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Effects of kinematic cuts on net-electric charge fluctuations

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The effects of kinematic cuts on electric charge fluctuations in a gas of charged particles are discussed. We consider a very transparent example of an ideal pion gas with quantum statistics, which can be viewed as a multi-component gas of Boltzmann particles with different charges, masses, and degeneracies. Cumulants of net-electric charge fluctuations χ_n^Q are calculated in a static and expanding medium with flow parameters adjusted to the experimental data. We show that the transverse momentum cut, $p_{t_{\min}} \leq p_t \leq p_{t_{\max}}$, weakens the effects of Bose statistics, i.e. contributions of effectively multi-charged states to higher order moments. Consequently, cuts in p_t modify the experimentally measured cumulants and their ratios. We discuss the influence of kinematic cuts on the ratio of mean and variance of electric charge fluctuations in a hadron resonance gas, in the light of recent data of the STAR and PHENIX Collaborations. We find that the different momentum cuts of $p_{t_{\min}} = 0.2$ GeV (STAR) and $p_{t_{\min}} = 0.3$ GeV (PHENIX) are responsible for more than 30% of the difference between these two data sets. We argue that the p_t cuts imposed on charged particles will influence the normalized kurtosis $\kappa_Q\sigma_Q^2 = \chi_4^Q/\chi_2^Q$ of the electric charge fluctuations. In particular, the reduction of $\kappa_Q\sigma_Q^2$ with increasing $p_{t_{\min}}$ will lead to differences between PHENIX and STAR data of $\mathcal{O}(6\%)$ which currently are buried under large statistical and systematic errors. We furthermore introduce the relation between momentum cut-off and finite volume effects, which is of relevance for the comparison between experimental data and lattice QCD calculations.

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I. INTRODUCTION

Fluctuations of conserved charges provide a unique opportunity to describe the state of matter created in heavy ion collisions [1–6]. In particular, higher order cumulants of the event-by-event charge multiplicity distribution are interesting quantities as they become increasingly sensitive to critical behavior near the phase boundary of QCD [4, 7–14]. In the Beam Energy Scan (BES) program at the Relativistic Heavy Ion Collider (RHIC) up to fourth order cumulants of net-proton fluctuations [15], as a proxy for net-baryon number fluctuations, and net-electric charge fluctuations have been measured [16, 17].

While cumulants of conserved charges exhibit singular behavior at the critical point [5, 10], they are also sensitive probes for pseudo-critical behavior in the vicinity of the chiral crossover transition [4, 9, 11, 14, 18]. Furthermore, fluctuations of conserved charges carry information on freeze-out conditions in heavy ion collisions [11, 19–24] and they can be influenced by different final state effects [25, 26].

The proximity of the chemical freeze-out, determined by a fit of statistical hadronization models to data for particle yields in heavy ion collisions [27], and the crossover transition determined in lattice QCD, suggests that one may gain access to critical phenomena at the QCD phase boundary by measuring fluctuations of conserved charges at the LHC and in the BES at RHIC [11, 20].

In order to identify critical properties of different fluctuations, one needs to understand a non-critical baseline for their probability distribution and resulting cumulants. Note that the often used “Poisson baseline”, i.e. the Skellam distribution, is not an adequate reference for the analysis of net-electric charge and net-strangeness fluctuations, due to contributions from multi-charged states and quantum statistics effects. This is not the case for net-baryon number fluctuations where due to the absence of multi-charged baryons and the large nucleon mass a Skellam distribution is indeed an appropriate reference [28].

Net-electric charge fluctuations are dominated[1] by pions. They are therefore strongly influenced by quantum statistics effects. Furthermore, experimental acceptance cuts can severely influence electric charge fluctua-

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[1] This is strictly speaking correct for quadratic and quartic charge fluctuations. We will argue later that baryons and kaons also give significant contributions to odd order cumulants, e.g. mean and skewness.

tions. Since the Skellam distribution describes the distribution for the difference of two independent quantities obeying Poisson statistics, limiting the momentum acceptance does not change the nature of the distribution. Thus, fluctuations of net baryon number should be rather weakly influenced by kinematic cuts [11]. However, due to contributions from multi-charged states and quantum statistic effects, this is not the case for electric charge fluctuations.

In this study, we discuss the effects of different kinematic cuts on the net-electric charge fluctuations in a static and expanding medium. We consider the effect of a transverse momentum cut-off and of a limited pseudo-rapidity acceptance on different ratios of the $n-$th order cummulants χ_n^Q of net-electric charge fluctuations. Similar studies, have been carried out in Ref. [29] within a hadron resonance gas model.

In the following, we will focus on the theoretical interpretation of these results, based on a very transparent example of an ideal gas of charged pions with quantum statistics, which can be viewed as a multi-component gas of particles with different charges, masses and degeneracies. Such a system contains all relevant properties of charge fluctuations that allow for a transparent implementation and interpretation of different kinematic cuts and their consequences.

In particular, we show that the substantial modification of the higher order cumulant ratios with momentum cuts , in the acceptance windows of STAR Collaboration, that have been discussed in Ref. [29] may be viewed as arising from the suppression of multi-charged states, reflecting the Bose momentum distribution in a pion gas. We will extend this discussion to different low and high transverse momentum cuts, as well as pseudo-rapidity windows. We will also focus on properties of the χ_1^Q/χ_2^Q ratio which is an important baseline to identify cut-off effects on observable which is non-critical with respect to the $O(4)$ universality class.

We perform a quantitative comparison of our analysis with data obtained by the PHENIX and STAR Collaborations. However, in this case we also include the contribution of other hadronic states using a hadron resonance gas (HRG) model. We will show, that the increase of the lower momentum cut from $p_{t_{\rm min}} = 0.2$ GeV (STAR) to $p_{t_{\rm min}} = 0.3$ GeV (PHENIX) does account for a large fraction of the differences present in the data for mean over variance, M_Q/σ_Q^2. Similar differences are expected to show up in data for the normalized kurtosis, $\kappa_Q\sigma_Q^2$.

Finally we will discuss the relation between non-zero momentum cut-off and finite volume effects in a non-interacting pion gas. This allows to argue that lattice QCD calculations of electric charge fluctuations performed in a finite volume are actually adequate to describe charge fluctuations determined experimentally and infinite volume extrapolation may even not be needed due to the similarity between finite volume and non-zero transverse momentum cut-off effects.

