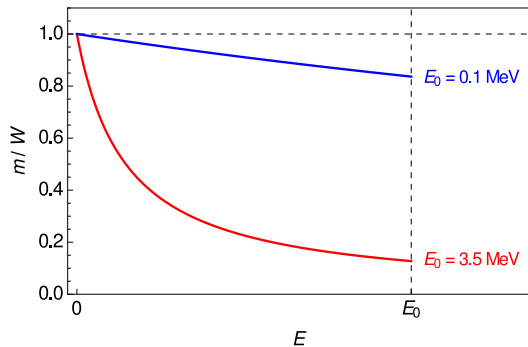

FIG. 3: (Color on-line) Energy dependence of the ratio m/W, for two different endpoint energies. Notice that the E-axis is rescaled to ease the comparison.

It is interesting to note that neutron decay and ^{6}He decay, which are being considered for precision measurements of the β energy spectrum [17, 18], have very comparable kinematic sensitivities (Fig. 2). It is worth stressing that transitions with endpoint energies in the range $E_0 = 200\text{-}300$ keV, such as ^{45}Ca, which are also currently the focus of new projects [10], are a factor of 2 less sensitive that those with $E_0 = 1\text{-}2$ MeV. Last but not least, it is observed that transitions with endpoint energies in the range 0.6-3.8 MeV have sensitivities that are within 20% of the optimal kinematic sensitivity and are therefore ideal to search for a non-zero Fierz term, at least on pure statistical grounds.

The observation of the loss of sensitivity toward smaller endpoint energies obtained in the Monte-Carlo study can be derived from simple arguments. For this we consider the central region of a β energy spectrum. If we divide the kinetic energy range in four equal intervals, the central values of the kinetic energies of the second and third intervals are respectively $E_2 = 3E_0/8$ and $E_3 = 5E_0/8$. Following Eq. (4), the ratio between the number of events in these two central bins is approximately proportional to

$$1 + b \cdot \left(\frac{m}{m + E_2} - \frac{m}{m + E_3} \right) . \qquad (6)$$

We further assume that, for a given spectrum, this ra-

tio dominates the sensitivity to b. With N events in the spectrum, the statistical uncertainty on the Fierz term extracted from such a ratio will approximately be determined from

$$\Delta b \cdot \left(\frac{m}{m + E_2} - \frac{m}{m + E_3} \right) \approx \frac{1}{\sqrt{N}} . \qquad (7)$$

The values of Δb extracted from Eq. (7), for $N = 10^8$, are shown by the dashed brown line in Fig. 2 as a function of the endpoint energy. This crude approximation gives the main trend of the sensitivity, independently of any reference to a fit. As the endpoint energy becomes smaller, the difference in Eq. (7) also becomes smaller resulting in the increase of Δb.

B. The recoil momentum spectrum

As a second example, we consider the extraction of b from the distribution of the recoil nucleus momentum r. The transformation of Eq. (2) in terms of the recoil momentum gives the distribution [11, 19]

$$N(W,r)dWdr = \frac{1}{2}rWq \left[1 + b\frac{m}{W} + a\frac{r^2 - p^2 - q^2}{2Wq} \right] dWdr , \qquad (8)$$

where, for a given value of r the total energy of the β particle varies between

$$W_{\min}(r) = \frac{(W_0 - r)^2 + m^2}{2(W_0 - r)} , \qquad (9)$$

$$W_{\max}(r) = \frac{(W_0 + r)^2 + m^2}{2(W_0 + r)} . \qquad (10)$$

The integration of Eq. (8) over the β energy gives the recoil momentum distribution

$$Q(r)dr = \frac{r}{12} \left[Q_0(r) + b\,Q_1(r) + a\,Q_2(r) \right] dr , \qquad (11)$$

where

$$Q_i(r) = [W F_i(W,r)]_{W_{\min}(r)}^{W_{\max}(r)} , \qquad (12)$$

and

$$F_0(W,r) = W(3W_0 - 2W) , \qquad (13)$$
$$F_1(W,r) = 3m(2W_0 - W) , \qquad (14)$$
$$F_2(W,r) = 3\left(r^2 + m^2 - W_0^2 + W_0 W - \frac{2}{3}W^2 \right) \qquad (15)$$

