

This is the accepted manuscript made available via CHORUS. The article has been published as:

Evidence for x-dependent proton color fluctuations in pA collisions at the CERN Large Hadron Collider

M. Alvioli, B. A. Cole, L. Frankfurt, D. V. Perepelitsa, and M. Strikman

Phys. Rev. C **93**, 011902 — Published 21 January 2016

DOI: [10.1103/PhysRevC.93.011902](https://doi.org/10.1103/PhysRevC.93.011902)

Evidence for x -dependent proton color fluctuations in pA collisions at the LHC

M. Alvioli,¹ B.A. Cole,² L. Frankfurt,³ D.V. Perepelitsa,⁴ and M. Strikman⁵

¹*Consiglio Nazionale delle Ricerche, Istituto di Ricerca per la Protezione Idrogeologica, via Madonna Alta 126, I-06128 Perugia, Italy*

²*Columbia University, New York, NY 10027, USA*

³*Tel Aviv University, Tel Aviv, Israel*

⁴*Brookhaven National Laboratory, Upton, NY 11973, USA*

⁵*104 Davey Lab, The Pennsylvania State University, University Park, PA 16803, USA*

The centrality-dependence of forward jet production in pA collisions at the LHC has been found to grossly violate the Glauber model prediction in a way that depends on the x in the proton. We argue that this modification pattern provides the first experimental evidence for x -dependent proton color fluctuation effects. On average, parton configurations in the projectile proton containing a parton with large x interact with a nuclear target with a significantly smaller than average cross section and have smaller than average size. Implementing the effects of fluctuations of the interaction strength and using the ATLAS analysis of how hadron production at backward rapidities depends on the number of wounded nucleons, we make quantitative predictions for the centrality dependence of the jet production rate as a function of the x -dependent interaction strength $\sigma(x)$. We find that $\sigma(x) \sim 0.6 \langle \sigma \rangle$ gives a good description of the data at $x = 0.6$. These findings support an explanation of the EMC effect as arising from the suppression of small size nucleon configurations in the nucleus.

PACS numbers: 14.20.Dh, 25.40.Ve, 13.85.-t, 25.75.

Studies of microscopic nucleon structure have progressed from probing single parton distributions to the study of generalized parton distributions (3D single parton distributions) and of parton-parton correlations in multi-parton interactions. Here we argue that correlations between soft and hard scattering processes in very high energy proton-nucleus (pA) collisions probe how the transverse area occupied by partons in a fast nucleon depends on the light cone momentum fraction (x) of the trigger parton. The projectile nucleon propagates through the nucleus in a frozen quark-gluon configuration, leading to the coherence of its interactions. Due to the color screening property of high energy QCD, the overall interaction strength of a color neutral configuration drops with a decrease in its transverse size [1]. Such phenomena have been observed directly in, for example, hard diffractive jet production [2] and in exclusive meson production [3, 4]. The presence of configurations of the nucleon which have smaller than average interaction strength results in new phenomena which are observable in the centrality dependence of single jet [5] or dijet [6] production in pA collisions at the Large Hadron Collider (LHC).

To visualize the origin of fluctuation phenomena in high energy processes, consider the propagation of an ultra-relativistic positronium atom through a slab of matter with density ρ and length L . From the uncertainty principle and Lorentz dilation of the interaction, the transverse distance r_t between the electron and positron remains constant over a longitudinal coherence distance $l_{coh} \sim (1/\Delta E) \sim \frac{2P_{pos}}{M^2 - m_{pos}^2}$, where P_{pos} is the momentum of the positronium and M is the mass of the intermediate (diffractive) state. In the limit $l_{coh} \gg L$, a new absorption pattern emerges. This is due to coherence effects and to the dependence of the interaction

cross-section σ of the e^+e^- dipole with a target on r_t , which at small transverse distance is $\sigma(r_t^2) \propto r_t^2$.