The paper is organized as follows: In the next section we discuss the momentum cut-off dependence of electric charge fluctuations in a pion gas. In Section III we introduce STAR and PHENIX data on cumulants of charge fluctuations χ_n^Q and their interpretation. In Section IV the effects of an expanding medium on fluctuations will be discussed. In Section V we compare the effects of finite size and momentum cuts on χ_n^Q. Our conclusions are presented in Section VI.

II. CHARGE FLUCTUATIONS IN A PION GAS

The thermodynamic pressure for a non-interacting pion gas at temperature T and electric charge chemical potential μ_Q is given by

$$p(T,\mu_Q) = -T\sum_{i=1}^{3}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\ln\left(1-e^{-(E_p-\mu_Q Q_i)/T}\right) \quad (1)$$

with

$$\int d^3\boldsymbol{p} = 2\pi \int_{\eta_{\rm min}}^{\eta_{\rm max}} d\eta \int_{p_{t_{\rm min}}}^{p_{t_{\rm max}}} dp_t p_t |\boldsymbol{p}| \quad (2)$$

where $E_p = \sqrt{|\boldsymbol{p}|^2+m^2}$ is the pion energy and $Q_i = 0,\pm 1$ denotes their electric charge.

The momentum integration in Eq. (2), is expressed in terms of the transverse momentum p_t and pseudo-rapidity $\eta = [\ln(|\boldsymbol{p}|+p_z)/(|\boldsymbol{p}|-p_z)]/2$. We consider the experimental acceptance window, 0.2 GeV $\le p_t \le$ 2 GeV and $|\eta| < 0.5$ for STAR [16], and 0.3 GeV $\le p_t \le$ 2 GeV and $|\eta| < 0.35$ for PHENIX measurements [17].

The cumulants of electric charge fluctuations are obtained from

$$\chi_n^Q \equiv \frac{\partial^n}{\partial(\mu_Q/T)^n}\frac{p(T,\mu_Q)}{T^4}\ . \quad (3)$$

To characterize the properties of the system, it is convenient to introduce different ratios of χ_n^Q/χ_m^Q. Hereafter we focus on two ratios, the mean over variance, $M_Q/\sigma_Q^2 = \chi_1^Q/\chi_2^Q$ and the normalized kurtosis $\kappa_Q\sigma_Q^2 = \chi_4^Q/\chi_2^Q$.

In the presence of a non-zero momentum cut-off the cumulants may be represented by the standard cluster expansion

$$\chi_n^Q = \frac{m^2}{\pi^2 T^2}\sum_{k=1}^{\infty} k^{n-2}\hat{K}_2(km/T)\cosh(k\mu_Q/T),\ \ n = \text{even}$$

$$\chi_n^Q = \frac{m^2}{\pi^2 T^2}\sum_{k=1}^{\infty} k^{n-2}\hat{K}_2(km/T)\sinh(k\mu_Q/T),\ \ n = \text{odd} \quad (4)$$

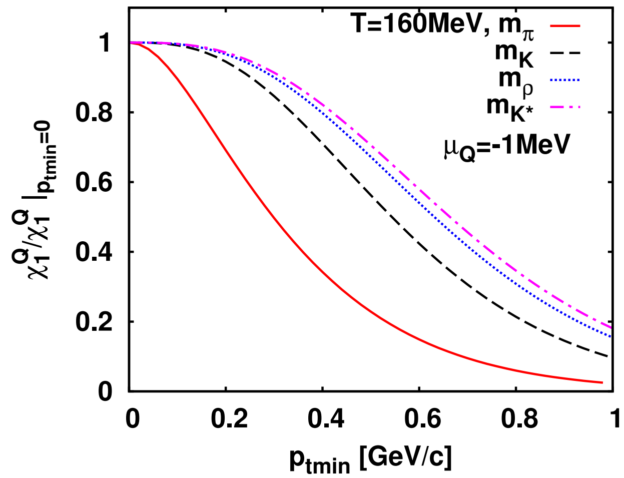

FIG. 1. The lower transverse momentum cut-off dependence of the first order cumulant χ_1^Q of the electric charge fluctuations in a Bose gas at different values of the particle mass, $m = m_\pi, m_K, m_\rho$ and m_{K^*} at $T = 160$ MeV and $\mu_Q = -1$ MeV. The results are normalized to the corresponding values at $p_{t_{\min}} = 0$.

where the function $\hat{K}_2(x)$ is defined as,

$$\hat{K}_2(km/T) = \frac{k}{2m^2T} \int_{\eta_{\min}}^{\eta_{\max}} d\eta \int_{p_{t_{\min}}}^{p_{t_{\max}}} dp_t p_t |\boldsymbol{p}| e^{-k\sqrt{|\boldsymbol{p}|^2+m^2}/T}. \tag{5}$$

It reduces to the modified Bessel function $K_2(km/T)$ after integration over the entire momentum space, $-\infty < \eta < \infty$ and $0 < p_t < \infty$. This makes the influence of the Bose statistics very transparent and highlights the similarity between cut-off effects on Bose particles and multi-charged hadrons.

From Eq. (4) it is clear that in a Bose gas χ_n^Q can be regarded as a sum over contributions from multi-charged particles with mass km, charge k and degeneracy k^{n-4}, where $k = 1, 2, \cdots, \infty$. The $k = 1$ term corresponds to the Boltzmann approximation for which cumulant ratios χ_{n+2}^Q/χ_n^Q=1, i.e., the normalized kurtosis $\kappa_Q\sigma_Q^2 = 1$. For $k = 1$ the corresponding probability distribution $P(N)$ for net charge N is the Skellam function [30, 31]. Contributions of multi-charged Boltzmann particles with $k > 1$, result in a wider $P(N)$, which has a broader tail than the Skellam function [28]. This leads to larger quartic (χ_4^Q) than quadratic (χ_2^Q) charge fluctuations. More generally, the higher order terms in k, which become increasingly important for large n owing to the degeneracy factor k^{n-4}, imply that $\chi_{n+2}^Q > \chi_n^Q$. Consequently, in the presence of multi-charged particles, as well as particles with Bose statistics, the normalized kurtosis $\kappa_Q\sigma_Q^2$ is always larger than unity in a gas of non-interacting hadrons. In Eq. (4) the momentum cut-off dependence