In order to determine the sensitivity to the Fierz term from a measurement of a recoil momentum spectrum we have again used a Monte-Carlo method. We have generated distributions following Eq. (11), with $b = 0$, for pure Fermi ($a = a_F = 1$) and pure Gamow-Teller ($a = a_{GT} = -1/3$) transitions, and for different values of the endpoint energy. Each spectrum contained 10^8

events. The spectra were then fitted between 5% and 95% of the momentum range, with a function given also by Eq. (11), with two free parameters: the overall normalization and b. Since the a coefficient depends on the nonstandard couplings but quadratically, the procedure above is equivalent to neglecting those contributions.
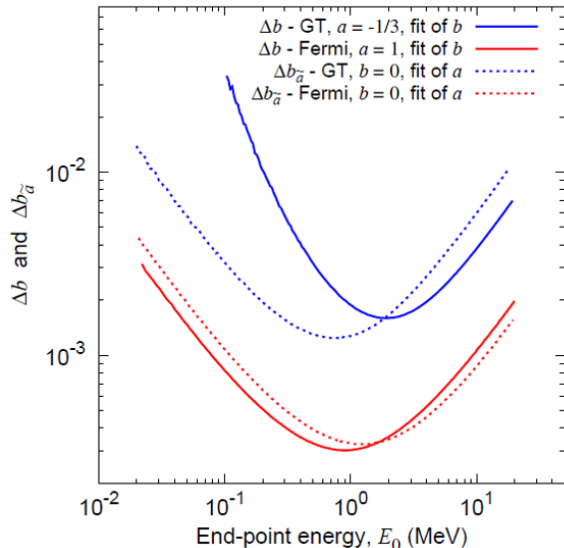

FIG. 4: (Color on-line) The 1σ statistical uncertainties on b obtained from fits of simulated recoil momentum spectra as a function of the endpoint energy E_0. The solid lines represent Δb and correspond to Fermi (red) and Gamow-Teller (blue) transitions for which the value of a was respectively fixed to a_F or a_{GT}. The dotted lines represent $\Delta b_{\tilde{a}}$, that is the uncertainty on b obtained by incorrectly re-interpreting a fit of a with $b = 0$, through the $\tilde{a}$-prescription, c.f. Eq. (18).

The solid lines in Fig. 4 show the 1σ statistical uncertainty on the Fierz term obtained from these fits as a function of the endpoint energy for pure Fermi (red curve) and pure Gamow-Teller (blue curve) transitions. For both types of transitions, the dependence is qualitatively the same as for the extractions of b from the β energy spectrum, and has the same origin. This behavior can again be understood analytically from simple arguments like those used to deduce Eq. (7), but the expressions are lengthier in this case. Figure 4 shows that the presence of the term proportional to a in Eq. (11) gives rise to different sensitivities for Fermi and Gamow-Teller transitions.

Obtaining an analogous sensitivity curve for mixed transitions is not possible since in these decays the correlation coefficient a is a function of the Gamow-Teller to Fermi mixing ratio, $a(\rho)$, and can therefore not be fixed[1]. More precisely, for pure Fermi and Gamow-Teller

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[1] This also means that one can extract both b and ρ (or equivalently b and a) simultaneously from the recoil spectrum.

transitions, the value of a is essentially determined by angular momentum arguments and can then be fixed to the SM value. In contrast, for mixed transitions, the expression of a depends on ρ that, in turn, is extracted from the measurement of another correlation coefficient such as $\tilde{A}$, or from the comparison between the ft-value of the mixed transition and $\mathcal{F}t(0^+ \to 0^+)$. However, these observables also receive a contamination due to b that affects the ρ extraction and that propagates to a. This "indirect" effect of the Fierz term, some times ignored in the literature (see e.g. Ref. [20]), is of the same order as the direct effect in the recoil spectrum, and therefore it has to be taken into account in the analysis, since it can lead to strong suppressions of sensitivity.

