Heavy quark correlations and the effective volume for quarkonia production

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Heavy quark correlations and the effective volume for quarkonia production

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Using the Boltzmann transport approach, we study the effect of initial spatial and momentum correlations between a heavy quark pair, such as that produced from a $p+p$ collision, on their collision rate in a partonic medium, which is relevant for their thermalization and the production of quarkonium from regeneration. Characterizing this effect by an effective volume given by the inverse of the ratio of their collision rate to the collision rate of a thermally equilibrated and spatially uniformly distributed heavy quark pair in a unit volume, we find that the effective volume is finite and depends sensitively on the momentum of the heavy quark and the temperature of the medium. Generally, it increases linearly with time t at the very beginning, thus an enhanced collision rate, and the increase then becomes slower due to multiple scattering, and finally it increases as $t^{3/2}$. We further find that the momentum distribution of the colliding heavy quark pair approaches a thermal distribution after about 5 fm/c with an effective temperature similar to that of the medium even though their transverse momentum spectra initially have a δ-like distribution.

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I. INTRODUCTION

It is well known that the relative abundance of different components in a multi-component atomic system depends on the volume of the system. For example, if same number of N_2O_4 is put into containers of different volumes that have same temperature, its number after reaching chemical equilibrium with NO_2 is larger in the container that has a smaller volume. The same happens to heavy quarks and quarkonium in heavy ion collisions if they reach thermal equilibrium and their numbers are fixed as assumed in the statistical model. This can also be understood from the dependence of the heavy quark collision rate on the volume of the system. To see this, we consider N_Q heavy quarks and $N_{\bar{Q}}$ heavy antiquarks that are uniformly distributed in a volume V and have a thermal distribution in momentum. Their collision rate R is then proportional to the densities ρ_Q of heavy quarks and $\rho_{\bar{Q}}$ of heavy antiquarks, that is

$$R = \langle \sigma v \rangle \rho_Q \rho_{\bar{Q}} V \propto \frac{N_Q}{V}\frac{N_{\bar{Q}}}{V} V = \frac{N_Q N_{\bar{Q}}}{V}, \qquad (1)$$

which is inversely proportional to the volume V of the system. In an infinitely large volume $V=\infty$, the collision rate R becomes zero, and no quarkonium can be produced in the system.

In relativistic heavy ion collisions, the initially produced heavy quarks and antiquarks are, however, correlated in space and momentum. This is especially the case if only one pair of heavy quark and antiquark are produced in a collision, as they would be produced essentially at the same position and have almost opposite momentum. In Ref. [1], it was argued that the spacial correlation between heavy quark and antiquark would lead to nonvanishing collision rate even if the volume V is infinitely large, and a semiempirical formula was suggested to take into account this effect in relativistic heavy ion collisions. Also, the back-to-back correlation between the momenta of produced heavy quark and antiquark makes them not likely to collide to form a heavy quarkonium if the medium is dilute, although they are initially close in space. The effect of the initial correlation between heavy quark and antiquark in both the coordinate space and the momentum space can be characterized by an effective volume $V_{\rm eff}$ such that their collision rate would be the same as that of thermally equilibrated heavy quarks that are uniformly distributed in this volume. We note that the effective volume is infinitely large in an open system without initial correlation between heavy quark and anitquark. Correlations reduce the effective volume and enhance the collision rate, leading to an increase in the production of quarkonium.

Besides their correlations in coordinates and in momenta, heavy quark and antiquark produced in heavy ion collisions are also strongly correlated in their yield, i.e. $N_Q = N_{\bar{Q}}$, which leads to an enhanced production of of quarkonia called the canonical ensemble effect [2, 3]. A similar effect due to the strangeness conservation in the production of rare strange particles in heavy ion collisions as well as in elementary proton-proton collision and e^+-e^- annihilation was found to be responsible for their suppressed production in these reactions. Such a canonical suppression effect has been extensively studied in Refs. [4–10]. We will focus in the present study on the effect due to correlations in the coordinates and momenta instead of the canonical ensemble effect on quarkonium production.

In the present paper, we consider the case of one pair of heavy quarks that are initially correlated in both the coordinate and the momentum space as produced in high energy collisions and study their scattering rate in terms of the effective volume defined in the above. To describe the dynamics of heavy quarks in a medium, we use a parton cascade model based on the Boltzmann equation since a recent study has shown that it gives a more accurate description [11] than the more widely used Langevin or Fokker-Planck equation [12–18].

The paper is organized as follow. In Sec. II, we evaluate the thermal collision rate and define the effective volume using the collision number calculated from a parton cascade simulation. We then show in Sec. III the numerical results from the simulation for both a close system and an open system. We further discuss in this Section the time behavior of the effective volume as well as that of the center of mass energy of the colliding heavy quark pair. Finally, conclusions are given in Sec. IV. In the appendices, Appendix A describes the criteria for heavy quark collisions and Appendix B shows the proof for the linear behavior of the effective volume at short times.

