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Theoretical analysis of isospin mixing with the β decay of ^{56}Zn

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We present a shell-model analysis of the β decay of ^{56}Zn. The calculations are performed using isospin-nonconserving Hamiltonians constructed on the basis of the GXPF1A and KB3G interactions. Our theoretical results reproduce the essential features of the decay of ^{56}Zn and explain the surprising competition between β-delayed proton and γ-ray emission from the isobaric analogue state.

Production of proton-rich nuclei in the last decades and advances in the development of experimental techniques made it possible to explore nuclear structure via exotic decay modes, such as direct or β-delayed emission of protons, di-protons or α-particles [1, 2]. In this context, β-delayed proton emission is of particular interest since it involves transitions proceeding via isospin-symmetry breaking and thus provides a perfect testing ground for the determination of isospin mixing in nuclear states and puts constraints on theoretical modeling.

In this article, we propose a theoretical study of the decay of the proton-rich pf shell nucleus ^{56}Zn. First experimental spectroscopic data were provided by Dossat *et al.* [3] and a comprehensive study of β-delayed proton and γ decay of ^{56}Zn has been reported recently in Ref. [4]. A striking feature of this decay is that proton and γ branches of almost equivalent intensity have been observed, in spite of a large amount of isospin mixing in the isobaric analogue state (IAS). Our work proposes a detailed theoretical analysis of this phenomenon.

A partial scheme of the β-decay of the 0^+, T=2 ground state of ^{56}Zn is shown in Fig. 1. The most intense branch populates the isobaric analogue 0^+ state at around 3.5 MeV excitation energy in the daughter nucleus, ^{56}Cu. Although this state is situated well above the proton separation threshold, its decay by proton emission to low-lying $T = 1/2$ states of ^{55}Ni is forbidden by isospin symmetry. Thus, if the isospin-symmetry were exact, the IAS would decay via γ emission only. The observation of proton emission from the IAS is possible solely if isospin symmetry is broken either in the proton emitting state or the final state. The IAS in the odd-odd daughter nucleus lying in a region of relatively high level density is expected to have much more isospin impurity than the ground state of the final nucleus.

Let us suppose that isospin mixing of the IAS can be modeled as due to the admixture of a single close-lying 0^+, T=1 state. Then we can express the IAS as $|IAS\rangle = \sqrt{1-\alpha^2}|T=2\rangle + \alpha|T=1\rangle$. Thus, the spectroscopic factor for proton emission from the IAS can be obtained as

$$S^{IAS} = \alpha^2 \, S^{T=1} \, , \qquad (1)$$

where $S^{T=1}$ is the allowed spectroscopic factor of the admixed state.

Indeed, it has been found experimentally [4] that the IAS of the ^{56}Zn ground state in ^{56}Cu is strongly mixed with another 0^+ (mainly T=1) state lying about 85 keV below. This conclusion was drawn on the basis of the Fermi strength $B(F)$ from the ^{56}Zn ground state, which turned out to be split between the two 0^+ states, resulting in α^2=33(10)% of isospin mixing in the IAS [4]. A puzzling feature is that in spite of this high isospin impurity of the IAS and a large probability for proton emission with an energy of E_p = 2948(10) keV, its proton decay does not represent the dominant decay, but is observed in competition with γ-ray emission with similar intensities, I_p=18.8(10)% and I_γ=19.2(50)%.