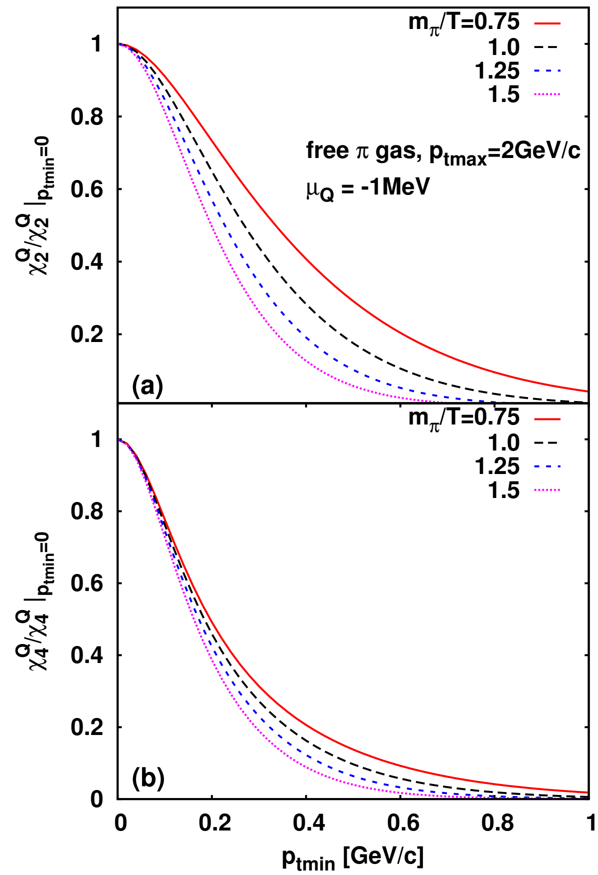

FIG. 2. The lower transverse momentum cut-off dependence of the quadratic [χ_2^Q, (a)] and quartic [χ_4^Q, (b)] cumulants of the electric charge fluctuations in a pion gas at different values of the temperature, 100 MeV $\lesssim T \lesssim$ 200 MeV and for $\mu_Q = -1$ MeV. The results are normalized to the corresponding values obtained at $p_{t_{\min}} = 0$.

enters through the function $\hat{K}_2(km/T)$ where the effect of $p_{t_{\min}} \neq 0$ quantitatively depends on m, T and k separately. A non-zero $p_{t_{\min}}$ thus modifies the relative contribution of different k-terms to χ_n^Q.

Fig. 1 displays the $p_{t_{\min}}$ dependence of the first cumulants at $T = 160$ MeV and $\mu_Q = -1$ MeV for various particle masses, $m = m_\pi, m_K, m_\rho$ and m_{K^*}, normalized to their values calculated with $p_{t_{\min}} = 0$.

The charge chemical potential is assumed to be negative and small, as expected in heavy ion collisions at high energies. For $m = m_{K^*}$, the mass is already so large that the quantum statistics effect is almost negligible, thus the result can be regarded as that of the Boltzmann approximation where the higher k-terms in Eq. (4) are neglected.

By decreasing the particle mass, one sees a stronger dependence of χ_1^Q on $p_{t_{\min}}$, reflecting the contribution from $k > 1$ terms which are also understood as the enhancement of the particle number at low p_t in the Bose distribution.

In Fig. 2 we show the cut-off dependence of the two even order cumulants χ_2^Q and χ_4^Q for a pion gas at $\mu_Q = -1$ MeV and at different temperatures. It is clear that cut-off effects are more severe for higher order moments and they are more pronounced at lower temperature. This temperature dependence can be deduced from the $\hat{K}_2(km/T)$ function by changing the integration variable from p_t to $x = kp_t/T$, which shifts the lower limit of the integration range to $x_{\rm min} = kp_{t_{\rm min}}/T$. An increase of the temperature, therefore effectively lowers the p_t cut. Consequently, the cumulants at higher temperature take a value closer to those without p_t cut.

The difference in the cut-off dependence of χ_2^Q and χ_4^Q will also influence their ratio, $\chi_4^Q/\chi_2^Q = \kappa_Q\sigma_Q^2$. However, the effect is weaker in $\kappa_Q\sigma_Q^2$, due to a partial cancelation of a similar dependence of quartic and quadratic fluctuations on p_t momentum cut-off. This is evident in Fig. 3, where we show the momentum cut-off dependence of χ_4^Q/χ_2^Q. The temperature dependence of $\kappa_Q\sigma_Q^2$ indicates, that there are stronger cut-off effects at higher temperature, where the multi-charged states in the cluster sum become increasingly important.

The above discussion of the temperature dependence of the cumulants and their ratios at different p_t cuts can be extended to more general cases. For $|\mu_Q| \ll T$ and $|\mu_Q| \ll m$, which is the case in heavy ion collisions at RHIC and LHC energies, the cluster expansion Eq. (4) implies, that

$$\chi_{2n+1}^Q/\chi_{2n}^Q \simeq \mu_Q/T \quad , \tag{6}$$

irrespective of the p_t cut. This is because, the $\hat{K}_2(km/T)$ functions cancel in these ratios. Consequently, in the pion gas, the lowest ratio χ_1^Q/χ_2^Q does not depend on the p_t-cut. Similarly, one finds

$$\frac{\chi_{2n+1}^Q}{\chi_{2n-1}^Q} \simeq \frac{\chi_{2n+2}^Q}{\chi_{2n}^Q}, \tag{7}$$

which implies that $\chi_4^Q/\chi_2^Q \simeq \chi_3^Q/\chi_1^Q$. These relations hold exactly for $\mu_Q = 0$, but deviations at $\mu_Q \neq 0$, in the range of RHIC and LHC energies, are found to be less than 1%.

Fig. 4 shows the normalized kurtosis, $\kappa_Q\sigma_Q^2$, calculated in a pion gas at $T = 160$ MeV as a function of the pseudo-rapidity cut $\eta_{\rm max}$, and for different p_t cuts corresponding to the STAR and PHENIX experiments, respectively.

