Large-N_{c} limit reduces the number of independent few-body parity-violating low-energy constants in pionless effective field theory

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Abstract

The symmetries of the Standard Model dictate that for very low energies, where nucleon dynamics can be described in terms of a pionless effective field theory (EFT($\not\pi$)), the leading-order parity-violating nucleon-nucleon Lagrangian contains five independent unknown low-energy constants (LECs). We find that imposing the approximate symmetry of QCD that appears when the number of colors N_c becomes large reduces the number of independent LECs to two at leading order in the combined EFT($\not\pi$) and large-N_c expansions. We also find a relation between the two isoscalar LECs in the large-N_c limit. This has important implications for the number of experiments and/or lattice calculations necessary to confirm this description of physics. In particular, we find that a future measurement of the PV asymmetry in $\vec{\gamma}d \rightarrow np$ together with the existing result for PV $\vec{p}p$ scattering would constrain all leading-order (in the combined expansion) LECs. This is considerably improved from the previous understanding of the system.

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I. INTRODUCTION

While the attempt to understand parity violation in few-nucleon systems has challenged scientists for decades and generated new experimental techniques and new theoretical paradigms, only more recently have weak interactions been recognized as a potential tool for probing QCD in few-nucleon systems. Weak interactions are well understood in isolation on the quark level, but how they are embedded in the nonperturbative environment of the nucleon/nucleus is not. The hope is that the lever of the weak interaction symmetry can expose QCD behavior. For reviews of parity violation in nucleon-nucleon interactions, see, e.g., Refs. [1–4].

Because QCD is nonperturbative at low energies, attempts to make rigorous predictions in this energy regime typically either involve discretizing QCD and attempting to solve it numerically on the lattice, or building effective field theories (EFTs) that, while applicable only in a narrow energy range, include QCD predictions. EFTs of QCD involve a number of unknown low-energy constants (LECs). The values of these LECs are not fixed by the symmetries of QCD but must be extracted from experiment or calculated on a lattice. Fortunately, up to the order we work in the EFT power counting, there are more observables available (even if not easily measurable) than LECs, which is why the EFT can have predictive power.

An EFT utilizes one or more small parameters obtained from ratios of disparate scales in the problem, which are then used as expansion parameters in a perturbative calculation. To increase calculational power, it is desirable to find as many small parameters as possible and to invoke as many symmetries (or near symmetries) of nature as possible. In addition to small parameters formed from ratios of masses and momenta, another expansion parameter available in QCD is $1/N_c$ in the limit where the number of colors N_c becomes large [5, 6]. Inclusion of this additional expansion typically results in relationships among LECs at a given order in the EFT power counting, thus reducing the number of LECs that occur at a given order in this combined expansion. An example is the heavy baryon chiral perturbation theory analysis of the interaction of the octet and decuplet of baryons with the octet of mesons. Expansion parameters include p or m_q in the numerator and Λ_χ and m_B in the denominator, with p a typical momentum transfer, m_q a light quark mass, Λ_χ the chiral symmetry breaking scale, and m_B the baryon mass. Imposing an SU(3) flavor symmetry

yields four LECs for describing leading strong interactions. However, when the approximate symmetry of large N_c is included, only one independent LEC remains in the combined expansion. Further, measured decay rates indicate that this dual expansion treatment works well [7].

At low enough energies, pionless EFT (EFT($\not{\pi}$)) is well established as a systematic and rigorous way to determine the restrictions that QCD symmetries place on few-nucleon observables in the parity-conserving (PC) sector. See, e.g., Refs. [8–10] for reviews and Ref. [11] for an example of a high-precision calculation. More recently the methods have been adapted to the parity-violating (PV) sector. At energies where the pion mass is not dynamical (i.e., $E < m_\pi^2/M_N$), QCD and weak interaction symmetries dictate that five LECs encode all of the physics of parity violation among two nucleons up to and including next-to-leading order (NLO) corrections expected at the 10 percent level [12–15].

Decades of experimental and theoretical effort have been dedicated to measuring parity violation and understanding its interplay with QCD. In the EFT framework, this can be viewed as an effort to determine whether the QCD-symmetry-based description is indeed consistent by overconstraining the values of the above LECs. Given the difficulties in gathering reliable experimental (or lattice QCD[1]) information on the LECs, any additional theoretical constraint is valuable.

A general classification of the PV potential in the large-N_c picture was given in Ref. [17], where they found that there are two leading-order (LO) terms, four terms at NLO, and six terms at next-to-next-to-leading order (NNLO) in the large-N_c counting. This information was used to analyze PV meson-nucleon couplings that appear in the meson-exchange picture of PV nucleon-nucleon forces [18]. Here, we consider the implications that a large-N_c analysis has on the LECs of EFT($\not{\pi}$). The benefit of the EFT($\not{\pi}$) formalism is that it is valid for the experiments we wish to describe, has fewer LECs than theories that contain mesons, and has a well-understood prescription for including higher-order corrections. The results of this paper continue the efforts of Ref. [4, 15, 19–22] to express two- and three-body PV processes in a unified description. We show that in a combined EFT($\not{\pi}$) and large-N_c expansion, the five EFT($\not{\pi}$) LECs are not equally important – only two are of leading order, as expected from the findings of Ref. [17]. Arriving at this result requires a careful analysis of the Fierz relations that are used when constructing a minimal form of the EFT($\not{\pi}$) Lagrangian (for an

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[1] See Ref. [16] for the first connected diagram calculation of the PV pion-nucleon coupling in lattice QCD.

example see Appendix A). One of the two dominant couplings is the isotensor LEC that may be accessible in the reaction $\vec{\gamma}d \to np$ at an upgraded High Intensity Gamma-Ray Source (HIGS) at the Triangle Universities Nuclear Laboratory [22, 23]. In addition, we find that the two isoscalar EFT($\not{\pi}$) LECs are equal up to order $1/N_c^2$, i.e., up to corrections expected to be at the 10% level. This presents a significant constraint on the EFT($\not{\pi}$) LECs because it matches or surpasses the expected precision of upcoming relevant experiments.

