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Independent measurement of the Hoyle state β feeding from ^{12}B using Gammasphere

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Using an array of high-purity Compton-suppressed germanium detectors, we performed an independent measurement of the β -decay branching ratio from ^{12}B to the second-excited state, also known as the Hoyle state, in ^{12}C . Our result is 0.64(11)%, which is a factor ~ 2 smaller than the previously established literature value, but is in agreement with another recent measurement. This could indicate that the Hoyle state is more clustered than previously believed. The angular correlation of the Hoyle state γ cascade has also been measured for the first time. It is consistent with theoretical predictions.

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I. INTRODUCTION

Carbon is the fourth most abundant element in the Universe and it plays a key role in stellar nucleosynthesis. It is mainly formed in stars at a temperature of 10^8 - 10^9 K in the triple- α fusion reaction, which proceeds via the second-excited state, also known as the Hoyle state, at 7.65 MeV in ^{12}C , famously proposed by Hoyle in 1953 [1].

The first attempt to theoretically explain the structure of the state was the linear alpha chain model by Morinaga in 1956 [2], where he, furthermore, conjectured a 2^+ state in the 9–10 MeV region. Several more sophisticated models have been developed since, as summarized in Ref. [3]. Most of these models predict a collective 2^+ excitation of the Hoyle state in the region of 0.8–2.3 MeV above it. Interestingly, the collective state increases the triple- α reaction rate at $T > 10^9$ K by a factor of 5-10 compared to the results of Caughlan *et al.* [4, 5]. This makes it highly relevant for core collapse supernovae [6–9].

Experimentally, it is challenging to investigate this energy region, since there are contributions from several

broad states and from the so-called Hoyle state “ghost anomaly” [10–12]. Using inelastic proton scattering, Freer *et al.* provided the first evidence for a broad 2^+ contribution at 9.6(1) MeV with a width of 600(100) keV [5]. Itoh *et al.* corroborated these results using inelastic α -scattering [13] and a simultaneous analysis was published as well [14]. Results from an experiment using the alternative $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$ reaction also identified a 2^+ state in this region, but at $10.13_{-0.05}^{+0.06}$ MeV and with a much larger width of 2080_{-260}^{+330} keV [15, 16]. The reason for this discrepancy is presently not understood. A natural explanation would be that several 2^+ resonances are present in the region, and that the different reaction mechanisms populate these with different strength.

An alternative experimental probe is the β decay of ^{12}B and ^{12}N . Due to the selection rules, decay of these 1^+ systems will predominantly populate states with spin and parity 0^+ , 1^+ or 2^+ and not the 3^- state at 9.64 MeV, which is the dominant channel in inelastic scattering experiments. This technique has been used in several studies of ^{12}C [17–22], but none of these has identified a 2^+ state at 10 MeV. The β decay populates a somewhat featureless excitation spectrum in ^{12}C which is analyzed with the R-matrix formalism in Ref. [22]. This analysis identified both 0^+ and 2^+ resonances in the 10.5 MeV to 12 MeV region with recommended energies for both resonances at 11 MeV. The R-matrix analysis includes a large contribution from the high energy tail of the Hoyle state, which is sometimes referred to as the “ghost anomaly” [10, 11]. This contribution is strongly dependent on the branching

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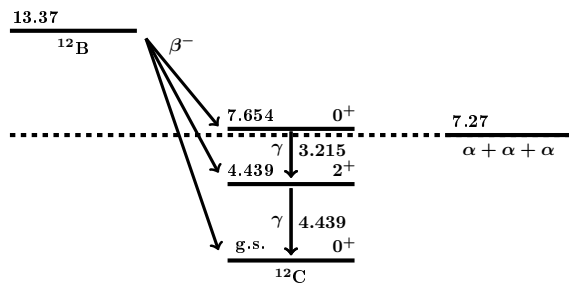


FIG. 1. Level scheme of ^{12}C also showing the α -threshold and the ^{12}B ground state. Energies are given in MeV relative to the ground state of ^{12}C [12].

ratio with which the Hoyle state is populated in the β decay.

In the most recent experimental study of the β decay, the beam was implanted in a silicon detector, which provided accurate normalization of the branching ratios, resulting in a revision of several of these [21]. More specifically, the branching ratio to the Hoyle state from the decay of ^{12}B was determined to be 0.58(2)%, which is inconsistent with the previously established value of 1.2(3)% [12, 23] (1.5(3)% is listed in Ref. [12], but this should be revised [23]). The reduced branching ratio for the population of the Hoyle state was used in the R-matrix analysis [22]. Furthermore, as the β decay to a pure 3α -cluster system is forbidden, a precise measurement of the branching ratio will provide insight into the strength of the cluster-breaking component of the Hoyle state [24]. It is therefore important to provide experimental confirmation of the reduced branching ratio measured in Ref. [21].

