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Prompted by recent measurements, we have surveyed world data and calculated radiative and isospin-symmetry breaking corrections for the superallowed $0^+ \rightarrow 0^+$ Fermi transitions from ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni. This increases the number of such transition with a complete set of calculated corrections from 20 to 23. The results are compared with their equivalents for the mirror superallowed transitions from ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co. The predicted ft-value asymmetries of these mirror pairs are sensitive to the correction terms and provide motivation for improving measurement precision so as to be able to test the corrections. To aid in that endeavor, we present a parameterization for calculating the f values for the new transitions to $\pm 0.01\%$.

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I. INTRODUCTION

At regular intervals over more than four decades, we have published critical surveys of world data on superallowed $0^+ \rightarrow 0^+$ Fermi β transitions and their impact on weak-interaction physics, with the last survey appearing in February 2015 [1]. In all, 20 transitions were included in this most-recent survey, of which 18 had a complete set of data, comprising in each case the Q_{EC} value, half-life and branching ratio. Of those 18, all but 4 had been measured to high precision. Our justification for including 20 cases, some of which were incomplete or poorly known, was that we deemed these 20 cases to encompass all those that were likely to be accessible to precision measurements in the near future.

By the time the survey was published, our prediction had already been proven wrong: In January 2015, Molina *et al.* [2] reported a measurement of the half-lives and Gamow-Teller branching ratios for the β decays of ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni. Although the ^{42}Ti transition was included in our survey, those of ^{46}Cr, ^{50}Fe and ^{54}Ni were not. In fact, the Q_{EC} values for the three latter transitions are still poorly known and even the new measurements of the half-lives and branching ratios have yet to reach the precision required to contribute meaningfully to any standard-model tests. Nevertheless, Molina *et al.* have convincingly demonstrated that these nuclei are indeed accessible and potentially amenable to more precise measurements.

This report is intended as an addendum to our 2015 survey [1], in which we extend the same evaluation of world data to the three new superallowed transitions and, more importantly, evaluate all the correction terms that are required to understand the results. At the same time, we take the opportunity to update results for ^{42}Ti to incorporate the new information.

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A β transition is characterized by its ft value, where f is the statistical rate function and t is its partial half-life. Three experimental quantities are required to establish the ft value: The total decay energy, Q_{EC}, is required to calculate f; and the half-life, $t_{1/2}$, and the branching ratio, R, combine as follows to produce the partial half-life:

$$t = \frac{t_{1/2}}{R}\left(1 + P_{EC}\right). \qquad (1)$$

Here P_{EC} is a small correction to account for competition from electron capture.

To the ft value, two theoretical corrections are applied to produce a corrected $\mathcal{F}t$ value, defined as

$$\begin{aligned}\mathcal{F}t &= ft(1+\delta_R)(1-\delta_C) \\ &= ft(1+\delta_R^{\prime})(1-\delta_C+\delta_{NS}). \qquad (2)\end{aligned}$$

Here δ_R is the nucleus-dependent part of the radiative correction, also called the "outer" radiative correction, and δ_C is an isospin-symmetry-breaking correction. It is convenient to subdivide δ_R further as $\delta_R = \delta_R^{\prime} + \delta_{NS}$ (see Sect. III A) and, since these quantities are small, rearrange the equation to the form displayed on the second line of Eq. (2), which is correct to first order in these corrections. This rearrangement places the nuclear-structure-dependent corrections together in the combination $\delta_C - \delta_{NS}$.

In what follows, we begin with a survey of world data for the superallowed β-decay branches of the $T_z = -1$ nuclei, ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni, from which the ft values are obtained. Next we calculate the correction terms $\delta_R^{\prime}$, δ_{NS} and δ_C, and hence obtain $\mathcal{F}t$ values for these 4 cases. These are then compared with results for the well-known superallowed decay branches from the $T_z = 0$ nuclei ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co, which are their mirror transitions. Finally we use the calculated correction terms to predict the ratio of ft values for each of the four pairs of mirror transitions. When the precision of world data is improved for the $T_z = -1$ cases, this will provide a stringent test of the correction terms [3].

TABLE I: Measured results from which the decay transition energies, Q_{EC}, have been derived for the superallowed β-decays of four $T_z = -1$ nuclei. In all cases only a single useful measurement has been made of each quantity. The lines giving the superallowed Q_{EC} values themselves are in bold print. Where no reference is given, the Q_{EC} value was determined from the difference between the measured parent and daughter mass excesses. (See Table V for the correlation between the alphanumeric reference code used in this table and the actual reference numbers.)

Parent/Daughter nuclei		Property[a]	Measured Energies used to determine Q_{EC} (keV)
^{42}Ti	^{42}Sc	$\boldsymbol{Q_{EC}(sa)}$	**7016.83 ± 0.25** [Ku09]
^{46}Cr	^{46}V	ME(p)	-29474 ± 20 [Zi72]
		ME(d)	-37074.55 ± 0.32[b]
		$\boldsymbol{Q_{EC}(sa)}$	**7600 ± 20**
^{50}Fe	^{50}Mn	ME(p)	-34489 ± 60 [Tr77]
		ME(d)	-42627.25 ± 0.90[b]
		$\boldsymbol{Q_{EC}(sa)}$	**8139 ± 60**
^{54}Ni	^{54}Co	ME(p)	-39223 ± 50 [Tr77]
		ME(d)	-48009.52 ± 0.56[b]
		$\boldsymbol{Q_{EC}(sa)}$	**8787 ± 50**

[a]Abbreviations used in this column are as follows: "sa", superallowed transition; "p", parent; "d", daughter; and "ME", mass excess; Thus, for example, "$Q_{EC}(sa)$" signifies the Q_{EC}-value for the superallowed transition, and "$ME(d)$", the mass excess of the daughter nucleus.

