Mass-number and excitation-energy dependence of the spin cutoff parameter

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Mass number and excitation energy dependence of the spin cutoff parameter

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The spin cutoff parameter determining the nuclear level density spin distribution $\rho(J)$ is defined through the spin projection as $< J_z^2 >^{1/2}$ or equivalently for spherical nuclei, $(\frac{<J(J+1)>}{3})^{1/2}$. It is needed to divide the total level density into levels as a function of J. To obtain the total level density at the neutron binding energy from the s-wave resonance count, the spin cutoff parameter is also needed. The spin cutoff parameter has been calculated as a function of excitation energy and mass with a super-conducting hamiltonian. Calculations have been compared with two commonly used semi-empirical formulas. A need for further measurements is also observed. Some complications for deformed nuclei are discussed. The quality of spin cut off parameter data derived from isomeric ratio measurement is examined.

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I. INTRODUCTION

Nuclear level density plays an important role in calculations of reaction cross sections and it is essential input in different reaction codes. The most uncertain parameter in level density calculations is the spin cutoff parameter σ determining the level density spin distribution. Bethe [1] introduced the spin cutoff parameter into the expansion for nuclear level density. He assumed that the distribution of nuclear states as a function of J_z at a given energy for a spherical nucleus had a Gaussian form.

$$\rho_s(U,J_z) = \frac{\rho_{st}(U)}{\sqrt{2\pi}\sigma}\exp(\frac{-J_z^2}{2\sigma^2}) \tag{1}$$

$\rho_{st}(U)$ is the total density of states of the nucleus at an energy U. σ is $< J_z^2 >^{1/2}$ and is also a function of energy. For a spherical nucleus, each level consists of $(2J+1)$ degenerate states with $-J <= J_z <= J$.

$$\begin{aligned}\rho_L(U,J) &= \rho_s(U,J) - \rho_s(U,J+1) \\ &\simeq -\frac{d\rho_s}{dJ}\mid_{J=J+1/2} \\ &= \frac{\rho_{st}(U)}{\sqrt{2\pi}}\frac{J+1/2}{\sigma^3}\exp(\frac{-(J+1/2)^2}{2\sigma^2})\end{aligned} \tag{2}$$

where $\rho_L(U,J)$ is the level density at excitation energy U and spin J. There are a number of ways that the value of σ and its energy dependence play a role in nuclear physics. Many level density compilations are based on level counting of s-wave resonances for neutron beams of low energy. S-wave neutrons populate compound nuclei of spin 1/2 if the target has spin 0 and $(J \pm 1/2)$ if the target has spin J. To convert this partial level density to the total level density, knowledge of σ is required. Changes in σ can also affect both cross sections and angular distributions calculated with Hauser-Feshbach codes [2]. The effects on angle-integrated cross sections are small, but the angular distribution effects are large. Finally, isomeric ratio measurements are quite sensitive to σ. As will be discussed later, they also have a sensitivity to other parameters as well.

There are two approaches to calculation of σ. The semiclassical procedure is to introduce the moment of inertia of the nucleus I. For the sphere of mass M and radius R, this would be $I = (2/5)MR^2$. In the semiclassical model, the energy of rotation would be the square of the angular momentum divided by twice the moment of inertia.

$$\frac{(J+1/2)^2}{2\sigma^2} = \frac{J(J+1)\hbar^2}{2I\theta} \tag{3}$$

where θ is the nuclear temperature. $\theta = \sqrt{U/a}$ in the Fermi-gas model. In an approximation of $(J+1/2)^2 \simeq J(J+1)$ (these two expressions differ by 1/4) the spin cutoff parameter can be expressed as

$$\begin{aligned}\sigma^2 = \frac{I\theta}{\hbar^2} = \\ \frac{2}{5}\frac{MR_0^2A^{2/3}}{\hbar^2}\sqrt{\frac{U}{a}} \\ = \frac{2}{5}\frac{m_pR_0^2A^{5/3}}{\hbar^2}\sqrt{\frac{U}{a}}\end{aligned} \tag{4}$$

M is the mass of the nucleus, m_p is the mass of the proton and the radius of the nucleus is R and is assumed to be

$$R = R_0A^{1/3} \tag{5}$$

a is the Fermi-gas level density parameter. This form assumes that the nucleus behaves like a classical "rigid body".