Here, we report on an independent measurement of this branching ratio through a measurement of the γ decay of the Hoyle state with an array of high-purity germanium detectors. The results of a preliminary analysis have been reported in [25].

II. METHOD

Figure 1 shows the lowest states in ^{12}C , the triple- α threshold and the ground state of ^{12}B . The first-excited state is below the α threshold and will only γ decay, whereas the Hoyle state cannot γ decay directly to the ground state as it is a 0^+ level. It can, however, decay via the first-excited state by emission of a 3215-keV photon.

The number of γ decays from the Hoyle state can be determined by counting the number of 3215-keV photons, and by furthermore requiring a simultaneous detection of a 4439-keV γ ray, the background is vastly reduced. The product of the branching ratio to the Hoyle state and its relative γ width can then be determined by normalizing to the decay of the first-excited state

$$\text{BR}(7.65) \frac{\Gamma_\gamma}{\Gamma} = \text{BR}(4.44) \frac{N_{\gamma\gamma}}{N_{4.44} \epsilon_{3.21} C_\theta}, \quad (1)$$

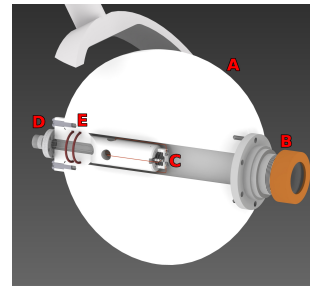


FIG. 2. (Color online) CAD drawing of the chamber manufactured from a Bonner sphere. (A) Bonner sphere, (B) vacuum flange, (C) target holder / Faraday cup, (D) electrical feed through, (E) O-rings

where $N_{\gamma\gamma}$ is the number of coincidence events, $\epsilon_{3.21}$ the efficiency for detecting a 3215-keV photon and C_θ corrects for the angular correlation between the two photons.

The relative γ width can be determined from all available data for the relative radiative width [26], excluding Seeger *et al.* [27], by subtracting the recommended relative pair width from [3]; yielding $\frac{\Gamma_\gamma}{\Gamma} = 4.07(11) \times 10^{-4}$. A conservative estimate of the branching ratio to the first excited state, $\text{BR}(4.44) = 1.23(5)\%$ has been published in [12].

Using this method, the branching ratio can be determined solely with γ -ray detectors, providing an experiment with entirely different systematic uncertainties than previous measurements based on detection of α or β particles.

III. EXPERIMENT

^{12}B was produced via the $^{11}\text{B}(d,p)^{12}\text{B}$ reaction in inverse kinematics, using a pulsed (40 ms on, 40 ms off) 40-MeV ^{11}B beam delivered by the Argonne Tandem-Linac Accelerator System (ATLAS) located at Argonne National Laboratory. A deuterated titanium foil (TiD_2), sufficiently thick to stop the beam, was used as target. The target was manufactured according to the method discussed in Ref. [28] and it contained approximately 1.5 mg/cm² deuterium (estimated by weight).

Photons were detected using Gammasphere [29], which is an array of 110 high-purity Compton-suppressed germanium detectors of which 98 were operational during the experiment. The array was operated in singles mode, where any of the detectors could trigger the data acquisition (DAQ). Data were only acquired during the beam-off period. Therefore, only delayed activity was measured (the half life of ^{12}B is 20.20(2) ms [12]). For each event, the time relative to beam-off as well as the energy and time for each γ ray in the detectors were recorded.

In order to minimize bremsstrahlung caused by high-energy β particles, a low-Z chamber was designed - see Figure 2. The chamber was manufactured from a Bonner sphere and was designed to minimize contribution

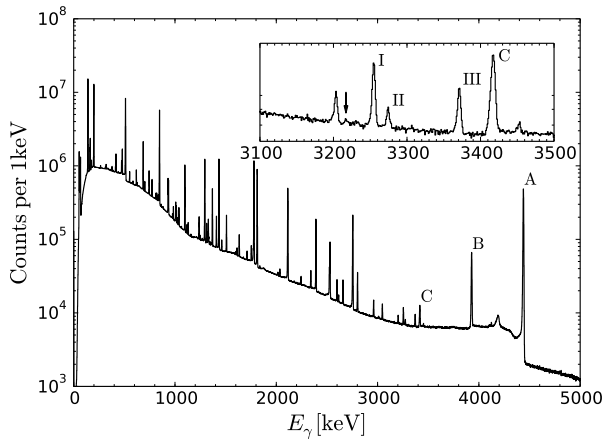


FIG. 3. The entire singles spectrum acquired during the beam-off period. The 4439-keV peak (A) and its escape peaks (B,C) are clearly visible. The insert shows the 3.1–3.5 MeV region. A small structure is visible around 3215 keV, indicated by an arrow.

from bremsstrahlung while maintaining high gamma-ray efficiency.