[b]Result obtained from the Q_{EC} value for the superallowed decay of the daughter d, which appears in Ref. [1], combined with the mass of the grand-daughter taken from [Wa12].

Our focus here is on providing information that will be useful to experimenters when such improvements have been achieved. In that context, we also tabulate the parameters needed to calculate easily the f values for the three new transitions – from ^{46}Cr, ^{50}Fe and ^{54}Ni – to high precision (±0.01%), which will be important once more precise Q_{EC} values are known. These parameters supplement those given in Ref. [4] for the 20 previously surveyed transitions.

II. EXPERIMENTAL DATA

We surveyed world data using exactly the same methods as in our 2015 survey [1] and, for consistency, we present the results here in a similar tabular format, even though relatively few references are involved. The Q_{EC} values appear in Table I, the half-lives in Table II and the branching ratios in Table III. Since the branching ratios for the decay of a $T_z = -1$ nucleus depend on a complete analysis of its spectrum of β-delayed γ rays, we give in Table IV the relative intensities of the γ rays for all four cases. As in the survey, each datum appearing in the tables is attributed to its original journal reference *via* an alphanumeric code made up of the initial two letters of the first author's name and the two last digits of the publication date. These codes are correlated with the actual reference numbers, Refs. [5]-[12], in Table V.

Several remarks can be made concerning the contents of the tables. Table I shows that the Q_{EC} value for ^{42}Ti decay has been directly measured quite recently; as reported in Ku09 [7], this was done with a Penning trap and is rather precisely known. The other three Q_{EC} values in the table had to be derived as differences between separately measured parent and daughter masses. Furthermore, all three parent masses were measured about 40 years ago, either from a reaction excitation function (see Zi72 [12]) or from the Q-values of (^{4}He, ^{8}He) reactions (see Tr77 [10]) and have large uncertainties by today's standards. Note also that the result for ^{42}Ti is the same as appeared in our 2015 survey [1].

In Tables II, III and IV the survey results for ^{42}Ti have been updated for new data from Mo15 [2]. In particular, the branching-ratio result has been changed significantly since Mo15 did not observe a β-delayed γ ray that had been attributed to ^{42}Ti. This is explained fully in footnote b of Table IV.

With the input data now settled, we can derive the ft values for the four superallowed transitions from the $T_z = -1$ nuclei, ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni. The results appear in the top four rows of Table VI, where we give the statistical rate functions, f, the electron-capture fractions, P_{EC}, the partial half-lives, t, obtained with Eq. (1), and finally the ft values. To facilitate later mirror comparisons, we also give the same information for the four mirror transitions from the $T_z = 0$ nuclei, ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co. These are identical to the results that appear in Table IX of Ref. [1].

The next step is to determine the theoretical correction terms δ'_R, δ_{NS} and δ_C. Their derivation is described in the next section.

III. THEORETICAL CORRECTIONS

A. Outer radiative correction, δ_R

As noted already, the nucleus-dependent outer radiative correction δ_R is conveniently divided into two components,

$$\delta_R = \delta'_R + \delta_{NS}. \tag{3}$$

The first comprises the bremsstrahlung and low-energy part of the γW-box graphs and is a standard QED calculation that depends only on the electron's energy and the charge, Z, of the daughter nucleus.

The calculation of δ'_R can be further broken down into four contributions [13]:

$$\delta'_R = \frac{\alpha}{2\pi}\left[\overline{g}(E_m) + \delta_2 + \delta_3 + \delta_{\alpha^2}\right]. \tag{4}$$

TABLE II: Half-lives, $t_{1/2}$, of four $T_z = -1$ superallowed β-emitters. (See Table V for the correlation between the alphabetical reference code used in this table and the actual reference numbers.)

Parent	Measured half-lives, $t_{1/2}$ (ms)				Average value	
nucleus	1	2	3	4	$t_{1/2}$ (ms)	scale
^{42}Ti	202 ± 5 [Ga69]	208.14 ± 0.45 [Ku09]	211.7 ± 1.9 [Mo15]	209.5 ± 5.2 [Mo15]	208.29 ± 0.79	1.8
^{46}Cr	224.3 ± 1.3 [Mo15]	223.9 ± 9.9 [Mo15]			224.3 ± 1.3	1.0
^{50}Fe	152.1 ± 0.6 [Mo15]	150.1 ± 2.9 [Mo15]			152.02 ± 0.59	1.0
^{54}Ni	114.2 ± 0.3 [Mo15]	114.3 ± 1.8 [Mo15]			114.20 ± 0.30	1.0

TABLE III: Measured results from which the branching ratios, R, have been derived for superallowed β-transitions from four $T_z = -1$ nuclei. The lines giving the average superallowed branching ratios themselves are in bold print. (See Table V for the correlation between the alphabetical reference code used in this table and the actual reference numbers.)

