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Phase space factors and half-life predictions for Majoron emitting $\beta^-\beta^-$ decay

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A complete calculation of phase space factors (PSF) for Majoron emitting $0\nu\beta^-\beta^-$ decay modes is presented. The calculation makes use of exact Dirac wave functions with finite nuclear size and electron screening and includes life-times, single electron spectra, summed electron spectra, and angular electron correlations. Combining these results with recent interacting boson nuclear matrix elements (NME) we make half-life predictions for the ordinary Majoron decay (spectral index $n=1$). Furthermore, comparing theoretical predictions with the obtained experimental lower bounds for this decay mode we are able to set limits on the effective Majoron-neutrino coupling constant $\langle g_{ee}^M \rangle$.

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I. INTRODUCTION

Double- β decay is a process in which a nucleus (A, Z) decays to a nucleus $(A, Z \pm 2)$ by emitting two electrons or positrons and, usually, other light particles

$$(A, Z) \rightarrow (A, Z \pm 2) + 2e^\mp + \text{anything}. \quad (1)$$

The mode where two antineutrinos or neutrinos are emitted is predicted by the standard model and has been observed in several nuclei (for a review, see e.g. [1]). The more exotic mode, neutrinoless double beta decay, is not allowed by the standard model, and once observed would offer new information on many fundamental aspects of elementary particle physics. As discussed in Ref. [2], several scenarios of neutrinoless double beta decay have been considered, most notably, light neutrino exchange, heavy neutrino exchange, and Majoron emission. After the discovery of neutrino oscillations, attention has been focused on the first scenario and the mass mode, where the transition operator is proportional to $\langle m_\nu \rangle / m_e$. Even though most current experimental efforts have been focused to the detection of this mode, interest on the mechanism predicting $0\nu\beta\beta$ decays through the emission of additional bosons called Majorons has also renewed lately.

Majorons were introduced years ago [3, 4] as massless Nambu-Goldstone bosons arising from a global $B - L$ (baryon number minus lepton number) symmetry broken spontaneously in the low-energy regime. These bosons couple to the Majorana neutrinos and give rise to neutrinoless double beta decay, accompanied by Majoron emission $0\nu\beta\beta M$ [5], as shown in Fig. 1 (a). Although these older models are disfavored by precise measurements of the width of the Z boson decay to invisible channels [6],

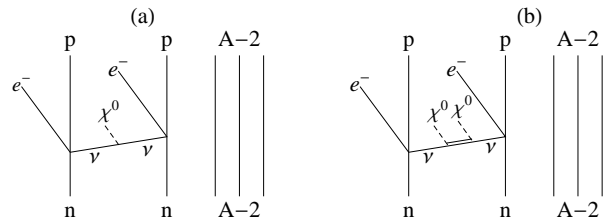


FIG. 1. Neutrinoless double beta decay accompanied by the emission of one or two Majorons.

several other models of $0\nu\beta\beta M$ decay have been proposed in which one or two Majorons, denoted by χ_0 , are emitted, (see Fig. 1)

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + \chi_0 \quad (2)$$

or

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\chi_0. \quad (3)$$

Table I lists some of the models proposed to describe these decays. The different models are distinguished by the nature of the emitted Majoron(s), i.e. is it a Nambu-Goldstone boson or not (NG), the leptonic charge of the emitted Majoron(s) (L), and the spectral index of the model (n), which characterizes the shape of the summed electron spectrum, as described in Sect. II.

In our previous articles we have studied phase space factors (PSF) and prefactors (PF) [12–14], and nuclear matrix elements (NME) [2, 13, 15–18] needed for the theoretical description of $0\nu\beta^-\beta^-$, $2\nu\beta^-\beta^-$, $0\nu\beta^+\beta^+$, $0\nu\beta^+EC^+$, $R0\nu ECEC$, $2\nu\beta^+\beta^+$, $2\nu\beta^+EC$, and $2\nu ECEC$ decay mediated in the case of neutrinoless decay by light or heavy neutrino exchange. In this article we continue our systematic evaluation by presenting phase space factors for the different Majoron emit-

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TABLE I. Different Majoron emitting models of $0\nu\beta\beta M$ decay [7–11]. The third, fourth, and fifth columns indicate whether the Majoron is Nambu-Goldstone boson or not, its leptonic charge L , and the model's spectral index n .

