Time-reversal-invariance-violating nucleon-nucleon potential in the 1/N_{c} expansion

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Abstract

We apply the large-N_c expansion to the time-reversal-invariance-violating (TV) nucleon-nucleon potential. The operator structures contributing to next-to-next-to-leading order in the large-N_c counting are constructed. For the TV and parity-violating case we find a single operator structure at leading order. The TV but parity-conserving potential contains two leading-order terms, which however are suppressed by $1/N_c$ compared to the parity-violating potential. Comparison with phenomenological potentials, including the chiral EFT potential in the TV parity-violating case, leads to large-N_c scaling relations for TV meson-nucleon and nucleon-nucleon couplings.

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I. INTRODUCTION

Time-reversal-invariance violation, or equivalently (assuming the validity of the CPT theorem [1–3]) CP violation, is an important component in the search for physics beyond the standard model (BSM). While the standard model contains CP-violating mechanisms in the complex phase of the Cabibbo-Kobayashi-Maskawa matrix and the QCD θ term, the predicted effects are much smaller than current experimental bounds on CP-violating observables. A signal of T violation beyond these predictions would be a clear indication of BSM physics. Among the considered time-reversal-invariance-violating (TV) observables, the neutron electric dipole moment (EDM) has received particular experimental interest, with the current upper limit $|d_n| < 3.0 \times 10^{-26}\, e\,\mathrm{cm}$ (90% C.L.) [4, 5]. However, more information than the measurement of a single observable is necessary to obtain detailed information about the underlying TV mechanisms.

Additional observables that have been considered include the EDMs of light nuclei, neutron-nucleus reactions, and nuclear decay parameters (see, e.g., Refs. [6–9]). In all of these processes, TV nucleon-nucleon (NN) forces play an important role. TV interactions can either be parity-conserving (PC) or parity-violating (PV), with the latter expected to give larger contributions to observables such as EDMs. These forces are the manifestation on the hadronic level of TV interactions among fundamental degrees of freedom. Because QCD is nonperturbative at low energies, a direct derivation of NN forces from the underlying theory is complicated, so various phenomenological parameterizations of TV NN forces have been developed. A general parameterization analogous to Wigner's approach to the T-conserving (TC) potential [10] was given in Ref. [11]. In phenomenological applications it is common to use a single-meson-exchange picture, with one strong (TC) and one TV meson-nucleon vertex [12–15]. More recently, TV interactions have been constructed in the effective field theory (EFT) framework, see, e.g., Refs. [16–19] and references therein. In all of these approaches, the TV short-distance physics is captured in the values of TV coupling constants: either meson-nucleon couplings and/or NN contact terms. However, the values of the couplings have not been derived from the underlying theory and couplings are constrained only weakly – if at all – by experiment.

In the following we apply the $1/N_c$ expansion of QCD [20, 21] to the TV NN potential, where N_c is the number of colors. The large-N_c analysis was first applied to the TCPC NN potential in Refs. [22, 23], and more recently to three-nucleon forces [24] and to the TCPV potential [25, 26]. The large-N_c expansion analysis allows us to capture dominant QCD effects of embedding the fundamental TV interactions in the nonperturbative environment of the nucleon. As a result, we find a hierarchy of terms in the TV potentials: In the TVPV case there is a single leading-order (LO) operator structure, with corrections suppressed by a single factor of $1/N_c$. For TVPC interactions we find two LO terms, with subleading corrections again suppressed by $1/N_c$. However, the leading TVPV and TVPC operators do not contribute at the same order: the dominant TVPV operator contributes at $\mathcal{O}(N_c)$, while the TVPC potential receives contributions starting at $\mathcal{O}(1)$. This hierarchy has to be superimposed on any suppression coming from the underlying BSM physics. At low energies it can be combined with the chiral suppressions that originate in the nonperturbative regime of QCD. The large-N_c and chiral suppressions are independent and complementary and, given the difficulty in obtaining experimental constraints, taken together they provide useful additional theoretical constraints that simplify the analysis of TV observables by reducing the number of unknowns that need to be considered in phenomenological applications.

The paper is organized as follows: Sec. II introduces the framework for analyzing NN potentials in the large-N_c expansion. In Sec. III we construct the TVPV and TVPC potentials at leading order (LO), next-to-leading order (NLO), and next-to-next-to-leading order (NNLO) in the large-N_c counting. These potentials are compared with phenomenological forms in Sec. IV, which allows us to extract the large-N_c scaling of the various TV couplings. We conclude in Sec. V.

II. THE NN POTENTIAL IN THE $1/N_c$ EXPANSION

Following Ref. [23], we define the NN potential as the matrix element

$$V(\mathbf{p}_-, \mathbf{p}_+) = \langle (\mathbf{p'}_1, C), (\mathbf{p'}_2, D) | H | (\mathbf{p}_1, A), (\mathbf{p}_2, B) \rangle. \tag{1}$$

Here, $A, \ldots, D$ collectively represent the spin and isospin components of the nucleons and $\mathbf{p}_i$ ($\mathbf{p'}_i$) denotes the incoming (outgoing) momentum of the ith nucleon, while

$$\mathbf{p}_\pm = \mathbf{p}' \pm \mathbf{p}\,, \tag{2}$$

where

$$\mathbf{p} = \frac{1}{2}\left(\mathbf{p}_1 - \mathbf{p}_2\right)\,, \qquad \mathbf{p}' = \frac{1}{2}\left(\mathbf{p'}_1 - \mathbf{p'}_2\right)\,. \tag{3}$$