II.   LARGE-N_c ANALYSIS OF THE PARITY-VIOLATING POTENTIAL

The terms contributing to the PV nucleon-nucleon potential up to NNLO in the large-N_c counting were determined in Ref. [17], which used the definition of the general nucleon-nucleon potential [24]

$$V(\mathbf{p}_-, \mathbf{p}_+) = \langle (\mathbf{p}'_1, \gamma, c), (\mathbf{p}'_2, \delta, d)|\hat{H}|(\mathbf{p}_1, \alpha, a), (\mathbf{p}_2, \beta, b)\rangle, \tag{1}$$

where Greek indices denote the spin components and Latin indices the isospin components of the nucleons, respectively.[2] The momenta $\mathbf{p}_\pm$ are given by

$$\mathbf{p}_\pm \equiv \mathbf{p}' \pm \mathbf{p}, \tag{2}$$

where $\mathbf{p}'$ and $\mathbf{p}$ are the outgoing and incoming relative momenta,

$$\mathbf{p}' = \mathbf{p}'_1 - \mathbf{p}'_2, \quad \mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2. \tag{3}$$

As defined, both $\mathbf{p}_+$ and $\mathbf{p}_-$ are P-odd, and $\mathbf{p}_+$ is T-odd, while $\mathbf{p}_-$ is T-even. The Hamiltonian $\hat{H}$ is the Hartree Hamiltonian, which can be written as [6, 24]

$$\hat{H} = N_c \sum_n \sum_{s,t} v_{stn} \left(\frac{\hat{S}}{N_c}\right)^s \left(\frac{\hat{I}}{N_c}\right)^t \left(\frac{\hat{G}}{N_c}\right)^{n-s-t}, \tag{4}$$

where

$$\hat{S}^i = \hat{q}^\dagger \frac{\sigma^i}{2} \hat{q}, \qquad \hat{I}^a = \hat{q}^\dagger \frac{\tau^a}{2} \hat{q}, \qquad \hat{G}^{ia} = \hat{q}^\dagger \frac{\sigma^i \tau^a}{4} \hat{q}. \tag{5}$$

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[2] This approach does not take into account the effects of virtual baryons, such as the Δ resonance. In the large-N_c limit nucleons and the Δ resonance become degenerate, and as shown in Ref. [25] inclusion of the Δ is important for the interpretation of the potential in the large-N_c limit to be consistent with meson exchange. Here we project onto the subspace of nucleons, assuming that this does not affect the large-N_c analysis. In a more detailed approach the Δ resonance would be integrated out [26, 27].

The $\hat{q}$'s denote the doublet of light quarks, which in the large-N_c analysis are colorless bosonic fields. The coefficients v_{stn} contain momenta. All vector, spin, and isospin indices (not shown in Eq. (4)) are contracted to give the desired symmetry properties. In the case considered here, we require the resulting potential to be invariant under rotations, P-odd, and T-even. The large-N_c scaling of the terms in the potential are determined from the matrix elements of the operators $\hat{S}$, $\hat{I}$, $\hat{G}$, and the identity operator $\mathbb{1}$ between nucleon states, which are given by [28, 29]

$$\langle N'|\hat{S}|N\rangle \sim \langle N'|\hat{I}|N\rangle \sim 1, \quad \langle N'|\hat{G}|N\rangle \sim \langle N'|\mathbb{1}|N\rangle \sim N_c. \tag{6}$$

Products of these operators acting on the same nucleon state can be reduced to single factors of the operators using operator identities and the Wigner-Eckart theorem [28]. In addition, the momenta scale as [24, 28, 29]

$$\mathbf{p}_- \sim 1, \quad \mathbf{p}_+ \sim N_c^{-1}. \tag{7}$$

The reason for the suppression of $\mathbf{p}_+$ is as follows [24]: it is consistent to interpret the potential derived from the Hartree Hamiltonian in terms of a meson-exchange picture [25]. In the t-channel, a factor of $\mathbf{p}_+$ can only appear as a relativistic correction and is therefore always accompanied by an inverse power of the nucleon mass M_N. Since $M_N \sim N_c$, $\mathbf{p}_+$ is taken to scale as N_c^{-1}. In the u-channel the roles of $\mathbf{p}_+$ and $\mathbf{p}_-$ are reversed, but it is common to take this exchange potential into account by considering matrix elements of the potential between states that are properly anti-symmetrized. A parametrization of the PV potential in terms of one-meson exchanges is given in Ref. [18]. Starting from the relativistic Hamiltonians for PC and PV meson-nucleon interactions given there, a nonrelativistic expansion of the resulting potential satisfies the above counting, i.e., each term proportional to $\mathbf{p}_+$ is suppressed by a factor of $1/M_N$. See Appendix B for details.