An alternative expansion which is quantum mechanical can also be obtained. Ericson [3] has shown that

$$n = g\theta = \frac{6}{\pi^2}a\sqrt{\frac{U}{a}} \tag{6}$$

where g is the single particle state density at the Fermi-level and a $= \pi^2$g/6, n is the number of excited nucleons

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and holes. The spin cutoff factor will be

$$\sigma^2 = n < m^2 >= \frac{6}{\pi^2}\sqrt{Ua} < m^2 > \tag{7}$$

Here $< m^2 >$ is the average angular momentum projection squared on the Z axis for single particle states at the Fermi level. It is interesting to note that equation (4) varies with $a^{-1/2}$ while equation (7) varies with $a^{1/2}$. This appears paradoxical, since the first form will show an increase in σ^2 at a closed nuclear shell where a is reduced relative to neiboring nuclei. On the other hand the second form Eq.7 would tend to decrease at a closed shell, although if $< m^2 >$ should show a significant increase at a closed shell this drop may be reduced or eliminated. A recent paper has been based on the assumption that the nucleus has "half rigid body" moment of inertia [4]. The result of this assumption is that the predicted behavior of σ^2 shows increases near A=90, 140, 208.

Since a normally increases roughly as A except near closed shells, it might be thought that the forms Eq.4 and Eq.7 might show a very different A dependence. This is not the case. If we assume that a equals αA then equation (4) becomes

$$\begin{aligned}\sigma^2 &= \frac{2}{5}\frac{m_p R_0^2 A^{5/3}}{\hbar^2}\sqrt{\frac{U}{\alpha A}} \\ &= \frac{2}{5}\frac{m_p R_0^2 A^{7/6}}{\hbar^2}\sqrt{\frac{U}{\alpha}}\end{aligned} \tag{8}$$

Similarly, Eq.7 would become

$$\sigma^2 = \frac{6}{\pi^2}\sqrt{UA\alpha} < m^2 > \tag{9}$$

If $< m^2 >$ increases with $A^{2/3}$, the dependence on A and U is the same for Eqs. (8) and (9). This is essentially a question of whether a nucleus behaves like a rigid body. Clearly, quantum mechanical effects causing a dependence of $< m^2 >$ which is not proportional to $A^{2/3}$ will destroy the rigid body behavior, but deviation of a from αA will also make Eq.(8) and Eq.(9) behave differently.

Table I shows $< m^2 >$ values for the lowest - lying orbits in the nucleus. Note very substantial fluctuations between the individual $< m^2 >$ values for neighboring orbitals. In table II, the $< m^2 >$ values are averaged over major shells. These averages do increase as the number of shells increases, but note that these averages when divided by $A^{2/3}$ do show a tendency to approach a constant.

In Ref.[4], an alternative derivation including an estimate for $< m^2 >$ based on a Fermi-gas average and an empirical form for a is presented in Eq.(58). We will compare this result with our microscopical calculations in the following section.

The purpose of this paper is to present calculations of σ^2 based on a superconducting hamiltonian as a function of excitation energy and mass number. These will be compared with rigid body predictions, formulas derived from fits to low-energy σ values and with measured values at somewhat higher energies.

II. CALCULATIONS

A. Semi-empirical models

It was pointed out in section 1 that two alternative forms exist for σ^2 which have a different dependence on the level density parameter a. The two have been compared in Ref.[4, 5]. If the nuclear radius is assumed to have a value $R = R_0 A^{1/3}$ where $R_0 = 1.25 fm$, then equation (8). becomes

$$\sigma^2 = 0.0151 A^{7/6}\sqrt{U}\alpha^{-1/2} \tag{10}$$

Similarly, Eq.9 can be shown to be

$$\sigma^2 = 0.6085\sqrt{UA\alpha} < m^2 > \tag{11}$$

If the nucleus is assumed to have a rigid-body moment of inertia, then equating Eq.10 and Eq.11 gives

