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The Role of Nucleon Strangeness in Supernova Explosions

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Recent hydrodynamical simulations of core-collapse supernova (CCSN) evolution have highlighted the importance of a thorough control over microscopic physics responsible for such internal processes as neutrino heating. In particular, it has been suggested that modifications to the neutrino-nucleon elastic cross section can potentially play a crucial role in producing successful CCSN explosions. One possible source of such corrections can be found in a nonzero value for the nucleon's strange helicity content Δs . In the present analysis, however, we show that theoretical and experimental progress over the past decade has suggested a comparatively small magnitude for Δs , such that its sole effect is not sufficient to provide the physics leading to CCSN explosions.

Supernova (SN) explosions have long been recognized as the means of populating the galaxies with heavier elements [1]. Despite much progress [2], the physics responsible for these events remains murky.

For core-collapse supernovae (CCSNe) specifically, the eventual explosion depends upon the evolution of the bounce shock produced by the implosion of a massive star's Fe core to proto-neutron star [2, 3]. Problematically, dissipative effects from nuclear dissociation/neutrino emission stall the advancing shock in numerical simulations, raising the question of what mechanism re-energizes the outwardly-moving front. Since the early work of [4], the conventional explanation is that delayed neutrinos reheat the post-shock region of the CCSN, reinvigorating the shock front's advance. This picture has placed a considerable premium upon controlling the various physics effects [5] that go into the neutrino-nucleon interactions inherent to the “delayed-neutrino mechanism.”

This is especially true in the vicinity of the so-called gain radius — the locus where cooling from neutrino emission is in approximate equilibrium with the corresponding heating due to neutrino absorption [3]. As such, even incremental modifications to the neutrino-nucleon cross section such as might follow from corrections to the nucleon's flavor structure could alter the explosive evolution of CCSNe. Effects of this kind might then in turn play a decisive role in numerical simulations that successfully produce CCSN explosions.

The importance of such considerations was brought to the fore by the recent publication of Melson *et al.* [6], which studied the impact of a large magnitude nucleon strange helicity $\Delta s = -0.2$ in a simulated CCSN originating from a $20 M_\odot$ progenitor star. As we shall describe, computing with $\Delta s = -0.2$ induces a $\sim 15\%$ reduction in the strength of the neutrino's axial coupling to the neutron, leading to the diminution of neutron opacities to neutrinos relative to a calculation using $\Delta s = 0$, for which no explosive behavior was obtained. Using $\Delta s = -0.2$ resulted in a calculation of a CCSN explosion ~ 300 ms after the shock bounce in Ref. [6]. In practice, this large amplitude for Δs served as a proxy for $\mathcal{O}(\sim 15\%)$ corrections to neutrino-nucleon opacities

that might arise from various effects; here for definiteness, however, we study the specific plausibility of such large values for Δs , given present knowledge of nucleon structure. The relevant physics is the neutrino-nucleon total elastic cross section, to lowest order in the weak coupling [5]:

$$\sigma_i(\epsilon) = \frac{2G_F^2 \epsilon^2}{3\pi} \left(c_{V_i}^2 + 5c_{A_i}^2 \right). \quad (1)$$

Here, ϵ is the energy of the incident neutrino, and i an isospin label. Electroweak physics specifies the values of the nucleon couplings of the weak vector and axial-vector (or, “axial” below) currents c_{V_i} and c_{A_i} , which for $SU(2)$ are known to be [7]

$$\begin{aligned} c_{V_p} &= \frac{1}{2} - 2\sin^2 \theta_W, & c_{V_n} &= -\frac{1}{2}; \\ c_{A_p} &= +\frac{g_A}{2}, & c_{A_n} &= -\frac{g_A}{2}, \end{aligned} \quad (2)$$

where $g_A \approx 1.26$ and $\sin^2 \theta_W \approx 0.2325$. These effective couplings are the main input to the neutrino-nucleon physics, and they potentially receive corrections from other flavor sectors — especially strange.