Using these ingredients, Ref. [17] determined the operator structures that appear in the PV NN potential up to NNLO in the large-N_c counting. At LO, i.e., $\mathcal{O}(N_c)$, they are

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2, \tag{8}$$

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, \mathcal{I}_{ab}\tau_1^a\tau_2^b, \tag{9}$$

where $\mathcal{I} = \text{diag}(1, 1, -2)$. The NLO terms $(\mathcal{O}(N_c^0) \sin^2 \theta_W)$ are

$$\mathbf{p}_+ \cdot (\vec{\sigma}_1 \tau_1^3 - \vec{\sigma}_2 \tau_2^3) \ , \tag{10}$$

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \, (\vec{\tau}_1 \times \vec{\tau}_2)^3 \ , \tag{11}$$

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \, (\vec{\tau}_1 + \vec{\tau}_2)^3 \ , \tag{12}$$

$$[(\mathbf{p}_+ \times \mathbf{p}_-) \cdot \vec{\sigma}_1 \, \mathbf{p}_- \cdot \vec{\sigma}_2 + (\mathbf{p}_+ \times \mathbf{p}_-) \cdot \vec{\sigma}_2 \, \mathbf{p}_- \cdot \vec{\sigma}_1] \, (\vec{\tau}_1 \times \vec{\tau}_2)^3 \ , \tag{13}$$

while at NNLO $(\mathcal{O}(N_c^{-1}))$

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \ , \tag{14}$$

$$\mathbf{p}_+^2 \, \mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \, \vec{\tau}_1 \cdot \vec{\tau}_2 \ , \tag{15}$$

$$\mathbf{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \ , \tag{16}$$

$$\mathbf{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \, \vec{\tau}_1 \cdot \vec{\tau}_2 \ , \tag{17}$$

$$\mathbf{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \, \mathcal{I}_{ab} \tau_1^a \tau_2^b \ , \tag{18}$$

$$\mathbf{p}_+^2 \, \mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \, \mathcal{I}_{ab} \tau_1^a \tau_2^b \ . \tag{19}$$

In the physical world, $N_c = 3$ and the expansion parameter is $1/3$. However, in some cases observables or relations between operators are protected from corrections at the next order, which leads to corrections of roughly 10%. Similarly, the power counting parameter in EFT($\not{\pi}$) is $\frac{p}{\Lambda_{\not{\pi}}} \sim 1/3$, with p a typical momentum and $\Lambda_{\not{\pi}} \lesssim m_\pi$, but the correction to the leading PV S-P wave mixing does not occur until two orders higher [15].

III. LARGE-N_c ANALYSIS OF PARITY-VIOLATING LOW-ENERGY CONSTANTS IN EFT($\not{\pi}$)

Our starting point is the LO EFT($\not{\pi}$) Lagrangian in the minimal form as given in Ref. [14],

$$\begin{aligned}
\mathcal{L}_{PV}^{\min} = \ & \mathcal{G}_1 (N^\dagger \vec{\sigma} N \cdot N^\dagger i \overleftrightarrow{D} N - N^\dagger N N^\dagger \vec{\sigma} \cdot i \overleftrightarrow{D} N) \\
& - \tilde{\mathcal{G}}_1 \epsilon_{ijk} N^\dagger \sigma^i N D^j (N^\dagger \sigma^k N) \\
& - \mathcal{G}_2 \epsilon_{ijk} \left[N^\dagger \tau^3 \sigma^i N D^j (N^\dagger \sigma^k N) + N^\dagger \sigma^i N D^j (N^\dagger \tau^3 \sigma^k N) \right] \\
& - \tilde{\mathcal{G}}_5 \mathcal{I}_{ab} \epsilon_{ijk} N^\dagger \tau^a \sigma^i N D^j (N^\dagger \tau^b \sigma^k N) \\
& + \mathcal{G}_6 \epsilon_{ab3} \vec{D} (N^\dagger \tau^a N) \cdot N^\dagger \tau^b \vec{\sigma} N \ ,
\end{aligned} \tag{20}$$

where

$$D_\mu N = \partial_\mu N + ie \frac{1 + \tau^3}{2} A_\mu N \tag{21}$$

is the nucleon covariant derivative and we define $a \, \mathcal{O}\overset{\leftrightarrow}{D}b$ to be[3]

$$a \, \mathcal{O}\overset{\leftrightarrow}{D}b = a \, \mathcal{O}\vec{D}b - (\vec{D}a)\mathcal{O}b, \tag{22}$$

with $\mathcal{O}$ some spin-isospin-operator. The $\mathcal{G}_i$ are related to the $\mathcal{C}_i$ of Ref. [14] by $\mathcal{G}_i = \mathcal{C}_i/\Lambda_\chi^3$, with Λ_χ the scale of chiral symmetry breaking. The potential derived from this Lagrangian is

$$
\begin{aligned}
V^{\min} = & - \mathcal{G}_1 \mathbf{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\
& - i\tilde{\mathcal{G}}_1 \mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \\
& - i\mathcal{G}_2 \mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)(\tau_1 + \tau_2)^3 \\
& - i\tilde{\mathcal{G}}_5 \mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
& + \frac{i}{2}\mathcal{G}_6 \mathbf{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)(\tau_1 \times \tau_2)^3 \ .
\end{aligned}
\tag{23}
$$

Comparing the terms in Eq. (23) with the terms given in Eqs. (8)-(19), a naive (and incorrect) assignment of large-N_c scaling to the PV LECs appears to be:

$$
\begin{aligned}
\tilde{\mathcal{G}}_5 & \sim N_c, \\
\mathcal{G}_2 \sim \mathcal{G}_6 & \sim N_c^0 \sin^2 \theta_W, \\
\mathcal{G}_1 \sim \tilde{\mathcal{G}}_1 & \sim N_c^{-1}.
\end{aligned}
\tag{24}
$$

This would suggest that in the combined EFT($\not{\pi}$) and large-N_c expansion the isoscalar couplings are suppressed compared to the isotensor and isovector ones. This seems to conflict with the general form of the potential in Eqs. (8)-(19), which contains an isoscalar part at LO in the large-N_c counting.