$$< m^2 >= \frac{0.0151}{0.6085\alpha}A^{2/3} = 0.1985A^{2/3} \tag{12}$$

If we assume $\alpha = 1/8$, this turns out to be between the estimates for $< m^2 >$ quoted in Ref.[6] and Ref.[7]. In Ref.[7] it is proposed that the constant in Eq.12 should be 0.146, while in Ref.[6] it is concluded that the early estimate included all levels and that focusing on those near the Fermi-level yielded a value of 0.24. This indicates that the value in Eq.12 is in a proper range, showing rough consistency with a rigid-body moment of inertia. The values tabulated in Table II are also consistent with the value in Eq.12 and with early estimates.

This leaves open the question of whether deviations from this rigid body limit are important below energies of 10 MeV. Two recent references in which a simple form for σ^2 is derived from data [4, 8] show a general fit to σ values over a range of A but the experimental values show some dispersion about the smooth line. This is no doubt due in part to the fact that the values have been obtained from tabulated levels at low energy. This technique may only be used at low energy where the level scheme is complete and may thus show fluctuations because of the small number of levels included. Another study [9] produced a fit to the spin-cutoff parameter (also based on information at low excitation energy) which has energy and A dependence which does not agree with energy and A dependance of rigid-body model.

In the fits in Ref.[8, 9], there are energy shifts in the excitation energy making the σ^2 values largest for odd-odd nuclei in a given mass region, slightly lower for odd A nuclei at the same energy in the same A range and smaller still for even-even nuclei.

TABLE I: $< m^2 >$ values for individual orbits

Orbit	$s_{1/2}$	$p_{3/2}$	$p_{1/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$g_{9/2}$
$< m^2 >$	0.25	1.25	0.25	2.917	0.25	1.25	5.25	1.25	2.917	0.25	8.25

TABLE II: $< m^2 >$ values averaged over major shells

Shell	Orbits	$< m^2 >$	$< m^2 > /A^{2/3}$
I	$s_{1/2}$	1/4	
II	$p_{3/2}p_{1/2}$	0.9167	0.1444
III	$d_{5/2}s_{1/2}d_{3/2}$	1.9167	0.1639
IV	$f_{7/2}p_{3/2}p_{1/2}f_{5/2}$	3.25	0.1750
V	$g_{9/2}d_{5/2}s_{1/2}d_{3/2}g_{9/2}$	4.9167	0.1824
VI	$h_{11/2}f_{7/2}p_{3/2}p_{1/2}f_{5/2}h_{9/2}$	6.7167	0.1875

Figure 1 presents σ^2 values as a function of U and A for the rigid body model. These have been evaluated for a radius of $R = R_0 A^{1/3}$ where $R_0 = 1.25$fm. Further, the a is assumed to be $a = A/8$. If shell effects are included in a, there will be enhancements of 30-40% in σ^2 at shell closures. Also, Figure 1 present predictions of the forms from Ref.[8] and Ref.[9]. For both Ref.[8] and Ref.[9] , there is an energy shift based on the ground state masses. In each case, the shift is close to zero for odd-A targets, a reduction in excitation energy is found for even-even A nuclei and an enhancement in effective energy for odd-odd nuclei. Thus, the σ^2 values are typically reduced by 35% at 2 MeV for A=20 and 15% for A=100 at 2 MeV for the results of Ref.7. At 10 MeV the drop is 15% and 8% respectively. The parameterizations of Ref. [9] have changes which are about 3/4 as large. In each case results for odd-odd nuclei are enhanced by the same factor.

The authors of Ref.[4] propose a half-rigid body value for σ^2. This would correspond to half magnitude shown in Fig. 1 although the modulations in A near closed shells produce additional peaks in these regions.

The predictions of Refs. [4, 8, 9] do not provide tight constraints on predicted values for σ^2 at low energy. None of the three fits was constrained to approach the rigid body value at higher excitation energy. This is because the level density at low excitation energies is small enough that oscillations make it difficult to determine systematic changes in σ as a function of A.