Differences of spin-conserving, axial current matrix elements can ultimately be related to the quark helicity content of the nucleon. That is, by considering nucleonic matrix elements of the weak axial current (here, in the light flavor $SU(2)$ sector with isospin label $i = p, n$)

$$\begin{aligned} &\langle P', \lambda'; i | J_A^\mu | P, \lambda; i \rangle \\ &= \bar{u}_i(P', \lambda') \left(\gamma^\mu \gamma_5 \frac{\tau_3}{2} G_A(Q^2) + \dots \right) u_i(P, \lambda), \end{aligned} \quad (3)$$

the axial form factor may be accessed through the appropriate helicity difference of the neutral current “+” components as defined on the light-front (for reviews of the light-front formalism, see [8–10]):

$$\begin{aligned} \pm \frac{1}{2} G_A(Q^2) &= \frac{1}{4P^+} \left(\langle P', \lambda' = \uparrow; i | J_A^+ | P, \lambda = \uparrow; i \rangle \right. \\ &\quad \left. - \langle P', \lambda' = \downarrow; i | J_A^+ | P, \lambda = \downarrow; i \rangle \right), \end{aligned} \quad (4)$$

in which P, P' are the 4-momenta of the initial/final state nucleon, and $\lambda, \lambda' = \uparrow, \downarrow$ their associated helicity designations; the \pm above corresponds to $i = p, n$. As usual, the form factor is scaled in terms of the elastic momentum-transfer $Q^2 = -(P' - P)^2$.

By merit of the parity-oddness of the $\gamma^+\gamma_5$ operator that weights $G_A(Q^2)$ in Eq. (3) for $\mu = +$, the axial form factor at $Q^2 = 0$ is directly related to the quark-level spin content of the nucleon as well as the effective neutrino-nucleon coupling [5] —

$$c_{Ai} = \frac{1}{2} \left(\pm G_A(0) - G_A^{s\bar{s}}(0) \right) = \frac{\pm g_A - \Delta s}{2}, \quad (5)$$

in the absence of electroweak radiative corrections [11]. The fact that it is the negative of Δs which enters Eq. (5) may be interpreted as resulting from the isoscalar nature of the strange axial form factor $G_A^{s\bar{s}}$ relative to the isovector G_A ; the former quantity is the analog of the $SU(2)$ axial form factor defined in Eqs. (3) and (4), but evaluated in a basis that couples the axial current to nucleon strangeness. In Eq. (5) we have also made the explicit identification $G_A^{s\bar{s}}(0) = \Delta s$.

From the result of Eq. (5) it is straightforward to infer the qualitative impact of nonzero strange helicity in the nucleon. Analyses generally prefer $\Delta s \leq 0$, such that

$$\begin{aligned} \left(c_{Ap}^{(\Delta s \leq 0)} \right)^2 &\geq \left(c_{Ap}^{(\Delta s = 0)} \right)^2, \\ \left(c_{An}^{(\Delta s \leq 0)} \right)^2 &\leq \left(c_{An}^{(\Delta s = 0)} \right)^2, \end{aligned} \quad (6)$$

which leads to the respective enhancement and suppression of the $\nu - p$ and $\nu - n$ cross sections given by Eq. (1) for $\Delta s \leq 0$ relative to the zero strange quark helicity scenario.

Notably, the question of the $Q^2 \sim 0$ behavior of $G_A^{s\bar{s}}(Q^2)$ is of central importance to the proton spin problem [12], wherein the total $1/2$ helicity of the proton is decomposed in the usual fashion among quark helicities, orbital angular momenta, and gluon total angular momentum according to

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2} \Delta \Sigma_q + L_q + J_g, \\ \Delta \Sigma_q &= \Delta u + \Delta d + \Delta s. \end{aligned} \quad (7)$$

In the expression above, Δs contains contributions from both s and \bar{s} , and we emphasize that $\Delta q \equiv \langle \Delta q + \Delta \bar{q} \rangle$, such that $\Delta \Sigma_q/2$ yields the quark helicity contribution to the proton spin.