From Figs. 2-4 we conclude that the dependence of electric charge cumulants on transverse momentum cuts is strong while the dependence on different pseudo-rapidity cuts is weak. We also checked the dependence of cumulant ratios on the high momentum cut-off, $p_{t_{\rm max}}$, and found that this is negligible for $p_{t_{\rm max}} >$ 1 GeV. (see Sec. IV)

The difference in the dependence of $\kappa_Q\sigma_Q^2$ on the transverse momentum and pseudo-rapidity cuts is transparent from the decomposition of the Bose integral in terms of the multi-charged cluster sum. A transverse momentum cut-off, $p_{t_{\rm min}}$, increases the effective mass of particles, $m_{\rm eff} = \sqrt{m^2 + p_{t_{\rm min}}^2}$, and thus further suppresses the statistical weight of the k-cluster contribution relative to the leading Boltzmann term. Consequently, increasing $p_{t_{\rm min}}$ reduces $\kappa_Q\sigma_Q^2$ towards unity, which is its value for the Boltzmann statistics. Cuts in pseudo-rapidity, on the other hand, do not restrict the minimal momentum in the particle dispersion relation and therefore do not

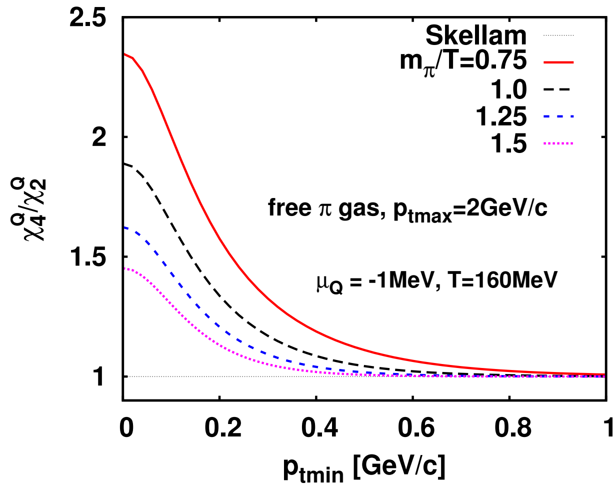

FIG. 3. The lower transverse momentum cut-off dependence of the ratio of quartic and quadratic electric charge fluctuations in a pion gas at different values of the temperature, 100 MeV $\lesssim T \lesssim$ 200 MeV for $\mu_Q = -1$ MeV.

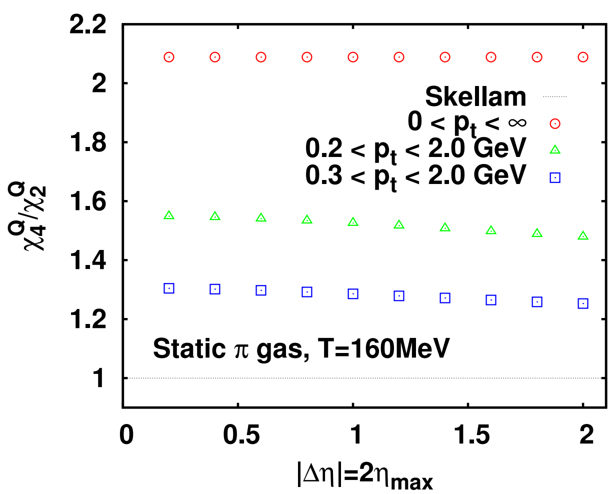

FIG. 4. Normalized kurtosis $\kappa_Q\sigma_Q^2 = \chi_4^Q/\chi_2^Q$ for an ideal pion gas at $T = 160$ MeV, $\mu_Q = 0$ and in different transverse momentum windows. Open circles stand for the fully integrated p_t range. Open triangles and squares show the result with the same p_t acceptance as STAR and PHENIX measurements, respectively [16, 17].

lead to an additional suppression of multi-charged clusters contributing to $\kappa_Q \sigma_Q^2$.

III. ELECTRIC CHARGE FLUCTUATIONS IN THE PHENIX AND STAR DATA

We have shown in the previous section, that a cut-off imposed on the low momentum part of the particle phase-space can have significant effects on different cumulants of net electric charge fluctuations and their ratios. Thus, the difference of 100 MeV in the transverse momentum cut-off used by the PHENIX and STAR Collaborations [16, 17], should naturally imply differences in their results on cumulants of electric charge fluctuations.

However, in order to quantify the influence of momentum cut-off effects on charge fluctuations in the context of heavy ion data, one needs to extend the pion gas calculation by including contributions from all known charged hadrons and resonances. In the following, we will apply a hadron resonance gas (HRG) model which is very successful in the description of the equation of state and different fluctuation observables calculated in lattice QCD in the hadronic phase [32–34].

We first consider the influence of a non-zero $p_{t_{\min}}$ on the ratio of mean and variance of net-electric charge fluctuations, $M_Q/\sigma_Q^2 = \chi_1^Q/\chi_2^Q$. This quantity is statistically well under control and a direct comparison between data obtained by STAR and PHENIX, as well as with model calculations, is meaningful.

The ratio M_Q/σ_Q^2 involves the first order cumulant χ_1^Q, which receives significant contributions from strange mesons as well as charged baryons. In fact, in the case relevant for heavy ion collisions, i.e. in a strangeness neutral system, $M_S = 0$, with a fixed ratio of net-electric charge and net-baryon number, $M_Q/M_B \simeq 0.4$, the electric charge chemical potential required to fulfill these constraints is negative. The contribution of charged, non-strange mesons to M_Q thus is negative and the mean becomes positive only due to contributions from charged baryons and strange mesons. Although in a single component Bose gas the ratio M_Q/σ_Q^2 does not depend on the p_t cut-off, Eq. (6), the cumulants themselves are modified depending on the mass of this component, as seen in Fig. 1. However, in a multi-component system, the change of M_Q and σ_Q^2 of each hadronic contribution does not cancel anymore in their ratio.

We have calculated M_Q/σ_Q^2 in a HRG model using three different scenarios for introducing the low momentum cut-off: (A) $p_{t_{\min}}$ is introduced only for the pion contribution, (B) an identical cut-off is introduce for all charged hadrons, and (C) a larger cut-off, $p_{t_{\min}} = 0.4$ GeV, is introduced for protons and anti-protons, as it is done in the electric charge measurements of the STAR Collaboration.