II. THE MODEL

Given two particle species A and B in a volume V at time t, their collision number ΔN_{AB} in a small time step Δt can be expressed as

$$\Delta N_{AB} = \Delta t \int_V d\mathbf{x} \int \frac{d\mathbf{p}_A d\mathbf{p}_B}{(2\pi)^3(2\pi)^3} f_A(\mathbf{x},\mathbf{p}_A,t) \cdot f_B(\mathbf{x},\mathbf{p}_B,t) v_{AB}\sigma_{AB}. \tag{2}$$

In the above, the distribution functions f_A and f_B are functions of the position $\mathbf{x}$ and momenta $\mathbf{p}_A$ and $\mathbf{p}_B$, while v_{AB} and σ_{AB} are their relative velocity and total scattering cross section.

For the process of quarkonium regeneration $Q+\bar{Q}\to(Q\bar{Q})+X$, we have $A=Q$ and $B=\bar{Q}$. If we assume that the distribution functions $f_Q=f_{\bar{Q}}$ are uniform in space and have a Boltzmann distribution in momentum with a temperature T, then the collision number is simply given by

$$\Delta N^{\rm th}_{Q\bar{Q}} = \frac{TV\Delta t}{4\pi^4}\frac{N_Q N_{\bar{Q}}}{N_{Q_0}N_{\bar{Q}_0}}\int_{\sqrt{s_0}}^{\infty} d\sqrt{s}\; sp^2\sigma_{Q\bar{Q}}(\sqrt{s}) \cdot K_1(\sqrt{s}/T), \tag{3}$$

where $\sqrt{s_0} = m_Q + m_{\bar{Q}}$ is the minimum center of mass energy $\sqrt{s}$ of the scattering pair, $p = \sqrt{[(s-m_Q^2-m_{\bar{Q}}^2)^2-4m_Q^2m_{\bar{Q}}^2]/(4s)}$ is their momentum in their center of mass frame, and K_1 is the modified Bessel function of the second kind. The number of Q in a volume V and its thermally equilibrated number with vanishing chemical potential are denoted by N_Q and $N_{Q_0} = m_Q^2 VTK_2(m_Q/T)/(2\pi^2)$, respectively, while those of $\bar{Q}$ are, respectively, $N_{\bar{Q}}$ and $N_{\bar{Q}_0}$. In the case of a constant cross section $\sigma_{Q\bar{Q}}$ and equal masses $m_Q = m_{\bar{Q}} = m$, the above expression can be further expressed as

$$\Delta N^{\rm th}_{Q\bar{Q}} = \frac{N_Q N_{\bar{Q}}\sigma_{Q\bar{Q}}\Delta t}{V} g(m^*) \tag{4}$$

with $g(z)\equiv 4K_3(2z)/[zK_2^2(z)]$ and $m^* = m/T$.

On the other hand, if only one pair of heavy quarks is produced in a heavy ion collision, they will initially be at the same position and have essentially same momentum but with opposite direction. As they propagate in the medium, they will gradually move apart and also become thermalized. Because of the large size of the medium and the relatively long relaxation time for heavy quarks and antiquarks in the medium, they are not likely to trace the whole volume of the fireball and to achieve kinetic equilibrium immediately. The deviation of the heavy quark distribution function from the uniform and thermal distribution used in obtaining Eq. (4) for rare heavy quark events in relativistic heavy ion collisions (e.g. charm at SPS energy and bottom at RHIC energy) can be characterized by an effective volume given by the ratio of the collision rates $R^{\rm th}_{Q\bar{Q}} = \Delta N^{\rm th}_{Q\bar{Q}}/\Delta t$ and $R_{Q\bar{Q}} = \Delta N_{Q\bar{Q}}/\Delta t$, i.e. $V_{\rm eff}\equiv R^{\rm th}_{Q\bar{Q}}V/R_{Q\bar{Q}}$ as the collision rate is inversely proportional to the volume in the thermal case. Since the product $R^{\rm th}_{Q\bar{Q}}V$ depends only on the temperature of the medium and the heavy quark and antiquark scattering cross section according to Eq. (4), taking the ratio $R^{\rm th}_{Q\bar{Q}}V/R_{Q\bar{Q}}$ removes the dependence on the heavy quark and antiquark scattering cross section. We can thus determine $V_{\rm eff}$ by following independently the motions of many similar pairs of heavy quarks and calculate the average number of heavy quark scatterings with infinitesimally small cross section, i.e.,

$$V_{\rm eff} \equiv \lim_{\substack{\Delta t\to 0,\sigma_{Q\bar{Q}}\to 0,\\ N_Q,N_{\bar{Q}}\to\infty}} \frac{N_Q N_{\bar{Q}}\sigma_{Q\bar{Q}}\Delta t}{\Delta N_{Q\bar{Q}}} g(m^*), \tag{5}$$

which then depends only on the distribution of the heavy quark pairs in the phase space and the temperature of the medium. We note that the effective volume approaches the volume of the medium when the heavy quarks are thermalized.