Thus, given its fundamental importance, Δs has been modeled/computed in various ways, and been the focus of multiple experimental efforts, as discussed below. Assuming it to be the sole correction to the $\nu - N$ cross section of Eq. (1) as in Ref. [6], the magnitude of Δs used to successfully produce an exploding CCSN is at odds with the latest hadron structure calculations and measurements. Citing the lower bound of the 2002 analysis by Horowitz [5], which obtained $\Delta s = -0.1 \pm 0.1$, the

hydrodynamical simulations of Ref. [6] were performed at the extreme lower value $\Delta s = -0.2$.

The result $\Delta s = -0.1 \pm 0.1$ found in Ref. [5] was obtained from the 1987 $\nu - p, \bar{\nu} - p$ elastic scattering measurements at BNL [13]. It must be stressed that this original analysis did not endeavor to determine Δs specifically, but rather simply included a correction term η , fitted to the neutrino-proton data, which was intended to parametrize potential modifications to the weak axial current regardless of origin. For this and other reasons pointed out by Kaplan and Manohar [14], the results of the analysis in Ref. [13] should be taken with caution regarding the nucleon's strange helicity.

The value $\Delta s = -0.2$ used in Ref. [6] lies well beyond the range of the analyses/measurements that have emerged in the decade following Ref. [5]. These determinations of Δs come from the complementary directions of theory and phenomenological analyses of relevant data, as we describe below.

Among newer sources of experimental information, global analyses of quark helicity distributions constrained by data (mainly, spin-polarized parity-violating deeply inelastic scattering [DIS] of electrons on nucleons) constrain the magnitude/sign of the total strange helicity, permitting a somewhat larger contribution to the proton spin. For example, the parton distribution function (PDF) analyses summarized in Ref. [15] include those of Glück, Reya, Stratmann, and Vogelsang [16]; Blümlein and Böttcher [17]; Leader, Sidorov, and Stamenov [18]; the Asymmetry Analysis Collaboration [19]; and de Florian, Navarro, and Sassot [20]. These yield moderate values for the total strange helicity, resulting in the average value $\Delta s = -0.120 \pm 0.021$. These analyses proceed by assuming a parametric form for the Bjorken x dependence of the quarks' helicity distributions $\Delta s(x, Q_0^2)$ and $\Delta \bar{s}(x, Q_0^2)$ at the boundary Q_0^2 of a numerical QCD evolution scheme. The total strange helicity asymmetry is then

$$\begin{aligned} \Delta s &= (s^+ - s^-) + (\bar{s}^+ - \bar{s}^-) \\ &= \int_0^1 dx \left[\Delta s(x) + \Delta \bar{s}(x) \right], \end{aligned} \quad (8)$$

and s^\pm represent spin-dependent distributions for strange quarks with helicities parallel/antiparallel to that of their parent nucleon. A persistent limitation of this approach is the lack of experimental constraints on the parametric form at Q_0^2 , as well as at the distribution endpoints, particularly $x \sim 0$ where the necessary extrapolation to evaluate Eq. (8) is especially fraught.

Since the PDF-based analyses described in [15], there have been other direct measurements of Δs , occasionally yielding somewhat smaller results. Of these, we briefly describe a representative sample.

Direct observations of spin asymmetries have enabled more precise extractions of Δs in the past several years. For instance, in 2007 HERMES measured the total strange helicity at intermediate $Q^2 = 10 \text{ GeV}^2$ [21], obtaining $\Delta s = -0.09 \pm 0.02$. Given its form in Eq. (8), the

Q^2 dependence of Δs is governed by flavor-singlet QCD evolution, and the corresponding moment at the perturbative starting scale Q_0^2 may therefore differ slightly. Moreover, these determinations were highly sensitive to the fragmentation function (FF) parametrization, which is inherently nonperturbative.