The minimal form of the potential of Eq. (23) is found by starting with the most general Lorentz-invariant form of the PV Lagrangian with a single derivative and applying Fierz identities as well as a nonrelativistic reduction [13, 14].[4] Because Fierz identities allow a non-unique set of operators to be expressed as linear combinations of other operators, there

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[3] This definition is not universal. Some authors use the opposite sign convention.

[4] In Ref. [14] the Fierz identities are applied to the relativistic forms of the operators in the basis of Dirac matrices. First performing the nonrelativistic reduction and then applying Fierz identities to the resulting Pauli matrices produces the same set of five nonrelativistic operators.

exists a freedom to choose which operators to eliminate in favor of others in the "minimal" form of the Lagrangian. From the EFT perspective, all of these choices are equivalent because the Fierz identities only apply to spin-isospin operators, but do not change the number of derivatives or dimensionful quantities such as masses. On the other hand, the large-N_c counting is related to the spin-isospin structure of the operators, and therefore it is possible to find operators at different orders in $1/N_c$ that are related by Fierz identities. Thus, different minimal forms of the Lagrangian, while equivalent in the EFT counting, can have different large-N_c scaling. To find the most conservative estimate of large-N_c behavior, we start from the most general form of the potential, identify the N_c scaling of the operator coefficients, and then apply Fierz identities to reduce the number of operators while maintaining the most dominant N_c scaling for each resulting coefficient.[5]

As discussed in Refs. [13, 14], the most general relativistic PV Lagrangian with one derivative contains 12 operators before the application of any Fierz identities. Two of these operators ($\mathcal{O}_6$ and $\tilde{\mathcal{O}}_6$ in the notation of Ref. [13]) result in the same leading nonrelativistic structure, while two of the other nonrelativistic operators (those proportional to $\tilde{C}_2$ and $\tilde{C}_4$ in Ref. [13]) are related by integration by parts.[6] Thus, the most general nonrelativistic Lagrangian contains 10 operators and is given by (cf. Ref. [13])

$$
\begin{aligned}
\mathcal{L}_{PV}^{\text{nonmin}} = {} & \mathcal{A}_1^+[N^\dagger N(N^\dagger \vec{\sigma} \cdot i\overset{\leftrightarrow}{D} N) - N^\dagger \vec{\sigma} N \cdot (N^\dagger i\overset{\leftrightarrow}{D} N)] \\
& + \mathcal{A}_1^- \epsilon_{ijk} N^\dagger \sigma^i N D^j (N^\dagger \sigma^k N) \\
& + \mathcal{A}_2^+[N^\dagger N(N^\dagger \tau^3 \vec{\sigma} \cdot i\overset{\leftrightarrow}{D} N) - N^\dagger \tau^3 \vec{\sigma} N \cdot (N^\dagger i\overset{\leftrightarrow}{D} N)] \\
& + \mathcal{A}_2^- \epsilon_{ijk} N^\dagger \sigma^i N D^j (N^\dagger \tau^3 \sigma^k N) \\
& + \mathcal{A}_3^+[N^\dagger \tau^a N(N^\dagger \tau^a \vec{\sigma} \cdot i\overset{\leftrightarrow}{D} N) - N^\dagger \tau^a \vec{\sigma} N \cdot (N^\dagger \tau^a i\overset{\leftrightarrow}{D} N)] \\
& + \mathcal{A}_3^- \epsilon_{ijk} N^\dagger \tau^a \sigma^i N D^j (N^\dagger \tau^a \sigma^k N) \\
& + \mathcal{A}_4^+[N^\dagger \tau^3 N(N^\dagger \vec{\sigma} \cdot i\overset{\leftrightarrow}{D} N) - N^\dagger \vec{\sigma} N \cdot (N^\dagger \tau^3 i\overset{\leftrightarrow}{D} N)] \\
& + \mathcal{A}_5^+ \mathcal{I}_{ab}[N^\dagger \tau^a N(N^\dagger \tau^b \vec{\sigma} \cdot i\overset{\leftrightarrow}{D} N) - N^\dagger \tau^a \vec{\sigma} N \cdot (N^\dagger \tau^b i\overset{\leftrightarrow}{D} N)] \\
& + \mathcal{A}_5^- \mathcal{I}_{ab} \epsilon_{ijk} N^\dagger \tau^a \sigma^i N D^j (N^\dagger \tau^b \sigma^k N) \\
& + \mathcal{A}_6^- \epsilon_{ab3} N^\dagger \tau^a N \vec{D} \cdot (N^\dagger \tau^b \vec{\sigma} N) \,,
\end{aligned}
\tag{25}
$$

where the $\mathcal{A}_i^+$ correspond to terms resulting in a $\mathbf{p}_+$ in the potential, while the $\mathcal{A}_i^-$ terms

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[5] We thank L. Girlanda for discussions on this point, see also [30].