Parent/Daughter		Daughter state	Measured Branching Ratio, R (%)		Average value	
nuclei		E_x (MeV)	1	2	R (%)	scale
^{42}Ti	^{42}Sc	0.611	51.1 ± 1.1 [Ku09]	55.9 ± 3.6 [Mo15]	51.5 ± 1.3	1.3
		gs			**48.1 ± 1.4**[a]	
^{46}Cr	^{46}V	0.994	21.6 ± 5.0 [On05]	13.9 ± 1.0 [Mo15]	14.2 ± 1.4	1.5
		gs			**76.7 ± 2.3**[a]	
^{50}Fe	^{50}Mn	0.651	22.5 ± 1.4 [Mo15]		22.5 ± 1.4	
		gs			**74.3 ± 1.6**[a]	
^{54}Ni	^{54}Co	0.937	22.4 ± 4.4 [Re99]	19.8 ± 1.2 [Mo15]	19.9 ± 1.2	1.0
.		**gs**			**78.9 ± 1.2**[a]	

[a]Result also incorporates data from Table IV

TABLE IV: Relative intensities of β-delayed γ-rays in the superallowed β-decay daughters. These data are used to determine the branching ratios presented in Table III. (See Table V for the correlation between the alphabetical reference code used in this table and the actual reference numbers.)

Parent/Daughter nuclei		daughter ratios[a]	Measured γ-ray Ratio	
^{42}Ti	^{42}Sc	$\gamma_{total}/\gamma_{611}$	0.0073 ± 0.0011[b]	[Mo15]
^{46}Cr	^{46}V	$\gamma_{total}/\gamma_{994}$	0.642 ± 0.026	[Mo15]
^{50}Fe	^{50}Mn	$\gamma_{total}/\gamma_{651}$	0.158 ± 0.015	[Mo15]
^{54}Ni	^{54}Co	$\gamma_{total}/\gamma_{937}$	0.0576 ± 0.0043	[Mo15]

[a]γ-ray intensities are denoted by γ_E, where E is the γ-ray energy in keV. The notation γ_{total} appearing in a numerator denotes the sum of all β-delayed γ rays feeding the daughter ground state, excluding the strongest γ ray, which is identified in the denominator.
[b]This result replaces the result appearing in our 2015 survey [1], which came from [Ga69] and [En90]. The 2223-keV γ ray identified in [Ga69] as originating from ^{42}Ti decay evidently originated from a contaminant since it was not observed in [Mo15].

TABLE V: Reference key, relating alphabetical reference codes used in Tables I-IV to the actual reference numbers.

Table code	Reference number	Table code	Reference number	Table code	Reference number
En90	[5]	Ga69	[6]	Ku09	[7]
Mo15	[2]	On05	[8]	Re99	[9]
Tr77	[10]	Wa12	[11]	Zi72	[12]

The leading order-α term contains the function $\overline{g}(E_m)$: It is the average over the β energy spectrum of the function $g(E, E_m)$, originally defined by Sirlin [14]. Here E is the total electron energy in the β-decay transition and E_m is its maximum value. The next two terms in Eq. (4), δ_2 and δ_3, represent corrections to order $Z\alpha^2$ and $Z^2\alpha^3$ respectively. The last term is a recently-added contribution [13] that gives a correction to order α^2.

Results for all four terms and their sums are recorded in Table VII for the superallowed decays of ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni, as well as for their mirror superallowed transitions. The differences in the radiative corrections for each pair of mirror transitions are given in the last four lines of the table. They are very small.

No uncertainties on δ'_R are listed in Table VII. This issue has been discussed in our recent survey [1], where it is argued that the uncertainty on δ'_R should be treated as a systematic, rather than a statistical one. We take the magnitude of the uncertainty to be one-third the contri-

TABLE VI: Results derived from Tables I-IV for the four superallowed Fermi beta decays from $T_z = -1$ nuclei. Also shown for comparison are the equivalent results for their mirror transitions from $T_z = 0$ nuclei; these are taken from Ref. [1].

Parent nucleus	f	P_{EC} (%)	Partial half life t(ms)	ft(s)	δ'_R(%)	$\delta_C - \delta_{NS}$(%)	$\mathcal{F}t$ (s)
$T_z = -1$							
^{42}Ti	7130.5 ± 1.4	0.087	433 ± 12	3090 ± 88	1.427	1.195 ± 0.066	3096 ± 88
^{46}Cr	10660 ± 150	0.092	292.6 ± 9.1	3120 ± 110	1.420	0.935 ± 0.090	3130 ± 110
^{50}Fe	14950 ± 600	0.100	204.8 ± 4.5	3060 ± 140	1.439	0.815 ± 0.053	3080 ± 140
^{54}Ni	21850 ± 670	0.104	144.9 ± 2.3	3170 ± 110	1.430	0.955 ± 0.070	3180 ± 110
$T_z = 0$							
^{42}Sc	4472.23 ± 1.15	0.099	681.44 ± 0.26	3047.5 ± 1.4	1.453	0.655 ± 0.050	3071.6 ± 2.1
^{46}V	7209.25 ± 0.54	0.101	$423.113^{+0.053}_{-0.056}$	$3050.32^{+0.44}_{-0.46}$	1.445	0.655 ± 0.063	3074.1 ± 2.0
^{50}Mn	10745.97 ± 0.50	0.107	283.68 ± 0.11	3048.4 ± 1.2	1.444	0.705 ± 0.034	3070.6 ± 1.6
^{54}Co	15766.7 ± 2.9	0.111	$193.493^{+0.063}_{-0.086}$	$3050.7^{+1.1}_{-1.5}$	1.443	0.805 ± 0.068	$3069.8^{+2.4}_{-2.6}$

TABLE VII: Calculated transition-dependent radiative corrections δ'_R in percent units, and their component contributions. As explained in the text, no uncertainty is given. The results for ^{46}Cr, ^{50}Fe and ^{54}Ni are presented here for the first time; the results for the other cases are the same as those appearing in Ref. [13]. The last four lines give the differences in radiative-correction terms for the designated mirror transitions.