This source of systematic uncertainty inspired additional dedicated efforts to extract Δs at COMPASS [22, 23]. Of these, Ref. [23] reported several values of Δs at $Q^2 = 3 \text{ GeV}^2$ using different extrapolation schemes. Employing a direct extrapolation to $x = 0,1$ as well as the DSSV parametrization, respectively, COMPASS obtained

$$\begin{aligned} \Delta s &= -0.02 \pm 0.02 & (\text{extrapolation}), \\ &= -0.1 \pm 0.02 & (\text{DSSV}). \end{aligned} \quad (9)$$

Finally, we also mention the result of a recent re-analysis of the spin-dependent strange PDFs $\Delta s(x)$ and $\Delta \bar{s}(x)$ [24] in light of new CLAS data on the proton spin structure function $g_1^p(x, Q^2)$ [25]. Computing the strange portion of the nucleon spin from PDFs constrained by these data, the result $\Delta s = -0.106 \pm 0.023$ follows [26]. Meanwhile, another comprehensive global analysis of these data was reported by Sato *et al.* [27], in this case making use of a novel, Monte Carlo-based fitting scheme. This calculation arrived at a comparable central value and range for the moment of the twist-2 strange spin-PDF, $\Delta s = -0.10 \pm 0.01$. We report the average of these two results ($\Delta s = -0.103 \pm 0.013$) as well as the other experimental information in Fig. 1, noting that the above determinations correspond to somewhat different Q^2 , which introduces some modest uncertainty.

While the precision of experimental data presently allows values of the strange helicity as large as $\Delta s \sim -0.1$, various theoretical calculations are considerably more stringent, several examples of which we highlight below.

In lattice gauge theory, contributions from strange quarks to nucleonic matrix elements inherently arise from disconnected diagrams whose evaluation is vastly more computationally expensive. Technical improvements, however, as well as the exponentiating availability of the necessary computational resources have now rendered such calculations more feasible.

Thus, in recent years Δs has become amenable to calculation using lattice techniques as carried out by, *e.g.*, the QCDSF collaboration [28]. This group obtains $\Delta s = -0.020 \pm 0.010 (\text{stat.}) \pm 0.004 (\text{sys.})$. Meanwhile, a separate lattice collaboration [29] has preliminarily calculated Δs (albeit *without* renormalization effects), also finding a small, negative result: $\Delta s = -0.019 \pm 0.011$. As opposed to these $N_f = 2$ analyses, however, work described in Ref. [30] employed the 2+1-flavor gauge configurations of the MILC Collaboration [35], yielding the renormalized value $\Delta s = -0.031 \pm 0.017$ at the physical pion mass. Ref. [31] found the slightly larger value $\Delta s = -0.0227 \pm 0.0034$ in a setting wherein the axial charges were comprehensively evaluated. Similarly, the analysis of Ref. [32] obtained $\Delta s = -0.018 \pm 0.006$ using