Model	Decay Mode	NG boson	L	n
IB	$0\nu\beta\beta\chi_0$	No	0	1
IC	$0\nu\beta\beta\chi_0$	Yes	0	1
ID	$0\nu\beta\beta\chi_0\chi_0$	No	0	3
IE	$0\nu\beta\beta\chi_0\chi_0$	Yes	0	3
IIB	$0\nu\beta\beta\chi_0$	No	-2	1
IIC	$0\nu\beta\beta\chi_0$	Yes	-2	3
IID	$0\nu\beta\beta\chi_0\chi_0$	No	-1	3
IIE	$0\nu\beta\beta\chi_0\chi_0$	Yes	-1	7
IIF	$0\nu\beta\beta\chi_0$	Gauge boson	-2	3
"Bulk"	$0\nu\beta\beta\chi_0$	Bulk field	0	2

ting mechanisms and combining the $n = 1$ results with recent interacting boson model (IBM-2) NMEs [18] to make half-life predictions for ordinary Majoron decay ($n = 1$). Furthermore, we compare our theoretical predictions with the obtained experimental lower bounds for this decay mode to set some limits on the effective Majoron-neutrino coupling constant $\langle g_{ee}^M \rangle$.

II. PHASE SPACE FACTORS IN MAJORON EMITTING DOUBLE- β DECAY

The key ingredients for the evaluation of phase space factors in single- and double- β decay are the scattering wave functions and for EC the bound state wave functions. The general theory of relativistic electrons and positrons can be found e.g., in the book of Rose [19]. The electron scattering wave functions of interest in $\beta^-\beta^-$ were given in Eq. (8) of [12].

In order to calculate PSFs for Majoron emitting $\beta^-\beta^-$ we use the formulation of Doi, Kotani, and Takasugi [20]. The differential rate for the decay is given by [20, 21]

$$dW_{m\chi_0 n} = \left(a^{(0)} + a^{(1)} \cos \theta_{12} \right) w_{m\chi_0 n} d\epsilon_1 d\epsilon_2 d(\cos \theta_{12}) \quad (4)$$

where ϵ_1 and ϵ_2 are the electron energies, θ_{12} the angle between the two emitted electrons, and $w_{m\chi_0 n}$ takes different values depending on the number of emitted Ma-

ajorons m and the spectral index n :

$$\begin{aligned} w_{1\chi_0 1} &= \frac{g_A^4 (G \cos \theta_C)^4}{64\pi^7 \hbar} \left(\frac{\hbar c}{2R} \right)^2 q(p_1 c)(p_2 c) \epsilon_1 \epsilon_2 \\ w_{1\chi_0 3} &= \frac{g_A^4 (G \cos \theta_C)^4}{64\pi^7 \hbar} q^3(p_1 c)(p_2 c) \epsilon_1 \epsilon_2 \\ w_{2\chi_0 3} &= \frac{g_A^4 (G \cos \theta_C)^4}{3072\pi^9 \hbar} (m_e c^2)^{-2} \left(\frac{\hbar c}{2R} \right)^2 q^3(p_1 c)(p_2 c) \epsilon_1 \epsilon_2 \\ w_{2\chi_0 7} &= \frac{g_A^4 (G \cos \theta_C)^4}{53760\pi^9 \hbar} (m_e c^2)^{-6} \left(\frac{\hbar c}{2R} \right)^2 q^7(p_1 c)(p_2 c) \epsilon_1 \epsilon_2. \end{aligned} \quad (5)$$

Here G is the Fermi constant, θ_C is the Cabibbo angle, and the Majoron energy q is determined as $q = Q_{\beta\beta} + 2m_e c^2 - \epsilon_1 - \epsilon_2$, and $R = r_0 A^{1/3}$ with $r_0 = 1.2$ fm, is the nuclear radius. The quantities $a^{(0)}$ and $a^{(1)}$ in Eq. (4) can be written as [21]

$$a^{(i)} = f_{11}^{(i)} |\langle g_{ee}^M \rangle|^{2m} |M_{0\nu M}^{(m,n)}|^2 \quad i = 0, 1, \quad (6)$$

where $|\langle g_{ee}^M \rangle|$ is the effective coupling constant of the Majoron to the neutrino, $m = 1, 2$ for the emission of one or two Majorons, respectively, and $M_{0\nu M}^{(m,n)}$ is the nuclear matrix element. The functions $f_{11}^{(0)}$, $f_{11}^{(1)}$ are defined as