Parent nucleus	$\frac{\alpha}{2\pi}\overline{g}(E_m)$	$\frac{\alpha}{2\pi}\delta_2$	$\frac{\alpha}{2\pi}\delta_3$	$\frac{\alpha}{2\pi}\delta_{\alpha^2}$	δ'_R
$T_z = -1$					
^{42}Ti	0.9051	0.4556	0.0501	0.0160	1.4269
^{46}Cr	0.8745	0.4734	0.0567	0.0154	1.4200
^{50}Fe	0.8489	0.5077	0.0675	0.0148	1.4390
^{54}Ni	0.8203	0.5205	0.0747	0.0144	1.4299
$T_z = 0$					
^{42}Sc	0.9392	0.4507	0.0467	0.0166	1.4533
^{46}V	0.9031	0.4720	0.0539	0.0159	1.4448
^{50}Mn	0.8728	0.4942	0.0620	0.0153	1.4444
^{54}Co	0.8440	0.5134	0.0707	0.0147	1.4427
^{42}Sc $-$ ^{42}Ti	0.0341	-0.0049	-0.0034	0.0006	0.0264
^{46}V $-$ ^{46}Cr	0.0286	-0.0014	-0.0028	0.0005	0.0248
^{50}Mn $-$ ^{50}Fe	0.0239	-0.0135	-0.0055	0.0005	0.0054
^{54}Co $-$ ^{54}Ni	0.0237	-0.0071	-0.0040	0.0003	0.0128

bution of the $Z^2\alpha^3$ term but apply it only to the final average $\overline{\mathcal{F}t}$ value, so that its influence is not reduced by statistical averaging.

The second component of the outer radiative correction, δ_{NS}, recognizes that the γW-box graph includes situations in which the γ-nucleon interaction in the nucleus does not involve the same nucleon as the one participating in the W-nucleon interaction. When this happens, two distinct nucleons are actively involved and a detailed shell-model calculation is required to evaluate δ_{NS}. Being nuclear-structure dependent, there is some uncertainty in the result, but fortunately δ_{NS} is smaller in magnitude than δ'_R so this is not a serious impediment. Our strategy has always been to mount several shell-model calculations with different effective interactions from the literature, adopt an average value of δ_{NS} from the results for each transition, and assign an uncertainty that embraces the range of results obtained. We follow that approach here too. We also use exactly the same sets of effective interactions that we used in Ref. [13], where they are described in more detail and fully referenced.

The calculation of δ_{NS} is based on the formula

$$\delta_{NS} = \frac{\alpha}{\pi}\left[C_{NS}^{\text{quenched}} + (q-1)C_{\text{Born}}^{\text{free}}\right], \qquad (5)$$

where the component terms are defined and discussed in Ref. [15]. We use quenched electroweak vertices in the nucleus [16], so q represents the quenching factor by which the product of the weak and electromagnetic coupling constants is reduced in the medium relative to its free-nucleon value. Detailed results are given in columns 2-5 of Table VIII, where we show contributions to C_{NS}^{quenched}

TABLE VIII: Calculated nuclear-structure-dependent radiative correction δ_{NS}. The four components that are summed to give C_{NS}^{quenched} characterize the four electromagnetic couplings: os = orbital isoscalar, ss = spin isoscalar, ov = orbital isovector, and sv = spin isovector. The table gives one sample shell-model result, while the adopted value gives an average over several different shell-model calculations, with an uncertainty that embraces the range. The last four lines give the difference in radiative corrections for mirror transitions. Note that the uncertainties of the mirror differences in δ_{NS} were not determined from the uncertainties on the two contributing δ_{NS} values but were independently evaluated to cover the spread in the calculated *differences*.

Parent	C_{NS}^{quenched}					$(q-1)C_{\text{Born}}^{\text{free}}$	$\delta_{NS}(\%)$	$\delta_{NS}(\%)$
nucleus	os	ss	ov	sv	total			adopted
$T_z = -1$								
^{42}Ti	−0.019	−0.160	−0.207	−0.388	−0.774	−0.241	−0.236	−0.235(20)
^{46}Cr	−0.004	−0.197	−0.099	−0.198	−0.498	−0.248	−0.173	−0.175(20)
^{50}Fe	−0.009	−0.185	−0.104	−0.153	−0.451	−0.254	−0.164	−0.155(20)
^{54}Ni	−0.012	−0.180	−0.133	−0.203	−0.528	−0.261	−0.183	−0.165(20)
$T_z = 0$								
^{42}Sc	−0.019	−0.160	0.207	0.388	0.416	−0.241	0.041	0.035(20)
^{46}V	−0.004	−0.197	0.099	0.198	0.096	−0.248	−0.035	−0.035(10)
^{50}Mn	−0.009	−0.185	0.104	0.153	0.063	−0.254	−0.044	−0.040(10)
^{54}Co	−0.012	−0.180	0.133	0.203	0.144	−0.261	−0.027	−0.035(10)
^{42}Sc − ^{42}Ti	0.000	0.000	0.414	0.776	1.190	0.000	0.276	0.270(30)
^{46}V − ^{46}Cr	0.000	0.000	0.198	0.396	0.594	0.000	0.138	0.140(10)
^{50}Mn − ^{50}Fe	0.000	0.000	0.208	0.306	0.514	0.000	0.119	0.115(20)
^{54}Co − ^{54}Ni	0.000	0.000	0.266	0.406	0.672	0.000	0.156	0.130(30)

from the various components of the electromagnetic interaction: orbital isoscalar (os), spin isoscalar (ss), orbital isovector (ov), and spin isovector (sv). Note that the spin contributions are larger than the orbital contributions.