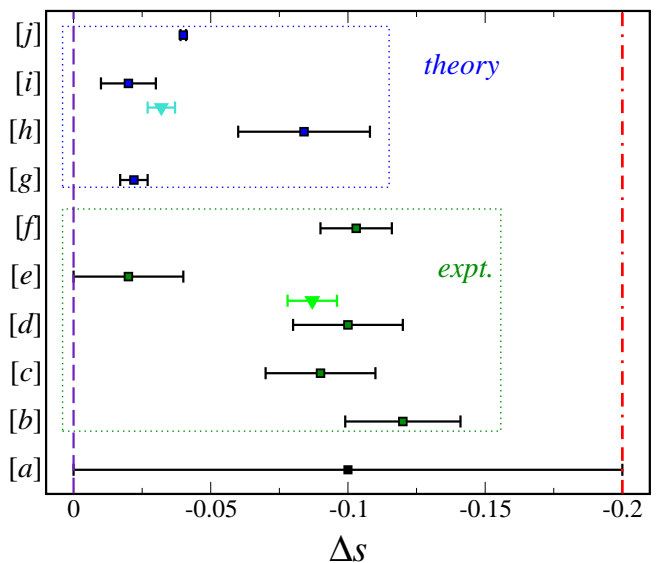


FIG. 1. A summary of Δs determinations in this analysis. Experimental/phenomenological findings subsequent to Ref. [5] (solid, black square, [a]) are contained in the lower rectangle. These include the averaged results in [15] [b], [21] [c], the extrapolation and DSSV-based methods of [23] [d and e, respectively], and the average of [24] and [27] [f]. The lower inverted triangle represents the average of these measurements: $\Delta s = -0.087 \pm 0.009$. Modern theoretical calculations are given in the top rectangle, including the averaged small- Δs lattice calculations [28–32] [g], the AWI-based computation of [26] [h], [33] [i], and [34] [j]. The upper inverted triangle depicts the theory average: $\Delta s = -0.032 \pm 0.005$, while the vertical dot-dashed line represents the value used in Ref. [6].

a numerical scheme motivated by the Feynman-Hellmann theorem adapted to the lattice, although this study and its predecessors [with the exception of [30]] were performed with unphysically large pion masses. These various lattice computations exhibit a general concordance, and for them we find an average of $\Delta s = -0.022 \pm 0.005$.

Unlike these efforts, the recent calculation of the χ QCD Collaboration [26] explicitly enforced the anomalous Ward identity (AWI) through a normalization factor κ_A weighting local axial-vector currents on the finite lattice. Following chiral extrapolation to the physical pion mass, this produced a comparatively large value relative to the other lattice works, $\Delta s = -0.084 \pm 0.024$.

Other model calculation are also possible under the auspices of various theoretical frameworks. For example, the Cloudy Bag Model calculation of Ref. [33], which followed the older calculation presented in [36], incorporated strange quarks into nucleon structure by including kaon-hyperon fluctuations into the extended pion cloud of the proton. Doing so, the flavor- $SU(3)$ axial charges could be evaluated in the context of the MIT Bag Model,

resulting in the values

$$\begin{aligned} g_A^{(0)} &= \Delta u + \Delta d + \Delta s = 0.37 \pm 0.02, \\ g_A^{(8)} &= \Delta u + \Delta d - 2\Delta s = 0.42 \pm 0.02, \end{aligned} \quad (10)$$

from which one may derive the similarly small $\Delta s = (g_A^{(0)} - g_A^{(8)})/3 = -0.02 \pm 0.01$. This method thus yields extremely close agreement with the aforementioned small- Δs lattice calculations, aside from [26].

In yet another model-based analysis, Hobbs *et al.* [34] investigated the strange content of the proton using the framework of light-front wave functions (LFWFs). Truncating the nucleon Fock expansion at a two-body quark/scalar tetraquark state containing s and \bar{s} , Ref. [34] obtained a universal wave function for the strange content of the nucleon with the ability to interpolate between elastic form factor measurements and DIS structure functions in the strange sector:

$$\begin{aligned} |\Psi_P^\lambda(P^+, \mathbf{P}_\perp)\rangle &= \frac{1}{16\pi^3} \sum_{q=s,\bar{s}} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}} \psi_{q\lambda_q}^\lambda(x, \mathbf{k}_\perp) \\ &\times |q; xP^+, x\mathbf{P}_\perp + \mathbf{k}_\perp\rangle, \end{aligned} \quad (11)$$

wherein the internal light-front 4-momentum of the strange quark interacting with the weak axial current is $(k^+, \mathbf{k}_\perp, k^-) = (xP^+, \mathbf{k}_\perp, k^-)$ in a frame with zero transverse momentum for the nucleon, $\mathbf{P}_\perp = \mathbf{0}_\perp$. By then constraining the helicity wave functions $\psi_{q\lambda_q}^\lambda(x, \mathbf{k}_\perp)$ to unpolarized DIS measurements, Ref. [34] found novel bounds on the elastic observables μ_s and ρ_s . In addition, with a phenomenological determination of the strange quark wave function of the nucleon, the strange sector matrix element analogous to Eq. (4) could be computed, which led to the result

$$-0.041 \leq \Delta s \leq -0.039, \quad (12)$$

in line with the small strange helicity magnitudes of the above-mentioned theoretical calculations. We summarize this sampling of theoretical calculations of Δs along with the above-mentioned experimental information in Fig. 1.

