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Abstract

In a single j-shell calculation we consider the effects of several different interactions on the values of Gamow-Teller (B(GT)'s) and magnetic moments. The interactions used are MBZE, J=0 pairing, J_{max} pairing and half and half. Care is taken when there are isospin crossings and/or degeneracies.

1 Introduction

In examining the spectrum of a system of a neutron and a proton beyond a closed shell one sees that not only the J=0 T=1 but also$J$=1 T=0 and J=J_{max}=2j lie low. For example in ^{42}Sc the matrix elements taken from experiment by Escuderos et al. [1] are shown in Table I:

Table I: Experimental two-body matrix elements

T=1	E	T=0	E
J		J	
0	0.0000	1	0.6111
2	1.5865	3	1.4904
4	2.8135	5	1.5101
6	3.2420	7	0.6163

In this work we will consider the above interaction which we call MBZE, as well as some exteme interactions.

a.J=0 pairing: the 8 matrix elements are respectively -1,0,0,0,0,0,0,0

b.J_{max} pairing: 0,0,0,0,0,0,0,-1

c. Half and half: -1,0,0,0,0,0,0,-1.

We will study how Gamow-Teller B(GT) values and magnetic moments in the $f_{7/2}$ shell respond to these different interactions.

2 Gamow-Teller B(GT) values

We start with the well known formula for the case where the Fermi matrix element vanishes.

$$ft = 6177/[B(F) + 1.583/B(GT)]$$

In an allowed Fermi transition neither the total anguar momentum nor the isospin can change. We will only consider cases where one or both change so that B(F)=0.

We then obtain

$$ft = 3902.0846497/B(GT)$$

$$log(ft) = 3.591266854 - log(B(GT))$$

We will be using bare operators throughout.

As an orientation we note that for a free neutron B(GT)=3.

With the interactions mentioned in the introduction we can go to more complex systems and obtain wave functions that are represented by amplitudes $D^I(J_p,J_n)$. The square of this amplitude is the probability that in a state I the protons couple to J_p and the neutrons toJ_n.

We first consider a simple case where we do not require the amplitude of the transition ^{42}Sc (I=7$^+$) $\rightarrow$ ^{42}Ca (I=6$^+$). The initial state has isospin T=0 and the final T=1.

The experimental value is B(GT)=0.2699, while the theoretical value, assuming a configuration $(f_{7/2})^2$ for both the initial and final states, is 0.2743. Thus, to agree with experiment, one needs a quenching factor of 0.992 for the GT operator. In ref [2] this quenching factor was used. However, in this third work we will stick with the bare operator. It is worth mentioning that in this case we have a proton changing into a neutron inside the nucleus and a positron and neutrino escaping.

We now show results in Table II which do depend on the amplitudes. The expression for B(GT) is given in 2 previous publications and is here repeated.

$$X_1 = \sum_{J_p,J_n} D^f(J_p,J_n)D^i(J_p,J_n)U(1J_pI_fJ_n;J_pI_i)\sqrt{J_p(J_p+1)}$$

$$X_2 = \sum_{J_p,J_n} D^f(J_p,J_n)D^i(J_p,J_n)U(1J_nI_fJ_p;J_nI_i)\sqrt{J_n(J_n+1)}$$

$$B(GT) = \frac{1}{2}\frac{2I_f+1}{2I_i+1}f(j)^2\left[\frac{\langle 1T_i1M_{T_i}|T_fM_{T_f}\rangle}{\langle 1T_i0M_{T_i}|T_fM_{T_i}\rangle}\right]^2(X_1-(-1)^{I_f-I_i}X_2)^2$$

where

$$f(j) = \begin{cases} \frac{1}{j} & j = l+\frac{1}{2} \\ \frac{-1}{j+1} & j = l-\frac{1}{2} \end{cases}$$

If $T_f \neq T_i$ or $I_f \neq I_i$, we find that $X_1 = -(-1)^{I_f - I_i} X_2$. We then get a simplified formula for B(GT):

$$B(GT) = 2\frac{2I_f+1}{2I_i+1} f(j)^2 \left[\frac{\langle 1T_i 1 M_{T_i} | T_f M_{T_f}\rangle}{\langle 1T_i 0 M_{T_i} | T_f M_{T_i}\rangle}\right]^2 (X_1)^2$$

This formula does not apply to the case of neutron decay because in that case, $I_f = I_i$ and $T_f = T_i$.