$$\begin{aligned} f_{11}^{(0)} &= |f^{-1-1}|^2 + |f_{11}|^2 + |f_1^{-1}|^2 + |f_1^{-1}|^2, \\ f_{11}^{(1)} &= -2\text{Re}[f^{-1-1} f_{11}^* + f_1^{-1} f_1^{-1*}]. \end{aligned} \quad (7)$$

with

$$\begin{aligned} f^{-1-1} &= g_{-1}(\epsilon_1) g_{-1}(\epsilon_2), \\ f_{11} &= f_1(\epsilon_1) f_1(\epsilon_2), \\ f_1^{-1} &= g_{-1}(\epsilon_1) f_1(\epsilon_2), \\ f_1^{-1} &= f_1(\epsilon_1) g_{-1}(\epsilon_2), \end{aligned} \quad (8)$$

where $g_{-1}(\epsilon)$ and $f_1(\epsilon)$ are obtained from the electron wave functions as explained in Ref. [12].

All quantities of interest are then given by integration of Eq. (4). Introducing

$$\begin{aligned} G_{m\chi_0 n}^{(i)} &= \frac{2}{g_A^4 \ln 2} \int_{m_e c^2}^{Q_{\beta\beta} + m_e c^2} \int_{m_e c^2}^{Q_{\beta\beta} + 2m_e c^2 - \epsilon_1} f_{11}^{(i)} \\ &\times w_{m\chi_0 n} d\epsilon_1 d\epsilon_2, \end{aligned} \quad (9)$$

where the axial vector coupling constant g_A is separated from the phase space factors for convenience, we can calculate:

(i) The half-life

$$[\tau_{1/2}^{0\nu}]^{-1} = g_A^4 G_{m\chi_0 n}^{(0)} |\langle g_{\chi_{ee}^M} \rangle|^{2m} |M_{0\nu M}^{(m,n)}|^2, \quad (10)$$

(ii) the single electron spectrum

$$\frac{dW_{m\chi_0 n}}{d\epsilon_1} = \mathcal{N}_{0\nu M}^{(m,n)} \frac{dG_{m\chi_0 n}^{(0)}}{d\epsilon_1} \quad (11)$$

where $\mathcal{N}_{0\nu M}^{(n,m)} = g_A^4 \ln 2 |\langle g_{\chi_{ee}^M} \rangle|^{2m} |M_{0\nu M}^{m,n}|^2$.

(iii) The summed electron spectrum, which shape makes the different Majoron emitting modes experimentally recognizable

$$\frac{dW_{m\chi_0 n}}{d(\epsilon_1 + \epsilon_2)} = \mathcal{N}_{0\nu M}^{(n,m)} \frac{dG_{m\chi_0 n}^{(0)}}{d(\epsilon_1 + \epsilon_2)}, \quad (12)$$

(iv) and the angular correlation between the two electrons

$$\alpha(\epsilon_1) = \frac{dG_{m\chi_0 n}^{(1)}/d\epsilon_1}{dG_{m\chi_0 n}^{(0)}/d\epsilon_1}. \quad (13)$$

TABLE II. Phase space factors $G_{m\chi_0 n}^{(0)}$ obtained using screened exact finite size Coulomb wave functions.

Nucleus	$G_{m\chi_0 n}^{(0)} (10^{-18} \text{ yr}^{-1})$			
	m=1,n=1	m=1,n=3	m=2,n=3	m=2,n=7
⁴⁸ Ca	1540	17.1	73.6	690
⁷⁶ Ge	44.2	0.073	0.22	0.420
⁸² Se	361	1.22	3.54	26.9
⁹⁶ Zr	905	4.21	11.0	128.
¹⁰⁰ Mo	598	2.42	6.15	50.8
¹¹⁰ Pd	94.1	0.205	0.487	0.946
¹¹⁶ Cd	569	2.28	5.23	33.9
¹²⁴ Sn	209	0.653	1.45	4.45
¹²⁸ Te	3.06	0.001	0.003	0.0003
¹³⁰ Te	413	1.51	3.21	14.4
¹³⁴ Xe	2.92	0.002	0.003	0.0002
¹³⁶ Xe	409	1.47	3.05	12.5
¹⁴⁸ Nd	197	0.505	0.986	1.72
¹⁵⁰ Nd	3100	21.1	40.8	538
¹⁵⁴ Sm	28.2	0.034	0.064	0.021
¹⁶⁰ Gd	1590	0.361	0.672	0.899
¹⁹⁸ Pt	60.7	0.068	0.110	0.021
²³² Th	82.4	0.073	0.105	0.009
²³⁸ U	337	0.532	0.756	0.213

