Short-range correlations in the deuteron: Chiral effective field theory, meson-exchange, and phenomenology

Francesca Sammarruca

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Short-range correlations in the deuteron: chiral effective field theory, meson-exchange, and phenomenology

Francesca Sammarruca

Physics Department, University of Idaho, Moscow, ID 83844-0903, U.S.A.

Abstract

High-momentum distributions and short-range correlation probabilities in the deuteron are studied with a variety of modern potentials based on chiral effective field theory up to fifth order in the chiral expansion. Conventional (meson-exchange and phenomenological) interactions are also considered. Predictions are examined in the context of short-range correlation probabilities as extracted from analyses of inclusive electron scattering data, with a discussion on whether modern interactions can be reconciled with the latter.

I. INTRODUCTION

High-momentum components in the nuclear wave function and in momentum distributions are a reflection of short-range correlations (SRC) in nuclei. The presence of high-momentum components is mainly due to the repulsive short-range central force and the tensor force.

Although high-quality meson-theoretic interactions continue to be employed in contemporary calculations of nuclear structure and reactions, since the 1990's there exists a general understanding that chiral effective field theory (EFT) [1, 2] is a superior framework. First, chiral EFT has a firm connection with quantum chromodynamics (QCD) through the symmetries of low-energy QCD. Second, it allows for a systematic expansion which makes possible a quantification of the theoretical error. At each order of chiral perturbation theory (χPT), the uncertainty associated with a particular prediction can be controlled and quantified. For these reasons, nuclear chiral effective theory is becoming increasingly popular as a model-independent approach.

The NN potentials constructed within chiral EFT are generally softer than "conventional" potentials, which makes them computationally more amenable to nuclear structure calculations. Also, potentials with a low resolution scale, obtained through a unitary transformation (that is, through renormalization group (RG) methods [3]) applied to a "harder" interaction, are very popular for many-body calculations. The resulting "low-k" potentials are equivalent to the original ones for all physical purposes, although essentially void of high momentum components.

On the experimental side, inclusive electron scattering measurements at high momentum transfer, on both light and heavy nuclei, have been analyzed with the purpose of extracting information on short-range correlations [4–6]. In a suitable range of Q^2 and x_B, the cross section is factorized in order to single out the probability of a nucleon to be involved in SRC, either two-body or three-body. When extended to nuclear matter, this probability is equivalent to the "wound integral", which measures the amount of correlations in the so-called defect function [7]. Information about two-body correlations can also be obtained in coincidence experiments involving knock-out of a nucleon pair with protons [8] or electrons [9–12].

Nuclear scaling and the plateaus seen in inclusive scattering cross section ratios [4] are

due to the dominance of SRC for momenta above approximately 2 fm^{-1}. In the same region, the momentum distribution in a nucleus relative to the one in the deuteron becomes almost flat, so that those distributions simply scale with A.

The discussion around some of these measurements which is presently going on in the literature is quite intriguing. The probabilities mentioned above are a manifestation of the off-shell nature of the potential, which cannot be determined uniquely from NN elastic data and is not an observable. Interactions may differ dramatically in their off-shell behavior while remaining phase-equivalent. The most striking example is provided by the RG-evolved potentials mentioned above, where the high-momentum structures of the original and the RG-evolved potential are obviously not the same.

Naturally, a low-resolution scale will impact the ability to resolve high momentum regions. With regard to this point, it has been noted that, if a unitary transformation is applied to both wave functions and operators, one regains the invariance of the cross section, as one should [13], thus attaining a consistent description of short- and long-range physics. This has been addressed recently by Neff *et al.*, who show how the short-range information can be recovered by transforming the density operators [14].

On the other hand, chiral potentials such as those developed in Refs. [15–17] are not low-momentum interactions in the sense of a V_{low-k}. In this paper, I examine those from the point of view of SRC. To broaden the discussion, I start with an analysis of SRC and conventional (that is, non-chiral) NN potentials, including high-precision potentials from the 1990's as well as a phenomenological one. I then move to a similar analysis with chiral interactions.