We show results from this analysis in Fig. 5, where M_Q/σ_Q^2 is calculated along the freeze-out line in central heavy ion collisions, with the energy dependent thermal

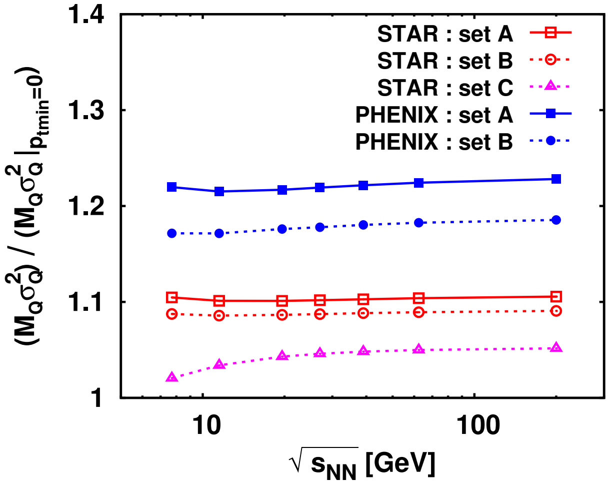

FIG. 5. The ratio of mean and variance, M_Q/σ_Q^2, of electric charge fluctuations for the three different scenarios (*see text*) with STAR acceptance (open symbols) and PHENIX acceptance (closed symbols) in a HRG model. The energy dependence of thermal parameters at chemical freeze-out are taken from Ref. [11]. The results are normalized with the values for $p_{t_{\min}} = 0$.

parameters, μ_B, μ_Q, μ_S and T from Ref. [11]. We find that the effect of a low momentum cut-off on M_Q/σ_Q^2 is almost independent of the collision energy. Comparing the different low momentum cut-off scenarios, one finds that the set (A) gives the largest effect, since it corresponds to cutting off the negative contribution of pions to M_Q and the pion contribution to σ_Q^2 is reduced. Incorporating the cut-off for other hadrons reduces the changes from the values without the cut-off. This is because the kaon contribution partly compensates the increase of M_Q due to the change of the pion contribution. The result of set (C) shows a weak $\sqrt{s_{NN}}$ dependence below $\sqrt{s_{NN}} \leq 20$ GeV, due to the dominance of the proton contribution. The difference among different scenarios is smaller than that observed between STAR and PHENIX acceptance.

We conclude that a non-zero transverse momentum cut-off increases the ratio M_Q/σ_Q^2 over the value obtained without any cuts. The larger $p_{t_{\min}}$ used by PHENIX suggests that their M_Q/σ_Q^2 ratio is about 10% larger than the STAR values. This accounts for about 1/3 of the difference between the PHENIX and STAR data. The ratio of these two data sets is, within large errors, constant over the entire energy range. A fit yields $(M_Q/\sigma_Q^2)_{\rm PHENIX}/(M_Q/\sigma_Q^2)_{\rm STAR} = 1.35(5)$. Note, however, that more than 99% of the error quoted by PHENIX is systematic.

In Fig. 6 we show the data measured by STAR [16] and PHENIX [17]. They have been normalized by the corresponding ratio for net-proton fluctuations measured by STAR [15]. As can be seen from the right hand panel of

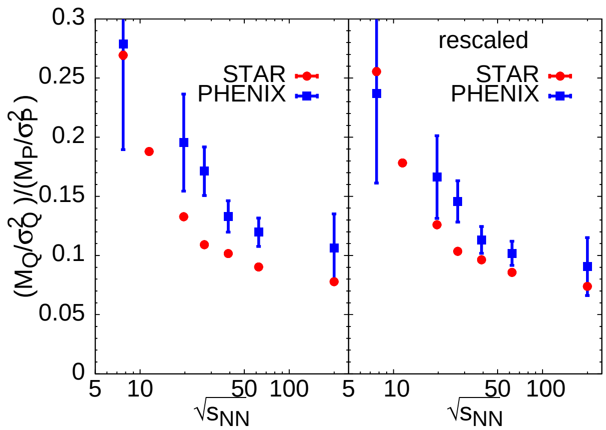

FIG. 6. The ratio of mean and variance, M_Q/σ_Q^2, of electric charge fluctuations obtained by STAR (circle) and PHENIX (squares) Collaboration at different beam energies [16, 17], (left panel). Data have been normalized to the corresponding ratio for net-proton number fluctuations, M_P/σ_P^2, obtained by STAR [15]. The right hand panel shows the rescaled data $(M_Q/\sigma_Q^2)_{p_{t_{\min}}=0} = r(M_Q/\sigma_Q^2)_{p_{t_{\min}}}$. Where the rescaling factor r is taken from the result of set (C) for the STAR data and set (B) for the PHENIX data shown in Fig. 5.

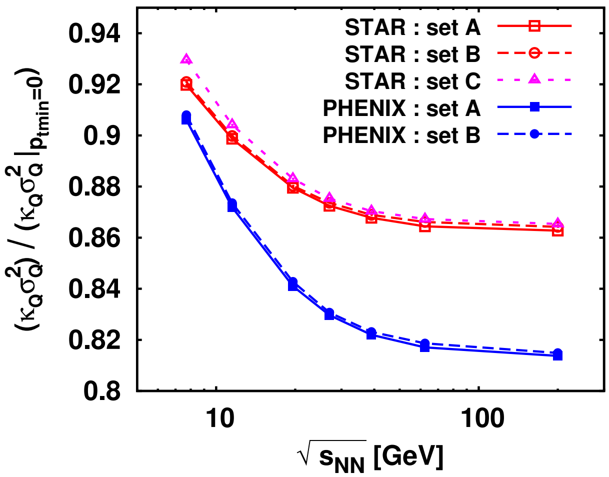

FIG. 7. Normalized kurtosis $\kappa_Q\sigma_Q^2$ for the three difference cut-off scenarios in a HRG model along the freeze-out line. The legends are the same as in Fig. 5.

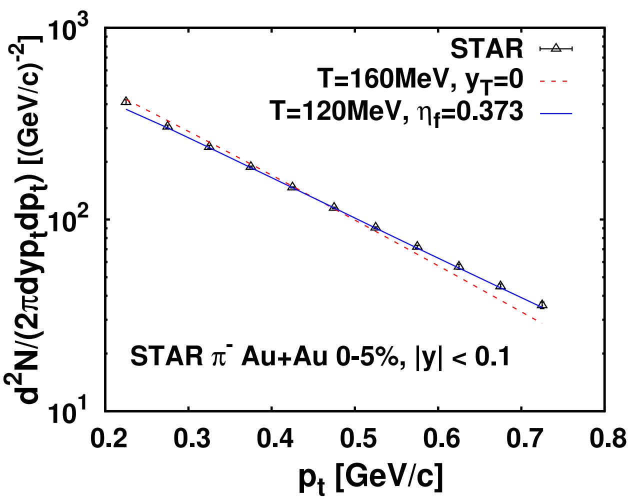

FIG. 8. The p_t distribution of charged pions. The spectrum of the STAR data [35] is shown as triangles. The dashed-line is the model results at chemical freeze-out ($T = 160$ MeV) from a longitudinally expanding fireball. The solid-line represents the distribution at thermal freeze-out for a longitudinally and transversally expanding fireball with parameters fitted to the STAR data [35].