In this limit, the probability of an inelastic interaction, $1 - \kappa$, is obtained by summing over a complete set of diffractive intermediate states with different r_t , weighted by the value of the positronium wave function at r_t ,

$$\kappa = \int dz d^2r_t \psi^2(z, r_t) \exp[-\sigma_{in}(r_t^2)\rho L]. \quad (1)$$

Here, $\psi(z, r_t)$ is the positronium wave function normalized as $\int \psi^2(r) d^3r = 1$, and σ_{in} is the inelastic positronium-target cross-section. Then, κ is the probability for positronium to transform to an e^+e^- pair without inelastic interactions. For $\sigma_{in}\rho L \gg 1$, the survival rate is $\kappa \approx 2/(\sigma_{in}\rho L)$ [7], which is larger than the naive expectation $\approx \exp(-\sigma_{in}\rho L)$. Since σ_{in} depends on r_t , positronium can be captured in a larger (smaller) configuration by selecting events with more (fewer) excited atoms in the target.

In QCD, fluctuations in the interaction strength of a hadron h with a nucleon originate from fluctuations in both the transverse size and in the number of constituents of the hadron. We refer to both generically as color fluctuations (CF). CF effects can be accounted for by introducing a probability distribution, $P_h(\sigma)$, for the hadron to be found in a configuration with total cross-section σ for the interaction. This probability distribution obeys the sum rules $\int P_h(\sigma) d\sigma = 1$ and $\int P_h(\sigma) \sigma d\sigma = \langle \sigma \rangle \equiv \sigma_{tot}^{hN}$ where $\langle \sigma \rangle$ is the configuration-averaged (total) cross section. The variance of the distribution divided by the mean squared, ω_σ , is given by the optical theorem [8, 9],

$$\omega_\sigma = (\langle \sigma^2 \rangle / \langle \sigma \rangle^2 - 1) = \left. \frac{\frac{d\sigma(h+p \rightarrow X+p)}{dt}}{\frac{d\sigma(h+p \rightarrow h+p)}{dt}} \right|_{t=0}, \quad (2)$$

where a sum over diffractively produced states X , including the triple Pomeron contribution [10], is implied. Fixed target and collider data [11] indicate that ω_σ for the proton first grows with energy, reaching $\omega_\sigma \sim 0.3$ for $\sqrt{s} \sim 100$ GeV, then decreases at higher energies to $\omega_\sigma \sim 0.1$ at the LHC [10].

Several considerations constrain the shape of $P_h(\sigma)$ [11]. For values of $\sigma \sim \langle \sigma \rangle$, $P_h(\sigma)$ is expected to be Gaussian due to small fluctuations in the number of, and in the transverse area occupied by, partons. This expectation is supported by an analysis [11] of coherent diffraction measurements in proton-deuteron collisions [12]. For $\sigma \ll \langle \sigma \rangle$, configurations with a small number constituents, n_q , localized in a small transverse area should dominate, leading to $P_h(\sigma) \propto \sigma^{n_q-2}$ [11]. For protons, the resulting form of $P_p(\sigma)$ and values of ρ, σ_0, Ω were chosen to smoothly interpolate between both regimes while reproducing measurements of the first three moments of the distribution. It is given by

$$P_p(\sigma) = \frac{\rho}{\sigma_0} \left(\frac{\sigma}{\sigma + \sigma_0} \right) \exp \left\{ -\frac{(\sigma/\sigma_0 - 1)^2}{\Omega^2} \right\}. \quad (3)$$

For the Gaussian distribution $\omega_\sigma = \Omega^2/2$.

To determine the inelastic cross-section σ_ν for the proton to interact with ν nucleons in pA collisions, the standard Gribov formalism [13] at high energies can be generalized to include CF effects [14]. When the impact parameters in nucleon-nucleon (NN) interactions are small compared to the typical distance between neighboring nucleons,

$$\sigma_\nu = \int d\sigma P_p(\sigma) \binom{A}{\nu} \times \int db \left[\frac{\sigma_{in}(\sigma)T(b)}{A} \right]^\nu \left[1 - \frac{\sigma_{in}(\sigma)T(b)}{A} \right]^{A-\nu}, \quad (4)$$