IV. CORRELATION COEFFICIENTS

Many past experiments extracted values of various correlation coefficients within an analysis that sets the Fierz term to zero. There are different reasons to do this. For instance, the extraction of the axial-vector coupling g_A from measurements of the asymmetry parameter A in neutron decay [21, 22], is usually performed within the SM framework so that nonstandard effects are simply ignored. Other experiments analyzed the data with the Fierz term set to zero, because existing bounds on it were strong enough, and they focused on the sensitivity to interactions entering mainly through quadratic contributions, as e.g. in Ref. [23].

It was noticed long ago [24] that, if a correlation coefficient X has been extracted from measurements of an asymmetry in counting rates assuming $b = 0$, it can be easily re-interpreted by including a non-zero Fierz term through the expression

$$\tilde{X} = \frac{X}{1 + b\langle m/W \rangle'} . \tag{16}$$

The factor $\langle m/W \rangle'$ denotes here the integration over the measured interval of the β energy spectrum. Such an expression was introduced for "standard" experimental determinations of parameters like A, B, G and D [24, 25]. This prescription has been used in global analyses of data in neutron and nuclear decays [1, 3, 26, 27] to take into account the various measurements where the Fierz term was not included in the original analysis.

Operationally, Eq. (16) was noticed to be valid when the measured correlation coefficient is deduced from an asymmetry resulting from the sign inversion of some kinematic variable or the inversion of the analysis direction of a variable, such as the direction of a magnetic field. Formally, the prescription is valid when the integration limits of the β-particle energy, W, and the relevant kinematic variable, θ, are independent, and then the W integration can be performed without introducing a genuine θ-dependence in the Fierz term. More precisely one would have

$$N(W,\theta)dW\,d\theta = G(\theta)H(W) \qquad (17)$$
$$\times \left[1 + b\,\frac{m}{W} + X\,R(W,\theta)\right]dW\,d\theta$$
$$\rightarrow N(\theta)d\theta \sim G(\theta)\left(1 + b\,\langle\frac{m}{W}\rangle\right)\left[1 + \tilde{X}\,R(\theta)\right]d\theta,$$

where $R(\theta) \equiv \langle R(W,\theta)\rangle_W$, and where we have also assumed that the W- and θ-dependences of the normalization function are factorizable. It is easy to see from this expression that the $\tilde{X}$-prescription will then apply not only for standard asymmetries, but also for measurements of the θ differential distribution [28].

However, the recoil momentum r and the β-particle energy W are not independent. As a result, the term $Q_1(r)$ in Eq. (11) has a different dependence on the recoil momentum than the term $Q_0(r)$ and cannot be factorized to produce a term of the form $(1+b\langle m/W\rangle)$. This is particularly relevant for measurements of the $\beta - \nu$ angular correlation coefficient. Unless this coefficient is extracted from an asymmetry in decay rates [29] or for a fixed β energy [30], the measured distribution will contain both b and a terms with different recoil momentum dependence.

Consequently, it is improper to use the prescription given in Eq. (16) to re-interpret previous extractions of the $\beta - \nu$ angular correlation coefficient where b was set to zero and only a was fitted. For the most precise measurement in a Gamow-Teller decay [23], a was actually extracted from a differential measurement of the recoil momentum distribution, as given by Eq. (11). The result was originally used to constrain possible tensor couplings through their quadratic contribution to a, assuming $b = 0$ and assigning an uncertainty $\Delta b = 0.012$ on the basis of a previous measurement in ^{22}Na. This uncertainty on b is furthermore included in the total uncertainty of the quoted value of a. This measurement of a in ^{6}He, with a later revision [31], has been used in several global fits [1, 3, 27], reviews [9, 32] and articles [20, 33], where a has been reinterpreted as $\tilde{a}$.