The deuteron is taken as a sample system. It is useful to recall that the high-momentum part of the momentum distribution shows similar features in nuclei with A=2 to 40 [18]. Thus, the deuteron offers representative features. Furthermore, deuteron SRC probabilities are a crucial element in the estimation of SRC probabilities in heavier nuclei as obtained in Ref. [4].

The central point of this paper are the momentum distribution and the probability of SRC calculated up to 5th order of chiral effective theory. Working with the A=2 system, one can go to any order of chiral EFT where NN potentials are available, without the need to worry about the corresponding three-nucleon forces (3NF). Although 3NF at N^{3}LO and N^{4}LO have been worked out [19–21], their application in few- and many-body systems

TABLE I: Probabilities of SRC and D-state probabilities for the potentials considered in Fig. 1.

Model	$a_{2N}(d)$	P_D
CD-Bonn	0.032	0.0485
AV18	0.042	0.0578
Nijmegen II	0.041	0.0565

still presents considerable challenges and requires unavoidable omissions/approximations. Calculations in the deuteron are free of those and thus well-controlled. Cutoff dependence and order-by-order convergence from lowest to 5th order of the chiral expansion is examined.

Some of the questions to be addressed are: To which extent are modern, *non-phenomenological* interactions (chiral or not) consistent with the information as extracted from $A(e,e')X$ measurements? What does one learn, on fundamental grounds, from the answer to this question? Are there characteristic differences among particular families of potentials from which one can obtain physical insight (beyond phenomenological observations)?

Results and conclusions are summarized in Section III.

II. HIGH-MOMENTUM DISTRIBUTION IN THE DEUTERON

A. Meson theory and phenomenology

This section begins with a step back into the 1990's by considering three members of the "high-precision" family of NN potentials, namely CD-Bonn [22], Nijmegen II [23], and AV18 [24]. The respective momentum distributions in the deuteron, with focus on high-momentum components, are shown in Fig. 1. $\rho(k)$ is the Fourier transform squared of the coordinate space wave function. There are noticeable differences between the (softer) predictions from CD-Bonn and those from the other two potentials, which are essentially indistinguishable.

To see how these differences carry into the probability of SRC, following Ref. [18] the

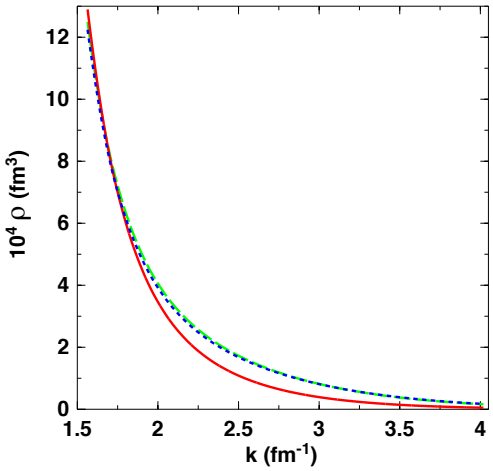

FIG. 1: (Color online) High momentum distributions in the deuteron as predicted by: CD-Bonn (solid red); Nijmegen II (dotted blue); and AV18 (dashed green).

probability of SRC in the deuteron is defined as

$$a_{2N}(d) = 4\pi \int_{k_{min}}^{\infty} \rho(k) k^2 \, dk \ , \tag{1}$$

where k_{min} is taken to be 1.4 fm^{-1} (276 MeV). This definition was adopted in Ref. [4], where the choice of the lower integration limit is suggested by the onset of scaling of the cross section, which should signal the dominance of scattering from a strongly correlated nucleon. In Ref. [4], the ratio of the per-nucleon probability of two nucleon (2N) SRC in a nucleus relative to ^{3}He is argued to be equal to the ratio of the inclusive electron scattering cross sections in the appropriate scaling region. The absolute per-nucleon probability in a nucleus can then be deduced if the absolute per-nucleon probability in ^{3}He is known. The latter is the product of the absolute per-nucleon probability in the deuteron, stated as 0.041$\pm$ 0.008 in Ref. [25], and the relative probability of 2N SRC in ^{3}He relative to the deuteron. Namely,

$$a_{2N}(A) = a_{2N}(A/^3He) a_{2N}(^3He) \quad \text{and} \quad a_{2N}(^3He) = a_{2N}(^3He/d) a_{2N}(d) \ . \tag{2}$$

It can be seen that the values for the deuteron and the ratio of ^{3}He to deuteron contain some model dependence from theoretical calculations (see Ref. [4] and Refs. [2,6,15,16] therein), likely to propagate in the predictions for heavier nuclei.