We have done a calculation of $G_{m\chi_0 n}^{(0)}$ and $G_{m\chi_0 n}^{(1)}$ in the list of nuclei shown in Table II. We also plot our results in Figs. 2-5, where they are compared with previous calculations [20–24]. For the comparison the values of [20, 22] have been multiplied by a missing factor of two and divided by g_A^4 , and values of [21] have been divided with factor $g_A^4 4R^2$. The factor of two is the correct choice, as was acknowledged in [25]. This factor of two is also included in Table II of Ref. [26], where the decay of ¹³⁶Xe have been studied. In their calculation for

the phase space factors they use Fermi functions $F(Z,E)$ that fully include nuclear finite size and electron screening and are evaluated at the nuclear radius. Their result with different $m = 1, 2$ and $n = 1, 3, 7$ are within 6% of the ones reported here.

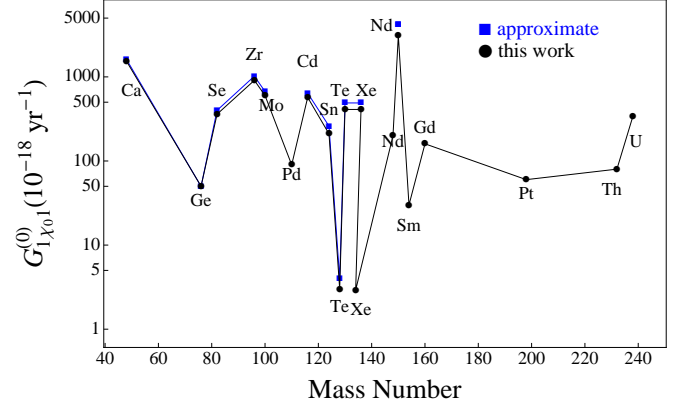


FIG. 2. (Color online) Phase space factors $G_{1\chi_0 1}^{(0)}$ in units (10^{-18} yr^{-1}). The label "approximate" refers to the results obtained by the use of approximate electron wave functions [20–22]. The figure is in semilogarithmic scale.

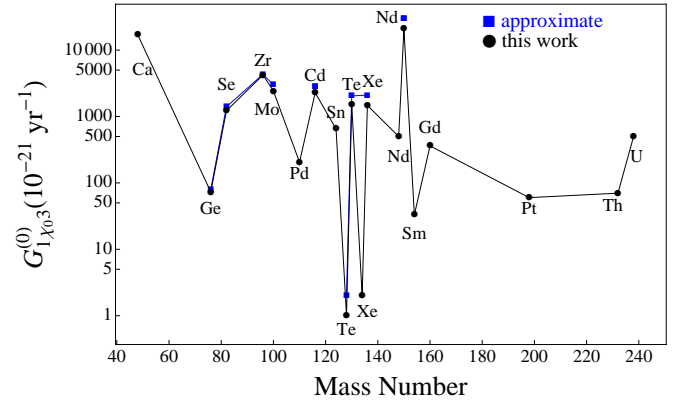


FIG. 3. (Color online) Phase space factors $G_{1\chi_0 3}^{(0)}$ in units (10^{-21} yr^{-1}). The label "approximate" refers to the results obtained by the use of approximate electron wave functions [23, 24]. The figure is in semilogarithmic scale.

We also have available upon request the single and summed electron electron spectra and angular correlation for all nuclei in Table II. An example, ¹³⁶Xe decay, is shown in Fig. 6. The shape of the spectra is determined by the spectral index n , i.e. for the case $n = 3$ it is the same for both the emission of one or two Majorons, but the overall scale is different (χ_0 result should be multiplied by $(\hbar c)^2 (2Rm_e c^2)^{-2} (48\pi^2)^{-1}$ to obtain $2\chi_0$ result). Also, since the angular correlation is obtained by dividing $dG_{m\chi_0 n}^{(1)}/d\epsilon_1$ by $dG_{m\chi_0 n}^{(0)}/d\epsilon_1$ the calculation becomes unstable for the higher n near the endpoint energy and that region is thus excluded from Fig. 6. The distinction between different values of n is most prominent in