Within a two-level mixing approach and using experimental values [4] for the square root of the ra-

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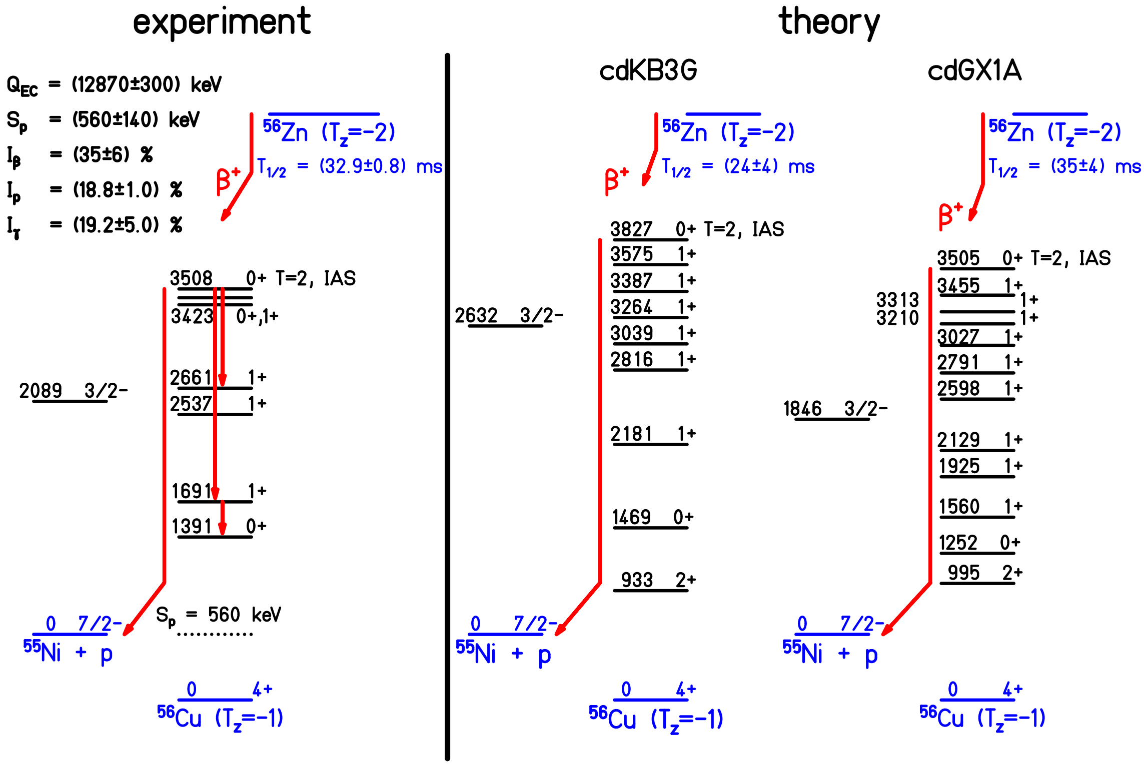

FIG. 1: (Color online) Experimental and theoretical partial decay schemes of ^{56}Zn. The experimental Q_{EC} value and the proton separation energy S_p are from AME2003 [12]. I_β is the theoretical β feeding of the IAS and I_p and I_γ are the experimental proton- and γ-emission branching ratios from the IAS. The theoretical calculations have been performed with two different effective interactions, cdKB3G and cdGX1A.

tio of the $B(F)$ values to the two 0^+ states in ^{56}Cu at 3432(150) keV and 3508(150) keV, R = $B(F)_{3432}/B(F)_{3508}$=0.69(20), and for the observed energy splitting of ΔE=85(10) keV, we can deduce the isospin-mixing interaction matrix element V as

$$V = \frac{\Delta E \times R}{1+R^2} = 40^{+22}_{-\ 9} \ \text{keV}. \qquad (2)$$

To understand the decay features and cross check theoretical description, we have performed large-scale shell-model calculations in the full pf shell using the NuShellX@MSU [6] and Antoine [5] shell-model codes with two different charge-dependent Hamiltonians. The first one, cdGX1A, is based on the GXPF1A [7] interaction with the addition of Coulomb, strong charge-asymmetry and charge-independence-breaking interactions from Ref. [8] and updated isovector single-particle energies from Ref. [9]. The other Hamiltonian, cdKB3G, was constructed on the basis of the KB3G interaction [10], with the addition of the Coulomb interaction and isovector single-particle energies scaled as $\sqrt{\hbar\omega(A)}$.