[6] This was indicated in Ref. [14].

give a factor of $\mathbf{p}_-$. The corresponding potential can be written as (cf. Ref. [13])

$$
\begin{aligned}
V^{\mathrm{nonmin}} = {}& \mathcal{A}_1^+ \mathbf{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\
& + \mathcal{A}_1^- \mathbf{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2) \\
& + \mathcal{A}_2^+ \, \mathbf{p}_+ \cdot (\vec{\sigma}_1 \tau_1^3 - \vec{\sigma}_2 \tau_2^3) \\
& + \frac{1}{2} \mathcal{A}_2^- \mathbf{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)(\tau_1 + \tau_2)^3 \\
& + \mathcal{A}_3^+ \, \mathbf{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2 \\
& + \mathcal{A}_3^- \, \mathbf{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2 \\
& + \mathcal{A}_4^+ \, \mathbf{p}_+ \cdot (\vec{\sigma}_1 \tau_2^3 - \vec{\sigma}_2 \tau_1^3) \\
& + \mathcal{A}_5^+ \, \mathbf{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
& + \mathcal{A}_5^- \mathbf{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
& - \frac{1}{2} \mathcal{A}_6^- \mathbf{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)i(\tau_1 \times \tau_2)^3 \ .
\end{aligned}
\tag{26}
$$

The nonminimal potential V^{nonmin} contains all operators of Eqs. (8)-(19) as identified in Ref. [17] with the exception of Eqs. (13), (15), and (19). These operators contain more than one power of momentum and therefore cannot be reproduced by starting from operators with a single derivative. In addition, the operator structure multiplied by $\mathcal{A}_4^+$ is of order N_c^{-2} and was not considered in Ref. [17]. We extract the following large-N_c scaling of the couplings:

$$
\begin{aligned}
\mathcal{A}_1^+ &\sim N_c^{-1} \ , & \mathcal{A}_1^- &\sim N_c^{-1} \ , \\
\mathcal{A}_2^+ &\sim N_c^0 \sin^2 \theta_W \ , & \mathcal{A}_2^- &\sim N_c^0 \sin^2 \theta_W \ , \\
\mathcal{A}_3^+ &\sim N_c^{-1} \ , & \mathcal{A}_3^- &\sim N_c \ , \\
\mathcal{A}_4^+ &\sim N_c^{-2} \ , & & \\
\mathcal{A}_5^+ &\sim N_c^{-1} \ , & \mathcal{A}_5^- &\sim N_c \ , \\
& & \mathcal{A}_6^- &\sim N_c^0 \sin^2 \theta_W \ .
\end{aligned}
\tag{27}
$$

Applying Fierz identities to arrive at the minimal form of Eq. (23), the relations between

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the nonminimal and minimal couplings are

$$\mathcal{G}_1 = -\mathcal{A}_1^+ + \mathcal{A}_3^+ - 2\mathcal{A}_3^- \ ,$$

$$\tilde{\mathcal{G}}_1 = -\mathcal{A}_1^- - 2\mathcal{A}_3^+ + \mathcal{A}_3^- \ ,$$

$$\mathcal{G}_2 = -\frac{1}{2}\left(\mathcal{A}_2^- + \mathcal{A}_2^+ + \mathcal{A}_4^+\right) \ , \qquad (28)$$

$$\tilde{\mathcal{G}}_5 = -\left(\mathcal{A}_5^- + \mathcal{A}_5^+\right) \ ,$$

$$\mathcal{G}_6 = -\mathcal{A}_6^- + \mathcal{A}_2^+ - \mathcal{A}_4^+ \ .$$

A detailed example is worked out in Appendix A. The scaling of these couplings is determined by the leading behavior of the LECs $\mathcal{A}_i^{\pm}$ of Eq. (27), which gives

$$\mathcal{G}_1 \sim N_c \ ,$$

$$\tilde{\mathcal{G}}_1 \sim N_c \ ,$$

$$\mathcal{G}_2 \sim N_c^0 \sin^2\theta_W \ , \qquad (29)$$

$$\tilde{\mathcal{G}}_5 \sim N_c \ ,$$

$$\mathcal{G}_6 \sim N_c^0 \sin^2\theta_W \ .$$

The isotensor coupling is LO and the isovector couplings again appear at NLO, as in the naive analysis, see Eq. (24). However, the isoscalar couplings are now also counted as LO in large N_c, unlike in the naive analysis. While it may seem that there are now three LO terms, the two isoscalar couplings are not independent. As seen from Eq. (28), the LO contribution to both $\mathcal{G}_1$ and $\tilde{\mathcal{G}}_1$ comes from the coupling $\mathcal{A}_3^-$, so that up to corrections of order $1/N_c^2$ the relation

$$\mathcal{G}_1 = -2\tilde{\mathcal{G}}_1 \qquad (30)$$

holds. This means that up to corrections on the order of approximately 10%, two of the five LECs that are independent from the EFT point of view are in fact related to each other. Given the difficulty in determining the LECs from either experiment or lattice QCD and the resulting uncertainties in such an extraction, this result presents a significant simplification.

The physics encoded in the minimal Lagrangian of Ref. [14] can also be expressed in a physically more intuitive basis, in which the incoming and outgoing two-nucleon states are