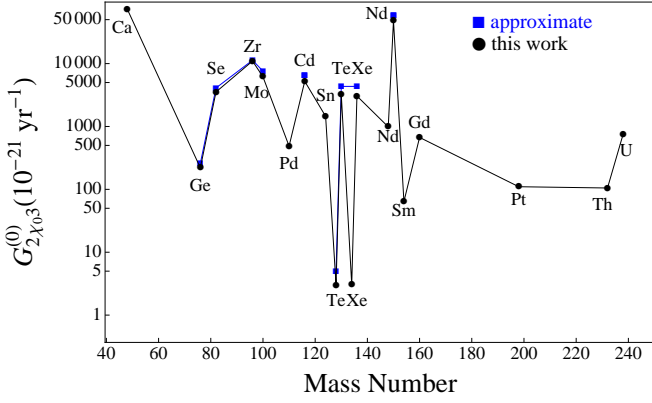


FIG. 4. (Color online) Same as Fig. 3 for $G_{2\chi_0 3}^{(0)}$.

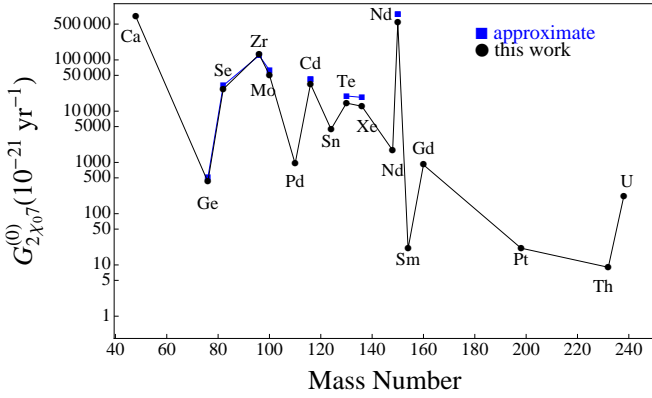


FIG. 5. (Color online) Same as Fig. 3 for $G_{2\chi_0 7}^{(0)}$. Due to a very small values, ^{128}Te and ^{134}Xe are excluded from this figure.

summed electron spectra, where as n increases the peak of the spectrum shifts from near the maximum kinetic energy to near the minimum kinetic energy. The difference between different values of n is also shown in single electron spectra but not as strongly as in the case of summed electron spectra.

III. EXPECTED HALF-LIVES AND LIMITS ON COUPLING CONSTANT: ORDINARY MAJORON EMITTING $\beta^-\beta^-$ DECAY

In the case of ordinary Majoron emitting double- β decay, $n = 1$, the nuclear matrix elements have the same form as in the $0\nu\beta\beta$ mediated by light neutrino exchange

$$M_{0\nu M}^{(1,1)} = g_A^2 \left(M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A} \right)^2 M_F^{(0\nu)} + M_T^{(0\nu)} \right). \quad (14)$$

The calculation of phase space factors can now be combined with updated nuclear matrix elements in IBM-2 [18] to produce predictions for half-lives for ordinary Majoron decay ($n = 1$) in Table III (left) and Fig. 7.

Judging by the predicted half-lives, the most prominent candidates are ^{150}Nd and ^{100}Mo . Furthermore, we can compare our predictions to half-life limits coming from experiments to set some limits on the effective Majoron-neutrino coupling constant. The obtained limits are shown on Table III (right). The most stringent limits for ordinary Majoron decay ($n = 1$) at the moment are for ^{136}Xe coming from KamLAND-Zen [34] and EXO-200 [26] experiments reaching the order of magnitude of 10^{-5} .

TABLE III. Left: Calculated half-lives for Majoron decay models where $n = 1$ (IB, IC, IIB) with $\langle g_{ee}^M \rangle = 10^{-4}$, $g_A = 1.269$ and recent IBM-2 nuclear matrix elements [18]. Right: Upper limit on Majoron coupling constant $\langle g_{ee}^M \rangle$ from current experimental limits.