An even more interesting observation from Table VIII is that the isoscalar and isovector contributions to δ_{NS} are in phase when the decaying nucleus has $T_z = -1$ and out of phase when it has $T_z = 0$. This leads to larger corrections for transitions from the $T_z = -1$ nuclei than for those from the $T_z = 0$ ones. As is made clear by the differences in mirror δ_{NS} values shown in the bottom four lines of the last column in Table VIII, this effect creates an asymmetry of between 0.1 and 0.3%. This asymmetry would of course contribute to the expected mirror asymmetry in the experimental ft values and, since current experiments aim at 0.1% precision, this effect is just at the edge of detectability.

B. Isospin-symmetry-breaking correction, δ_C

The isospin-symmetry-breaking correction is defined as the reduction in the square of the Fermi matrix element, $|M_F|^2$, from its symmetry-limit value, $|M_F^0|^2$. Thus,

$$|M_F|^2 = |M_F^0|^2(1-\delta_C). \tag{6}$$

For calculational convenience, we separate δ_C into two components [1, 13]

$$\delta_C = \delta_{C1} + \delta_{C2}. \tag{7}$$

The idea is that δ_{C1} follows from a tractable shell-model calculation that does not include significant nodal mixing, while δ_{C2} corrects for the nodal mixing that would be present if the shell-model space were much larger.

For δ_{C1}, a modest shell-model space (usually one major oscillator shell) is employed, in which Coulomb and other charge-dependent terms have been added to the charge-independent effective Hamiltonian customarily used for the shell model. However, the most-important Coulomb force is long range and its influence in configuration space extends much further than a single major oscillator shell. The principal impact of multi-shell mixing is to change the radial wave function of the proton through mixing with radial functions that have more nodes. In the β-decay matrix element, M_F, there is an overlap between the radial functions of the proton and neutron that participate in the transition, and it is the reduction from unity of the overlap integral that leads to the correction δ_{C2}.

The details of the calculations for δ_{C1} are described in Ref. [13]. If isospin were an exact symmetry then the decay of the parent $0^+, T = 1$ state would proceed exclusively to its 0^+ analog state in the daughter nucleus. Fermi transitions to all other 0^+ states in the daughter would be expressly forbidden. But when charge-dependent terms are added to the shell-model Hamiltonian there is some depletion of the analog transition strength, with the missing strength appearing in weak transitions to excited 0^+ states. Significantly, in many cases the bulk of the analog-state depletion shows up in

TABLE IX: Shell-model calculation of the isospin-symmetry-breaking correction, δ_{C1}. The table gives one sample shell-model result, while the adopted value gives an average over several different shell-model calculations, with an uncertainty that embraces the range. The results for ^{46}Cr, ^{50}Fe and ^{54}Ni are presented here for the first time; the results for the other cases are the same as those appearing in Ref. [13]. The last four lines give the difference in isospin-symmetry-breaking corrections for mirror transitions. Note that the uncertainties of the mirror differences in δ_{C1} were not determined from the uncertainties on the two contributing δ_{C1} values but were independently evaluated to cover the spread in the calculated *differences*.

Parent nucleus	$E_x(0^+)$ expt	$E_x(0^+)$ SM	$\delta_{C1}(\%)$ unscaled	$\delta_{C1}(\%)$ scaled	$\delta_{C1}(\%)$ adopted
$T_z = -1$					
^{42}Ti	1.84	3.16	0.038	0.113	0.105(20)
^{46}Cr	3.57[a]	4.86	0.012	0.023	0.045(20)
^{50}Fe	3.69	3.62	0.021	0.020	0.025(20)
^{54}Ni	2.56	2.26	0.030	0.023	0.065(30)
$T_z = 0$					
^{42}Sc	3.30[a]	5.05	0.007	0.017	0.020(10)
^{46}V	3.57[a]	4.86	0.040	0.075	0.075(30)
^{50}Mn	3.69	3.62	0.057	0.054	0.035(20)
^{54}Co	2.56	2.26	0.058	0.045	0.050(30)
^{42}Sc − ^{42}Ti			−0.031	−0.096	−0.080(15)
^{46}V − ^{46}Cr			0.028	0.052	0.030(20)
^{50}Mn − ^{50}Fe			0.036	0.035	0.010(15)
^{54}Co − ^{54}Ni			0.028	0.022	−0.015(60)

[a]Second excited 0^+state; shell-model calculations indicate this state takes up most of the depletion from the analog state.

feeding a single excited 0^+ state, usually (but not always) the lowest excited one. In the limit of two-state mixing, perturbation theory would indicate that

$$\delta_{C1} \propto \frac{1}{(\Delta E)^2} \tag{8}$$

where ΔE is the energy separation of the analog and non-analog 0^+ states. Since the calculated energy separation in the shell model, $(\Delta E)_{\rm theo}$, does not exactly match the experimental value, $(\Delta E)_{\rm expt}$, we refine our model calculation of δ_{C1} by scaling its value by a factor $(\Delta E)^2_{\rm theo}/(\Delta E)^2_{\rm expt}$.

Our δ_{C1} results for the decays of ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni are found in Table IX, where they can be compared with the mirror decays of ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co. In each case, columns 2 and 3 give the experimental and calculated excitation energies of the non-analog 0^+ state that takes the bulk of the Fermi strength depleted from the analog states. Columns 4 and 5 give δ_{C1} without and with scaling by $(\Delta E)^2_{\rm theo}/(\Delta E)^2_{\rm expt}$.

For each nucleus, we performed several shell-model calculations with several different charge-independent effective Hamiltonians – the same as those described and referenced in Ref. [13]. Only one of these calculations is recorded in the table but the adopted value, which appears in the sixth column, represents an average over all calculations, with an uncertainty assigned to span the range of results obtained.