In Table I, I show the probability as defined in Eq. (1) for the interactions used in Fig. 1. As an additional, related information, I also show the corresponding D-state probabilities. As expected in light of Fig. 1, there is a significant difference between CD-Bonn and the other two cases, with the AV18 and Nijmegen II predictions closer to the value used in the analysis from Ref. [25].

The differences noted above are due to the non-local nature of CD-Bonn, which adopts fully relativistic momentum-space expressions for the one-pion-exchange. More precisely, the off-shell nature of CD-Bonn is based upon the relativistic Feynman amplitudes for meson exchange. This determines well-founded non-localities in the tensor force, whereas Nijmegen II and AV18 make use of the non-relativistic, static one-pion-exchange which generates a local tensor force. The characteristically softer nature of a relativistic momentum-space potential reflected in Table I is a desirable feature for the purpose of applications in nuclear structure.

This point will be revisited for a more complete discussion after the next section.

B. Interactions based on chiral EFT

In spite of the good theoretical foundation behind meson-exchange Feynman amplitudes, meson theory does not provide a systematic approach to constructing nuclear forces. As mentioned in the Introduction, chiral EFT presents the opportunity for such systematic development.

Crucial for a nuclear EFT are the processes of regularization and renormalization. Concerning the former, all chiral interactions are multiplied by a regulator function which typically has the form:

$$f(p',p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}] \,, \tag{3}$$

where Λ is known as the cutoff parameter.

Nucleon-nucleon potentials have been developed at different orders and cutoff values [16, 17]. Chiral EFT predictions allow for the quantification of uncertainties that stem from the truncation error and cutoff variations (as well as additional sources of errors).

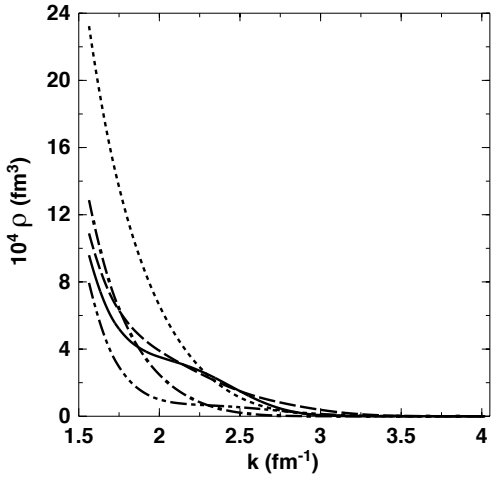

FIG. 2: Momentum distributions in the deuteron predicted with chiral potentials at: LO (dotted); NLO (dash-double dot); N^{2}LO (dash-dot); N^{3}LO (dash); and N^{4}LO (solid). The cutoff is fixed at 500 MeV.

Consistent with that philosophy, in Fig. 2 I show the momentum distribution in the deuteron, including five orders of the chiral expansion. The potential at N^{4}LO is a preliminary version of a high-precision nucleon-nucleon potential at fifth order [27]. (Some wavy structures noticeable in the figure are most likely due to the polynomial nature of the EFT contacts.) Huge variations can be seen at the lowest orders, particularly from LO to NLO, and a clear convergence pattern with increasing order.

Table II shows the integrated probabilities corresponding to Fig. 2. At the higher, converging orders, the SRC probabilities are not very different from the one predicted by CD-Bonn. That is, chiral potentials with suitable cutoff can be constructed with excellent fit to the NN data and off-shell nature similar to the highest-quality *non-local* meson-exchange forces.