The on-shell condition is given by $\mathbf{p}_+ \cdot \mathbf{p}_- = 0$. The momenta are taken to be independent of N_c, i.e., $p \sim \Lambda_{QCD}$. Our analysis does not depend on a low-momentum expansion of the potential, unlike in chiral or pionless EFTs. The Hamiltonian H is the nuclear Hamiltonian in the Hartree expansion, which in the large-N_c limit can be written as [23, 27]

$$H = N_c \sum_{s,t,u} v_{stu} \left(\frac{S}{N_c}\right)^s \left(\frac{I}{N_c}\right)^t \left(\frac{G}{N_c}\right)^u \,, \tag{4}$$

where the coefficients v_{stu} are functions of the momenta $\mathbf{p}_\pm$. The operators S, I, and G are given by

$$S^i = q^\dagger \frac{\sigma^i}{2} q \,, \quad I^a = q^\dagger \frac{\tau^a}{2} q \,, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q \,, \tag{5}$$

and when evaluated between single-nucleon states scale as [23]

$$\langle N'|S^i|N\rangle \sim 1 \,, \quad \langle N'|I^a|N\rangle \sim 1 \,, \quad \langle N'|G^{ia}|N\rangle \sim N_c \,. \tag{6}$$

In the large-N_c formalism, it is consistent to interpret the potential as originating from one-meson exchanges [23, 28, 29]. In this picture, a factor of $\mathbf{p}_+$ arises from relativistic corrections and is therefore suppressed by the nucleon mass m_N. Since m_N scales as N_c and we consider momenta $\sim N_c^0$, each power of $\mathbf{p}_+$ introduces a suppression by $1/N_c$. The coefficients v_{stu} are constructed such that the resulting Hamiltonian has specific symmetry properties. In the following, H is rotationally invariant, even under particle interchange, time-reversal odd, and we consider both parity-odd and parity-even cases. The transformation properties under time reversal (T), parity (P), and particle interchange (P_{12}) of the various building blocks are given in Tables I and II. There, $[AB]_2^{ij}$ denotes the symmetric and traceless rank-two tensor, constructed from the vector quantities A^i, B^j as

$$[AB]_2^{ij} \equiv A^i B^j + A^j B^i - \frac{2}{3}\delta^{ij} A \cdot B \,. \tag{7}$$

For a review of how to construct the TCPC NN potential see [24]. In the next Section we apply those methods to obtain the TV NN potentials.

III. THE TIME-REVERSAL-INVARIANCE-VIOLATING POTENTIALS

A. The TVPV potential

We first consider the TVPV potential. By using the $1/N_c$-counting rules for the momenta, spin, and isospin operators, as well as their transformation properties under time reversal,

	$\mathbf{p}_+$	$\mathbf{p}_-$	$\vec{\sigma}_1\cdot\vec{\sigma}_2$	$(\vec{\sigma}_1+\vec{\sigma}_2)^z$	$(\vec{\sigma}_1-\vec{\sigma}_2)^z$	$\vec{\sigma}_1\times\vec{\sigma}_2$	$[\sigma_1\sigma_2]_2^{ij}$
T	-	+	+	-	-	+	+
P	-	-	+	+	+	+	+
P_{12}	-	-	+	+	-	-	+

TABLE I. Transformation properties of momenta and spin operators under time reversal (T), parity (P), and particle interchange (P_{12}).

	$\vec{\tau}_1\cdot\vec{\tau}_2$	$(\vec{\tau}_1+\vec{\tau}_2)^z$	$(\vec{\tau}_1-\vec{\tau}_2)^z$	$(\vec{\tau}_1\times\vec{\tau}_2)^z$	$[\tau_1\tau_2]_2^{zz}$
T	+	+	+	-	+
P	+	+	+	+	+
P_{12}	+	+	-	-	+

TABLE II. Transformation properties of isospin operators under time reversal (T), parity (P), and particle interchange (P_{12}).

parity, and particle interchange, we construct the TVPV potential up to NNLO in the large-N_c counting. There is one operator structure at LO, $\mathcal{O}(N_c)$,

$$V_{N_c}^{\not{T}\not{P}} = N_c\, U^1_{\not{T}\not{P}}(\mathbf{p}_-^2)\,\mathbf{p}_-\cdot\,(\vec{\sigma}_1\,\tau_1^z-\vec{\sigma}_2\,\tau_2^z)\,. \tag{8}$$

At NLO, $\mathcal{O}(N_c^0)$, five additional operators contribute,

$$\begin{aligned} V_{N_c^0}^{\not{T}\not{P}} =& U^2_{\not{T}\not{P}}(\mathbf{p}_-^2)\,\mathbf{p}_-\cdot\,(\vec{\sigma}_1-\vec{\sigma}_2) \\ &+U^3_{\not{T}\not{P}}(\mathbf{p}_-^2)\,\mathbf{p}_-\cdot\,(\vec{\sigma}_1-\vec{\sigma}_2)\,\vec{\tau}_1\cdot\,\vec{\tau}_2 \\ &+U^4_{\not{T}\not{P}}(\mathbf{p}_-^2)\,\mathbf{p}_-\cdot\,(\vec{\sigma}_1-\vec{\sigma}_2)\,[\tau_1\,\tau_2]_2^{zz} \\ &+U^5_{\not{T}\not{P}}(\mathbf{p}_-^2)\,\mathbf{p}_+\cdot\,(\vec{\sigma}_1\times\vec{\sigma}_2)\,[\tau_1\,\tau_2]_2^{zz} \\ &+U^6_{\not{T}\not{P}}(\mathbf{p}_-^2)\,\mathbf{p}_+\cdot\,(\vec{\sigma}_1\times\,\vec{\sigma}_2)\,\vec{\tau}_1\cdot\,\vec{\tau}_2\,. \end{aligned} \tag{9}$$