Fig. 6, rescaling the STAR and PHENIX data to vanishing transverse momentum cut-off values removes a large part of the differences between these data. In fact, within the large systematic errors of the PHENIX data it yields consistent results.

In Fig. 7 we show the influence of a non-zero transverse momentum cut-off on the normalized kurtosis, $\kappa_Q\sigma_Q^2$. Compared to the pion gas results shown in Fig. 3, the reduction of $\kappa_Q\sigma_Q^2$ is smaller and turns out to be 10-20% due to a smaller cut-off dependence of the heavier additional degrees of freedom in a HRG. The small discrepancy in the different scenarios nevertheless implies that the main reason for the reduction is the pion effect. Contrary to M_Q/σ_Q^2, this result shows a stronger dependence of $\kappa_Q\sigma_Q^2$ on $\sqrt{s_{NN}}$. Thus, we expect that the difference in $p_{t_{\min}}$ between STAR and PHENIX acceptance will lead to about 6% difference in $\kappa_Q\sigma_Q^2$ at $\sqrt{s_{NN}} = 200$ GeV, and it will be negligible at $\sqrt{s_{NN}} = 7.7$ GeV. We note, that similar results have been obtained in Ref. [29] where a further reduction of $\kappa_Q\sigma_Q^2$ due to effects of resonance decays is also discussed.

IV. EFFECTS OF EXPANSION

Until now, we have discussed the influence of momentum cuts imposed in a thermal medium at rest. In heavy ion experiments, however, the kinematic cuts on measured fluctuations are applied to hadrons that move freely only after kinetic freeze-out. Consequently, for the comparison of experimental data with theoretical calculations, it is important to verify how momentum cuts implemented in different stages of the fireball evolution can influence the measurement of fluctuations of conserved charges [29].

Here we employ an expanding source model with parameters chosen such that the pion p_t spectrum measured by the STAR Collaboration [35] is reproduced. We compare the effect of a p_t cut imposed either at the time of chemical or thermal freeze-out on the measured values for the kurtosis of net-electric charge fluctuations.

Assuming boost invariance along the collision axis and transverse expansion with a linear rapidity profile, as well as an instantaneous emission at $\tau = \tau_f$ hyper-surface, we parametrize the phase space distribution of pions as [36]

$$S(x, \boldsymbol{p}) = \frac{m_T \cosh(y - \eta_s)\delta(\tau - \tau_f)}{(2\pi)^3} n_B(u \cdot p) e^{-\frac{x^2+y^2}{2R^2}}. \tag{8}$$

Here $y = \ln[(E_p + p_z)/(E_p - p_z)]/2$ is the rapidity of the pions, $\eta_s = \ln[(t+z)/(t-z)]/2$ is the space-time rapidity which is equal to the longitudinal flow rapidity y_L, and R is the Gaussian transverse size of an expanding fireball. The flow 4-velocity is parametrized as

$$u^\mu = (\cosh y_T \cosh \eta_s, \sinh y_T \cos\phi, \sinh y_T \sin\phi, \cosh y_T \sinh \eta_s). \tag{9}$$

The transverse flow rapidity is assumed to be $y_T = \eta_f r/R$, with $r = \sqrt{x^2 + y^2}$ and a flow control parameter η_f which is adjusted so as to reproduce the p_t spectrum of pions.

Since the invariant spectrum is obtained from the source function from Eq. (8), as

$$E_p \frac{dN}{d^3\boldsymbol{p}} = \int d^4x S(x, \boldsymbol{p}), \tag{10}$$

the p_t spectrum can be obtained by integrating over space-time variables, as $\int d^4x = 2\pi\tau_f \int d\tau \int d\eta_s \int r dr$. Consequently, cumulants of charge fluctuations are given by

$$\chi_n^Q \propto \frac{\partial^{n-1}}{\partial \mu_Q^{n-1}} \int \frac{d^3\boldsymbol{p}}{E_p} S(x, \boldsymbol{p}). \tag{11}$$

The proportionality factor is canceled out when taking ratios such as χ_4^Q/χ_2^Q. Note that Eq. (11) reduces to Eq. (3) when considering a static source, i.e. by choosing $y_L = 0$ and $y_T = 0$.

Fig. 8 displays the p_t spectrum of π^- measured by the STAR Collaboration [35]. We fit the data with the above source model with $T = 120$ MeV and $R = 6$ fm, and obtain $\eta_f = 0.373$. Although the overall scale factor is also fitted, our result still underestimates the yields at the lowest p_t bin. For comparison we also plot the spectrum at $T = 160$ MeV without transverse flow, i.e. for $y_T = 0$ but at finite y_L. One sees that it does not reproduce the slope of the data. Therefore, it is expected that the influence of a p_t cut on fluctuation observables can be different at chemical and thermal freeze-out due to differences in the p_t spectrum.

Fig. 9 shows the normalized kurtosis $\kappa_Q\sigma_Q^2$ calculated in an expanding and static source with and without transverse flow for different $p_{t_{\min}}$. The pseudo-rapidity window is fixed to $|\eta| < 0.35$ and the upper momentum cut to $p_{t_{\max}} = 2$ GeV. For $p_{t_{\min}} < 0.2$ GeV there is a clear reduction of χ_4^Q/χ_2^Q relative to its value obtained with a static source from Eq. (1), when the longitudinal and/or transverse expansion is included. However, for

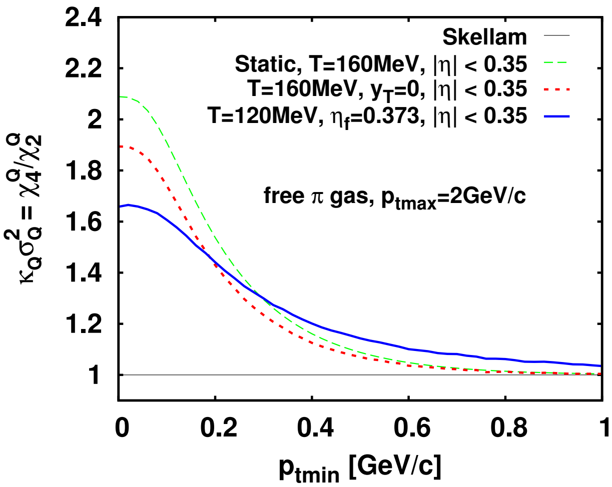

FIG. 9. The kurtosis $\kappa_Q\sigma_Q^2 = \chi_4^Q/\chi_2^Q$ as a function of $p_{t_{\min}}$. The long-dashed line stands for the static source from Eq. 1. The short-dashed and solid lines denote the results obtained from the spectra shown in Fig. 8.