Next, we consider δ_{C2}. For its computation, the radial functions we use in the overlap integral are eigenfunctions of a Woods-Saxon potential, as justified in our survey article [1]. The methods of calculation have been described in detail in [13, 15]. Much benefit is gained from a very strong constraint: The asymptotic forms of all radial functions must match the measured separation energies S_p and S_n, where S_p is the proton separation energy in the decaying nucleus and S_n is the neutron separation energy in the daughter nucleus. The Woods-Saxon potential for a nucleus of mass A and charge $Z+1$ is taken to be the following:

$$V(r) = -V_0 f(r) - V_s g(r) \mathbf{l}\cdot\boldsymbol{\sigma} + V_C(r) - V_g g(r) - V_h h(r) \tag{9}$$

where

$$\begin{aligned} f(r) &= \left\{ 1 + \exp\left(\frac{r-R}{a}\right)\right\}^{-1}, \\ g(r) &= \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{a_s r} \exp\left(\frac{r-R_s}{a_s}\right) \\ &\quad \times \left\{1 + \exp\left(\frac{r-R_s}{a_s}\right)\right\}^{-2}, \\ h(r) &= a^2 \left(\frac{df}{dr}\right)^2, \\ V_C(r) &= Ze^2/r \quad \text{for } r \geq R_c, \\ &= \frac{Ze^2}{2R_c}\left(3 - \frac{r^2}{R_c^2}\right) \quad \text{for } r < R_c, \end{aligned} \tag{10}$$

with $R = r_0(A-1)^{1/3}$ and $R_s = r_s(A-1)^{1/3}$. The first three terms in Eq. (9) are the central, spin-orbit and

TABLE X: Calculations of δ_{C2} with Woods-Saxon radial functions for three methodologies (δ_{C2}^{II}, δ_{C2}^{III}, δ_{C2}^{IV}) for one sample shell-model interaction. The adopted values and uncertainties reflect the spread in results for several shell-model interactions, different methodologies, and the uncertainty in the radius parameter, r_0. The last four lines give the differences in isospin-symmetry-breaking corrections for the four mirror transitions. Note that the uncertainties of the mirror differences in δ_{C2} were not determined from the uncertainties on the two contributing δ_{C2} values but were independently evaluated to cover the spread in the calculated *differences*.

Parent nucleus	Radius parameters (fm) $\langle r^2\rangle^{1/2}$	r_0	$\delta_{C2}^{II}(\%)$	$\delta_{C2}^{III}(\%)$	$\delta_{C2}^{IV}(\%)$	Adopted $\delta_{C2}(\%)$
$T_z = -1$						
^{42}Ti	3.616(5)	1.323(2)	0.901	0.869	0.800	0.855(60)
^{46}Cr	3.70(10)	1.316(44)	0.764	0.723	0.658	0.715(85)
^{50}Fe	3.58(6)	1.206(24)	0.674	0.613	0.615	0.635(45)
^{54}Ni	3.68(5)	1.201(21)	0.784	0.684	0.710	0.725(60)
$T_z = 0$						
^{42}Sc	3.570(24)	1.319(11)	0.704	0.681	0.632	0.670(45)
^{46}V	3.60(7)	1.285(31)	0.587	0.542	0.506	0.545(55)
^{50}Mn	3.712(20)	1.273(8)	0.657	0.621	0.615	0.630(25)
^{54}Co	3.83(7)	1.275(29)	0.760	0.688	0.706	0.720(60)
^{42}Sc − ^{42}Ti			−0.197	−0.188	−0.168	−0.185(20)
^{46}V − ^{46}Cr			−0.177	−0.181	−0.152	−0.170(80)
^{50}Mn − ^{50}Fe			−0.017	0.008	0.000	−0.005(40)
^{54}Co − ^{54}Ni			−0.024	0.004	−0.004	−0.005(60)

Coulomb terms respectively. The fourth and fifth are additional surface terms whose role we discuss shortly.

Most of the parameters are fixed at standard values, $V_s = 7$ MeV, $r_s = 1.1$ fm and $a = a_s = 0.65$ fm, and the radius of the Coulomb potential, R_c, is determined from the root-mean-square charge radius, $\langle r^2\rangle^{1/2}$, of the decaying nucleus. Likewise the radius parameter of the central potential, r_0, is determined by requiring that the charge density constructed from the proton eigenfunctions of the potential yields a root-mean-square charge radius $\langle r^2\rangle^{1/2}$ in agreement with the known experimental value. The radius parameters used in our calculations of δ_{C2} appear in the second and third columns of Table X.

Our results for δ_{C2} itself, calculated with three different methodologies, are given in columns 4-6 of Table X, with the ultimately adopted values in column 7. The shell model enters these computations because the initial and final A-particle states are expanded in a complete set of $(A-1)$-particle states and single-particle states. The shell model provides the expansion coefficients. For a state in the $(A-1)$ system at an excitation energy E_x, the proton and neutron separation energies assigned to the single particle for this term in the expansion are $S_p + E_x$ and $S_n + E_x$. For the methodology labeled II, the strength of the central potential V_0 was continually readjusted for each term in the parentage expansion to reproduce these separation energies. With the radial overlap integral obtained from these eigenfunctions, the isospin-symmetry-breaking correction is labelled δ_{C2}^{II}. Alternatively, the adjustment to the Woods-Saxon potential can be accomplished with the surface terms: For δ_{C2}^{III} we adjusted V_g and for δ_{C2}^{IV} it was V_h that was adjusted. Further details of this approach are given in Refs. [13, 15].