in particular partial waves [15],

$$\mathcal{L}_{PV} = - \Big[\mathcal{C}^{(^3S_1-^1P_1)} \left(N^T \sigma^2 \vec{\sigma} \tau^2 N \right)^\dagger \cdot \left(N^T \sigma^2 \tau^2 i \overset{\leftrightarrow}{D} N \right)$$
$$+ \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} \left(N^T \sigma^2 \tau^2 \vec{\tau} N \right)^\dagger \left(N^T \sigma^2 \vec{\sigma} \cdot \tau^2 \vec{\tau} i \overset{\leftrightarrow}{D} N \right)$$
$$+ \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)} \, \epsilon_{3ab} \left(N^T \sigma^2 \tau^2 \tau^a N \right)^\dagger \left(N^T \sigma^2 \vec{\sigma} \cdot \tau^2 \tau^b \overset{\leftrightarrow}{D} N \right) \qquad (31)$$
$$+ \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} \, \mathcal{I}_{ab} \left(N^T \sigma^2 \tau^2 \tau^a N \right)^\dagger \left(N^T \sigma^2 \vec{\sigma} \cdot \tau^2 \tau^b i \overset{\leftrightarrow}{D} N \right)$$
$$+ \mathcal{C}^{(^3S_1-^3P_1)} \, \epsilon_{ijk} \left(N^T \sigma^2 \sigma^i \tau^2 N \right)^\dagger \left(N^T \sigma^2 \sigma^k \tau^2 \tau^3 i \overset{\leftrightarrow}{D}{}^j N \right) \Big] + \text{H.c.} \, ,$$

where ΔI is the isospin change involved in the process. The relations between the couplings in the two formalisms are [15, 31]

$$\mathcal{C}^{(^3S_1-^1P_1)} = \frac{1}{4}(\mathcal{G}_1 - \tilde{\mathcal{G}}_1) \, ,$$
$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} = \frac{1}{4}(\mathcal{G}_1 + \tilde{\mathcal{G}}_1) \, ,$$
$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)} = \frac{1}{2}\mathcal{G}_2 \, , \qquad (32)$$
$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} = -\frac{1}{2}\tilde{\mathcal{G}}_5 \, ,$$
$$\mathcal{C}^{(^3S_1-^3P_1)} = \frac{1}{4}\mathcal{G}_6 \, .$$

These relations can be established using Fierz identites, which as discussed earlier can hide the large-N_c scaling of the associated couplings. However, since we have kept the leading terms for the couplings $\mathcal{G}_i$, the relations of Eq. (32) also establish the leading scaling behavior for the partial-wave couplings $\mathcal{C}^{(X-Y)}$,

$$\mathcal{C}^{(^3S_1-^1P_1)} \sim N_c \, ,$$
$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} \sim N_c \, ,$$
$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)} \sim N_c^0 \sin^2 \theta_W \, , \qquad (33)$$
$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} \sim N_c \, ,$$
$$\mathcal{C}^{(^3S_1-^3P_1)} \sim N_c^0 \sin^2 \theta_W \, .$$

As before, the two isoscalar terms are not independent at leading order in the large-N_c counting, but up to $1/N_c^2$ corrections are related by

$$\mathcal{C}^{(^3S_1-^1P_1)} = 3\,\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} \, . \qquad (34)$$

Again, the large-N_c analysis shows a relation between two of the five LECs, which is valid at the $\approx 10\%$ level. A determination of one of the isoscalar couplings from either experiment or

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a future lattice QCD calculation constrains the second isoscalar LEC significantly without need of further experimental or lattice QCD input.

The relations derived above apply to renormalized LECs. As common in EFT($\not{\pi}$), we assume that dimensional regularization in combination with the power divergence subtraction renormalization scheme [32] is applied to obtain any observable. As shown in Refs. [15, 19], the dependence of the PV LECs on the subtraction point μ is the same as the corresponding S-wave PC LECs, given by [32]

$$\mathcal{C}^{(^1S_0)} \sim \frac{1}{-1/a^{(^1S_0)} - \mu} \,, \quad \mathcal{C}^{(^3S_1)} \sim \frac{1}{-1/a^{(^3S_1)} - \mu} \,. \tag{35}$$

In the large-N_c limit the LO PC interactions are Wigner-SU(4) symmetric [33], which implies that $\mathcal{C}^{(^1S_0)} = \mathcal{C}^{(^3S_1)}$ in this limit [29]. As a result the μ dependence of all PC and PV LO LECs becomes identical. In general, since we are considering the scaling of the LECs with N_c, the relations should hold for any renormalization condition that itself is independent of the number of colors. However, as pointed out in Ref. [29], the approximate Wigner-SU(4) symmetry is hidden for the physical case of $N_c = 3$ if the strong LECs are fit close to threshold, which corresponds to choosing μ close to zero as the renormalization condition. The S-wave scattering lengths are unnaturally large and choosing $\mu = 0$ means that the applicability of the EFT($\not{\pi}$) expansion is very limited [32]. From the EFT point of view it is therefore beneficial to use a renormalization condition in which μ is of the order of momenta significantly above threshold. In these cases, the ratio of PC LECs also gives a much better indication of the approximate Wigner-SU(4) symmetry. Because physical observables relevant to hadronic parity violation always involve an interplay of PC and PV interactions, i.e., both PC and PV LECs appear, renormalization conditions in which the consequences of the large-N_c limit are manifest in the PC LECs should be chosen.

IV. APPLICATION TO EXISTING MEASUREMENTS

There has been only one nonzero measurement of a PV two-nucleon observable: the longitudinal asymmetry in $\vec{p}p$ scattering.[7] The lowest energy result was found in Ref. [35–37]:

$$A_L^{\vec{p}p}(E = 13.6 \text{ MeV}) = (-0.93 \pm 0.21) \times 10^{-7} \,. \tag{36}$$

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[7] The NPDGamma [34] experiment has finished taking data and a result for the PV angular asymmetry in $\vec{n}p \to d\gamma$ is expected to be announced shortly.