Table II: B(GT) values

Transition	I_i	I_f	MBZE	J=0	Half	J=7	Experiment
^{43}Sc $\to ^{43}$Ca	3.5	2.5	0.1181	0	0.0592	0.2434	0.0326
^{43}Sc $\to ^{43}$Ca	3.5	3.5	0.1682	0.5713	0.2747	0.0397	
^{43}Sc $\to ^{43}$Ca	3.5	4.5	8.31×10^{-6}	0	3.29×10^{-4}	0.00136	
^{44}Sc $\to ^{44}$Ca	2	2	0.0505	0.0613	0.0142	0.0259	0.01962
^{45}Sc $\to ^{45}$Ca	3.5	2.5	0.0094	0	0.0094	2.32×10^{-5}	
^{45}Ca $\to ^{45}$Sc	3.5	3.5	0.0552	0.4571	0.1423	4.49×10^{-4}	
^{45}Sc $\to ^{45}$Ca	3.5	4.5	1.64×10^{-4}	0	3.16×10^{-4}	1.03×10^{-5}	
^{45}Ti $\to ^{45}$Sc	3.5	3.5	0.1466	0.1499	0.1732	5.89×10^{-4}	0.0980
^{46}Ti $\to ^{46}$V	4	4	0.0065	0.0166	0.2898	2.03×10^{-4}	0.0025
^{46}Ti $\to ^{46}$V	4*	4	0.0058	0.5458	0.0018	6.36×10^{-4}	0.0025
^{46}Ti $\to ^{46}$V	1	0	0.0789	0	0.0367	0.2332	0.0196
^{46}Ti $\to ^{46}$V	1*	0	0.0184	0.1523	6.73×10^{-4}	0	

Table III: log(ft) values

Transition	I_i	I_f	MBZE	J=0	Half	J=7	Experiment
^{43}Sc $\to ^{43}$Ca	3.5	2.5	4.519	∞	4.819	4.205	5.0
^{43}Sc $\to ^{43}$Ca	3.5	3.5	4.365	3.834	4.152	4.992	4.9
^{43}Sc $\to ^{43}$Ca	3.5	4.5	8.672	∞	7.074	6.458	
^{44}Sc $\to ^{44}$Ca	2	2	4.888	4.804	5.440	5.178	5.3
^{45}Sc $\to ^{45}$Ca	3.5	2.5	5.619	∞	5.619	8.226	
^{45}Ca $\to ^{45}$Sc	3.5	3.5	4.849	3.931	4.438	7.948	
^{45}Sc $\to ^{45}$Ca	3.5	4.5	7.376	∞	7.092	8.578	
^{45}Ti $\to ^{45}$Sc	3.5	3.5	4.425	4.415	4.353	6.821	4.6
^{46}Ti $\to ^{46}$V	4	4	5.779	5.370	4.130	7.284	6.2
^{46}Ti $\to ^{46}$V	4*	4	5.828	3.854	6.336	6.788	6.2
^{46}Ti $\to ^{46}$V	1	0	4.694	∞	5.027	4.224	5.3
^{46}Ti $\to ^{46}$V	1*	0	5.326	4.409	6.763	∞	

Consider first the behaviour in going from$J$=0 pairing to$J$=7 pairing via half and half. For the case ^{43}Sc (I=7/2 T=1/2) $\to$ ^{43}Ca (T=3/2) we find that when I_f is 5/2 or 9/2, B(GT) vanishes for J=0 pairing. For this interaction, seniority v is a good quantum number. We can classify the states by (v,T,t) where t is the reduced isospin. The initial I=7/2 state has v=1 and the final states have v=3. The reduced isospins are also different, t=1/2 and t=3/2 respectively. It is not correct to say that seniority must be conseved – that is not the case. As discussed by Harper and Zamick [5,6], with a J=0 pairing interaction one cannot have both the senority and reduced isospin change at the same time.

As we go from J=0 pairing to J=7 pairing we get a steady increase in B(GT) in the $7/2 \rightarrow 9/2$ and $7/2 \rightarrow 5/2$ cases. The former values are (0, 3.29×10^{-4}, 0.00136) whilst for $7/2 \rightarrow 5/2$ the values are (0, 0.0592, 0.2434). We next consider $7/2 \rightarrow 7/2$ in ^{43}Sc. Now we have an opposite behaviour. The J=0 case yields the largest value for B(GT).

In ^{45}Sc we have two examples of non-monotonic behaviour. This is for the cases $7/2 \rightarrow 9/2$ and $7/2 \rightarrow 5/2$. The 3 values are (0, 3.16×10^{-4}, 1.03×10^{-5}) and (0, 9.4×10^{-3}, 2.32×10^{-5}) respectively.

In general, the values of B(GT) in ^{45}Sc are smaller than in ^{43}Sc. It should be mentioned that systematics of B(GT)'s in the $f_{7/2}$ region can be explained by the Lawson K selection rule [7].