In fact, by making use of the same numerical approach employed by [34] to explore the parametric LFWF model space, we find that a strange helicity asymmetry as large as $\Delta s = -0.2$ as in Ref. [6] would require magnitudes for nucleon strangeness in the DIS sector as ruinously large as $xS^+ \sim 0.1$, where we define

$$xS^\pm \equiv \int_0^1 dx x [s(x) \pm \bar{s}(x)]. \quad (13)$$

Such a sizable value is highly excluded by global analyses of the world's high energy data [37, 38]. We obtain this by performing systematic scans of the parameter space of the model in Ref. [34] and admitting for consideration only those combinations consistent within 5% of $\Delta s = -0.2$. The resulting locus of points representing these specific model parameter combinations may then

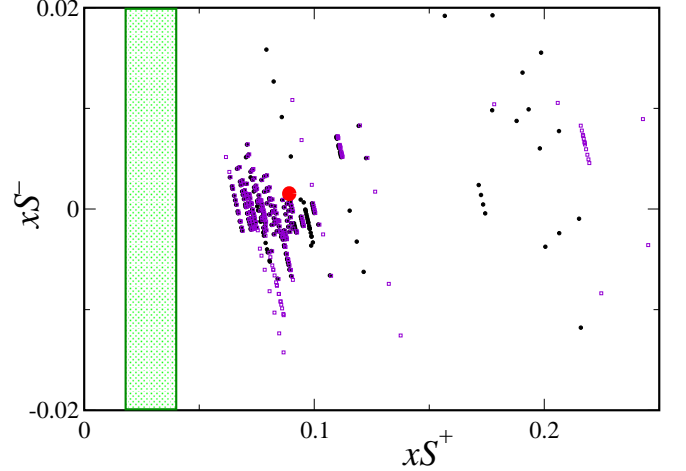


FIG. 2. Parameter scans for the [34] model consistent with $\Delta s = -0.20 \pm 0.01$. Results using wave functions with Gaussian (dots) and dipole-like (squares) momentum dependence are displayed. Parameter combinations satisfying $\Delta s \sim -0.2$ require large wave function normalizations, generating the lower bound $xS^+ \geq 0.062$. This substantially overshoots the range (vertical band) for xS^+ determined by CTEQ [37]. The large disk near $(xS^+, xS^-) = (0.09, 0.002)$ represents the model used for the strange PDFs plotted in Fig. 3.

be plotted in a two-dimensional space spanned by xS^- vs. xS^+ as shown in Fig. 2. We note that while the smallest value of xS^+ tolerated by the model for $\Delta s = -0.2$ occurs for $xS^+ = 0.062$, the mean of the full set corresponds to $xS^+ = 0.1$. In either case this far exceeds the range determined by global analyses of DIS data such as a recent dedicated CTEQ strangeness [37], which found the 90% limit $0.018 \leq xS^+ \leq 0.04$ indicated by the green band in Fig. 2. Similarly, a simple estimate using the fitted distributions of MSTW at the QCD evolution starting scale $Q_0^2 = 1 \text{ GeV}^2$ leads to the comparable 68% range $0.017 \lesssim xS^+ \lesssim 0.033$ [38].

As a further demonstration, we consider a particular model taken from the space plotted in Fig. 2, and use it to compute the unpolarized quark PDF combination $x[s(x) + \bar{s}(x)]$. This has been measured by various experimental collaborations, including HERMES [39], and we compare a model prediction associated with $\Delta s = -0.2$ with these data at $Q^2 = 2.5 \text{ GeV}^2$ in Fig. 3. According to expectation given the large values of xS^+ shown in Fig. 2 for $\Delta s = -0.2$, the evolved model plotted in Fig. 3 as the dot-dashed curve egregiously overhangs the experimental data obtained via kaon production in semi-inclusive DIS.