IV. ANALYSIS

A. Yield

During the experiment $\sim 10^9$ γ rays were collected in 67 hours. The singles spectrum is displayed in Fig. 3, where the transition from the first-excited state in ^{12}C at $4439.5 \pm 0.7(\text{sys})$ keV (A) together with the first (B) and second escape (C) peaks at lower energy are clearly seen. The insert shows the region from 3.1–3.5 MeV in which a structure around 3215 keV is visible, as indicated by an arrow. However, the region is dominated by a peak at 3200 keV.

The 4439-keV peak was fitted with a sum of a Gaussian distribution, a skewed Gaussian distribution, a linear background and a smoothed step function [30]. In order to minimize systematic effects, the fit was performed with the Poisson log likelihood ratio [31] using the MINUIT minimizer [32]. From this procedure, the area of the peak was determined to be $N_{4.44} = 9.20(2) \times 10^6$, where the error was dominated by uncertainties in the functional form of the peak.

B. Coincidence spectrum

To obtain a coincidence spectrum, a gate was placed on the relative time between the two γ rays and on the energy of the 4439-keV transition. The widths of these gates were chosen to minimize any systematic effects..

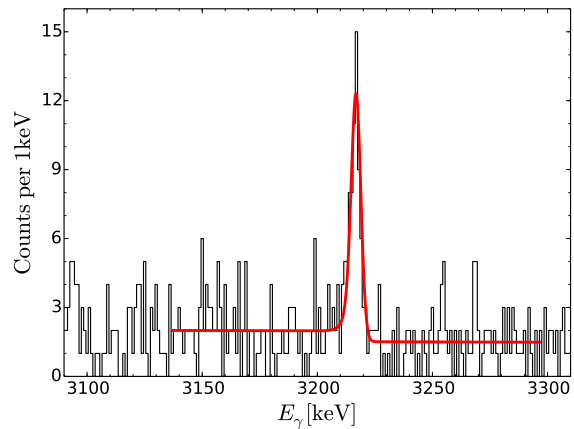


FIG. 4. (Color online) Coincidence spectrum, acquired by gating on the 4439 keV peak and the time difference. A clear peak centered at 3217 keV is consistent with the Hoyle state decaying via the first-excited state.

The coincidence spectrum is given in Fig. 4, where a clear peak centered at $3216.9^{+0.7(\text{sys})}_{-0.4(\text{stat})}$ keV is visible. This is consistent with a cascade decay of the Hoyle state via the first-excited level. The peak was fitted with the same functional form as in the previous section, but the parameters for the skewed Gaussian are determined from peaks I-III in Fig. 3. Peak I-III originate from ^{56}Mn and ^{56}Co produced in beam by reactions with Ti. The area of the peak, determined from the fit, is $N_{\gamma\gamma} = 58(9)$.

C. Efficiency

The relative efficiency was determined using the standard calibration sources ^{152}Eu and ^{56}Co mounted at the target position. This provides calibration points, both at low energy and in the important 3-MeV region. The absolute efficiency was calculated using the coincidence method, including a correction for random coincidence events, for both a ^{60}Co source and ^{24}Mg , which was produced by in-beam reactions [33]. From this procedure, the absolute efficiency at 3217 keV was determined to be $\epsilon_{3.21} = 2.94(2)\%$.

D. Angular correlation

Due to the excellent angular coverage of Gammasphere, it is possible to measure the angular correlation of the two γ rays, which had not been measured previously. Using the gates described above and in addition requiring the energy of the second γ ray to be within 10 keV of 3217 keV, it is possible to extract the true coincidence events plus some background. The shape of the background was determined by gating outside the peak, and was found to be flat.

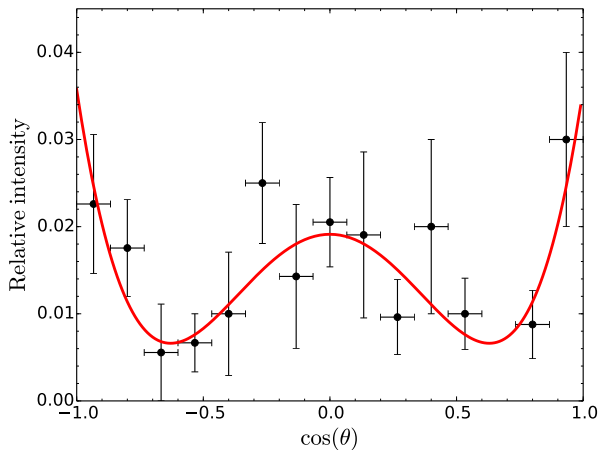


FIG. 5. (Color online) Angular correlation of the Hoyle state γ cascade corrected for the geometric efficiency (number of detector pairs with a given angle between them). The solid line shown is the best fit to equation (2).