This gives the following constraint on a linear combination of the EFT($\not{\pi}$) LECs [15]

$$\frac{4(\mathcal{C}_{(\Delta I=0)}^{(^1S_0-^3P_0)} + \mathcal{C}_{(\Delta I=1)}^{(^1S_0-^3P_0)} + \mathcal{C}_{(\Delta I=2)}^{(^1S_0-^3P_0)})}{\mathcal{C}^{(^1S_0)}} = (-1.5 \pm 0.3) \times 10^{-10} \text{ MeV}^{-1} \quad , \tag{37}$$

where $\mathcal{C}^{(^1S_0)}$ is the strong interaction LEC. The ratio of the PV and strong LECs is subtraction point independent. Because $\mathcal{C}_{(\Delta I=0)}^{(^1S_0-^3P_0)}$ and $\mathcal{C}_{(\Delta I=2)}^{(^1S_0-^3P_0)}$ dominate in the large-N_c limit, we will neglect $\mathcal{C}_{(\Delta I=1)}^{(^1S_0-^3P_0)}$. In particular, one immediate result is that in the large-N_c limit, the longitudinal asymmetry of $\vec{p}p$ scattering and $\vec{n}n$ scattering is the same [15]. Imposing the large-N_c relationship between $\mathcal{C}^{(^3S_1-^1P_1)}$ and $\mathcal{C}_{(\Delta I=0)}^{(^1S_0-^3P_0)}$ yields

$$\frac{4(\mathcal{C}^{(^3S_1-^1P_1)}/3 + \mathcal{C}_{(\Delta I=2)}^{(^1S_0-^3P_0)})}{\mathcal{C}} = (-1.5 \pm 0.3) \times 10^{-10} \text{ MeV}^{-1} \quad , \tag{38}$$

where $\mathcal{C}$ denotes the large-N_c limit of $\mathcal{C}^{(^1S_0)}$. With only one equation and two unknowns, it is not possible to make any statement about the relative size of the two LECs.

However, the reason for expressing $\mathcal{C}_{(\Delta I=0)}^{(^1S_0-^3P_0)}$ in terms of $\mathcal{C}^{(^3S_1-^1P_1)}$ is that an experimental limit exists on the induced circular polarization in unpolarized neutron capture, $np \to d\vec{\gamma}$ [38, 39], which also depends on $\mathcal{C}^{(^3S_1-^1P_1)}$. Imposing the large-$N_c$ results on the expression for this process [19] yields

$$P_\gamma = -\frac{16M_N}{\mathcal{C}} \frac{1}{\kappa_1(1-\gamma a^{(^1S_0)})} \left(\mathcal{C}^{(^3S_1-^1P_1)}(1 - \frac{5}{9}\gamma a^{(^1S_0)}) - \frac{2}{3}\gamma a^{(^1S_0)} \mathcal{C}_{(\Delta I=2)}^{(^1S_0-^3P_0)} \right)$$
$$= (1.8 \pm 1.8) \times 10^{-7} \quad , \tag{39}$$

where we have also set $\mathcal{C}^{(^3S_1)} \sim \mathcal{C}$ as dictated by the large N_c limit [29]. γ is the binding momentum of the deuteron, κ_1 is the isovector anomalous magnetic moment, and $a^{(^1S_0)}$ is the singlet channel scattering length. This relationship allows us to say with some confidence that $\mathcal{C}_{(\Delta I=2)}^{(^1S_0-^3P_0)}$ is of the same size as $\mathcal{C}^{(^3S_1-^1P_1)}$. If it were not, but negligible compared to $\mathcal{C}^{(^3S_1-^1P_1)}$, then the $\vec{p}p$ scattering measurement would yield

$$\frac{\mathcal{C}^{(^3S_1-^1P_1)}}{\mathcal{C}} \approx (-1.1 \pm 0.2) \times 10^{-10} \text{ MeV}^{-1} \quad , \tag{40}$$

and predict a P_γ larger than its present bound.

V. CONCLUSIONS

We have shown that a large-N_c symmetry imposed on the EFT($\not{\pi}$) description of low-energy parity violation in two-nucleon systems through NLO in EFT($\not{\pi}$) power counting

reduces the number of independent LECs from five to two at LO in the combined expansion. This is in agreement with the general large-N_c analysis of PV nucleon-nucleon forces of Ref. [17]. However, we find the result expressed in the EFT($\not{\pi}$) formalism to be more useful for interpreting experimental measurements in few-nucleon systems, both because there are fewer LECs and because analyzing higher-order corrections is straightforward. Further, the result in the EFT($\not{\pi}$) formalism is not obvious from a naive application of the large-N_c counting rules to the minimal form of the EFT($\not{\pi}$) potential. The reason is that the Fierz identities used in reducing the most general Lagrangian to its minimal form can hide the order in large-N_c counting at which a given term first contributes. We have also found a relation between the two isoscalar LECs $\mathcal{C}^{(^3S_1-^1P_1)}$ and $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)}$ that is expected to hold up to corrections of order $1/N_c^2$, i.e., on the order of 10%.

This could have important implications for how quickly parity violation in two- and few-nucleon systems can be understood. In particular, the existing measurement of the longitudinal asymmetry in $\vec{p}p$ scattering along with the limit of the $np \to d\vec{\gamma}$ PV asymmetry suggests that the two PV LECs that are of LO in the combined EFT($\not{\pi}$) and large-N_c expansion, $\mathcal{C}^{(^3S_1-^1P_1)}$ and $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}$, are in fact of the same size. That is, current experiments support the leading-order analysis. One linear combination of these two LECs can be determined from the existing experimental result on $\vec{p}p$ scattering. If the result that these two LECs are of the same size holds when higher-order EFT($\not{\pi}$) and large-N_c corrections are included, it is a motivation to measure $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}$ at the potential HIGS2 facility [23] and gives further motivation to perform a lattice calculation of $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}$ [40]. Because of the large-N_c relation between $\mathcal{C}^{(^3S_1-^1P_1)}$ and $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)}$, knowledge of $\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}$ would then constrain both isoscalar LECs.