A general and robustly motivated light-front model is therefore difficult to reconcile with large strange quark helicities without simultaneously predicting implausibly large values for the total strange momentum xS^+ and x -dependent distribution $x[s(x) + \bar{s}(x)]$. This finding is in step with the comparatively small magnitudes of Δs obtained in the most well-supported theoretical calcula-

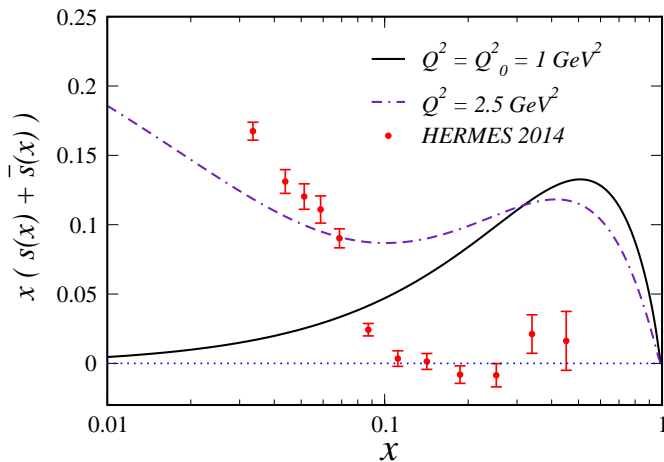


FIG. 3. $x[s(x) + \bar{s}(x)]$ consistent with $\Delta s = -0.2$ according to the model of [34]. The model parameters leading to the above correspond to the large dot at $xS^+ \sim 0.09$ in Fig. 2. The model has been evolved from $Q_0^2 = 1 \text{ GeV}^2$ (solid line) to $Q^2 = 2.5 \text{ GeV}^2$ (dot-dashed line) for comparison with HERMES [39] (red circles). A model constrained to $|\Delta s|$ comparable to that of Ref. [6] thus hugely over-estimates the strange PDF — especially for $x \gtrsim 0.1$.

tions, as well as the minimal strangeness permitted by global analyses of high energy data.

While slight ambiguity remains as to whether $\Delta s \neq 0$, significant improvements in the last decade on the dual fronts of experimental measurement and theory have strongly excluded the large magnitudes $\Delta s \sim -0.2$ barely allowed at the lower reaches of the earliest analyses. As demonstrated above and summarized in Fig. 1, more re-

cent experiments suggest $\Delta s \gtrsim -0.1$, whereas theoretical analyses impose $\Delta s \gtrsim -0.04$. Most lattice and Bag Model-based calculations especially prefer still smaller strange helicity magnitudes, $\Delta s \sim -0.02$ — fully an order-of-magnitude shallower than the critical value assumed in the simulations of Ref. [6].

We therefore conclude that, while the nucleon’s strange spin is an important consideration in the microphysics of CCSN simulations, it cannot contribute at the higher level employed in, *e.g.*, Ref. [6]; by extension, Δs cannot represent the single, decisive effect generative of CCSN explosions. Rather, for the sake of future numerical simulations of CCSNe, we advocate the use of more moderate values of Δs such as would be supported by the most up-to-date calculations and measurements. Giving precedence to experimental limits, we suggest $\Delta s = -0.1$ as a well-motivated figure for which hydrodynamical calculations would be on firm ground. At the same time, improved experiments may eventually track toward the smaller magnitudes of most theoretical computations, and we therefore recommend $\Delta s = -0.04$ as a more conservative, auxiliary value in line with such work.

Ultimately, to complement these reduced magnitudes for Δs , we further urge the exploration of other potential mechanisms, which might enter at the microscopic level of the relevant nuclear physics or in the form of heretofore unexplored dynamical effects in the macroscopic structure of CCSNe themselves.

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