In Table X, the δ_{C2} results for the three different methodologies are given for one sample shell-model interaction. The adopted value is an average over the different shell-model calculations and different methodologies with an uncertainty that covers the spread in the results and the uncertainty associated with the experimental root-mean-square charge radius.

The question of what is the appropriate root-mean-square charge radius had to be revisited for these calculations following the recent compilation of experimental results by Angeli and Marinova [17], which were not incorporated into our 2015 survey [1]. Considering first the $T_z = 0$ parent nuclei, we find that for two of them, ^{46}V and ^{54}Co, there have been no updates in charge radii, so the results given in Table X for these nuclei are identical to those published in 2008 [13] and used in 2015. However, for ^{42}Sc and ^{50}Mn, new experimental charge radii have appeared so the δ_{C2} values for these nuclei have had to be recomputed. Their new δ_{C2} results, shown in Table X, are slightly higher than before and have smaller uncertainties compared to those assigned in 2008, reflecting the greater precision of the new charge radii. The reduction is limited, though, by contributions from uncertainties arising from the spread in results among the different methodologies and different shell-model interactions, which remains unchanged from before. Reassuringly, the new results for δ_{C2} agree well with those published in 2008 [13] within the latter's stated uncertainties.

As to the $T_z = -1$ parents, charge radii are not known for ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni, although they are for

TABLE XI: Calculated ft^a/ft^b ratios for the four mirror doublets.

Decay pairs $a;b$	$\delta_R^b - \delta_R^a$(%)	$\delta_C^b - \delta_C^a$(%)	ft^a/ft^b
^{42}Ti → ^{42}Sc ; ^{42}Sc → ^{42}Ca	0.296(30)	−0.265(25)	1.00561(39)
^{46}Cr → ^{46}V ; ^{46}V → ^{46}Ti	0.165(10)	−0.140(82)	1.00305(83)
^{50}Fe → ^{50}Mn ; ^{50}Mn → ^{50}Cr	0.120(20)	0.005(43)	1.00115(47)
^{54}Ni → ^{54}Co ; ^{54}Co → ^{54}Fe	0.143(30)	−0.020(85)	1.00163(90)

heavier isotopes of each element, typically for those with masses A+4, A+6 and A+8. In each case, we have done a quadratic fit to the known charge radii and then extrapolated four mass units back to the isotope of interest. A generous error is assigned to charge radii obtained in this manner. Table X lists the root-mean-square charge radii we have used for these nuclei. For ^{42}Ti, the new results here represent a modest update to those published in 2008 [13], although the new adopted value of δ_{C2} is well within the previous uncertainty.

The last four lines of Table X give the differences in δ_{C2} values for the four mirror pairs of transitions. It is interesting to observe that the differences for the mass-42 and 46 pairs are about 0.2%, significantly larger than those for masses 50 and 54, which are nearly zero. This is something that could be tested in future higher-precision experiments.

IV. THE $\mathcal{F}t$ VALUES

In Sec. II, world data were evaluated for transitions from four $T_z = -1$ parent nuclei, and the results were entered into Table VI, where the equivalent (previously evaluated [1]) information for their mirror transitions from $T_z = 0$ nuclei also appear. The derived ft values for all eight transitions are also given. With the theoretical corrections, δ_R', δ_{NS}, δ_{C1} and δ_{C2} that appear in Tables VII-X respectively, we are now in a position to use Eq. (2) to obtain the $\mathcal{F}t$ values for all eight transitions. Columns 5 and 6 of Table VI give the theoretical corrections combined as they appear in Eq. (2), and column 7 lists the final $\mathcal{F}t$ values.

It is well known that the $\mathcal{F}t$ values for superallowed transitions provide valuable tests of weak-interaction physics. In accordance with Conservation of the Vector Current (CVC), all the $\mathcal{F}t$ values should be the same irrespective of the particular nuclei in which they are determined. Once consistency is established among the measured $\mathcal{F}t$ values, the resulting average $\overline{\mathcal{F}t}$ value can then be used to determine $V_{\rm ud}$, the up-down element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The $V_{\rm ud}$ result is a key ingredient in the most-definitive available test of CKM matrix unitarity, a fundamental principle of the standard model.

The four $\mathcal{F}t$ values in Table VI for transitions from $T_z = 0$ parents have already been incorporated in the most recent evaluation of $V_{\rm ud}$ [1]. Their $\sim 0.06\%$ precision is representative of the 14 transitions used in that evaluation. Clearly the four $\mathcal{F}t$ values for the $T_z = -1$ cases currently lack the precision to contribute to this picture. However, that could change in future as experimental improvements are made, especially in the measurement of Q_{EC}. For now, though, we can declare that the $\mathcal{F}t$ values for the $T_z = -1$ cases are consistent with the current best value for the average [1]: $\overline{\mathcal{F}t} = 3072.27 \pm 0.62$ s

V. MIRROR ASYMMETRY

The addition of three new proton-rich $T_z = -1$ β emitters whose superallowed Fermi branches are the isospin mirrors of already well-known $T_z = 0$ β emitters gives us the opportunity to examine the ratio of ft values for these mirror transitions and to discuss their asymmetry in terms of isospin-symmetry breaking. This approach has already been advanced for the mirror Fermi decays of ^{38}Ca and ^{38m}K by Park *et al.* [18, 19]. If we accept the CVC requirement that all the $T = 1$ superallowed transitions must have the same $\mathcal{F}t$ values, then obviously this requirement applies to each mirror pair and, from Eq. (2), we can derive the following expression for the ratio of experimental ft values for such a pair:

$$\frac{ft^a}{ft^b} = 1 + (\delta_R^b - \delta_R^a) - (\delta_C^b - \delta_C^a), \qquad (11)$$

where superscript "a" denotes the decay of the $T_z = -1$ parent and "b" denotes the decay of the mirror $T_z = 0$ parent. Here $\delta_R = \delta_R' + \delta_{NS}$ and $\delta_C = \delta_{C1} + \delta_{C2}$ and their mirror differences are already listed in Tables VII, VIII, IX and X. The advantage offered by Eq. (11) is that the theoretical uncertainty on a difference like $\delta_C^b - \delta_C^a$ is less than the uncertainties on δ_C^b and δ_C^a individually.