The NNLO, $\mathcal{O}(N_c^{-1})$, operators are given by

$$\begin{aligned} V_{N_c^{-1}}^{\not{T}\not{P}} = N_c^{-1}\Big[& U_{\not{T}\not{P}}^{7}(\mathbf{p}_-^2)\, \mathbf{p}_- \cdot (\vec{\sigma}_1\, \tau_2^z - \vec{\sigma}_2\, \tau_1^z) \\ & + U_{\not{T}\not{P}}^{8}(\mathbf{p}_-^2)\, \mathbf{p}_+^2\, \mathbf{p}_- \cdot (\vec{\sigma}_1\, \tau_1^z - \vec{\sigma}_2\, \tau_2^z) \\ & + U_{\not{T}\not{P}}^{9}(\mathbf{p}_-^2)\, \mathbf{p}_+ \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)\, (\vec{\tau}_1 \times \vec{\tau}_2)^z \\ & + U_{\not{T}\not{P}}^{10}(\mathbf{p}_-^2)\, \mathbf{p}_+ \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, (\vec{\tau}_1 + \vec{\tau}_2)^z \\ & + U_{\not{T}\not{P}}^{11}(\mathbf{p}_-^2)\, [(\mathbf{p}_+ \times \mathbf{p}_-)\mathbf{p}_-]_2^{ij}\, [\sigma_1\sigma_2]_2^{ij}\, (\vec{\tau}_1 - \vec{\tau}_2)^z \\ & + U_{\not{T}\not{P}}^{12}(\mathbf{p}_-^2)\, [(\mathbf{p}_+ \times \mathbf{p}_-)\mathbf{p}_+]_2^{ij}\, [\sigma_1\, \sigma_2]_2^{ij}\, (\vec{\tau}_1 \times \vec{\tau}_2)^z \Big] . \end{aligned} \tag{10}$$

The $U_{\not{T}\not{P}}^{i}(\mathbf{p}_-^2)$ are arbitrary functions of $\mathbf{p}_- \sim N_c^0$ and do not change the large-N_c scaling of the corresponding operator structures. While corrections to the LO term in the potential are suppressed by single powers of $1/N_c$, for a given isospin sector the first correction is suppressed by $1/N_c^2$: the LO term in the potential is an isovector, while the $1/N_c$-suppressed terms are purely isoscalar and isotensor pieces. The NNLO contributions are again only of isovector form.

B. The TVPC potential

The TVPC potential can be constructed analogously. In this case, the LO contribution appears at $\mathcal{O}(N_c^0)$, and is therefore suppressed compared to the LO terms of the TVPV potential. There are two LO operators,

$$\begin{aligned} V_{N_c^0}^{\not{T}P} = & U_{\not{T}P}^{1}(\mathbf{p}_-^2)\, \mathbf{p}_-^i\, \mathbf{p}_+^j\, [\sigma_1\sigma_2]_2^{ij}\, \vec{\tau}_1 \cdot \vec{\tau}_2 \\ & + U_{\not{T}P}^{2}(\mathbf{p}_-^2)\, \mathbf{p}_-^i\, \mathbf{p}_+^j\, [\sigma_1\sigma_2]_2^{ij}\, [\tau_1\, \tau_2]_2^{zz} . \end{aligned} \tag{11}$$

The NLO, $\mathcal{O}(N_c^{-1})$, operators are given by

$$\begin{aligned} V_{N_c^{-1}}^{\not{T}P} = N_c^{-1}\Big[& U_{\not{T}P}^{3}(\mathbf{p}_-^2)\, (\mathbf{p}_- \times \mathbf{p}_+) \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, (\vec{\tau}_1 - \vec{\tau}_2)^z \\ & + U_{\not{T}P}^{4}(\mathbf{p}_-^2)\, (\mathbf{p}_- \times \mathbf{p}_+) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\, (\vec{\tau}_1 \times \vec{\tau}_2)^z \\ & + U_{\not{T}P}^{5}(\mathbf{p}_-^2)\, \mathbf{p}_-^i\, \mathbf{p}_+^j\, [\sigma_1\sigma_2]_2^{ij}\, (\vec{\tau}_1 + \vec{\tau}_2)^z \Big] . \end{aligned} \tag{12}$$

For completeness, we also show the result for the NNLO, $\mathcal{O}(N_c^{-2})$ operators, even though this order is not considered for the TVPV case,

$$
\begin{aligned}
V^{\not{T}P}_{N_c^{-2}} = N_c^{-2}\Big[& U^{6}_{\not{T}P}(\mathbf{p}_-^2)\,\mathbf{p}_-^i\,\mathbf{p}_+^j\,[\sigma_1\sigma_2]_2^{ij} \\
& + U^{7}_{\not{T}P}(\mathbf{p}_-^2)\,\mathbf{p}_+^2\,\mathbf{p}_-^i\,\mathbf{p}_+^j\,[\sigma_1\sigma_2]_2^{ij}\,\vec{\tau}_1\cdot\vec{\tau}_2 \\
& + U^{8}_{\not{T}P}(\mathbf{p}_-^2)\,\mathbf{p}_+^2\,\mathbf{p}_-^i\,\mathbf{p}_+^j\,[\sigma_1\sigma_2]_2^{ij}\,[\tau_1\,\tau_2]_2^{zz}\Big]\,.
\end{aligned}
\tag{13}
$$

The $U^i_{\not{T}P}(\mathbf{p}_-^2)$ are again arbitrary functions that do not scale with N_c. As in the TVPV case, for a given isospin sector the first corrections are suppressed by $1/N_c^2$, e.g., here the LO isoscalar and isotensor terms only get corrections at NNLO.