B. Microscopical model

Calculated spin cutoff parameters were obtained using a formalism proposed by Sano and Yamasaki [10] and Morretto [11]. The nucleus is assumed to have a BCS (superconductor) Hamiltonian.

$$H = \sum_k e_k(a_k^+ a_k + a_{-k}^+ a_{-k}) - G\sum_{kk'} a_k^+ a_{-k'}^+ a_k a_{-k'} \tag{13}$$

Where e_k is the energy of the k-th doubly degenerate single particle level and a_k^+ and a_k are the creation and annihilation operators for the k-th particle state. $E_k = [(e_k - \lambda)^2 + \Delta^2]^{1/2}$ where E_k is the quasi-particle energy and Δ is the pairing gap. The following equations must be satisfied at each β value, where the β is a reciprocal of the temperature.

$$\frac{2}{G} = \sum_k \frac{tanh(\frac{1}{2}\beta E_k)}{E_k} \tag{14}$$

$$N = \sum_k (1 - \frac{e_k - \lambda}{E_k}) tanh(\frac{1}{2} p E_k) \tag{15}$$

Each of these equations is separately solved for protons and neutrons. Finally

$$\sigma^2 = \frac{1}{2}\sum_k \frac{m^2 k}{cosh^2(\beta E_k/2)} \tag{16}$$

This equation is separately summed for protons and neutrons and two sums are combined. Finally, the energy E is

$$E = \sum_k c_k[1 - \frac{e_k - \lambda}{E_k} tanh(\beta E_k/2)] \tag{17}$$

This equation also consists of a proton and neutron sum. G is a constant which is determined at zero temperature to give a solution to Eq.14 of $\Delta = 12/\sqrt{A}$. As the temperature is increased, solution of Eq.14 with the fixed value of G gives a value for Δ which decreases until at and above the critical temperature the only solution to Eq.14 is for $\Delta = 0$.

Single particle energies were taken from Refs.[12–14]. Calculations were done for A=20,...,100 in steps of ten and A=100,...., 240 in steps of 20. For each A the Z and N were chosen to be even and such that they are in the valley of stability. In general, the σ^2 values were about 30% higher for adjacent odd-A at low energy (about 2 MeV) and 15% higher at 10 MeV for A about 30. At A=200, the differences are 15% and 8%, respectively. In each case, the σ^2 for odd-odd nuclei was higher by about twice these amounts than for even-even nuclei of similar mass.

σ values are shown in Fig. 1 under calculated points. For each A the σ^2 values were calculated for each single particle set and then averaged. The dispersion for low-A was about 25% but more typically 15% for large A. As the energy is increased, the dispersion dropped to about half of these values. Exceptional cases were A=160,180,220 and 240. In these cases the single particle energies of Ref.[12] were used with the appropriate deformation parameters.

Note that the values tend to oscillate as a function of U and A relative to the rigid body values. As the energy approached 20 MeV, the values tended to converge to the rigid body values. At this energy the differences between even-even, odd-A and odd-odd nuclei in a given mass region were about 5-10%.

The behavior shown in Figure 1 is consistent with the following conclusions:

- There is a tendency for σ^2 values at energies of 10 MeV and below to reflect the shell structure of nuclei. In some regions of the A range, the σ^2 values are low relative to the rigid body value while in other regions the spin cutoff parameter is above the rigid body.

- Particularly at low energy (U<6 MeV), even-even nuclei have the lowest spin cut-off factors, with odd-A somewhat above them and odd-odd nuclei are highest of all. As A and U increase, these differences diminish.

- There is a general tendency for the spin cut off parameter to approach the rigid body value at 20 MeV

- There is no clear tendency for σ to be larger for closed shell nuclei then for neighboring nuclei. At low energies, σ values tend to have minima at A=30, 60,90,140 and 200. Of these minima, A=30 is not a near closed shell but is where the $s_{1/2}$ orbital is at the Fermi-level. The other values for A listed for minima are at or near closed shells but one closed shell at A of about 120 has a broad maximum. The microscopic model predicts a drop in σ^2 at magic numbers because of the $a^{1/2}$ factor, but if $< m^2 >$ increases at this point, the σ^2 could show only slight modulation or could increase.