The nonequilibrium dynamics of heavy quarks can be described by the Boltzmann equation for their phase space distribution function $f_Q(\mathbf{x},\mathbf{p},t)$,

$$\partial_t f_Q(\mathbf{x},\mathbf{p},t) + \mathbf{v}_Q\cdot\nabla f_Q(\mathbf{x},\mathbf{p},t) = C[f_Q], \tag{6}$$

where $\mathbf{v}_Q$ is the velocity of the heavy quark, and $C[f_Q]$ is the collision term. For a given initial position and momentum of a heavy quark, this equation can be solved using the heavy quark collision rate with thermal partons in the medium, given by

$$R = \frac{mT^3m_g^{*2}N_g}{2\pi^2E}\int_0^\infty dy\; e^{-m_g^*\cosh y\cosh y_r}\sinh^2 y \cdot\sinh(m_g^*\sinh y_r\sinh y)\sigma_{gQ}(y), \tag{7}$$

which is obtained directly from Eq. (2) by using the Boltzmann distribution for the thermal partons. In the above, $\sigma_{gQ}(y)$ is the cross section for scattering between the heavy quark and a parton of rapidity y in the heavy quark frame; N_g is the degeneracy of the parton, which is taken to be 16 if we include only gluons and neglect quarks as their scattering cross sections with heavy quarks are small [14]; $y_r = \text{acosh}(p \cdot u/m)$ is the rapidity of the heavy quark relative to the medium with four-velocity u^μ; $m_g^* = m_g/T$ where m_g is the mass of gluons; and E is the energy of the heavy quark. For a constant cross section and massless partons, the heavy quark collision rate in the rest frame of the medium can be simplified to $R = N_g T^3 \sigma_{gQ}/\pi^2$, resulting in an average time between collisions, defined here as the mean free time, that is given by the inverse of the collision rate, i.e., $\tau_{\text{mean}} = 1/R$. As the heavy quark traverses through the medium, the probability for it to collide with a parton during a small time step $dt \ll \tau_{\text{mean}}$ is then dt/τ_{mean}. A collision occurs if a random number generated between 0 and 1 is smaller than dt/τ_{mean}. With the parton momentum randomly selected according to the thermal distribution, the momentum of the heavy quark after the collision can be determined from the energy and momentum conservations once its direction is obtained from the differential cross section. For simplicity, we take the cross section to be isotropic with the magnitude $\sigma_{gQ} = 4$ mb, which is the approximate value expected from the pQCD for a high energy heavy quark scattering with a thermal parton [14]. The treatment to $\bar{Q}$ is identical to Q, except that the directions of their initial momenta are opposite.

For the scattering between the heavy quark and antiquark, it is treated by comparing their impact parameter with the scattering cross section and is described in detail in Appendix A. Although only one pair of heavy quarks is initially produced at the same position with opposite momentum in a heavy ion collision event, we can study their mean dynamics in the hot medium by following independently the motions of many similar pairs of heavy quarks and calculate the average number of heavy quark scatterings as in Eq. (5).

III. RESULTS AND DISCUSSIONS

A. Heavy quarks in a closed thermal system

To illustrate the method used in our study, we first consider the collision dynamics of a pair of heavy quarks in a periodic cubic box of length $L = 10$ fm on each side. The heavy quarks are initially located at same position and have opposite momentum of 5 GeV/c along z direction. With the temperature of the medium taken to be 0.3 GeV, the time evolution of the effective volume V_{eff} of the heavy quark pair calculated according to Eq. (5) is shown in Fig. 1 by its ratio to the volumeV_{th} of the box, i.e., $V_{\text{eff}}/V_{\text{th}}$. It is seen that the ratio is initially less than one as the heavy quark pair is more likely to collide with each other due to their closeness in phase space than in the case that they are uniformly distributed in space. As expected, the effective volume V_{eff} approaches the volume of the medium as time t approaches infinity, and the time scale for the heavy quarks to spread uniformly in the box is about 30 fm/c.