Appendix A: Example of the application of Fierz identities

As an example of how to reduce the nonminimal EFT($\not{\pi}$) Lagrangian to its minimal form, consider the term proportional to $\mathcal{A}_3^-$,

$$\epsilon_{ijk}N^\dagger\tau^A\sigma^i N D^j(N^\dagger\tau^A\sigma^k N)\,, \tag{A1}$$

which in components reads

$$(\tau^A_{ab}\tau^A_{cd})(\epsilon_{ijk}\sigma^i_{\alpha\beta}\sigma^k_{\gamma\delta})N^\dagger_{\alpha,a}N_{\beta,b}D^j(N^\dagger_{\gamma,c}N_{\delta,d})\,. \tag{A2}$$

Applying the Fierz identity

$$\tau_{ab}^A \tau_{cd}^A = 2\delta_{ad}\delta_{cb} - \delta_{ab}\delta_{cd}, \tag{A3}$$

the second term immediately gives

$$-\epsilon_{ijk} N^\dagger \sigma^i N D^j \cdot (N^\dagger \sigma^k N) , \tag{A4}$$

i.e., the operator structure proportional to $\mathcal{A}_1^-$. For the remaining term, the Fierz identity in spin space

$$\sigma_{\alpha\beta}^i \sigma_{\gamma\delta}^k = \frac{1}{2} \left[\sigma_{\alpha\delta}^i \sigma_{\gamma\beta}^k + \sigma_{\alpha\delta}^k \sigma_{\gamma\beta}^i + \delta^{ik}(\delta_{\alpha\delta}\delta_{\gamma\beta} - \sigma_{\alpha\delta}^l \sigma_{\gamma\beta}^l) + i\epsilon_l^{ik}(\sigma_{\alpha\delta}^l \delta_{\gamma\beta} - \delta_{\alpha\delta}\sigma_{\gamma\beta}^l) \right] \tag{A5}$$

can be applied. In addition, using integration by parts and dropping the term proportional to a total derivative, the combination of nucleon fields can be rewritten as

$$\begin{aligned} N_{\alpha,a}^\dagger N_{\beta,b} D^j (N_{\gamma,c}^\dagger N_{\delta,d}) &= \frac{1}{2} \left[N_{\alpha,a}^\dagger N_{\beta,b} D^j (N_{\gamma,c}^\dagger N_{\delta,d}) - D^j (N_{\alpha,a}^\dagger N_{\beta,b})(N_{\gamma,c}^\dagger N_{\delta,d}) \right] \\ &= \frac{1}{2} \left[N_{\alpha,a}^\dagger N_{\delta,d}(N_{\gamma,c}^\dagger \overset{\leftrightarrow}{D} N_{\beta,b}) - (N_{\alpha,a}^\dagger \overset{\leftrightarrow}{D} N_{\delta,d}) N_{\gamma,c}^\dagger N_{\beta,b}) \right]. \end{aligned} \tag{A6}$$

The resulting term has the same form as the operator structure proportional to $\mathcal{A}_1^+$. Therefore, the term proportional to $\mathcal{A}_3^-$ can be removed by a redefinition of $\mathcal{A}_1^+$ and $\mathcal{A}_1^-$.

Appendix B: Momentum dependence in the PV meson-exchange potential

As an example of how the suppression of terms proportional to $\mathbf{p}_+$ with a factor of $1/M_N$ arises in the meson-exchange picture, we consider the PV potential as given in Ref. [18], which parameterizes the PV interactions in terms of π, ρ, and ω exchange. Here we further restrict the discussion to ω exchange, but analogous arguments apply to π and ρ exchanges. The PC Hamiltonian is given by [18]

$$\mathcal{H}_{PC}^\omega = g_\omega \bar{\Psi} \left(\gamma^\mu + i\frac{\chi_S}{2\Lambda_\chi} \sigma^{\mu\nu} k_\nu \right) \omega_\mu \Psi , \tag{B1}$$

where compared to Ref. [18] we have replaced a factor of $M_N \sim N_c$ by $\Lambda_\chi \sim 1$ in the denominator (see the discussion in Ref. [17]) because the origin of this scale is not dynamical and should not introduce spurious N_c dependence. The PV Hamiltonian reads

$$\mathcal{H}_{PV}^\omega = \bar{\Psi} \left(h_\omega^0 \omega_\mu + h_\omega^1 \tau^3 \omega_\mu \right) \gamma^\mu \gamma_5 \Psi . \tag{B2}$$

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Under a nonrelativistic expansion, the terms proportional to g_ω are LO for $\mu = 0$ and suppressed by $1/M_N$ for $\mu = i$, while terms proportional to $g_\omega \chi_S$ scale as $1/M_N$ for $(\mu\nu) = (0i)$ and are LO for $(\mu\nu) = (ij)$. Analogously, the PV terms scale as $1/M_N$ for $\mu = 0$ and are LO for $\mu = i$. Combining these expressions, the terms proportional to $\mathbf{p}_+$ arise in combination with $g_\omega h_\omega^{0,1}$ and are suppressed by $1/M_N$, while terms proportional to $\mathbf{p}_-$ come with $g_\omega \chi_S h_\omega^{0,1}$ and are LO in the nonrelativistic expansion. Analogous relations hold for π and ρ exchanges.

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