IV. COMPARISON WITH PHENOMENOLOGICAL TV POTENTIALS

In the following we compare our results with existing parameterizations of the TV potentials and extract the large-N_c scaling of the corresponding couplings. If available at all, experimental constraints on the TV couplings are very weak (see, e.g., Ref. [30]), so we are unable to compare our results to data. However, the hierarchy of couplings established in our analysis should prove helpful in identifying the most relevant couplings on which to focus in future TV studies.

A. General parameterization

1. *TVPV potential*

A general parameterization of the TVPV and TVPC Hamiltonians to first order in $\mathbf{p}_+$ was given in Ref. [11]. We follow the notational conventions of Ref. [31], but adapt them to our definition of the potential as a function of $\mathbf{p}_-$ and $\mathbf{p}_+$. The resulting potential can be written as

$$
\begin{aligned}
V_{\not{T}\not{P}} = &\Big[\bar{g}_1(\mathbf{p}_-^2) + \bar{g}_2(\mathbf{p}_-^2)\,\vec{\tau}_1\cdot\vec{\tau}_2 + \bar{g}_3(\mathbf{p}_-^2)\,[\tau_1\,\tau_2]_2^{zz}\Big]\,\mathbf{p}_-\cdot(\vec{\sigma}_1-\vec{\sigma}_2)\\
&+\big(\bar{g}_4(\mathbf{p}_-^2)+\bar{g}_5(\mathbf{p}_-^2)\big)\;\mathbf{p}_-\cdot(\vec{\sigma}_1\tau_1^z-\vec{\sigma}_2\tau_2^z)\\
&+\big(\bar{g}_4(\mathbf{p}_-^2)-\bar{g}_5(\mathbf{p}_-^2)\big)\;\mathbf{p}_-\cdot(\vec{\sigma}_1\tau_2^z-\vec{\sigma}_2\tau_1^z)\\
&+\Big[\bar{g}_6(\mathbf{p}_-^2)-\bar{g}_{10}(\mathbf{p}_-^2)+\big(\bar{g}_7(\mathbf{p}_-^2)-\bar{g}_{11}(\mathbf{p}_-^2)\big)\,\vec{\tau}_1\cdot\vec{\tau}_2\\
&\quad+\big(\bar{g}_8(\mathbf{p}_-^2)-\bar{g}_{12}(\mathbf{p}_-^2)\big)\,[\tau_1\,\tau_2]_2^{zz}+\big(\bar{g}_9(\mathbf{p}_-^2)-\bar{g}_{13}(\mathbf{p}_-^2)\big)\,(\vec{\tau}_1+\vec{\tau}_2)^z\Big]\mathbf{p}_+\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\\
&+\bar{g}_{14}(\mathbf{p}_-^2)\,[(\mathbf{p}_+\times\mathbf{p}_-)\mathbf{p}_-]_2^{ij}\,[\sigma_1\sigma_2]_2^{ij}\,(\vec{\tau}_1-\vec{\tau}_2)^z\\
&+\big(\bar{g}_{15}(\mathbf{p}_-^2)-\bar{g}_{16}(\mathbf{p}_-^2)\big)\,\mathbf{p}_+\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\,(\vec{\tau}_1\times\vec{\tau}_2)^z\,. \qquad (14)
\end{aligned}
$$

The functions $\bar{g}_i(\mathbf{p}_-^2)$ are related to Fourier transforms of the functions $g_i(r)$ of Ref. [31]. Because $\mathbf{p}_-$ is independent of N_c, the Fourier transform does not alter the large-N_c scaling and the relations derived below for the $\bar{g}_i(\mathbf{p}_-^2)$ should also hold for the corresponding $g_i(r)$. Comparison with Eqs. (8)-(10) shows that these structures are reproduced in the large-N_c analysis up to NNLO, with the exception of the term proportional to $(\bar{g}_6-\bar{g}_{10})$, which is suppressed even further. On the other hand, Eq. (10) contains an additional term, proportional to $U^{12}_{\not{T}\not{P}}(\mathbf{p}_-^2)$, which is not included in Eq. (14) because it is second order in $\mathbf{p}_+$. The following large-N_c scaling relations for the couplings can be extracted:

$$
\begin{aligned}
&\bar{g}_1\sim N_c^0\,, && \bar{g}_2\sim N_c^0\,, && \bar{g}_3\sim N_c^0\,,\\
&(\bar{g}_4+\bar{g}_5)\sim N_c\,, && (\bar{g}_4-\bar{g}_5)\sim N_c^{-1}\,, && \\
&(\bar{g}_6-\bar{g}_{10})\sim N_c^{-2}\,, && (\bar{g}_7-\bar{g}_{11})\sim N_c^0\,, && (\bar{g}_8-\bar{g}_{12})\sim N_c^0\,,\\
&(\bar{g}_9-\bar{g}_{13})\sim N_c^{-1}\,, && \bar{g}_{14}\sim N_c^{-1}\,, && (\bar{g}_{15}-\bar{g}_{16})\sim N_c^{-1}\,. \qquad (15)
\end{aligned}
$$

In the large-N_c limit, the order-N_c TVPV interactions proportional to $\bar{g}_4+\bar{g}_5$ dominate. From the two relations containing $\bar{g}_4$ and $\bar{g}_5$ it follows that these two couplings are equal up to corrections of relative order $1/N_c^2$, i.e., up to corrections expected to be of order 10%:

$$
\bar{g}_4=\bar{g}_5\left(1+\mathcal{O}(1/N_c^2)\right)\,. \qquad (16)
$$

Terms proportional to $\mathbf{p}_+$ are absent at LO and start to contribute at NLO, leading to the order-N_c^0 scaling of $(\bar{g}_7-\bar{g}_{11})$ and $(\bar{g}_8-\bar{g}_{12})$, the same order as some of the terms in the static potential.