In Fig.2, we present a comparison of our calculated microscopic σ values with those obtained from Eq.(58) of Ref. [4]. Generally good agreement is seen, but the present results show sharper structure where minima are predicted (A $\sim$ 30, 60 and 200). This difference reflects the fact that the present calculations include shell model values for $< m >^2$ while those of Ref.[4] are based on Fermi-gas averages for $< m >^2$ as a function of A.

Other techniques have also been used to calculate the spin cutoff parameter. Authors have used [5, 15–20] techniques which use the full two-body interaction to calculate both level densities and spin cutoff parameters. These have been concentrated in a mass region below A=65 but at least one paper presents calculations near A=160 [20].

Ref.[5] and [15] focus on spin cutoff parameters for nuclei near A=30. Both the measurements [15] and two body model calculations in Ref. [5]-[15] show a tendency for σ not to increase at given U as fast with A near A=30 as the rigid body prediction ($A^{7/6}$). This agrees with the predictions in this mass region from calculations presented in Fig. 1. Also, the calculations for ^{20}Ne [16], ^{24}Mg [17] and ^{28}Si [21] show a slower rise with A at low U than the rigid body model predicts. Spinella and Johnson [18] present two body calculations of level densities and spin cutoff parameters between mass 22 and 47 which show magnitudes for spin cutoff parameters similar to those in Fig. 1. In Ref.[19], spin cutoff parameters for Fe isotopes are presented, while in Ref.[20], deformed nuclei are calculated. These studies will show how important two-body forces are in determining the spin distribution of the nuclear level density.

The present calculations use single particle states in a deformed bases for nuclei which are deformed. This modifies the σ^2 values from what would be obtained if spherical levels had been used. There is an additional change in σ^2 caused by the addition of rotational bands. This correction was about 15% but was not included in the quoted values. As has been shown in Ref.[22], for deformed nuclei a modified equation expressing the relative spin distribution rather than Eq.2 must be used to calculate the J distribution. The revised formula produces changes larger than the above σ^2 correction for deformed nuclei.

Other work [4] has included level density enhancement factors for both vibrational and rotational levels. While both of these factors produce a significant enhancement in the level density, the forms traditionally used [4] only evaluate the enhancement factors as a function of excitation energy. Since they do not vary with J, they leave σ^2 unchanged. A more recent treatment of the rotational enhancement [22] not only modifies σ^2 somewhat with the inclusion of the rotational levels but also changes the basic formula relating σ^2 to the J distribution of the levels.

III. EXPERIMENTAL DATA

A. Angular distributions

In addition to the values obtained from resolved levels, spin cutoff parameters can be obtained from the angular distributions of compound nuclear reactions.

Four papers which report values for σ^2 from reaction angular distributions are based on α-induced reactions [2, 15, 23, 24].

Refs. [2, 15, 23] present σ values deduced from (α, n) angular distribution measurements while Ref.[24] focused on (α, α'), (α, p) and (p, α). In principle, there is no reason why more targets could not be studied with these reactions. Compound nuclear reactions of the types (p, p'), (n,n') and (p,n) show small anisotropy and are not useful for obtaining σ values. As A increases beyond 100, the anisotropy for α-particle reactions is reduced. It may be necessary to study reactions induced by ^6Li, ^7Li or ^{12}C

projectiles to obtain more information about σ values beyond A=100.

Examination of the σ values in Fig. 1. shows general consistency between the various measurements. Near A=50, the σ values are somewhat above the rigid body estimate, while for A between 50 and 65 the σ values fall below the rigid body estimate at excitation energy of 8 MeV. It is also found that the one value near A=118 (near Z=50 closed shell) the σ value tends to be lower than the rigid body value. This contradicts the rigid body model predictions that closed shell nuclei will have σs which are above the rigid body values. The same conclusion may be drawn from the data at Z=28, where the σ values tend not to be above the rigid body estimates.