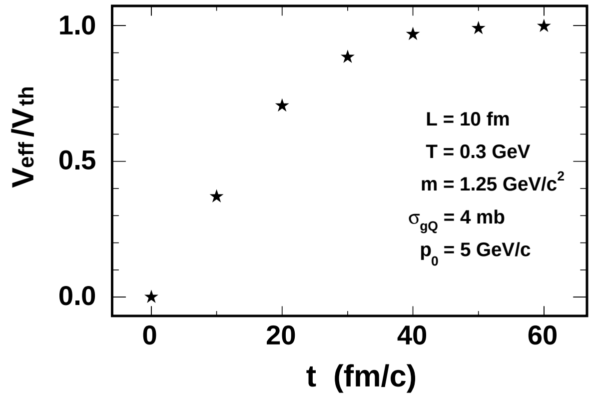

FIG. 1: Time evolution of the effective volume V_{eff}, expressed as its ratio to the volume V_{th} of the system, of a pair of heavy quarks with mass m and back-to-back momentum p_0 in a periodic cubic box of length L on each side, which consists of a gluonic matter at temperature T, and undergo scattering with gluons with the cross section σ_{gQ}.

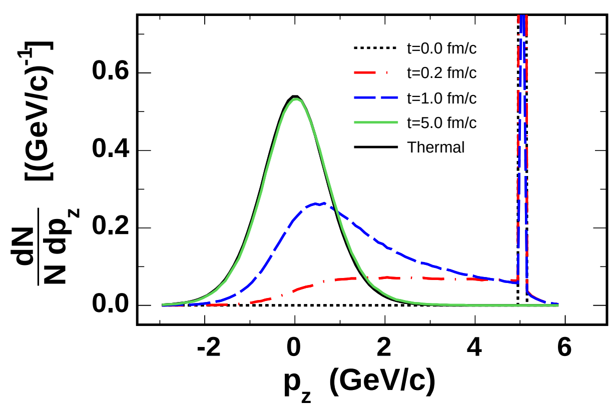

FIG. 2: (Color on line) Longitudinal momentum p_z distribution of heavy quarks with initial momentum $p_0 = 5$ GeV/c along the z direction at different times in a finite gluonic matter of temperature $T = 0.3$ GeV. The thermal p_z distribution calculated from Eq. (8) is reached at $t = 5$ fm/c.

To see how heavy quarks approach thermal equilibrium, we show in Fig. 2 their p_z distribution at different times. It is seen that the initial δ-like peak at $p_z = 5$ GeV/c initially moves down in p_z and gradually approaches a thermal distribution. During early times, the distribution deviates significantly from a Gaussian function, in contrast with the prediction based on the Langevin approach because of the comparable average parton kinetic energy and heavy quark mass, which is beyond the region where the Langevin approach is applicable [19] . Since heavy quarks have a mean free time $\tau_{\text{mean}} = 0.44$ fm/c, their p_z distribution essentially becomes the thermal distribution

$$f_{P_z}(p_z) = \frac{dN}{N\ dp_z} \ = \ \frac{(m_L^* + 1)e^{-m_L^*}}{2Tm^{*2}K_2(m^*)} \qquad (8)$$

with $m_L^* \equiv \sqrt{m^2 + p_z^2}/T$ at $t = 5$ fm/c $\gg \tau_{\rm mean}$, which is comparable to the lifetime of the partonic fireball produced in heavy ion collisions at RHIC and LHC.

B. Heavy quarks in an open thermal system

In this subsection, we consider the case that the heavy quarks move in a medium of infinite volume, using the same heavy quark mass $m = 1.25$ GeV/c^2 [20], heavy quark-gluon scattering cross section $\sigma_{gQ} = 4$ mb, and the medium temperature $T = 0.3$ GeV (except in Section III B 1) as in the previous subsection. Results on the time evolution of $V_{\rm eff}$ are shown in Figs. 3, 4, and 5 for the three time stages $t \ll \tau_{\rm mean}$, $t \sim \tau_{\rm mean}$, and $t \gg \tau_{\rm mean}$ relative to the mean free time $\tau_{\rm mean} = 0.44$ fm/c. In all three cases, the effective volume $V_{\rm eff}$ depends sensitively on the initial momentum with the larger one resulting in a larger effective volume.

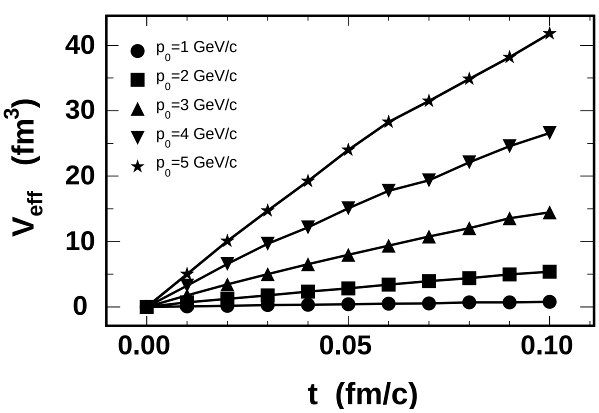

FIG. 3: Time evolution of $V_{\rm eff}$ at early times $t \ll \tau_{\rm mean}$ for different initial heavy quark momentum p_0 in an infinite medium of temperature $T = 0.3$ GeV.