First, we have calculated theoretical spectra at low energies of the nuclei of interest, ^{56}Cu and ^{55}Ni, as well as the ground state properties of ^{56}Zn to check the validity of the predictions. Table I summarizes the theoretical β-decay half-life of ^{56}Zn, the excitation energy of the 0^+ IAS and of the admixed 0^+ state in ^{56}Cu, as well as their electromagnetic and proton decay characteristics. The γ-decay widths have been calculated using standard effective charges $e_p = 1.5e$, $e_n = 0.5e$ and optimized empirical g-factors as given in Ref. [7] for the cdGX1A interaction and in Ref. [10] for the cdKB3G interaction. The experimental energy E_γ=1835 keV of the most intense γ transition from the IAS to the 1^+_1 state at 1691 keV has been used and, therefore, we suppose E_γ=1750 keV for a possible decay from the 0^+ state which is 85 keV below the IAS. A quenching factor q_F=0.74 was applied to the Gamow-Teller (GT) operator to calculate the β-decay strength distribution. The theoretical half-lives are evaluated on the basis of the twenty lowest 1^+ states due to a very large model space needed for the calculations. We note a good agreement between both sets of calculations and existing experimental data (see Fig. 1).

The experimentally known levels of ^{56}Cu are well reproduced by the charge-dependent versions of the GXPF1A and KB3G interactions. The IAS of the ^{56}Zn ground state is found to be the 4th (cdGX1A) and the 3rd 0^+ state (cdKB3G) in ^{56}Cu. Both interactions confirm that this state is mixed with a lower-lying 0^+, T=1 state by a strong isospin-nonconserving (INC) matrix el-

TABLE I: Comparison of experimental and theoretical quantities of ^{56}Zn. The β-decay half-life of ^{56}Zn, the excitation energy of the 0^+ IAS and of the admixed 0^+ state in ^{56}Cu are shown together with their electromagnetic and proton decay characteristics.

	Exp	cdGX1A	cdKB3G
^{56}Zn			
$T_{1/2}$ [ms]	32.9(8)	35(4)	24(4)
^{56}Cu, 0^+,IAS			
E^{IAS} [MeV]	3.508(140)	3.505	3.827
S^{IAS}	$0.12(4) \times 10^{-3}$	1.5×10^{-3}	3.1×10^{-3}
Γ_p^{IAS} [eV]	0.13(4)	1.6(1)	3.2(2)
Γ_γ^{IAS} [eV]		0.16	0.11
^{56}Cu, 0^+, T=1			
$E^{T=1}$ [MeV]	3.423(140)	2.910	3.456
$S^{T=1}$	$0.4(1) \times 10^{-3}$	4.4×10^{-3}	9.4×10^{-3}
$\Gamma_p^{T=1}$ [eV]	0.32(8)	3.6(2)	7.7(3)
$\Gamma_\gamma^{T=1}$ [eV]		0.04	0.02
α^2 (%)	33(10)	11	34

ement of V_{INC}=20 keV (cdGX1A) and V_{INC}=48 keV (cdKB3G). In first order perturbation theory, the magnitude of mixing is proportional to $(V/\Delta E)^2$, where V is the mixing matrix element and ΔE is the energy difference between the admixed states. The latter value calculated in an odd-odd nucleus may carry a significant uncertainty, which influences the amount of the isospin mixing [13]. While the mixing matrix element is well reproduced, the shell-model energy spacings between the IAS and its lower 0^+, T=1 neighbor, 595 keV (cdGX1A) and 372 keV (cdKB3G), are much larger than the experimental energy difference, resulting in a very small amount of mixing, 0.23% and 1.8%, respectively. Performing a scaling of these values as $\Delta E_{th}^2/\Delta E_{exp}^2$, we get α^2=11% (cdGX1A) and α^2=34% (cdKB3G). These numbers point to an enhanced amount of the isospin mixing, in good agreement with experiment.