The truncation error at order n is defined as the difference between the predictions at orders $n+1$ and n. Thus, the error of $a_{2N}(d)$ at N^{3}LO is ± 0.004. I have also considered variations of the cutoff parameter between 500 and 600 MeV. At N^{3}LO, the value of $a_{2N}(d)$

TABLE II: Probabilities of SRC and D-state probabilities for the chiral interactions considered in Fig. 2.

Model	$a_{2N}(d)$	P_D
LO	0.047	0.0757
NLO	0.015	0.0313
N^2LO	0.022	0.0417
N^3LO	0.030	0.0451
N^4LO	0.026	0.0414

with cutoff of 600 MeV was found to be nearly the same as with 500 MeV. Therefore, cutoff uncertainty is below the truncation error, and the final result at N^3LO can be stated as 0.030 ± 0.004. Concerning the uncertainty of the N^4LO result, the prediction for the next higher order is unknown. Therefore, assuming (pessimistically) the same truncation error as at N^3LO, the prediction at N^4LO can be stated as 0.026 ± 0.004.

C. Discussion

The deuteron is the simplest system where off-shell behavior can be explored. Characteristic differences exist between meson-theoretic potentials using fully relativistic one-pion exchange amplitudes (that is, non-local tensor forces) and those which use static one-pion-exchange. *Off-shell behavior is not observable and thus cannot be uniquely determined by measurements.* The best one can do is to have a good theoretical foundation for it. In meson theory, this is provided by relativistic meson-exchange amplitudes.

In Section II A it was shown that the SRC probability predicted with CD-Bonn is roughly 25% below the value of $a_{2N}(d)$ cited in Ref. [4] and used to evaluate absolute probabilities in heavier nuclei. A discrepancy of qualitatively similar nature exists for the wound integral in nuclear matter. More precisely, conventional non-local potentials are known to predict

about 10-15% for the wound integral (at normal density) [7], whereas a value of about 25% is cited from extrapolation to nuclear matter of the empirical information [28]. In fact, the ratio of the probability for nucleus A to the deuteron, extrapolated to infinite symmetric matter, is given to be 6.5±1.0 [28]. With the absolute probability for the deuteron taken equal to 0.04, the value cited above is obtained. However, when the $a_{2N}(d)$=0.032 value from CD-Bonn is used, one obtains 20%, and with $a_{2N}(d)$=0.026 as from the converged chiral results, a value of 17% is obtained for empirical short-range correlations in nuclear matter, which is getting closer to the predicted wound integrals.

In summary, the question of consistency between description of short and long range physics seems to go beyond the (intrinsically) "low-momentum" nature of some potentials. Instead, it points to non-locality in the tensor force, a feature which has been found since a long time to be very attractive in nuclear structure. So, this is an issue which has resurfaced and is being revisited in the light of new experiments. It is important to take a broad view of it, combining our former and present understanding of microscopic nuclear forces and their development. Former studies of the differences among predictions of the deuteron and the two-nucleon correlation function from phase-equivalent NN potentials and the effect of localizing the one-pion exchange contribution can be found, for instance, in Refs. [29, 30].

III. CONCLUSIONS AND OUTLOOK

The deuteron is a beautifully simple benchmark for theories of nuclear forces. From this study, one may conclude that predictions of high-momentum distributions in the deuteron with high-quality non-local meson-exchange forces or state-of-the-art chiral forces are systematically lower than what is used to extract empirical information for heavier nuclei. Taking those results into account leads to a better agreement between SRC in nuclear matter and theoretical predictions. However, the above statement assumes that the ratio of inclusive cross sections is indeed equal to the ratio of SRC probabilities. One must keep in mind, though, that cross section ratios may be sensitive to other mechanisms (such as, for instance, final-state interaction), and may not be identified with the ratio of SRC probabilitities in a quantitative way. In fact, such quantitative association should be taken with care.

I plan to extend this microscopic analysis to the A=3 system using a broad spectrum of

interactions as in the present study. I hope this will shed more light on how to reconcile theory and empirical analyses.

Finally, some caution should be excercised in the interpretation of the empirical information discussed above as an experimental constraint on the off-shell behavior.

Acknowledgments

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