As shown in Fig. 3, the effective volume $V_{\rm eff}$ at earlier times $t \ll \tau_{\rm mean}$ is proportional to time, which is true in general, and the proof is given in Appendix B.

When the time t becomes comparable to the mean free time $\tau_{\rm mean}$, heavy quarks are more likely to turn around and collide with each other after undergoing successive collisions. As a result, the increase of $V_{\rm eff}$ with time becomes slower as shown in Fig. 4.

For time t much longer than the mean free time $\tau_{\rm mean}$, when heavy quarks have collided many times with partons in the medium, their behavior can be described by random walks. In this picture, the distance traveled by a heavy quark is proportional to $t^{1/2}$, and the effective volume for heavy quarks to collide is thus proportional to $t^{3/2}$ as shown in Fig. 5. Although the magnitude of the

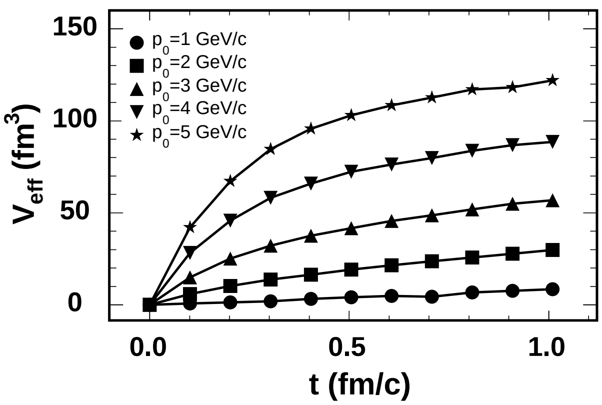

FIG. 4: Same as Fig. 3 for intermediate times $t \sim \tau_{\rm mean}$.

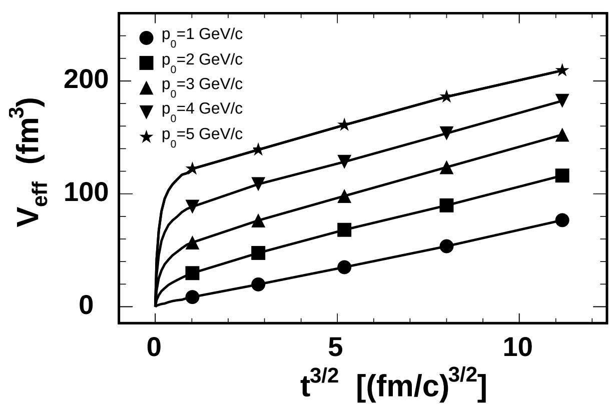

FIG. 5: Same as Fig. 3 for long times $t \gg \tau_{\rm mean}$ but with the horizontal axis changed from t to $t^{3/2}$.

effective volume increases with heavy quark initial momentum, as it depends on its nonequilibium dynamics, the coefficient of the proportionality or the slope of the lines in the figure is independent of the initial momentum, since it only depends on their thermal motions.

Since the initially produced heavy pair are distributed in a certain volume due to their quantum nature, our classical calculation based on the Boltzmann equation is valid only after certain time t_0 when the wave packets of the heavy quark and antiquark are separated.[1] The total number of Q-$\bar{Q}$ collisions is then given by

$$N_{Q\bar{Q}} \ = \ \int_{t_0}^{\infty} dt\ \frac{\sigma_{Q\bar{Q}}}{V_{\rm eff}(t)} g(m^*). \qquad (9)$$

Because of the long time behavior of the heavy quark effective volume, the integral in Eq. (9) converges at $t = \infty$. Therefore, the heavy quark pair hardly have the chance

[1] A detailed discussion on the value of t_0 requires the spatial distribution of the heavy quark when it is produced, which is beyond the scope of study in this paper.

to collide with each other long after they are produced even if the lifetime of the medium is infinitely long.[2]

1. Temperature dependence of the effective volume

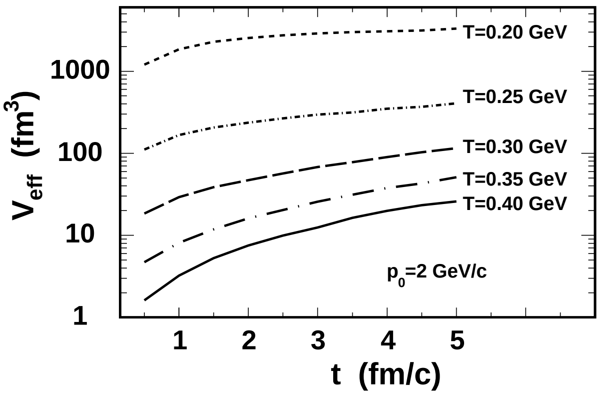

FIG. 6: Time evolution of the effective volume $V_{\rm eff}$ of heavy quarks with initial charm momentum $p_0 = 2$ GeV/c for different medium temperatures.