Table II summarizes excitation energies of the five lowest 0^+ states in ^{56}Cu, obtained from the two interactions. For each state, different from the IAS, within a two-level mixing approximation we have also estimated an INC matrix element (V_{INC}) between a given state and the IAS. This can be done using Eq. (2) and a theoretical distribution of the Fermi strength among various 0^+ states due to the isospin mixing. If a state is fed only by a small fraction of the Fermi strength, the corresponding R-value is small and therefore $V_{INC} \approx \Delta E \times R$.

In the calculations with cdGX1A the next 0_5^+ state comes out to be almost degenerate with the IAS, leading to a strong mixing between the two states by, however, a very small mixing matrix element of V_{INC}=3 keV with respect to the experiment value. Hence, to estimate the interaction matrix elements V_{INC} between the first three 0^+ states and the IAS, we used an unperturbed T=2 state.

In the same table, we also present the spectroscopic factors for an $f_{7/2}$ proton emission from the five lowest 0^+ states in ^{56}Cu to the $7/2^-$ ground state of ^{55}Ni obtained from a large-scale shell-model diagonalization. It is seen that, except for the first 0^+ state, all other states have rather small spectroscopic factors. In particular, the allowed spectroscopic factors of the T=1 states admixed to the IAS by a strong mixing matrix element are small, $S^{T=1}$= 4.4×10^{-3} and $S^{T=1}$= 9.4×10^{-3} for cdGX1A and cdKB3G, respectively.

Spectroscopic factors of the IAS that come out of the shell-model diagonalization are S_{diag}^{IAS}=2.1×10^{-3} (cdGX1A) and S_{diag}^{IAS}=0.2×10^{-3} (cdKB3G). Let us remark that these values arise from quantities of mixing different from the experimental one. A relatively large spectroscopic factor from cdGX1A is mainly due to the mixing with the closely lying 0_5^+, as we mentioned above. A small value of the cdKB3G spectroscopic factor comes from a small amount of isospin mixing as obtained in diagonalization. Taking into account the uncertainty of the positions of the unknown T=1 states we cannot use these results directly.

Therefore, we propose to estimate the spectroscopic factors for the isospin-forbidden proton emission from the IAS as about 1/3 of the $S^{T=1}$ values, based on the experimentally determined amount of mixing. This gives S^{IAS}=1.5×10^{-3} (cdGX1A) and S^{IAS}=3.1×10^{-3} (cdKB3G), shown in Table I.

TABLE II: Excitation energies, interaction mixing matrix element and spectroscopic factors of the lowest 0^+ states in ^{56}Cu with respect to proton emission to the $7/2^-$ ground state of ^{55}Ni. The values corresponding to the IAS are shown in bold.

State	cdGX1A			cdKB3G		
	E^* [MeV]	$V_{\rm INC}$ [keV]	S_p	E^* [MeV]	V_{INC} [keV]	S_p
0_1^+	1.253	25	0.0590	1.469	16	0.0336
0_2^+	2.675	20	0.0083	3.456	48	0.0094
0_3^+	2.910	20	0.0044	**3.827**	-	**0.0002**
0_4^+	**3.505**	-	**0.0021**	4.007	16	0.0076
0_5^+	3.511	3	0.0035	4.611	1	0.0044

Interestingly, these numbers can be compared with experimentally deduced values. To show this let us note that branching ratios I_x^{IAS} of the IAS decay are related to the ratio of its partial decay widths Γ_x^{IAS} as follows:

$$\frac{I_p^{IAS}}{I_\gamma^{IAS}} = \frac{\Gamma_p^{IAS}}{\Gamma_\gamma^{IAS}}\,. \tag{3}$$