2. TVPC potential

The general parameterization of the TVPC Hamiltonian up to first order in the relative momentum contains 18 terms [11]. Here we only show those that have a corresponding term in Eqs. (11)-(13), following some of the notational conventions of Ref. [32]. The terms proportional to $\tilde{g}_1$ through $\tilde{g}_8$ vanish because of the on-shell condition $\mathbf{p}_- \cdot \mathbf{p}_+ = 0$. The potential can then be written as

$$\begin{aligned} V_{\not{T}P} = \Big[& \tilde{g}_9(\mathbf{p}_-^2) - \tilde{g}_{13}(\mathbf{p}_-^2) + \big(\tilde{g}_{10}(\mathbf{p}_-^2) - \tilde{g}_{14}(\mathbf{p}_-^2)\big)\, \vec{\tau}_1 \cdot \vec{\tau}_2 \\ & + \big(\tilde{g}_{11}(\mathbf{p}_-^2) - \tilde{g}_{15}(\mathbf{p}_-^2)\big)\, [\tau_1\, \tau_2]_2^{zz} + \big(\tilde{g}_{12}(\mathbf{p}_-^2) - \tilde{g}_{16}(\mathbf{p}_-^2)\big)\, (\vec{\tau}_1 + \vec{\tau}_2)^z \Big]\, \mathbf{p}_-^i\, \mathbf{p}_+^j\, [\sigma_1 \sigma_2]_2^{ij} \\ & + \tilde{g}_{17}(\mathbf{p}_-^2)\, (\mathbf{p}_- \times \mathbf{p}_+) \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)(\vec{\tau}_1 - \vec{\tau}_2)^z \\ & + \tilde{g}_{18}(\mathbf{p}_-^2)\, (\mathbf{p}_- \times \mathbf{p}_+) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)(\vec{\tau}_1 \times \vec{\tau}_2)^z . \end{aligned} \tag{17}$$

Identifying the operators structures with those of Eqs. (11)-(13), the following large-N_c scalings for the functions $\tilde{g}_i(\mathbf{p}_-^2)$ (we use the tilde to distinguish them from the TVPV functions $\bar{g}_i(\mathbf{p}_-^2)$) are extracted:

$$\begin{aligned} &(\tilde{g}_9 - \tilde{g}_{13}) \sim N_c^{-2} , && (\tilde{g}_{10} - \tilde{g}_{14}) \sim N_c^0 , && (\tilde{g}_{11} - \tilde{g}_{15}) \sim N_c^0 , \\ &(\tilde{g}_{12} - \tilde{g}_{16}) \sim N_c^{-1} , && \tilde{g}_{17} \sim N_c^{-1} , && \tilde{g}_{18} \sim N_c^{-1} . \end{aligned} \tag{18}$$

Contrary to what was observed in the TVPV case, in the TVPC potential terms proportional to $\mathbf{p}_+$ are already present at LO. This leads to a relative suppression of $1/N_c$, so that the dominant TVPC interactions proportional to $\tilde{g}_{10} - \tilde{g}_{14}$ and $\tilde{g}_{11} - \tilde{g}_{15}$ are of order N_c^0. Again, the next-order terms are only suppressed by a single factor of $1/N_c$. The terms proportional to $U^7_{\not{T}P}(\mathbf{p}_-^2)$ and $U^8_{\not{T}P}(\mathbf{p}_-^2)$ in Eq. (13) contain more than one power of $\mathbf{p}_+$ and thus were not considered in Ref. [11].

B. One-meson exchange potential

1. TVPV Potential

The TVPV potential is commonly parameterized in terms of one-meson exchanges with one TCPC and one TVPV meson-nucleon coupling [14, 15, 33, 34]. Following Ref. [13], we consider π, η, ρ, and ω exchanges. The Lagrangian describing the TCPC meson-nucleon interactions is given by

$$\begin{aligned}\mathcal{L}_{\rm st} &= g_\pi \bar{N} i\gamma_5 \tau^a \pi^a N + g_\eta \bar{N} i\gamma_5 \eta N \\ &- g_\rho \bar{N}\left(\gamma^\mu - i\frac{\xi_V}{2\Lambda}\sigma^{\mu\nu}q_\nu\right)\tau^a\rho^a_\mu N - g_\omega \bar{N}\left(\gamma^\mu - i\frac{\xi_S}{2\Lambda}\sigma^{\mu\nu}q_\nu\right)\omega_\mu N \,, \end{aligned} \tag{19}$$

where $q_\nu = p_\nu - p'_\nu$, while the TVPV Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\not{T}\not{P}} &= \bar{N}\left(\bar{g}^{(0)}_\pi \tau^a\pi^a + \bar{g}^{(1)}_\pi \pi^0 + \bar{g}^{(2)}_\pi (3\tau^z\pi^0 - \tau^a\pi^a)\right) N \\ &+ \bar{N}\left(\bar{g}^{(0)}_\eta \eta + \bar{g}^{(1)}_\eta \tau^z \eta\right) N \\ &+ \bar{N}\left(\bar{g}^{(0)}_\rho \tau^a \rho^a_\mu + \bar{g}^{(1)}_\rho \rho^0_\mu + \bar{g}^{(2)}_\rho (3\tau^z\rho^0_\mu - \tau^a\rho^a_\mu)\right)\frac{\sigma^{\mu\nu}q_\nu\gamma_5}{2\Lambda} N \\ &+ \bar{N}\left(\bar{g}^{(0)}_\omega \omega_\mu + \bar{g}^{(1)}_\omega \tau^z\omega_\mu\right)\frac{\sigma^{\mu\nu}q_\nu\gamma_5}{2\Lambda}\, N \,. \end{aligned} \tag{20}$$