We next carefully discuss the case$I$=$1^+$$\rightarrow$ I=0^+ in ^{46}Ti. This was discussed by Harper and Zamick [6] but in the context of an M1 transition B(M1). However, that makes no difference because it was shown that B(GT) and the corresponding B(M1) were proportional. There is, nonetheless, an apparent difference in the behaviour as we go fromJ_{max} pairing to$J$=0 pairing. Harper et al.[6] state that there is non-monotonic behavour – J=7 is relatively large, half and half small, and$J$=0 pairing large again. But in the second last row of the present work we get a monotonic decrease as we go from$J$=7 to J=0.

The difference is that Harper et al. [6] always chose the state of lowest energy whilst in the present work we take the state of lowest energy for a fixed isospin. As we go to the J=0 pairing limit the T=2 J=1^+ state in ^{46}Ti state starts coming below a T=1 J=1^+ state. The B(GT) (or B(M1)) to the T=2 state is relatively large and this explains why the value of B(GT), which first decreases in going from$J$=7 to half and half, suddenly increases. If, as we do in this work, we constrain the isospin to be unchanged, we get the simpler monotonic behaviour. To get the Harper et al result [6] we take the J=7 pairing and the half value from the second last row, 0.0307, and the J=0 result from the last row, 0.1532. The I =1^+ state in this last row has isospin T=2, whereas in the second last row the 1^+ state is the lowest with T=1.

For B(GT) ^{46}Ti 4 to 4 we have to take care since for J=0 pairing the lowest 4^+ T=1 states are degenerate. We therefore slightly remove the degeneracy by considering an interaction 0.9 J=0 pairing and 0.1 J=7 pairing. We see that one of the B(GT)'s is small and the other large. With MBZE the B(GT)'s to the lowest two I =4^+ states are both small.

We next compare the 'realistic' MBZE results with experiment. Although things are in the right ballpark, there are significant deviations, indicating the need for configuration mixing.

3 Magnetic moments

In table IV we show a corresponding study of magnetic moments.

Table IV: Magnetic moments

Nucleus	Spin	MBZE	J=0	Half	J=7	Experiment
^{43}Sc	3.5	4.324	3.614	4.204	4.328	+4.62
^{44}Sc	2	1.990	0.592	1.779	2.268	+2.56
^{45}Sc	3.5	4.646	4.468	4.703	4.158	+4.76
^{45}Ti	2.5	-0.764	0.041	-0.905	-0.751	-0.133
^{45}Ti	3.5	-0.604	-0.891	-0.779	-0.377	0.095
^{46}Ti	2	0.991	1.990	1.152	0.613	-0.98

It should be noted that since 1964 a new magnetic moment has been measured experimentally – that of ^{45}Ti. The value is 0.095, but the sign is undetermined. All our interactions yield negative magnetic moments. The closest is the case of J_{max} pairing which gives -0.377, still a big discrepancy.

We lastly note that there has been considerable activity with the (^{3}He,t) reaction by Y.Fujita et al. [8,9,10] .The targets in these reactions include ^{44}Ca [8]and ^{42}Ca [9,10] and ^{54}Fe [10]. We also note theoretical work of C.L. Bai et al. [11] where GT transitions are calculated with "the isoscalar spin-triplet pairing interaction included in QRPA on top of the isovector spin-singlet one in the HFB method."

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References

[1] A. Escuderos, L.Zamick and B.F. Bayman, Wave functions in the $f_{7/2}$ shell, for educational purposes and ideas, arXiv: nucl-th 05060

[2] B.F. Bayman, J.D. McCullen and L. Zamick, Phys Rev Lett 10, (1963) 117

[3] J.D. McCullen, B.F. Bayman, and L. Zamick, Phys Rev 134, (1964) 515

[4] J.N. Ginocchio and J.B. French Phys. Lett 7, (1963) 137

[5] M.Harper and L.Zamick, Phys. Rev. C 91,014304 (2015)

[6] M. Harper and L.Zamick Phys. Rev. C 91, 054310 (2015)

[7] R.D. Lawson, Phys. Rev. 124,1500 (1961)

[8] Y. Fujita et al., Phys. Rev. C88,014368 (2013)

[9] Y. Fujita et al.,Phys.Rev. Lett. 112 ,112502 (2014)

[10] Y. Fujita et al. ,Phys. Rev. C 91, 064316 (2015)

[11] C.L. Bai, H.Sagawa, G.Colo,Y.Fujita, H.Q.Zhang,X.Z. Zhang and F.R.Xu, Phys. Rev.C 90, 054335 (2014)