Comment on “Nonidentical protons”

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Ref. [1] examines the question of whether the proton radius puzzle can be explained if protons have a distribution of radii rather than a fixed size. The authors claim that averaging over an ensemble of protons with different sizes yields a significantly smaller radius when examining electron scattering data. They conclude that a $\sim$20% variation in proton radii can resolve the discrepancy between electron scattering extractions of the proton charge radius [2–5] and the muonic hydrogen result [6, 7].

Questions related to the fitting approach and uncertainties were raised in Ref. [8], and we do not address these further. This comment focuses two issues, both of which undermine the conclusion that averaging over a range of proton sizes brings the electron scattering radius into agreement with the muonic hydrogen measurements:

1. The radius obtained before averaging over a range of proton sizes is already consistent with the muonic hydrogen. Thus, their resolution of the discrepancy is related to the assumptions that go into their fit rather than the effect of ensemble averaging.

2. The authors incorrectly evaluate the RMS radius of their ensemble averaged fit and, in fact, the RMS radius from this analysis is larger than the simple dipole fit for a fixed proton radius.

The first issue can be seen clearly, as a charge radius of 0.84 fm is obtained for the fit with a fixed dipole mass parameter Λ (Fig. 3 of Ref. [1]), so the 0.04 fm reduction in the radius compared to the Mainz extraction is not related to a variation of proton radii. The fact that the radius from the simple dipole fit agrees with the muonic hydrogen radius is not by itself particularly meaningful as the fit neglects the normalization uncertainties which have been shown to have a significant impact on the radius [4, 9]. In addition, the simple dipole parameterization is insufficient to give a good fit to the data up to 0.25 GeV^2, as seen in Fig. 4 of Ref. [1] which shows $\chi^2/N \approx 4$ even for their more flexible two-parameter fit.

The second issue arises in the analysis that includes a range of proton radii. Instead of fitting with a fixed dipole mass parameter, they use an ensemble of radii with Λ varied uniformly over the range $\Lambda_1 \pm \Delta\Lambda$ and the parameters Λ_1 and $\Delta\Lambda$ are determined from a fit to the data. Allowing non-zero values of $\Delta\Lambda$ modifies the fit function from the simple dipole form, allowing a better fit to the data. However, the authors' claim of a reduction in radius for this fit is incorrect. While the radius corresponding to the best-fit value of Λ_1 is reduced when allowing a range of Λ values, the RMS radius of such an ensemble does not correspond to the central Λ value. Increasing $\Delta\Lambda$ while keeping Λ_1 fixed increases the radius, as can be seen in Figs. 1 and 2 of Ref. [1] where the slope of the form factor increases as $\Delta\Lambda$ is increased. It can also be seen in a simple example: an ensemble of protons with dipole mass parameters $\Lambda = 0.6$, 0.8, and 1.0 GeV, corresponding to $\langle r^2 \rangle$ values of 1.298, 0.730, and 0.467 fm^2. Averaging these values gives $\langle r^2 \rangle = 0.832$ fm^2 for the ensemble, yielding an RMS radius of 0.912 fm which is significantly larger than the RMS radius of 0.854 fm corresponding to the central Λ value.

The final RMS charge radii quoted in Ref. [1] are 0.840 fm for $\Delta\Lambda = 0$, and 0.833 fm for their best fit value of $\Lambda_1 = 0.8203$ GeV ($\Delta\Lambda/\Lambda_1 = 21.5\%$). However, the RMS radius for this ensemble is actually 0.853 fm, as determined from either the slope of the ensemble-averaged form factor or by averaging the mean-square radii of the ensemble as in the example above. It is thus the slight deviation from the simple dipole Q^2 dependence, rather than a decrease in the radius, that allows for a better fit with a non-zero value for $\Delta\Lambda$.

In conclusion, the analysis of Ref. [1] yields a radius consistent with muonic hydrogen only because they perform an incomplete analysis of the data. The observed reduction does not depend on averaging over a range of proton radii and, in fact, the true RMS charge radius associated with their ensemble of proton radii yields a small increase in the proton radius compared to their simple dipole fit, rather than the claimed decrease.

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