In comparison to Ref. [13] we have replaced $\chi_{V,S}/m_N \to \xi_{V,S}/\Lambda$ in $\mathcal{L}_{\rm st}$ and $1/m_N \to 1/\Lambda$ in $\mathcal{L}_{\not{T}\not{P}}$, where $\Lambda \sim 1\,$GeV is independent of N_c. This prevents spurious factors of $m_N \sim N_c$ from appearing in the expression for the potentials; see Ref. [25] for an analogous discussion for the TCPV case. The TVPV potential derived from these Lagrangians is given in Refs. [13, 31]. Using our conventions and transforming to momentum space it takes the form

$$\begin{aligned} V^{\rm meson}_{\not{T}\not{P}} &= \left[-\frac{\bar{g}^{(0)}_\eta g_\eta}{2m_N}\,Y^{(\eta)}(\mathbf{p}_-^2) + \frac{\bar{g}^{(0)}_\omega g_\omega}{2\Lambda}\,Y^{(\omega)}(\mathbf{p}_-^2)\right](\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\mathbf{p}_- \\ &+ \left[-\frac{\bar{g}^{(0)}_\pi g_\pi}{2m_N}\,Y^{(\pi)}(\mathbf{p}_-^2) + \frac{\bar{g}^{(0)}_\rho g_\rho}{2\Lambda}\,Y^{(\rho)}(\mathbf{p}_-^2)\right]\vec{\tau}_1\cdot\vec{\tau}_2\,(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\mathbf{p}_- \\ &+ \left[-\frac{\bar{g}^{(2)}_\pi g_\pi}{2m_N}\,Y^{(\pi)}(\mathbf{p}_-^2) + \frac{\bar{g}^{(2)}_\rho g_\rho}{2\Lambda}\,Y^{(\rho)}(\mathbf{p}_-^2)\right]\frac{3}{2}[\tau_1\tau_2]^{zz}_2\,(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\mathbf{p}_- \\ &+ \left[-\frac{\bar{g}^{(1)}_\pi g_\pi}{2m_N}\,Y^{(\pi)}(\mathbf{p}_-^2) + \frac{\bar{g}^{(1)}_\omega g_\omega}{2\Lambda}\,Y^{(\omega)}(\mathbf{p}_-^2)\right](\vec{\sigma}_1\,\tau_1^z - \vec{\sigma}_2\,\tau_2^z)\cdot\mathbf{p}_- \\ &+ \left[\frac{\bar{g}^{(1)}_\eta g_\eta}{2m_N}\,Y^{(\eta)}(\mathbf{p}_-^2) - \frac{\bar{g}^{(1)}_\rho g_\rho}{2\Lambda}\,Y^{(\rho)}(\mathbf{p}_-^2)\right](\vec{\sigma}_2\,\tau_1^z - \vec{\sigma}_1\,\tau_2^z)\cdot\mathbf{p}_- \,, \end{aligned} \tag{21}$$

where $Y^{(a)}(\mathbf{p}_-^2) = \frac{1}{\mathbf{p}_-^2 + m_a^2}$.

Comparison of Eq. (21) with Eqs. (8)-(10) shows that the meson-exchange potential contains the LO term of Eq. (8), as well as three of the five NLO terms of Eq. (9) and one NNLO term of Eq. (10). Because the meson-exchange potential in the form of Eq. (21) is linear in the momenta and does not include any relativistic corrections, it does not contain any of the operator structures that are proportional to a single factor of $\mathbf{p}_+$, nor terms that contain tensor structures of $\mathbf{p}_-$ and $\mathbf{p}_+$.

Now, using the known large-N_c scalings of the strong couplings, it is possible to determine the constraints that the large-N_c analysis places on the TVPV meson-nucleon couplings. The N_c scaling of the strong couplings is [23, 25, 28]

$$\begin{aligned}
g_\pi &\sim N_c^{3/2}\,, & g_\eta &\sim N_c^{1/2}\,. \\
g_\omega &\sim N_c^{1/2}\,, & g_\omega\,\xi_S &\sim N_c^{-1/2}\,, \\
g_\rho &\sim N_c^{-1/2}\,, & g_\rho\,\xi_V &\sim N_c^{1/2}\,.
\end{aligned} \tag{22}$$

As stated above, the scale Λ is independent of N_c, $\Lambda \sim N_c^0$. The same holds for the momentum $\mathbf{p}_-$ and the meson masses m_a $(a = \pi, \eta, \omega, \rho)$, so we also have

$$Y^{(a)}(\mathbf{p}_-^2) \sim N_c^0\,. \tag{23}$$

Requiring the coefficient functions $U^i_{\not T \not P}(\mathbf{p}_-^2)$ to be of order N_c^0 and not further suppressed, Eq. (21) allows to set constraints on the N_c scalings of the TVPV meson-nucleon couplings. Because there are contributions of more than one TVPV coupling to a single operator structure in Eq. (21), in principle only upper limits can be extracted for their scaling. However, at large distances pions dominate compared to the heavier meson exchanges. Therefore, pion couplings should saturate the upper limits and we obtain

$$\begin{aligned}
\bar{g}_\pi^{(0)} &\sim N_c^{-1/2}\,, & \bar{g}_\rho^{(0)} &\lesssim N_c^{1/2}\,, \\
\bar{g}_\pi^{(1)} &\sim N_c^{1/2}\,, & \bar{g}_\omega^{(1)} &\lesssim N_c^{1/2}\,, \\
\bar{g}_\pi^{(2)} &\sim N_c^{-1/2}\,, & \bar{g}_\rho^{(2)} &\lesssim N_c^{1/2}\,, \\
\bar{g}_\eta^{(0)} &\lesssim N_c^{1/2}\,, & \bar{g}_\omega^{(0)} &\lesssim N_c^{-1/2}\,, \\
\bar{g}_\eta^{(1)} &\lesssim N_c^{-1/2}\,, & \bar{g}_\rho^{(1)} &\lesssim N_c^{-1/2}\,.
\end{aligned} \tag{24}$$