Decay	$\tau_{1/2}^{0\nu M} (10^{21} \text{yr})$	$\tau_{1/2, exp}^{0\nu M} (\text{yr})$	$\langle g_{ee}^M \rangle (\text{eV})$
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	8.19	$> 7.2 \times 10^{20\text{a}}$	$< 3.4 \times 10^{-3}$
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	39.8	$> 6.4 \times 10^{22\text{b}}$	$< 7.9 \times 10^{-4}$
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	7.68	$> 1.5 \times 10^{22\text{c}}$	$< 7.2 \times 10^{-4}$
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	5.32	$> 1.9 \times 10^{21\text{c}}$	$< 1.7 \times 10^{-3}$
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3.62	$> 3.9 \times 10^{22\text{d}}$	$< 3.0 \times 10^{-4}$
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	25.0		
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	7.06	$> 8 \times 10^{21\text{e}}$	9.4×10^{-4}
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	18.1		
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	765	$> 2 \times 10^{24\text{f}}$	$< 6.2 \times 10^{-4}$
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	6.82	$> 1.6 \times 10^{22\text{g}}$	$< 6.5 \times 10^{-4}$
$^{134}\text{Xe} \rightarrow ^{134}\text{Ba}$	805		
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	10.1	$> 2.6 \times 10^{24\text{h}}$	$< 6.2 \times 10^{-5}$
		$> 1.2 \times 10^{24\text{i}}$	$< 9.2 \times 10^{-5}$
$^{148}\text{Nd} \rightarrow ^{148}\text{Sm}$	36.8		
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	1.74	$> 1.5 \times 10^{21\text{c}}$	$< 1.1 \times 10^{-3}$
$^{154}\text{Sm} \rightarrow ^{154}\text{Gd}$	173		
$^{160}\text{Gd} \rightarrow ^{160}\text{Dy}$	14.5		
$^{198}\text{Pt} \rightarrow ^{198}\text{Hg}$	132		
$^{232}\text{Th} \rightarrow ^{232}\text{U}$	28.7		
$^{238}\text{U} \rightarrow ^{238}\text{Pu}$	4.94		

^a Ref. [27].

^b Ref. [28].

^c Ref. [29].

^d Ref. [30].

^e Ref. [31].

^f Ref. [32], geochemical.

^g Ref. [33].

^h Ref. [34].

ⁱ Ref. [26].

IV. CONCLUSIONS

In this article, we have reported a complete calculation of phase space factors for $0\nu\beta^-\beta^-$ decay proceeding through emission of one or two Majorons. The reported results include half-lives, single electron spectra, summed electron spectra, and electron angular correlations, to be used in connection with the calculation of nuclear matrix