Since both the parton density and the parton energy depend on the temperature of the medium, the heavy quark effective volume also depends on the temperature of the medium, and this is shown in Fig. 6. The effective volume is seen to depend sensitively on temperature. For the temperature $T = 0.2$ GeV, the effective volume already exceeds 1000 fm^3 at $t = 0.5$ fm/c due to the strong back to back correlation, while for $T = 0.4$ GeV, the effective volume is less than 10 fm^3 at $t = 2$ fm/c as a result of faster thermalization of the heavy quarks. As discussed in the previous subsection, the $V_{\rm eff}$ increases with time monotonically for all temperatures.

2. Center of mass frame energy distribution

Since a heavy quark pair is produced from hard collisions of nucleons at high energies, their initial momenta are large and opposite in direction. As they diffuse in a medium and collide with thermal partons, their momenta gradually approach a thermal distribution. Because the cross section for quarkoinum production from a heavy quark pair depends on their center of mass energy $\sqrt{s}$, it is of interest to know how the latter changes with time in the quark-gluon plasma. Shown in Fig. 7 by symbols

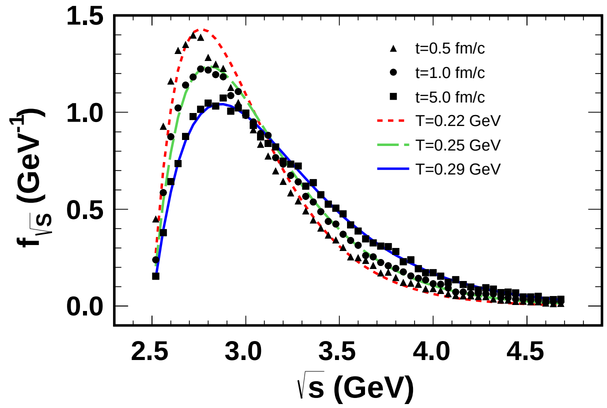

FIG. 7: Center of mass frame energy $\sqrt{s}$ distribution of colliding heavy quark pairs with initial back-to-back momentum of 5 GeV/c in a medium of temperature $T = 0.3$ GeV at different times (shown by symbols). Lines are collision rates from calculations based on thermalized heavy quarks.

are the distribution $f_{\sqrt{s}}$ at different times for a pair of heavy quarks with an initial back-to-back momentum of 5 GeV/c in a medium of temperature $T = 0.3$ GeV. It is seen that the peak of the distribution increases to larger center of mass energy $\sqrt{s}$ with increasing time. Also shown in the figure by lines is the center of mass energy distribution of heavy quark pairs that have a thermal distribution. According to Eqs. (3) and (4), the latter is given by

$$f^{\rm th}_{\sqrt{S}}(\sqrt{s}, T) \equiv \frac{1}{N^{\rm th}_{Q\bar{Q}}} \frac{dN^{\rm th}_{Q\bar{Q}}}{d\sqrt{s}} = \frac{s(s-4m^2)K_1(\sqrt{s^*})}{2^4 T^2 m^3 K_3(2m^*)} \quad (10)$$

with $s^* = s/T^2$, and $N^{\rm th}_{Q\bar{Q}}$ is the collision number between thermal Q and $\bar{Q}$ within a certain time. It shows that the distribution of $\sqrt{s}$ at $t = 0.5$ fm/c, which is comparable to the mean free time for the heavy quarks, can be roughly described by a thermal distribution at a lower temperature $T = 0.22$ GeV than that of the medium. This is because the heavy quarks carry very little momenta after reversing their direction of motion at this time. At $t = 1.0$ fm/c, the distribution of $\sqrt{s}$ can well be described by $T = 0.25$ GeV since the heavy quarks are partially thermalized. At an even later time $t = 5$ fm/c, the distribution of $\sqrt{s}$ approaches the equilibrium one and can thus be described by an effective temperature of $T = 0.29$ GeV close to that of the medium. The approaching of the $\sqrt{s}$ distribution to the thermal distribution from a lower temperature is very different from that of the p_z distribution, which approaches the thermal distribution gradually from an initial hard distribution as

[2] The space dimension $d = 3$ is important here. If the dimension is $d = 1$ or $d = 2$, the heavy quark pair will collide with each other at a certain time since the integral in Eq. (9) diverges at $t = \infty$, and the equilibrium between the heavy quark pair and quarkonia can be established even if the system is infinitely large.

shown in Fig. 2[3].