In the last two pairs of bounds obtained for $\bar{g}_\eta^{(0)}$, $\bar{g}_\omega^{(0)}$ and $\bar{g}_\eta^{(1)}$, $\bar{g}_\rho^{(1)}$ at least one of each pair of couplings must saturate the bound. In the pion sector a clear hierarchy between the various couplings is predicted. The isovector coupling $\bar{g}_\pi^{(1)}$ dominates, while $\bar{g}_\pi^{(0)}$ and $\bar{g}_\pi^{(2)}$ are both suppressed by a factor of $1/N_c$, which agrees with the $(\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)}) \sim N_c^{-1/2}$ scaling found in the Skyrme model [35].

2. TVPC Potential

Constraints exist on the spin and parity of the exchanged bosons in the TVPC potential, and these exclude, e.g., one-pion exchange [12]. Here we consider the potential of Ref. [32], which includes $\rho(770)$ and $h_1(1170)$ exchanges. These are the lightest mesons that contribute to the TVPC potential. However, our analysis can straightforwardly be extended if additional/different mesons are considered. The relevant interactions are [32]

$$\begin{aligned}
\mathcal{L}_{\mathrm{st}} &= -g_\rho \bar{N}\left(\gamma^\mu - i\frac{\xi_V}{2\Lambda}\sigma^{\mu\nu}q_\nu\right)\tau^a\rho^a_\mu N - g_h\bar{N}\gamma^\mu\gamma_5 h_\mu N,\\
\mathcal{L}_{\not{T}P} &= -i\frac{\tilde{g}_\rho}{2\Lambda}\bar{N}\sigma^{\mu\nu}q_\nu(\vec{\tau}\times\vec{\rho}_\mu)^z N - \frac{\tilde{g}_h}{2\Lambda}\bar{N}\sigma^{\mu\nu}\gamma_5 q_\nu h_\mu N,
\end{aligned} \tag{25}$$

with the same replacements of $\chi_V/m_N \to \xi_V/\Lambda$ and $1/m_N \to 1/\Lambda$ for the vector meson couplings as in the TVPV case. The potential in momentum space then reads (cf. Ref. [32])

$$\begin{aligned}
V^{\mathrm{meson}}_{\not{T}P} =& \frac{\tilde{g}_\rho g_\rho}{2m_N\Lambda}Y^{(\rho)}(\mathbf{p}_-^2)(\vec{\tau}_1\times\vec{\tau}_2)^z\,(\mathbf{p}_-\times\mathbf{p}_+)\cdot(\vec{\sigma}_1-\vec{\sigma}_2)\\
&+\frac{\tilde{g}_h g_h}{2m_N\Lambda}Y^{(h)}(\mathbf{p}_-^2)\mathbf{p}_-^i\,\mathbf{p}_+^j\,[\sigma_1\sigma_2]_2^{ij}\,.
\end{aligned} \tag{26}$$

To extract the large-N_c scaling of the TVPC meson-nucleon couplings we take the strong hNN coupling to scale as [23, 28]

$$g_h \sim N_c^{-1/2}\,. \tag{27}$$

Comparison with Eqs. (11)-(13) shows that the ρ-meson exchange term corresponds to the NLO term proportional to $U^4_{\not{T}P}(\mathbf{p}_-^2)$, while the h_1-meson term corresponds to the NNLO term proportional to $U^6_{\not{T}P}(\mathbf{p}_-^2)$. The TVPC meson-nucleon couplings therefore scale as

$$\tilde{g}_\rho \sim N_c^{1/2}\,, \qquad \tilde{g}_h \sim N_c^{-1/2}\,. \tag{28}$$

The potential of Eq. (26) does not contain any of the LO terms in the large-N_c counting. These are related to the exchange of additional mesons. For example, inclusion of the isovector a_1 meson results in a term that matches the operator structure of the $U^1_{\not{T}P}(\mathbf{p}_-^2)$ term [32]. Given that the mass of the $a_1(1260)$ is close to that of the $h_1(1170)$ meson, the large-N_c analysis suggests that a_1 exchange should not be neglected in phenomenological applications.