where $T(b) = \int_{-\infty}^{\infty} dz \rho(z, b)$ and ρ is the nuclear density distribution normalized such that $\int \rho(r) d\mathbf{r} = A$. $\sigma_{in}(\sigma)$ is the inelastic cross-section for a configuration with the given total cross-section, which following Ref. [10, 11] is taken to be a fixed fraction of σ . In the limit of no CF effects, $P_p(\sigma) = \delta(\sigma - \langle \sigma \rangle)$, and Eq. 4 reduces to the Glauber model expression. The distribution over ν can be calculated with a Monte Carlo Glauber procedure, which includes NN correlations and finite size effects [15]. For $\nu \leq 2\langle \nu \rangle$ the distribution over ν depends mainly on ω_σ and only weakly on the exact form of $P_p(\sigma)$ [15]. Although the Glauber approximation ignores energy-momentum conservation in the inelastic interaction of the proton with multiple nucleons, this does not modify the calculation of σ_ν , or of the hadron multiplicity close to the nuclear fragmentation region [10]. (However, energy-momentum conservation effects may be important in the evaluation of multiplicities at forward and central rapidities.) This approach also accounts

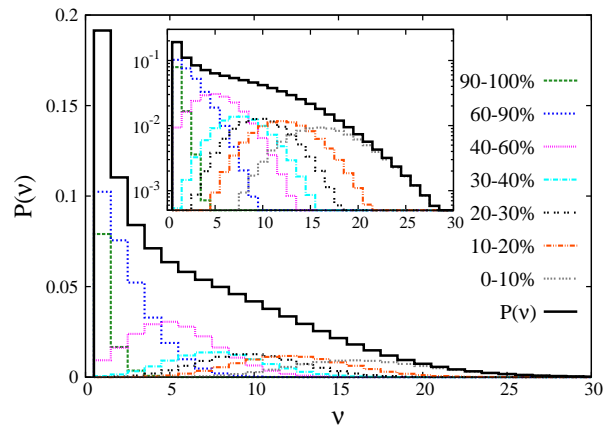


FIG. 1: (Color online) Probability distributions of ν proton-nucleon collisions in all pA collisions and in those selected by different ΣE_T , or centrality, ranges. The inset shows the distributions on a log scale.

both for inelastic shadowing [16] and for the possibility of intermediate diffractive states between successive collisions [10, 11].

ATLAS has studied the role of CF effects in interpreting the correlation between hadron production at central rapidities and at $-4.9 < \eta < -3.2$ in the nucleus-going direction in pA collisions at $\sqrt{s} = 5$ TeV [17]. The total transverse energy, ΣE_T , near the nuclear fragmentation region is not expected to be influenced by energy conservation effects (due to the approximate Feynman scaling in this region), or to be strongly correlated with the activity in the rapidity-separated central and forward regions. This expectation is validated by a recent measurement of ΣE_T as a function of hard scattering kinematics in pp collisions [18]. In Ref. [17], ΣE_T distributions were constructed as a function of the number of participating nucleons $\nu + 1$. Neglecting CF effects in calculating ν resulted in ΣE_T distributions narrower than those observed in data. Using the CF approach, which generally leads to a broader ν distribution from the $\sigma > \langle \sigma \rangle$ tail of $P_p(\sigma)$ [14], with the value $\omega_\sigma = 0.1$ estimated for LHC energies gives a reasonable description of the ΣE_T data, while $\omega_\sigma \sim 0.2$ produces an overly broad distribution. The resulting $\Sigma E_T(\nu)$ parameterization was used to calculate the relative contributions from collisions with different ν values to the pA centrality classes (bins in ΣE_T) used by ATLAS. Fig. 1 demonstrates that these centrality-selected distributions have well separated mean values.

A challenging question is whether the fluctuations are modified or amplified when a parton carrying a fraction x of the projectile momentum is present in the parton configuration, and, in particular, whether large x partons originate from configurations with smaller than average σ . In the positronium example above, let the $e^-(e^+)$ momentum in the rest frame be $\pm \vec{k}^e$, and apply a boost to a fast frame. The light cone fraction carried by the elec-

tron is $x_e = 1/2 + k_3^e/2m_e$. If $|x_e - 1/2| \gg \langle |k_3^e|/2m_e \rangle$, the positronium is squeezed, leading to the cancellation of the photon field discussed above.