In Table XI we list values of $\delta_R^b - \delta_R^a$ and $\delta_C^b - \delta_C^a$ and hence values for ft^a/ft^b. These values differ from unity by amounts ranging from 0.1% to 0.6% with radiative-correction and isospin-symmetry-breaking differences contributing comparably. With future experimental precision at the $\sim 0.1\%$ level, it would become possible to test the corrections for these pairs in the way first demonstrated by Park *et al.* [18] for the mirror superallowed decays of ^{38}Ca and ^{38m}K. Particularly attractive

TABLE XII: Values of the coefficients a_0 and a_1 that yield the statistical rate function f_0 from Eq. (13), and coefficients b_0, b_1, b_2 and b_3 that yield the correction δ_S from Eq. (14). Coefficients a_2 and a_3 are held fixed at the values: $a_2 = -2/15$ and $a_3 = 1/4$.

Parent nucleus	a_0	a_1	$b_0(\%)$	$b_1(\%)$	$b_2(\%)$	$b_3(\%)$
^{46}Cr	0.0207203	−0.0797342	0.29193	0.17401	0.26989	−0.00219
^{50}Fe	0.0200743	−0.0845341	0.34970	0.17589	0.27937	−0.00199
^{54}Ni	0.0191989	−0.0398293	0.42003	0.20090	0.31418	−0.00216

is the mass-42 mirror pair, for which the ft-value ratio is expected to differ from unity by nearly 0.6%.

VI. PARAMETERIZATION OF f FOR ^{46}CR, ^{50}FE AND ^{54}NI

To hone the $\mathcal{F}t$ values for the decays of ^{46}Cr, ^{50}Fe and ^{54}Ni to the precision required to compete effectively with the currently well-known superallowed transitions, the Q_{EC} values in particular will have to be improved considerably. When this happens, the statistical rate function, f, will have to be calculated with a precision to match. We recently published [4] a parameterization of f that allows a user to easily calculate the f value to high precision (±0.01%) for the 20 transitions included in our survey [1]. For completeness, we give in Table XII the parameters required to calculate f for the three transitions we have added here.

We follow the parameterization developed in Ref. [4], in which

$$f = f_0(1 + \delta_S), \tag{12}$$

where

$$f_0 = a_0 W_0^4 p_0 + a_1 W_0^2 p_0 + a_2 p_0 + a_3 W_0 \ln(W_0 + p_0) \tag{13}$$

and

$$\delta_S = b_0 + b_1 W_0 + b_2/W_0 + b_3 W_0^2, \tag{14}$$

where W_0 is the maximum total energy of the decay positron in electron rest-mass units and $p_0 = (W_0^2 - 1)^{1/2}$ is the corresponding momentum. Two of these parameters are fixed: $a_2 = -2/15$ and $a_3 = 1/4$. The other six are listed in Table XII.

Note that, as in Ref. [4], this parameterization is only valid for the transitions identified and only for a limited range of energies (±60 keV) around the currently accepted Q_{EC} values.

VII. CONCLUSIONS

Prompted by new measurements from Molina *et al.* [2], we have thoroughly examined the superallowed Fermi decays of ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni, the latter three of which having never before been included in our periodic surveys of world data for such decays.

We began this report by assembling all pertinent references and arriving at recommended results for the Q_{EC} values, half-lives and branching ratios for all four transitions; next, we presented calculations of their radiative and isospin-symmetry-breaking corrections. From this input we obtained their ft and $\mathcal{F}t$ values.

The results have all been presented in such a way that these four transitions from $T_z = -1$ nuclei could be compared with their mirror superallowed transitions from the $T_z = 0$ nuclei ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co. This also gave us the opportunity to update the δ_C values for ^{42}Ti, ^{42}Sc and ^{50}Mn in order to incorporate an update in the recommended values for the root-mean-square charge radii, $\langle r^2 \rangle^{1/2}$, of these nuclei as tabulated by Angeli and Marinova [17].

By presenting our results in terms of comparisons of mirror pairs of transitions with A=42, 46, 50 and 54, we demonstrate the importance of measuring the $T_z = -1$ members of these mirror pairs with improved precision. The difference in the ft values between the two members of each mirror pair is sensitive to the calculated correction terms, and can be used to test, and possibly improve, them.

Although the ft-value uncertainties for the decays of ^{42}Ti, ^{46}Cr, ^{50}Fe and ^{54}Ni are still too large for this purpose, we take the view that, with experimental accessibility now demonstrated, there is sufficient motivation to proceed with improving the precision. An obvious place to begin is with modern re-measurements of the 40-year-old Q_{EC} values for the decays of ^{46}Cr, ^{50}Fe and ^{54}Ni with a precision to match the recent Penning-trap measurement of the ^{42}Ti Q_{EC} value.

To aid in this endeavor, we have also provided the means to easily calculate f values for the superallowed transitions from ^{46}Cr, ^{50}Fe and ^{54}Ni to the required ±0.01% precision.

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