The values of σ^2 presented in [2, 15, 23, 24] are subject to an error of 15-20%. Of this, a substantial part comes from the fitting of the angular distributions. Not only statistics but also possible non-compound reactions contribute to the error. More subtle effects come from the fact that each study only looked at one reaction channel for each compound nucleus. There is some coupling between the σ values inferred for the (a,a') reaction and σ values assumed for final nuclei populated by neutrons and protons. There is an advantage to using the (a,n) reaction for these studies. Because the neutron decay channel is usually dominant, the σ values derived from (a,n) studies are less affected by assumptions made about the σ values in the other channels than is the case for (a,a'), (a,p) and (p,a) reactions. There seems to be a general consistency between the values obtained in four studies. Clearly, additional data would be valuable.

The results from Monte-Carlo Shell Model calculations presented in [19] for ^{55}Fe are compared with the data of Ref.[2] (shown in Fig. 1). The agreement is generally good, but the data show a slightly lower slope in $4 \leq U \leq 8$ MeV region. The present microscopic calculations are also close to the measurements but also show a slightly steeper slope than the measurements. There appears to be some sensitivity in the microscopic results to the $f_{7/2}$ - $p_{3/2}$ single particle energies splitting. This could also affect the calculations of Ref.[19], although the uncertainties on the measurements Ref.[2] do not rule out the slightly steeper slope of the two calculations.

B. Isomeric ratios

Experimental values of σ have also been deduced from isomeric ratios. These are ratios between the cross sections for the populations of an isomeric states to the populations of the ground state. The isomeric state has a long lifetime because it has a J value which differs substantially from low-lying states which requires a γ-transition of high multipolarity. Changing σ causes there to be more large J states if σ is increased, while the opposite is true if the σ is reduced. One of the early compilations of σ values deduced by calculating isomeric ratios which fit measurements was provided by Ref. [25]. A more recent study which includes references to many other recent papers on isomeric ratios has been provided by Ref. [26]. Although the calculations are quite sensitive to the spin cutoff parameter, there are other parameters which influence the calculated ratios significantly. It is clearly important to include M1 and E2 γ-strength in addition to E1 strength in calculating the γ-decays. Both the integral and the energy dependence of each γ-strength function influenced calculations. Similarly, there is often a propensity for the levels at low excitation energy to be predominantly of one parity [8]. This will tend to reduce the contribution of E1 decays and make M1 and E2 decays more important. Calculations of isomeric ratios have not always included this parity ratio effect. There also is a need to determine wether the reaction mechanism is compound nuclear or includes an important contribution from pre-equilibrium processes. [27]. Finally, many isomeric ratio measurements have been made for deformed nuclei. It has recently been shown [22] that for deformed nuclei a modified HF formula is needed and that the previous formalism substantially overestimates the population of large J levels relative to small J levels. It should also be noted that for deformed nuclei Ref.[22] proposes that use of the traditional spin distribution Eq. 2 causes errors.

The values for σ^2 in Ref. [25] show a reasonable consistency with the present calculations for A $<$ 100. Above this value they are consistently smaller than the present calculations. Because nuclear temperature drops at a give energy as A increases, measurements reported in Ref. [25] (at $12 \leq$ En ≤ 16 MeV) were probably more affected by pre-equilibrium components for large A than for small A. Analyzing the ratios which have contributions from pre-equilibrium reactions as if there are entirely compound gives a σ which is too small. Similarly, some of the ratios at large A were for deformed nuclei. If these are analyzed with a spherical Hauser-Feshbach code, the results of Ref.[22] indicates the σ values will be too small.

C. Applications

As has previously been pointed out, a very important use of the spin cutoff parameter is in the conversion of the neutron s-wave resonance count at the binding energy to the total level density. If we calculate the $\rho(U, 1/2^{+})$ from 2 and separately sum 2 over J and parity, we obtain the ratio of $2\sigma^2$, where the 2 comes from the parity sum and the σ^2 term from the J sum.