C. Effective field theory

TVPV interactions have also been analyzed in effective field theory, see, e.g., Refs. [16–19] and references therein. In a chiral EFT the interactions are parameterized in terms of pion exchanges and nucleon-nucleon contact terms. The LO potential is [19, 36]:

$$
\begin{aligned}
V_{\not{T}\not{P}}^{\mathrm{EFT}} = & - i\frac{\bar{C}_1}{2}\,(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\mathbf{p}_- - i\left(\frac{g_A[\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)}]}{2F_\pi}\frac{1}{(\mathbf{p}_-^2 + M_\pi^2)} + \frac{\bar{C}_2}{2}\right)\vec{\tau}_1\cdot\vec{\tau}_2\,(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\mathbf{p}_- \\
& - i\frac{g_A\bar{g}_\pi^{(1)}}{2F_\pi}\frac{1}{(\mathbf{p}_-^2 + M_\pi^2)}(\vec{\sigma}_1\tau_1^z - \vec{\sigma}_2\tau_2^z)\cdot\mathbf{p}_-\,.
\end{aligned}
\tag{29}
$$

Here $\bar{C}_{1,2}$ are NN contact terms, $F_\pi = 92.4$ MeV is the pion decay constant, and $\bar{g}_\pi^{(0,1,2)}$ are the TVPV pion-nucleon couplings defined in Eq. (20). The term proportional to $\bar{g}_\pi^{(1)}$ in $V_{\not{T}\not{P}}^{\mathrm{EFT}}$ reproduces the LO term in the large-N_c analysis. $V_{\not{T}\not{P}}^{\mathrm{EFT}}$ also contains two terms that are NLO in the $1/N_c$ expansion. This suggests that, even though all three terms appear at the same order in chiral EFT, the one-pion exchange contribution proportional to $\bar{g}_\pi^{(1)}$ is dominant in a combined chiral and large-N_c analysis. $\bar{g}_\pi^{(0,1,2)}$ are all assumed to be natural (i.e., of order 1) in the chiral EFT analysis, but in fact $\bar{g}_\pi^{(0)}$ and $\bar{g}_\pi^{(2)}$ are suppressed compared to $\bar{g}_\pi^{(1)}$ by a factor of $1/N_c$. Comparison with Eqs. (8)-(10) also leads to $\bar{C}_1 \sim N_c^0$, $\bar{C}_2 \lesssim N_c^0$ for the NN contact terms. However, since naturalness is difficult to define quantitatively and $1/N_c = 1/3$ in the physical world, it seems reasonable to retain all terms in Eq. (29) in phenomenological applications.

The fact that the LO chiral EFT potential in the TVPC case contains the leading term in the $1/N_c$ expansion is different from the TCPV case. There pion exchange constitutes the sole LO contribution to the potential in the chiral counting, but the analysis of Ref. [25] shows it is actually suppressed by $\sin^2\theta_W/N_c$ compared to other mechanisms.

V. CONCLUSIONS

We applied the $1/N_c$ expansion to the TVPV and TVPC NN potentials. In the TVPV case, the LO terms are of order N_c, while the LO contributions in the TVPC case are of order N_c^0. In both cases first corrections are suppressed by a single power of $1/N_c$. However, to the order we considered, the expansion in a given isospin sector is in $1/N_c^2$, as it is in the TCPV and TCPC cases [25]. In terms of a meson-exchange picture, the LO in N_c TVPV potential corresponds to π and ω exchanges. Using the known large-N_c scaling of the strong meson-nucleon couplings, we derived bounds on the scaling of the TVPV meson-nucleon couplings. In the pion sector, we find that the isovector coupling $\bar{g}_\pi^{(1)}$ scales as $N_c^{1/2}$, while both isoscalar and isotensor couplings $\bar{g}_\pi^{(0)}$ and $\bar{g}_\pi^{(2)}$ are smaller by a factor of $1/N_c$. The NLO potential also contains terms that are not reproduced in the meson-exchange picture. These terms are proportional to $\mathbf{p}_+$ and correspond to relativistic corrections. In the TVPC case, the commonly considered ρ and h_1 exchanges only start to contribute at NLO in the large-N_c counting. The LO potential is generated by the exchange of additional mesons, e.g., the a_1 meson. While these are heavier than the ρ and h_1 mesons, from the large-N_c point of view all meson masses scale as N_c^0 and the a_1 contribution should be considered.

Comparison with the TVPV potential $V_{\not{T}\not{P}}^{\mathrm{EFT}}$ derived at LO in chiral EFT shows that it reproduces the leading large-N_c operator, together with some subleading terms in the large-N_c expansion. In particular, the pion-exchange term proportional to $\bar{g}_\pi^{(1)}$ contributes to the leading large-N_c operator. This is in contrast to the TCPV case, where the pion-exchange contribution, despite being the LO term in the chiral power counting, only generates subleading terms in the $1/N_c$ expansion. The extracted large-N_c scalings of the pion-nucleon couplings show that the TCPV pion-nucleon coupling $h_\pi^{(1)}$ is $1/N_c$-suppressed relative to the TVPV pion coupling $\bar{g}_\pi^{(1)}$. This has the effect that the LO chiral TVPV single-pion exchange potential is enhanced compared to the LO chiral TCPV single-pion exchange. It is interesting to note that, according to the recent analysis of Ref. [9], this strong-interaction enhancement of the isovector TV pion exchange may increase the sensitivity of experiments involving neutron scattering on nuclear targets to TV effects.

The chiral suppressions discussed in Refs. [16, 17] and the large-N_c scalings derived here are independent and complementary. Possible sources of T violation within the Standard Model and beyond can be studied in an EFT framework, where the new physics mechanisms and their characteristic energy scales enter through dimension six operators. Taken together with the combined effect of chiral symmetry breaking, isospin breaking, and large N_c suppressions, they give rise to a rich structure of hierarchies that can be accessed by studying their implications for observables like the EDMs of the nucleon and light nuclei [18]. We plan to address these phenomenological issues in connection with the $1N_c$ expansion in a future publication.

Given the difficulty of obtaining experimental constraints on the TV couplings, future lattice QCD calculations, while themselves highly complex, could contribute significantly to a better understanding of CP-violating effects in nuclear systems. In particular, calculations of the pion-nucleon couplings $\bar{g}_\pi^{(I)}$ $(I = 0, 1, 2)$ could check the hierarchy predicted by our large-N_c analysis.

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