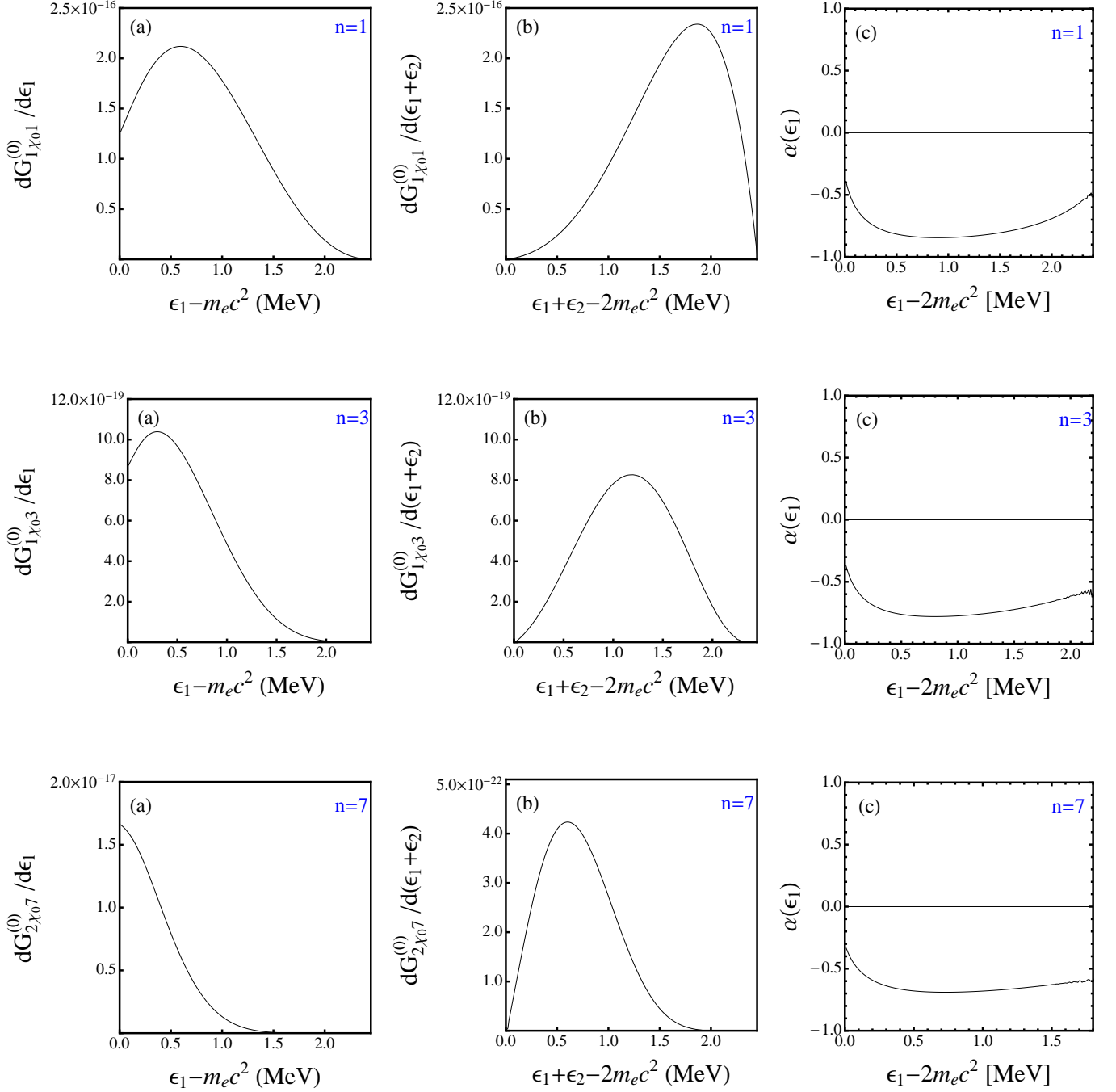


FIG. 6. (Color online) Single electron spectra (panel a), summed electron spectra (panel b), and angular correlations between the two outgoing electrons (panel c) for the $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ $0\nu\beta\beta M$ -decay with different spectral indices $n = 1, 3, 7$. The scale of the panels (a) and (b) should be multiplied by $\mathcal{N}_{0\nu M}^{(n,m)}$ for a realistic estimate.

elements. Furthermore, we have combined our results with recent IBM-2 nuclear matrix elements to produce predictions of half-lives in the case of ordinary Majoron decay, spectral index $n = 1$. Comparing these predic-

tions with experimental lower bounds, we have set some limits on the effective Majoron-neutrino coupling constant $\langle g_{ee}^M \rangle$. At the moment the best limits are coming from ^{136}Xe experiments reaching the order of mag-

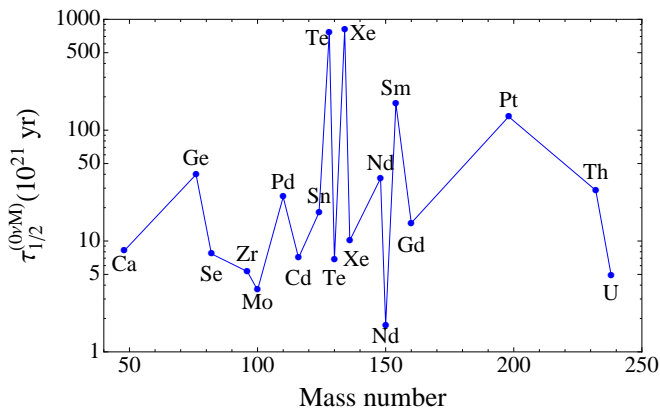


FIG. 7. (Color online) Expected half-lives for Majoron decay models where $n = 1$ (IB,IC,IIB) with $\langle g_{ee}^M \rangle = 10^{-4}$, $g_A = 1.269$ and IBM-2 isospin restored nuclear matrix elements. The figure is in semilogarithmic scale.

nitude of 10^{-5} . Also, the results in Table III are for $g_A = 1.269$. If g_A is renormalized to $g_{A,eff}$, $\tau_{1/2}^{0\nu M}$ should be multiplied by $(1.269/g_{A,eff})^4$ and limits on $\langle g_{ee}^M \rangle$ by $(1.269/g_{A,eff})^2$.

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