An analogous effect is present in the models of hadrons where few quarks in the rest frame interact with each other through a potential which is Coulomb-like at short distances, cf. the discussion in section 5.1 of Ref. [19]. In these models, the size of a given configuration decreases with increasing quark momentum. Thus quarks with large x in the fast frame arise from configurations with large relative momenta in the rest frame and, thus, a smaller size. The density of the gluon field in these configurations is necessarily reduced. Additionally, in QCD the main contribution to the parton density in configurations with $x \gg \langle x \rangle$ is from configurations with the minimal number of partons, leading to the quark counting rules [20]. Hence for such configurations, the nucleon's $q\bar{q}$ cloud is suppressed, leading to a reduction of the soft cross-section even on the non-perturbative scale [21]. This picture is corroborated through a body of experimental evidence such as, for example, measurements of coherent dijet production in pion-nucleus collisions [2].

Jet production measurements [5, 6] in inclusive pA collisions at the LHC confirmed the pQCD expectations for the total production rate. In the following, we focus on the data from ATLAS which is directly adaptable to our analysis, although qualitatively similar effects were observed by CMS [6]. For centrality-selected pA collisions, the jet production rate, σ_ν^{hard} for collisions with ν wounded nucleons, was compared to the Glauber model expectation through the ratio

$$R_\nu^{hard} = (\sigma_\nu^{hard}/\sigma_\nu) / (\nu \cdot \sigma_{NN}^{hard}/\langle \sigma_{in} \rangle), \quad (5)$$

where σ_{NN}^{hard} is the jet rate in NN collisions, and $\langle \sigma_{in} \rangle = \sigma_{in}^{NN}$. Large deviations from the expected $R_\nu^{hard} = 1$ were observed for jets produced along the proton-going direction: namely, an enhancement for peripheral (small ν) collisions and a suppression for central (large ν) collisions, which compensate each other in the inclusive cross section. These findings are not sensitive either to finite size effects [10] in the Glauber modeling nor, as explained above, to energy-conservation effects. In the ATLAS data [5], R_ν^{hard} is presented as a function of the fraction of the energy of the proton carried by the jet $z = E_{jet}/E_p$, which, in the forward kinematics of interest, coincides with the x of the parton in the proton involved in the hard interaction¹. The data demonstrate that for fixed energy release in the nuclear hemisphere (ΣE_T) R_ν^{hard} is predominantly a function of z and not of the jet p_T or rapidity alone.

¹ Studying the correlation between the initial parton-parton kinematics and the final state kinematics of the produced jets in MC event generators confirms that events with a forward jet with energy E have a distribution of x values narrowly peaked around $x = 2E/\sqrt{s}$.

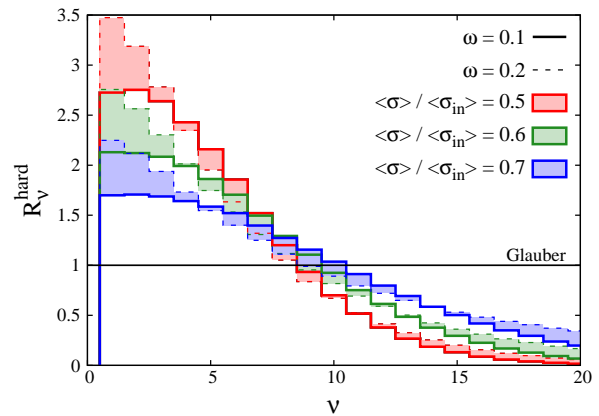


FIG. 2: (Color online) Probability of a hard process corresponding to a small $\lambda = \langle \sigma_{in}(x) \rangle / \langle \sigma_{in} \rangle$ trigger selection relative to that for a generic hard processes, as given in Eq. 5 for $\omega_\sigma(x) = 0.1, 0.2$. $R_\nu^{hard} = 1$ is the expectation of the Glauber model.