The total proton width can be expressed as a single-particle proton width multiplied by the corresponding spectroscopic factor [11], i.e. $\Gamma_p = \Gamma_{sp}\,S$. The single-particle proton width has been calculated from the proton scattering cross-section in a Woods-Saxon potential with the potential depth adjusted to reproduce known proton energies. Thus, if we provide the electromagnetic width of a decaying state from the shell model, this relation can be used to extract from experimental data a spectroscopic factor for an isospin-forbidden proton emission from the IAS [14]:

$$S^{IAS}_{exp} = \frac{\Gamma^{IAS}_{\gamma}}{\Gamma^{IAS}_{sp}} \frac{I^{IAS}_{p}}{I^{IAS}_{\gamma}} \,. \qquad (4)$$

The calculated γ decay width of the IAS is Γ^{IAS}_{γ}=0.135(30) eV (an average from the two interactions). The γ width of the lower T=1 state (accounting for a phase space factor when it is shifted closer to the T=2 state) is about 0.03(1) eV (an average from the two interactions again). The calculated single-particle proton decay width is Γ_{sp}=1040(50) eV. Assuming a two-level mixing and that the γ width of the T=1 state is small compared to that of the T=2 state, we can extract a proton decay width of the IAS, Γ_p=0.13(5) eV, and a spectroscopic factor, $S^{IAS}_{\rm exp}$=0.13(5)$\times 10^{-3}$. From the experimentally determined amount of mixing, 34(10)%, the allowed spectroscopic factor for the T=1 state can be estimated as $S^{T=1}_{\rm exp} = 0.4(1) \times 10^{-3}$. This value is a factor of 10 smaller than the theoretical estimates which can be partially explained by the fact that a two-level mixing model might not be fully justified and that the errors on these small spectroscopic factors of T=1 states might be large. However, the main conclusion of the analysis is clear: the hindrance of the proton decay from the IAS is thus due to a very small overlap between the admixed 0^+, T=1 state of ^{56}Cu and the ground state of ^{55}Ni plus an $f_{7/2}$ proton. Proton emission from the admixed 0^+, T=1 state is allowed by the isospin quantum number selection rule, however, it is hindered by nuclear structure effects. So, a large admixture of that state to the IAS makes the proton width of the IAS comparable to but not much larger than its γ decay width.

The properties of the 0^+, T=1 state which is admixed to the IAS are also well understood. From the experimental proton decay energy E_p=2683(10) keV, we get the proton single-particle width of this state of Γ_{sp}=819(30) eV. Thus, using the experimentally deduced spectroscopic factor, we get the total proton width of Γ_p=0.32(8) eV (the factor of 10 difference from the theoretical width is due to the difference in spectroscopic factors, as pointed out above). The shell-model electromagnetic width is much smaller, about 0.02-0.04 eV. This result is in agreement with the fact that the electromagnetic decay of the state at 3423 keV has not been observed in the past experiments.

Finally, we have examined the GT strength distribution from ^{56}Zn beta decay. The GT feeding of the 1^+_1 state is found to be around 23% (cdGX1A) and 25% (cdKB3G), perfectly matching the measured proton branching ratio of 23.8(11)%. The shell model predicts a few other excited 1^+ states below the IAS, while only two additional states have been seen experimentally [4]. The summed GT strength to those theoretical 1^+ states is about 17% (cdGX1A) and 24% (cdKB3G), which can be compared to the experimental proton branching ratio for the $1^+_{2,3}$ states of 21.7(12)%.

In conclusion, we have studied β-delayed proton and γ emission from the 0^+, T_z=−2 pf-shell precursor ^{56}Zn. This theoretical analysis allows us to understand the highly retarded proton decay of the IAS in ^{56}Cu, largely mixed with a close-lying 0^+ T=1 state. The observed hindrance is due to a very small spectroscopic factor of the admixed state indicating a small overlap between this excited 0^+ state of ^{56}Cu and the ground state of ^{55}Ni plus an $f_{7/2}$ proton. We demonstrated that the spectroscopic factor for an isospin-forbidden proton emission can be deduced with good accuracy, if the experimental data is supplemented by a simple shell-model input.

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