In order to illustrate the impact on the extraction of b from this incorrect interpretation when applied to allowed pure Fermi and Gamow-Teller transitions, we have performed additional fits of the recoil momentum spectra, this time with $b = 0$ and a left as free parameter. If the fitted value of a is reinterpreted as $\tilde{a}$ one can extract the uncertainty on the Fierz term using Eq. (16),

$$\Delta b_{\tilde{a}} = \frac{\Delta\tilde{a}}{|a_{SM}|}\langle\frac{m}{W}\rangle^{-1}. \qquad (18)$$

The dotted lines in Fig. 4 show the 1σ statistical uncertainty on the Fierz term obtained from these fits as a function of the endpoint energy for pure Fermi (red curve) and pure Gamow-Teller (blue curve) transitions. The dependence on the endpoint energy is again qualitatively similar to the direct extraction of b, with a loss of sensitivity toward small endpoint values. This is so because the loss of sensitivity in the extraction of a from

the differential distribution dominates over the mild increase of the factor $\langle m/W\rangle$. For Fermi transitions the relative differences between the two curves are small, of about 25% at small endpoint values and of 20% at high values. However, for Gamow-Teller transitions the differences between the two results diverge for low endpoint values. It can be shown analytically that

$$\lim_{E_0 \to 0} \frac{\Delta b}{\Delta b_{\tilde{a}}} \approx \left|\frac{3a}{1 + 3a}\right|, \qquad (19)$$

where Δb is the uncertainty extracted from direct fits of b. This limit tends to infinity for $a = a_{GT}$ and explains the divergence observed between the values of Δb and $\Delta b_{\tilde{a}}$. If the $\tilde{a}$-prescription would have been applied to a Gamow-Teller transition with an end-point of $E_0 = 100$ keV the error on the uncertainty of b would have been of about an order of magnitude. From this analysis, the relative difference of the two Gamow-Teller curves at the ^{6}He endpoint, $E_0 = 3.5$ MeV, is 30%. The position between the two curves, to the right of the intersection point, indicates that the values of the Fierz term extracted from an incorrect reinterpretation of a are less precise than the values extracted from a direct fit of b using the same data. This result should however be taken with the proper caution, as the above simplified analysis neglects systematic effects and other details in the data analysis.
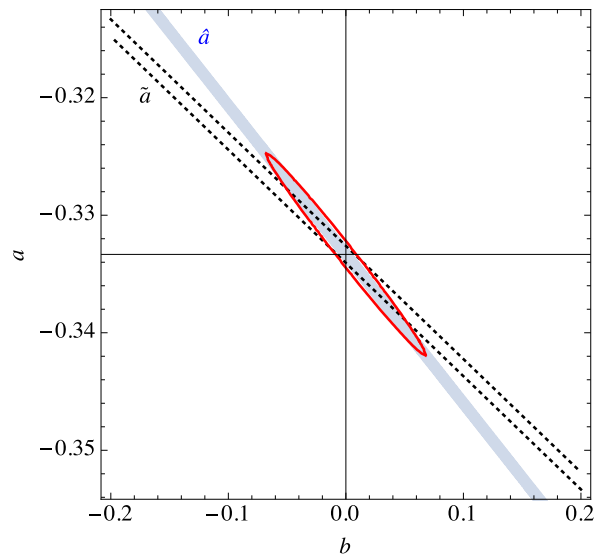

FIG. 5: (Color on-line) The solid red ellipse shows the 1σ region obtained from a fit of simulated recoil momentum spectra with 10^7 events, for the ^{6}He decay, where both a and b were left as free parameters. The blue filled band shows the 1σ bound on the combination $\hat{a} = a + 0.127\,b$, whereas the black dotted lines represent the 1σ bound obtained using the $\tilde{a}$-prescription.

A somewhat different way of analyzing the error of the $\tilde{a}$-prescription is obtained by performing a fit of the differential recoil distributions with both a and b as free parameters. Fig. 5 shows the result we obtained for the

[6]He decay with 10^7 events in the spectrum. One observes that there is indeed a large correlation between a and b, i.e. that a certain linear combination of them, namely $\hat{a} = a + 0.127\,b$, is strongly constrained. We see, however, that the $\hat{a}$-band is not aligned with the one obtained using $\tilde{a} \approx a(1 - \langle m/W \rangle\,b) \approx a(1 - 0.286\,b)$.