In our quantitative analysis, we combine the ATLAS model for the ν -dependence of ΣE_T with the distribution of ν given by a CF Monte Carlo approach [10, 15]. The strength of the interaction σ_{in} at given impact parameter, b , is generated with the measure $P_h(\sigma)$ with inelastic profile function $\Gamma_{in}(b)$ evaluated using the optical theorem. In the evaluation, the elastic amplitude is taken to be proportional to $\exp(Bt/2)$ with the t -slope B fixed by the requirement that for small NN impact parameter the interaction is nearly black as it is for the total pp amplitude. The transverse spread of partons in the colliding nucleons were generated using generalized parton densities which take into account a much stronger localization of hard interactions relative to soft ones as well as the spatial NN correlations in the nucleus [22]. (For a detailed description of the calculation of the hard collision rate as a function of ν see Ref. [10].)

Fig. 2 shows R_ν^{hard} as a function of the ratio λ of the average strength of the inelastic interaction for a trigger with a given x , $\langle \sigma_{in}(x) \rangle$ to that averaged over all configurations, $\langle \sigma_{in} \rangle$,

$$\lambda = \langle \sigma_{in}(x) \rangle / \langle \sigma_{in} \rangle. \quad (6)$$

For the generic hard collisions we used Eq. 3 with $\omega_\sigma = 0.1$ which, within ATLAS, provides a good description of soft production. For the small $\sigma_{in}(x)$ triggers in Fig. 2, we considered a range of λ values and $\omega_\sigma(x)$ values between 0.1 and 0.2. The relative jet production rate corresponding to small $\langle \sigma_{in}(x) \rangle$ is enhanced at small ν and strongly suppressed at large ν . With the exception of the most peripheral and most central collisions, R_ν^{hard} depends primarily on the mean $\langle \sigma_{in}(x) \rangle$ and not on the choice of $\omega_\sigma(x)$. Thus we fix $\omega_\sigma(x) = 0.1$ and reduce the dependence of R_ν^{hard} on ν to only one free x -dependent parameter, λ .

R_ν^{hard} values were then calculated for the specific centrality bins used by ATLAS and compared to the R_ν^{hard}

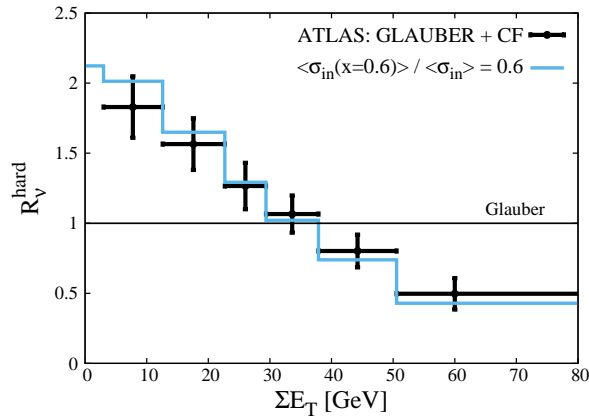


FIG. 3: (Color online) R_V^{hard} for pA collisions with $x = E_{jet}/E_p = 0.6$, and $\lambda = 0.6$ for centrality bins extracted from the ATLAS data [5] and using ν distributions given by the CF model [17]. Errors are combined statistical and systematic errors. The solid line is the Glauber model expectation.

extracted from data. For each centrality bin, we consider the measured $R_{pPb}(p_T \cosh y)$ values from the four rapidity intervals in the range $0.3 < y < 2.8$. These were fit to a linear function in $\log(x/0.6)$ in the range $0.04 < x < 1$, with $x \equiv 2p_T \cosh y / \sqrt{s}$ and $y > 0$ denoting the proton-going direction, and the value at $x = 0.6$ was extracted. Statistical uncertainties estimated by evaluating the RMS deviation of the data points from the linear function in the region of the fit were combined with systematic uncertainties on the data points to yield total uncertainties. Figure 3 shows that $\lambda \sim 0.6$ gives a good description of the data at $x = 0.6$. We emphasize that a naive interpretation of the data due to jet energy loss cannot explain either the modification pattern in the centrality-dependent R_V^{hard} , which features both enhancement and suppression, or the observation of $R^{hard} = 1$ for inclusive collisions, which follows from QCD factorization.