The angular correlation, corrected for the geometric efficiency (number of detector pairs with a given angle between them), is shown in Fig. 5, together with the best fit to the equation

$$W(\theta) = k [1 + a_2 \cos^2(\theta) + a_4 \cos^4(\theta)], \quad (2)$$

where θ is the angle between the two γ rays. The result of the fit is $a_2 = -3.3(7)$ and $a_4 = 4.2(9)$, which is consistent with the theoretical expectations $a_2 = -3$ and $a_4 = 4$ for a $0 \rightarrow 2 \rightarrow 0$ cascade [34].

With the theoretical angular correlation confirmed, it can be used to estimate the correction factor C_θ from eq. (1). This is done with a simple Monte Carlo simulation of the detector setup, which gives $C_\theta = 1.00(1)$, as was expected from the large angular coverage by Gamma-sphere.

E. Extraction of branching ratio

The property directly measured in this experiment is the product of the relative γ width and the β feeding of the Hoyle state

$$\text{BR}(7.65) \frac{\Gamma_\gamma}{\Gamma} = 2.6(4) \times 10^{-4}. \quad (3)$$

Inserting the calculated value for the relative γ width into equation (3) gives

$$\text{BR}(7.65) = 0.64(11) \%, \quad (4)$$

which is clearly inconsistent with the previous literature value of 1.2(3) % [12, 23], but agrees with that of 0.58(2) % found in Ref. [21]. Therefore, the feeding of the Hoyle state from ^{12}B is roughly a factor of 2 smaller than indicated by Refs. [12, 23].

V. DISCUSSION

The branching ratio from ^{12}B and ^{12}N to the Hoyle state is a sensitive way to probe the clustering of this state, as the β -decay matrix element to the pure 3α system is exactly zero due to the Pauli principle [24]. The fact that β decay is possible means that the Hoyle state must contain some α -cluster breaking component. Theoretically, this is obtained by mixing shell-model-like states with cluster states as it is done; e.g. in Fermionic Molecular Dynamics (FMD) [35, 36] and Antisymmetrized Molecular Dynamics (AMD) approach [24]. Alpha-cluster breaking was explicitly investigated in Ref. [37] using a hybrid shell/cluster model, where it was found that the spin-orbit force significantly changes the excited 0^+ states.

Here, we compute the $\log ft$ value, which can be directly compared with these models. The available phase space (f factor) for β decay from the ground state of ^{12}B to the Hoyle state was computed using the method in [38], with the excitation energy and half life from [12]. With this input our result is

$$\log ft = 4.50(7). \quad (5)$$

Due to the large change of the measured branching ratio compared to previous results [12], the theoretical prediction of the AMD model, $\log ft = 4.3$ [24], is no longer compatible with the experiment.

Hence, our branching ratio, together with the branching ratio for both ^{12}B and ^{12}N from Hyldegaard *et al.* [21], indicate that the α clustering of the Hoyle state is more pronounced than previously believed.

VI. CONCLUSION AND OUTLOOK

The β -decay branching ratio from ^{12}B to the second-excited state of ^{12}C has been measured using an array of high-purity Compton-suppressed germanium detectors. The branching ratio was determined by counting the Hoyle state γ decay, and normalizing to the decay of the first-excited state. The result is 0.64(11) %, consistent with the value found in [21], but is a factor ~ 2 smaller than the previously-established value from [12]. The updated branching was used to compute $\log ft = 4.50(7)$, which is not consistent with latest results from AMD calculations [24]. Our results indicate that the clustering of the Hoyle state is more pronounced than previously thought.

The angular correlation between the two photons emitted in the decay of the Hoyle state has also been measured. The distribution was consistent with theoretical expectations [34].

The errors on the present measurement are dominated by the uncertainty on the number of coincidence events, which contributes 91 % of the total error, while 6 % and 2 % come from the branching ratio to the first-excited state and the relative γ width of the Hoyle state, respectively.

Therefore, it is possible to make a $\sim 6\%$ measurement of either the γ width or the β -branching ratio by increasing statistics.

During the experiment, the beam current was limited to 2 pA in order to minimize neutron damage to Gamma-sphere. The main source for these neutrons was reactions with titanium since the beam energy is above the Coulomb barrier. Exchanging titanium with hafnium permits running with higher beam currents which, when combined with digital Gamma-sphere [39], would make it possible to accumulate sufficient statistics. Research into production of such a target is ongoing.

VII. ACKNOWLEDGMENTS

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