$p_{t_{\min}} \geq 0.2$ GeV the effect of a p_t-cut on $\kappa_Q\sigma_Q^2$ is similar in all cases. The values of the kurtosis do not differ much for $p_{t_{\min}} \geq 0.2$ GeV and the $p_{t_{\min}}$ dependence is only slightly modified in the presence of transverse flow. As can be seen in Fig. 8 the p_t spectrum at thermal freeze-out, $T = 120$ MeV and $\eta_f = 0.373$, clearly has a larger slope than that calculated at $T = 160$ MeV without transverse flow. This difference in the spectral shapes leads to a somewhat slower convergence of $\kappa_Q\sigma_Q^2$ to unity with increasing $p_{t_{\min}}$. Nonetheless, the resulting decrease of $\kappa_Q\sigma_Q^2$ in the interval $p_{t_{\min}} \in [0.2\text{ GeV}, 0.3\text{ GeV}]$ is as large as that found for the static source.

For completeness we also show in Fig. 10 the influence of $p_{t_{\max}}$ on $\kappa_Q\sigma_Q^2$ at fixed $p_{t_{\min}} = 0.3$ GeV and $|\eta| < 0.35$. There is no notable change of the kurtosis when decreasing $p_{t_{\max}}$ from 2 GeV to 1 GeV. For $p_{t_{\max}} <$ 1 GeV, the relative contribution of lower momentum particles with charges $k > 1$ in Eq. (4), i.e. higher order terms appearing due to Bose statistics, start to dominate fluctuations, consequently $\kappa_Q\sigma_Q^2$ is increasing with decreasing $p_{t_{\max}}$. The dependence of $\kappa_Q\sigma_Q^2$ on $p_{t_{\max}}$ is similar for a static and transversally expanding medium.

The above results on flow effects were derived in a rather simplified system of a non-interacting pion gas. However, due to leading contributions of pions to electric charge fluctuations and a large mass of multi-charged baryons, our conclusions should also hold in a hadron resonance gas which accounts for contributions of all known hadrons and resonances.

The sensitivity to kinematic cuts may still be modified if charge fluctuations are influenced by the critical chiral dynamics [37], which is clearly not included in this study. For the case of charge fluctuations in the crossover region, however, we conclude that flow effects can safely

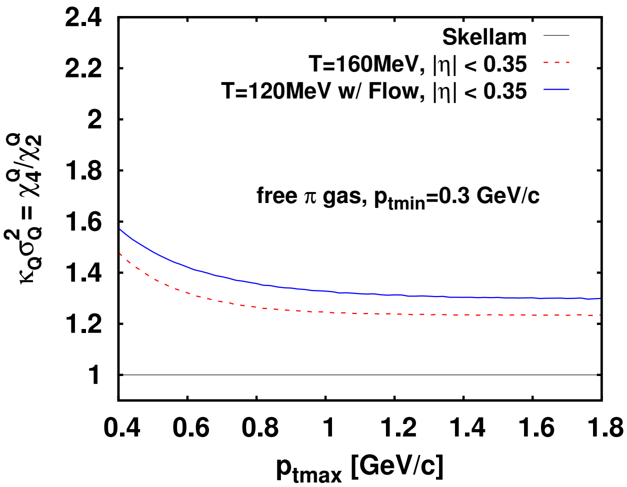

FIG. 10. Dependence of the ratio of quartic (χ_4^Q) and quadratic (χ_2^Q) cumulants of electric charge fluctuations on $p_{t_{\max}}$ for fixed $p_{t_{\min}} = 0.3$ GeV and $|\eta| = 0.35$. The results correspond to pion spectra shown in Fig. 8.

be ignored in the analysis of charge fluctuations.

V. TRANSVERSE MOMENTUM CUT-OFF AND FINITE-SIZE EFFECTS

In the previous sections we have seen that a quantitative comparison between experimental data taken in specific acceptance windows, in particular with a non-zero transverse momentum cut-off, with the thermal resonance gas model requires also to take into account corresponding cut-offs in the model calculation.

In a non-interacting hadron gas a non-zero p_t cut reduces the available phase space, which may be viewed as an increase of the effective mass of particles, and thus leads to corresponding changes in thermodynamic observables. This effect is similar to the thermodynamics of particles in a finite volume [38, 39]. The finite size of a system, e.g. a box of size L^3, suppresses low momentum states.

When considering a box with periodic boundary conditions the lowest non-zero momentum of bosons in such a box is $2\pi/L$. In the case of a non-interacting scalar field theory, which describes the thermodynamics of spin zero bosons, i.e. pions, it is straightforward to analyze the volume dependence of thermodynamic observables [38]. In particular, one can calculate the volume dependence of conserved charge fluctuations and compare the influence of finite volume effects with those of a transverse momentum cut-off.

In Fig. 11 we show a calculation of quadratic and quartic charge fluctuations of a pion gas in a finite volume. Indeed, there is a clear correspondence between finite volume and momentum cut-off effects in such a non-interacting gas. As can be seen from Fig. 11, all quadratic and quartic charge fluctuations in a volume of size L^3 can be reproduced by a calculation of charge fluctuations in an infinite box, but with a transverse momentum cut-off

$$p_{t_{\min}} = 2\pi k/L \;\text{ with }\; k = 0.707\,. \qquad (12)$$

The proportionality factor k is found to be rather weakly depend on the pseudorapidity acceptance. Indeed, for $|\Delta\eta| = 0.35$ one gets $k = 0.707$, while for $|\Delta\eta| = 5$ one finds $k \simeq 0.5$.