There are three special concerns in the use of this equation. At low energies the parity distribution is typically asymmetric. A fit to the parity ratio [8] at low energy shows that this asymmetry vanishes as the energy increases but could still be a concern at the binding energy if A is less than 90. A further more subtle concern should also be examined. Calculations of the nuclear level density made by Goriely, Hilaire and Koning [28] indicate that even when the total number of levels of each parity are equal at a given energy, there may still be a spin fractionation, for a few MeV, i.e. the σ^2 values may not be the same for positive and negative parity levels. Finally, the formula in Eq.2 was derived specifically for a spherical nucleus. It has been shown [22] that the spin distribution for a deformed nucleus does not have the same functional form as for the spherical nucleus. This is different than has normally been assumed [29], which has the number of levels of each J for a deformed nucleus multiplied by the same factor $\sigma_\perp^2$ relative to the spherical results. The corrected formulas show a rotational enhancement which is larger for large J but decreases with K for the given J (Eq.(20) in [22]). It also depends on both $\sigma_\perp^2$ and $\sigma_\parallel^2$. The overall effect is to reduce enhancement somewhat below $\sigma_\perp^2$ for the sum. On the other hand, since the measured densities are for low J the total level density derived from resonance measurement will be increased. Finally, the new form does not have $< J_z^2 >= \frac{1}{3}J(J+1)$ unlike the previous results. This relation is only valid for spherical nuclei.

Some recent measurements [30, 31] have deduced the level density for a specific spin and parity at a given excitation energy for specific nuclei. These measurements are analogous to neutron s-wave resonance measurements. If values of σ are available, they can give the total level density. Until another measurement can give the density of levels of a different J for that nucleus, they do not yield values for σ.

The recent study of Koehler et al [32] finds levels with a range of spins and both parities for ^{96}Mo. In this case, the focus of the work was γ-strength distributions and the authors do not give a summary of the number of levels for each J and π, precluding of calculations of σ.

These measurements [30–32] are obviously of great value in comparing with microscopic level density calculations even without the σ values.

IV. SUMMARY

An examination of the predictions of the rigid body model and the microscopic model shows that the conditions for convergence of the two are met at higher energies. For energies near the neutron binding energy, it appears that quantum effects reflecting the spins of the individual single particle orbits have not yet been averaged out. This leads to spin cutoff factor values which in specific A regions are above or below rigid body values. Comparison with measured values shows that, despite the limited number of measurements, some indication of the oscillation of sigma values about the rigid body value at the binding energy is seen. Some nuclei have been studied using a two-body Hamiltonian. Again, these cases are limited but there seems to be good general agreement with data and with microscopic model calculations.

It is noted that more data are needed. Further, reanalysis of neutron resonance data for deformed nuclei should be undertaken using a corrected factor to convert s-wave resonance level density to the total level density.

Finally, values for σ obtained from isomeric ratios should be re-examined. Some earlier publications do not include M1 and E2 γ -decays, parity ratio differences in level densities, pre-equilibrium reactions or were analyzed with a spherical Hauser-Feshbach code even though the nucleus was deformed.