Figure 4 shows the predictions of our model for R_V^{hard} in each centrality bin as a function of λ . These predictions could be tested by extending the current analysis of the LHC pA data to $x < 0.6$ as well as by analyzing the RHIC dA data [23]. The magnitude of the deviations from $R_V^{hard} = 1$ increase smoothly with decreasing $\lambda < 1$. For larger than average size configurations, corresponding to $\lambda > 1$, the modification pattern reverses, producing an enhancement and suppression in central and peripheral events respectively.

The agreement of the data for $x = 0.6$ with our calculation using $\lambda = 0.6$ has a number of implications. It demonstrates that large x configurations have a weaker than average interaction strength. More generally, it confirms the presence of CF effects in pA interactions and suggests that they should contribute to the dynamics of central AA collisions [14]. It is in line with the QCD quark counting rules which assume that large x partons belong to configurations with a minimal number

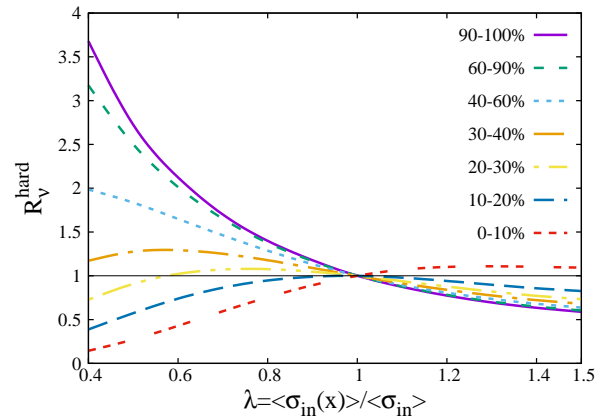


FIG. 4: (Color online) R_V^{hard} for different centralities as a function of λ .

of constituents interacting via hard gluon exchanges [20]. However, it is in tension with approaches which neglect the short range correlations between hadron constituents, such as the model in [24], and with those in which the transverse size of the hadron is not squeezed at large x .

To explore the energy dependence of this effect, the value of λ at fixed x can be determined at two different energies $\sqrt{s_1}$ and $\sqrt{s_2}$ through the probability conservation of $P_h(\sigma)$: $\int_0^{\sigma(\sqrt{s_1})} P_h(\sigma, \sqrt{s_1}) d\sigma = \int_0^{\sigma(\sqrt{s_2})} P_h(\sigma, \sqrt{s_2}) d\sigma$. At 30 GeV, $\lambda \approx 1/4$, a factor of two smaller than at the LHC. This follows from the fact that in pQCD, the cross-section of small-size configurations grows faster with increasing collision energy than that of average configurations.

A weaker interaction strength for configurations with $x \geq 0.5$ has implications for our understanding of the EMC effect. This follows from the analysis of Ref. [21], in which the Schrodinger equation for the bound state of the nucleus included a potential term which depends on the internal coordinates of the nucleons. In this potential, the overall attractive nature of the NN interaction results in a smaller binding energy for nucleons in small configurations. Thus, by the variational principle, the probability of such configurations is suppressed. The magnitude of this suppression is comparable to the strength of the EMC effect at $x \geq 0.5$, as further discussed in Ref. [25].

Future pA data in which the dijet kinematics are used to determine x and x_A on an event-by-event basis would allow for a more detailed study of nucleon structure. In particular, studying the modification pattern for hard processes to which gluons with $x \geq 0.3$ significantly contribute would probe whether the squeezing of the transverse size is also present for configurations with large- x gluons. Similarly, studying dijet production for $x \leq 0.01$ would allow for a study of configurations which have a higher than average interaction strength. Measurements at lower energies would reveal the energy dependence of $\langle\sigma(x)\rangle$ and test whether it grows much faster with energy

than σ as suggested by our analysis above. Finally, a comparison of W^+, W^- production at large x_p would allow for a comparison of the transverse structure of proton configurations with leading u and d quarks, respectively.

Acknowledgments M.A.'s research was supported by grants provided by the Regione Umbria, under contract POR-FESR Umbria 20072013, asse ii, attività a1, azione 5, and by the Dipartimento della Protezione Civile, Italy. B.A.C.'s research was supported by the US Department

of Energy Office of Science, Office of Nuclear Physics under Award No. DE-FG02-86ER40281. L.F.'s research was supported by the Binational Scientific Foundation Grant No. 0603216203. D.V.P.'s research was supported by the US Department of Energy under contract DE-AC02-98CH10886. M.S.'s research was supported by the US Department of Energy Office of Science, Office of Nuclear Physics under Award No. DE-FG02-93ER40771.