Similarly, the distribution of the total momentum $|\mathbf{P}|$ of colliding charm pairs, which can also be approximately described by a thermal distribution, is found to have an effective temperature of $0.36, 0.33$ and 0.30 GeV at time $t = 0.5, 1.0$, and 5.0 fm/c, respectively. The thermal like distributions of $\sqrt{s}$ and $|\mathbf{P}|$ implies that the regeneration of heavy quarkonia from a medium is always dominated by heavy quarks with low momentum when they are rare. For example, neither the high p_T J/ψ at SPS nor the high p_T Υ at RHIC are expected to be produced from regeneration although the heavy quarks may not be thermalized.

IV. CONCLUSIONS

Based on the Boltzmann equation, we have studied the effective volume $V_{\rm eff}$ of a correlated classical heavy quark pair in a hot medium based on their collision rate for rare heavy quark events, which is more realistic than simply considering the volume of the fireball as in the statistical model and also used in essentially most studies based on the kinetic approach. The $V_{\rm eff}$ is finite due to their initial spatial and momentum correlations even though the system is an open one like in heavy ion collisions. We have found that $V_{\rm eff}$ is proportional to the time t when it is much shorter than their mean free time $\tau_{\rm mean}$ between collisions with the medium partons, i.e., $t \ll \tau_{\rm mean}$. The increase becomes slower for $t \sim \tau_{\rm mean}$, and eventually $V_{\rm eff}$ increases as $t^{3/2}$ for $t \gg \tau_{\rm mean}$. Consequently, the chance for a heavy quark pair to collide with each other per unit time decreases monotonically with time t. Also, the chance for the heavy quarks to collide again depends sensitively on their initial momentum and the temperature of the medium. Heavy quarks of lower initial momentum in a medium of higher temperature have a larger chance to collide. Furthermore, the distribution of heavy quark pair center of mass energy corresponds to an effective temperature which is lower than the actual temperature of the medium. All these properties are important for quarkonium regeneration in collisions where heavy quarks are rarely produced. We have considered in the present study an initial heavy quark pair that have exact opposite momenta as given in the leading order QCD. Including the contribution from the next-to-leading order would make the momenta of the heavy quark pair less back-to-back, thus decreasing their effective volume. On the other hand, including more heavy quark pairs in a heavy ion collision event would increase the effective volume. Finally, we have studied the correlation effect using heavy quark scattering cross sections with partons and among themselves that are not from specific model calculations. Although the above results are not expected to qualitatively change, a quantitative study requires more accurate cross sections, which we leave as a future work. Also, it is important to include the initial correlations of heavy quark pairs in the transport model to study the regeneration contribution to quarkonia production in heavy ion collisions from heavy quarks in the produced quark-gluon plasma.

[3] The periodic condition there has no effect in the momentum space.

Acknowledgements

We thank Ralf Rapp and Nu Xu for helpful discussions. This work was supported by the U.S. National Science Foundation under Grant No. PHY-1068572, the US Department of Energy under Contract No. DE-FG02-10ER41682, and the Welch Foundation under Grant No. A-1358.

Appendix A: Conditions for heavy quark collisions

Consider one pair of free heavy quark Q and antiquark $\bar{Q}$ with their 4-dimensional coordinates x_{Q0} and $x_{\bar{Q}0}$, and velocities u_Q and $u_{\bar{Q}}$, respectively. Their trajectories in space are

$$\begin{aligned} \mathbf{x}_Q(t_Q) &= \mathbf{x}_{Q0} + \frac{\mathbf{u}_Q}{u^0_Q}(t_Q - t_{Q0}), \\ \mathbf{x}_{\bar{Q}}(t_{\bar{Q}}) &= \mathbf{x}_{\bar{Q}0} + \frac{\mathbf{u}_{\bar{Q}}}{u^0_{\bar{Q}}}(t_{\bar{Q}} - t_{\bar{Q}0}). \end{aligned} \tag{A1}$$

Then

$$\begin{aligned} s^2(t_Q, t_{\bar{Q}}) &\equiv (x_Q(t_Q) - x_{\bar{Q}}(t_{\bar{Q}}))^2 \\ &= (t_Q - t_{\bar{Q}})^2 - [\mathbf{x}_Q(t_Q) - \mathbf{x}_{\bar{Q}}(t_{\bar{Q}})]^2. \end{aligned} \tag{A2}$$