V. ACKNOWLEDGMENTS

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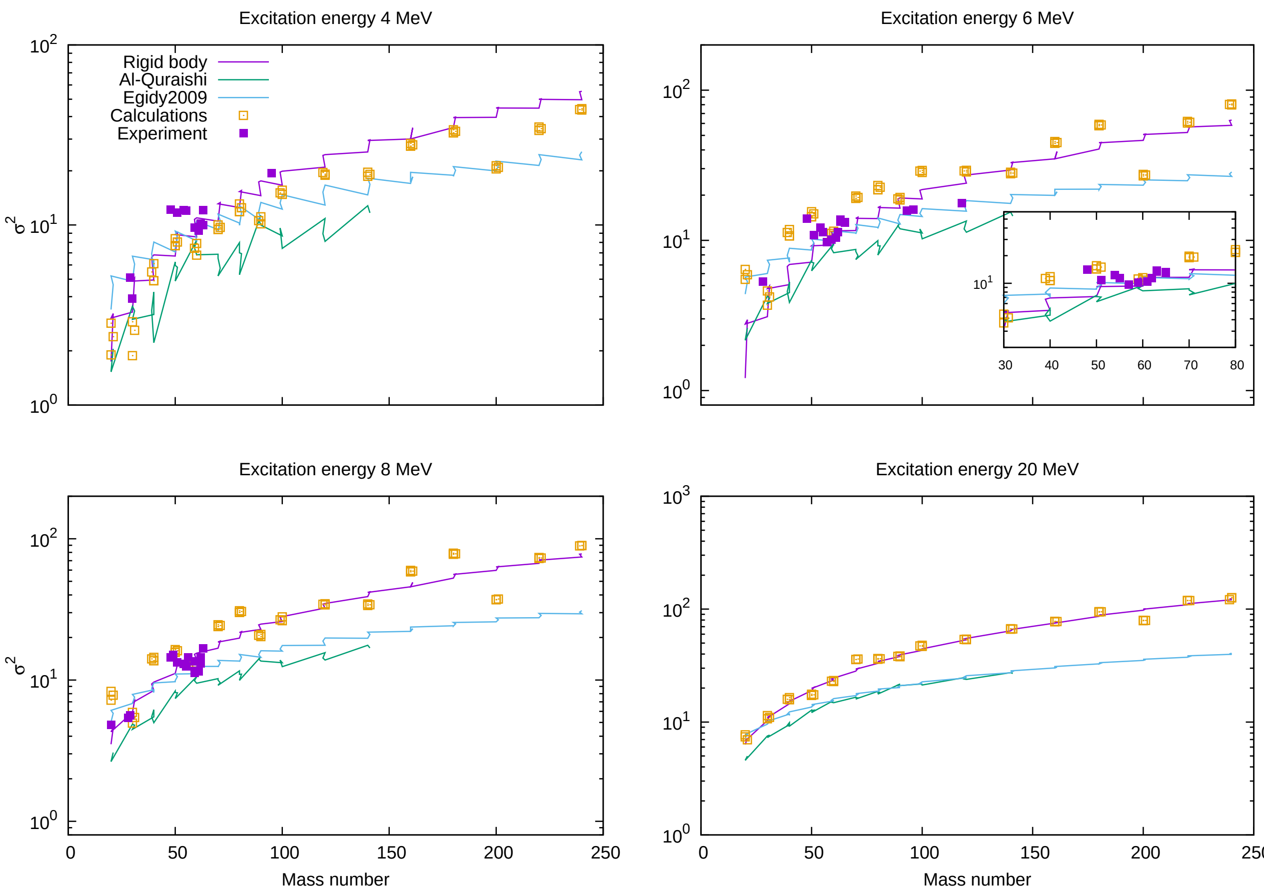

FIG. 1: Comparison of experimental spin cutoff parameters from Refs.[2, 23] with calculations based on different models: Rigid-body, Al-Quarishi [8], Egidy2009 [9], microscopical calculations from this work. Error bars for experimental points are typically 15% in σ^2.

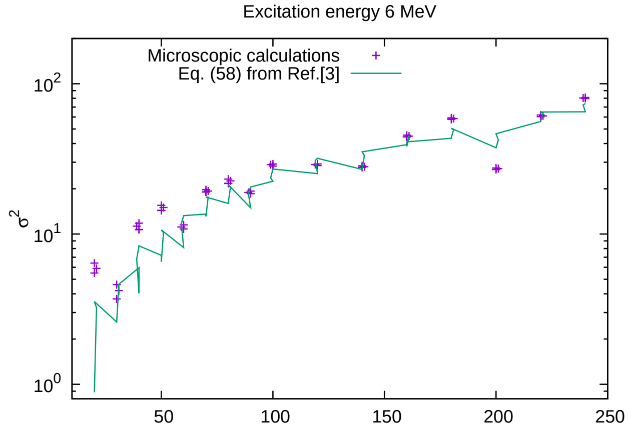

FIG. 2: Comparison of present (microscopic) calculations of σ^2 with those of Ref.[4] (Eq.(58)) at 6 MeV of excitation energy.

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