-
- [1] F. E. Low, Phys. Rev. D **12**, 163 (1975). L. L. Frankfurt and V. A. Khoze. in Proceedings of 10th LNPI Winter School, Leningrad, USSR, 1975, v2, pp 196-408. A.H. Mueller, in Proceedings of 17th rencontre de Moriond, Moriond, 1982 Van (Editions Frontieres, Gif-sur-Yvette, France, 1982) Vol. I p.13. S.J. Brodsky in Proceedings. World Scientific. of the 13th Intl Symposium on Multiparticle Dynamics, W. Kittel, W. Metzger and A. Stergiou (eds.) Singapore 1982, p.963. L. Frankfurt, G. A. Miller and M. Strikman, Phys. Lett. B **304**, 1 (1993)
 - [2] E. M. Aitala *et al.* [E791 Collaboration], Phys. Rev. Lett. **86**, 4773 (2001)
 - [3] B. Clisie *et al.*, Phys. Rev. Lett. **99**, 242502 (2007)
 - [4] L. El Fassi *et al.* [CLAS Collaboration], Phys. Lett. B **712**, 326 (2012)
 - [5] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **748**, 392 (2015)
 - [6] S. Chatrchyan *et al.* [CMS Collaboration], Eur. Phys. J. C **74**, no. 7, 2951 (2014)
 - [7] L. Frankfurt and M. Strikman, Prog. Part. Nucl. Phys. **27**, 135 (1991).
 - [8] H. I. Miettinen and J. Pumplin, Phys. Rev. D **18**, 1696 (1978).
 - [9] M. L. Good and W. D. Walker, Phys. Rev. **120** (1960) 1857.
 - [10] M. Alvioli, L. Frankfurt, V. Guzey and M. Strikman, Phys. Rev. C **90**, no. 3, 034914 (2014)
 - [11] B. Blaettel, G. Baym, L. L. Frankfurt and M. Strikman, Phys. Rev. Lett. **70**, 896(1993); Phys. Rev. D **47**, 2761(1993).
 - [12] Y. Akimov, L. Golovanov, S. Mukhin, V. Tsarev, E. Malamud, R. Yamada, P. Zimmerman and R. Cool *et al.*, Phys. Rev. D **14**, 3148 (1976).
 - [13] L. Bertocchi and D. Treleani, J. Phys. G **3** (1977) 147.
 - [14] H. Heiselberg, G. Baym, B. Blaettel, L. L. Frankfurt and M. Strikman, Phys. Rev. Lett. **67**, 2946 (1991); Phys. Rev. C **52**, 1604 (1995)
 - [15] M. Alvioli and M. Strikman, Phys. Lett. B **722**, 347 (2013)
 - [16] V. N. Gribov, Sov. Phys. JETP **29** (1969) 483.
 - [17] G. Aad *et al.* [ATLAS Collaboration], arXiv:1508.00848 [hep-ex].
 - [18] G. Aad *et al.* [ATLAS Collaboration], arXiv:1512.00197 [hep-ex].
 - [19] L. L. Frankfurt, G. A. Miller and M. Strikman, Ann. Rev. Nucl. Part. Sci. **44**, 501 (1994)
 - [20] S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).
 - [21] L. L. Frankfurt and M. I. Strikman, Nucl. Phys. B **250**, 143 (1985).
 - [22] M. Alvioli, H. -J. Drescher and M. Strikman, Phys. Lett. B **680**, 225 (2009)
 - [23] A. Adare *et al.* [PHENIX Collaboration], arXiv:1509.04657 [nucl-ex].
 - [24] S. J. Brodsky and G. F. de Teramond, Subnucl. Ser. **45**, 139 (2009)
 - [25] A. J. Freese, M. M. Sargsian and M. I. Strikman, Eur. Phys. J. C **75**, no. 11, 534 (2015)