In a non-interacting pion gas the scale for all dimensionfull quantities is set by the temperature. Consequently, the size of a finite volume or a non-zero momentum becomes meaningful in units of T. We thus may interpret the above results in terms of LT, which

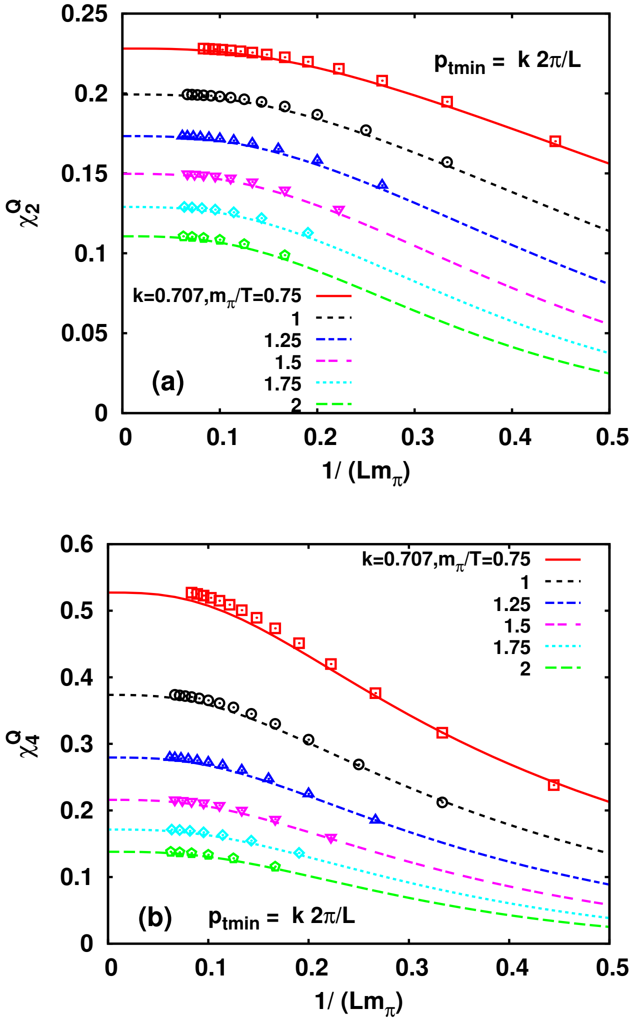

FIG. 11. Quadratic [χ_2^Q, (a)] and quartic [χ_4^Q, (b)] cumulants of electric charge fluctuations in a non-interacting pion gas calculated in a finite box of size L^3 for different m_π/T (symbols). Results are compared to calculations in an infinite box but with a non-zero transverse momentum cut-off $p_{t_{\min}} = 2\pi k/L$ with $k = 0.707$ (lines).

is conventionally used in lattice field theory. In particular, lattice QCD calculations are performed on lattices of size $L^3T^{-1} \equiv (N_\sigma a)^3 N_\tau a$ and the continuum limit ($N_\sigma \to \infty, N_\tau \to \infty$) is taken for fixed aspect ratio $N_\sigma/N_\tau \equiv LT$. Typically $LT \simeq 4$ in such calculations. We may compare this to the momentum cut-off $p_{t_{\min}} \simeq (0.2 - 0.3)$ GeV currently used in heavy ion experiments. At a freeze-out temperature $T \simeq 160$ MeV these momentum cut-offs expressed in units of temperature are, $p_{t_{\min}}/T \simeq (1.25 - 1.875)$. Using Eq. (12), which relates $p_{t_{\min}}/T$ to a box size LT, we find that these cut-offs correspond to aspect ratios, $LT \simeq 2.4-3.6$. Thus, lattice calculations performed with an aspect ratio $LT = 4$ seem to be appropriate to characterize the thermodynamics probed with transverse momentum cut-offs similar to those used by the STAR Collaboration.

VI. CONCLUDING REMARKS

We have discussed the influence of kinematic cuts in transverse momentum and pseudo-rapidity on electric charge fluctuations in a static and expanding medium.

Our basic arguments were given by considering a very transparent example of an ideal gas of charged pions with quantum statistics. Such a system can be viewed as a multi-component gas of Boltzmann particles with different charges, masses, and degeneracy factors, thus it contains all relevant properties to study charge fluctuations, with a transparent implementation of different kinematic cuts.

We demonstrate, that a non-vanishing lower p_t cut in the net-electric charge fluctuations, can substantially reduce the second and fourth order cumulants of net electric charge fluctuations, as well as, their ratio quantified by the normalized kurtosis $\kappa_Q\sigma_Q^2$. These conclusions are valid in a static and expanding medium. The effect of pseudo-rapidity cuts was shown to be rather negligible. Consequently, the observed differences between STAR and PHENIX data on cumulants of electric charge fluctuations could be traced back to their different p_t acceptance.

The experimental data on the ratio of mean and variance, M_Q/σ_Q^2, and normalized kurtosis $\kappa_Q\sigma_Q^2$, were analyzed in a hadron resonance gas. It was shown, that in heavy ion collisions, the influence of a non-zero transverse momentum cut-off, $p_{t_{\min}}$, on M_Q/σ_Q^2 is almost independent of the collision energy, and that it increases, with increasing $p_{t_{\min}}$. Thus, the larger $p_{t_{\min}}$ used by PHENIX, suggests that their M_Q/σ_Q^2 should be larger than the corresponding STAR values. We have shown, that indeed, by rescaling the STAR and PHENIX data to vanishing transverse momentum, makes these data compatible within the still large systematic errors of the PHENIX data. Furthermore, it was argued that the reduction of $\kappa_Q\sigma_Q^2$ due to the shift of $p_{t_{\min}}$ will lead to differences between PHENIX and STAR data of $\mathcal{O}(6\%)$, which currently are buried under large statistical and systematic errors.

Having in mind a possible comparison between experimental data on fluctuations of conserved charges and lattice QCD results, we have discussed the relation between momentum cut-off and finite volume effects. We have shown, in the context of an ideal pion gas, that cumulants of electric charge fluctuations in a finite box of size L^3, can be reproduced by a calculation of charge fluctuations in an infinite box, but with a transverse momentum cut-off $p_{t_{\min}} = 2\pi k/L$, where k is a constant.

Finally, considering typical box sizes used in lattice QCD calculations, and the lower transverse momentum cut-off scale in the experimental data on electric charge fluctuations, we have argued, that their direct comparison at a chiral crossover temperature, might be justified.

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