When the minimum distance b between Q and $\bar{Q}$ in their center of mass frame is reached, it is a saddle point of $s^2(t_Q, t_{\bar{Q}})$. Requiring

$$\partial_{t_Q} s^2 = \partial_{t_{\bar{Q}}} s^2 = 0, \tag{A3}$$

we find the minimum distance b and the corresponding times t_Q and $t_{\bar{Q}}$ to be

$$\begin{aligned} b &= \Big(-(\Delta x)^2 - \big[(\Delta x \cdot u_Q)^2 + (\Delta x \cdot u_{\bar{Q}})^2 \\ &\quad - 2(\Delta x \cdot u_Q)(\Delta x \cdot u_{\bar{Q}})u_{Q\bar{Q}}\big] / \left(u^2_{Q\bar{Q}} - 1\right)\Big)^{1/2}, \\ t_Q &= t_{Q0} + \frac{-\Delta x \cdot u_Q + (\Delta x \cdot u_{\bar{Q}})u_{Q\bar{Q}}}{u^2_{Q\bar{Q}} - 1} u^0_Q, \\ t_{\bar{Q}} &= t_{\bar{Q}0} + \frac{\Delta x \cdot u_{\bar{Q}} - (\Delta x \cdot u_Q)u_{Q\bar{Q}}}{u^2_{Q\bar{Q}} - 1} u^0_{\bar{Q}}, \end{aligned} \tag{A4}$$

with $\Delta x = x_{\bar{Q}0} - x_{Q0}$ and $u_{Q\bar{Q}} = u_Q \cdot u_{\bar{Q}}$. During the time interval $(t, t + \Delta t)$, the two particles are regarded as undergoing a collision if and only if $b \le \sqrt{\sigma_{Q\bar{Q}}/\pi}$ and $t < (t_Q + t_{\bar{Q}})/2 < t + \Delta t$ are satisfied.

Appendix B: Proof of the linear behavior of the effective volume $V_{\rm eff}$ at short times

To investigate the time dependence of $V_{\rm eff}$, we consider a pair of heavy quarks in a medium and follow their motions. When the time t is much smaller than the mean free time τ, a heavy quark can have at most one collision with the partons in the medium. Therefore, if we slow down the time by a factor λ to $t' = \lambda t$ and also stretch the coordinates by λ to $l' = \lambda l$, so that the velocities of the particles remain unchanged, then the number of heavy quark collisions in the original and the scaled space-time are related by

$$\begin{aligned} & \Delta N'_{Q\bar{Q}}(t', \Delta t', \sigma'_{Q\bar{Q}}, \sigma'_{gQ}, f'_g) \\ = \; & \Delta N_{Q\bar{Q}}(t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g) \end{aligned} \tag{B1}$$

with $\Delta t' = \lambda \Delta t$, $\sigma'_{Q\bar{Q}} = \lambda^2 \sigma_{Q\bar{Q}}$, $\sigma'_{gQ} = \lambda^2 \sigma_{gQ}$, and $f'_g = \lambda^{-3} f_g$. On the other hand, since the number of heavy quark collisions at a given time is, up to a constant, given by

$$\Delta N_{Q\bar{Q}} \propto \Delta t \sigma_{Q\bar{Q}} (\sigma_{gQ} f_g)^2. \tag{B2}$$

we have

$$\begin{aligned} & \Delta N'_{Q\bar{Q}}(t', \Delta t', \sigma'_{Q\bar{Q}}, \sigma'_{gQ}, f'_g) \\ = \; & \frac{\Delta t' \sigma'_{Q\bar{Q}} (\sigma'_{gQ} f'_g)^2}{\Delta t \sigma_{Q\bar{Q}} (\sigma_{gQ} f_g)^2} \Delta N_{Q\bar{Q}}(t', \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g) \\ = \; & \lambda \Delta N_{Q\bar{Q}}(\lambda t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g). \end{aligned} \tag{B3}$$

Combining Eqs. (B1) and (B3), we obtain

$$\begin{aligned} & \Delta N_{Q\bar{Q}}(\lambda t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g) \\ = \; & \lambda^{-1} \Delta N_{Q\bar{Q}}(t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g), \end{aligned} \tag{B4}$$

and therefore

$$V_{\rm eff}(\lambda t) \; = \; \lambda V_{\rm eff}(t). \tag{B5}$$

This proves our claim that the effective volume $V_{\rm eff}$ is linearly proportional to t, as long as t is much smaller than the mean free time τ between the collisions of heavy quarks with medium partons. Because the scaling in Eq.(B1) does not change either the velocities of particles or the angular distribution after their collisions, this proof is independent of the details of the cross sections σ_{gQ} or $\sigma_{Q\bar{Q}}$.

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