Concerning pure Fermi transitions, the two most precise extractions of a are those in ^{32}Ar [34] and in ^{38m}K decay [35], which were also used in global fits [1, 3]. The direct observable was the delayed proton spectrum following ^{32}Ar decay and the time-of-flight spectra of ^{38}Ar^{n+} ions in ^{38m}K decay, where the β particle spectrum was either totally or partly integrated. Thus, according to the results presented above, the $\tilde{a}$-prescription is not applicable in such measurements. Although the corresponding analyses were performed using a parameter called $\tilde{a}$ [34–36], it is important to notice that this parameter does not follow the standard definition of $\tilde{a}$, c.f. Eq. (16), also used in the present work. Instead, it corresponds to what we called $\hat{a}$ above, i.e. a linear combination of a and b that is strongly constrained by the fit, with coefficients that have to be calculated for each transition a posteriori [37, 38]. For example, in the ^{32}Ar experiment the value of the coefficient that multiplies b in $\hat{a}$, namely 0.1913 [34], is 11% smaller than the value of $\langle m/W \rangle = 0.214$ expected for this transition in the $\tilde{a}$ expression [34, 37]. As can be inferred from Fig. 4, the numerical expressions of $\tilde{a}$ and $\hat{a}$ are quite similar for pure Fermi transitions independently of the endpoint energy. It is however important to notice that conceptually these two parameters are completely different.

V. CONCLUDING REMARKS

The Fierz term, b, is one of the few parameters in β decays that is linearly sensitive to nonstandard interactions and its precise measurement represents a competitive New-Physics probe even in the LHC era [5, 6, 9]. In this work we have analyzed a few aspects that are relevant for (i) the selection of sensitive nuclear decays for future experiments, and (ii) the extraction of precise and correct bounds on b from past and future measurements.

In Sec. II-III we have analyzed how the statistical sensitivity to the Fierz term changes with the endpoint energy of the decay. We showed that although the effect in the overall normalization is maximal for very low endpoints, its effect on the β-energy and recoil momentum differential distribution goes to zero in that limit. For each case we identified the window of endpoint values where the sensitivity to b is maximal. These results are relevant for the selection of the most sensitive transitions in measurements of b. It is important to stress that the kinematic sensitivity is only one, and possibly the simplest, among several criteria for such a selection. Other criteria for measurements of β spectrum shapes are related with the size and accuracy of: 1) Coulomb and radiative corrections to the β spectrum [14, 15]; 2) the form factors which enter the weak hadronic currents in recoil terms [13]; and 3) instrumental effects such as the scattering of β particles in matter. For example, atomic effects due to screening are known to be large for low energy β particles from transitions in medium and heavy mass nuclei [39] and require therefore particular attention in precision measurements. On the instrumental side, the effects of scattering of electrons from matter in the setup and their back-scattering from detector surfaces also increases toward lower energies and the description of the processes with current simulation tools [40, 41] is not yet sufficiently accurate for competitive measurements of the β spectrum shape. These two criteria tend also to disfavor transitions with small endpoint energies in the selection of candidates.

In Sec. IV we discussed what we called the $\tilde{X}$-prescription and its relation to the Fierz term. We have shown explicitly that the prescription cannot be applied to values of a extracted from differential measurements of the recoil momentum distributions and we have explained under which conditions the procedure is justified. The prescription has been applied however, in a somewhat undiscriminated way, in several recent global fits [1, 3, 26, 27] for the re-interpretation of values of a extracted in ^{6}He and in neutron decays. The numerical impact of this misinterpretation on the constraints of exotic couplings extracted in global fits is in most cases quite small, simply because the precision achieved so far in measurements of recoil distributions is moderate. The associated constraints are therefore not competitive with determinations of b from other observables that dominate the fits. However it is important to notice that this might change in the near future with new generation measurements of the recoil spectrum [42].

Finally, in most of the numerical analyses performed in this work we neglected quadratic nonstandard effects that contribute to a. We would like to emphasize that it would be suitable that future analyses of new measurements of differential recoil distributions fit both a and b simultaneously, and provide their correlation, as in Fig. 5. One could then study specific cases with left-handed and right-handed couplings separately. For instance, for exotic interactions involving right-handed neutrinos, the linear terms are absent and then the quadratic terms become the leading ones.

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