Nonperturbative lepton-sea fermions in the nucleon and the proton radius puzzle

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Non-Perturbative Lepton Sea Fermions in the Nucleon and the Proton Radius Puzzle NT@UW-14-27

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A potential explanation [U. D. Jentschura, Phys. Rev. A **88**, 062514 (2013)] of the proton radius puzzle originating from the non-perturbative lepton-pair content of the proton is studied. Well-defined quantities that depend on this lepton-pair content are evaluated. Each is found to be of the order of $10(\frac{\alpha}{\pi})^2$, so that we find such a lepton-pair content exists in the proton. However, we argue that this relatively large result and general features of loop diagrams rule out the possibility of lepton-pair content as an explanation of the proton radius puzzle. The contributions of a class of potential explanations of the proton radius puzzle (for which the dependence on the μp relative distance is as contact interaction) are shown to be increase very rapidly with atomic number.

I. INTRODUCTION

Recent high precision experimental studies of muonic hydrogen [1, 2] obtain a value of the proton radius that is about 4% smaller than that obtained from ordinary electronic hydrogen. The problem of understanding this difference has become known as the proton radius puzzle, and has generated a vast array of possible solutions, see the review [3]. One of the novel suggested solutions [4] is that a non-perturbative feature of the proton's structure, namely the possible presence of light sea fermions as constituent components of the proton, could account for the difference in the extracted radii. In particular, it is argued that the assumption that the presence of 2.1×10^{-7} light sea positrons per quark, leads to an an extra term in the electron-proton versus muon-proton interaction, which has the right sign and magnitude to explain the proton radius puzzle .

The basic idea is that a bound electron may annihilate with a positron that is part of the non-perturbative e^-e^+ cloud of the proton. The annihilation leads to a virtual photon, which in turn decays to a bound electron and a positron that is also part of the e^-e^+ cloud of the proton. See Fig. 1. The term non-perturbative here refers to a component of the proton Fock-space wave function that can be seen at small values of momentum transfer. If one could take a snapshot of a proton in isolation one would see electron-positron or muon-anti-muon pairs pop in and out of existence. This effect is therefore different from the generation of pairs by evolution in momentum transfer that is akin to the source of the $q\bar{q}$ sea of perturbative QCD.

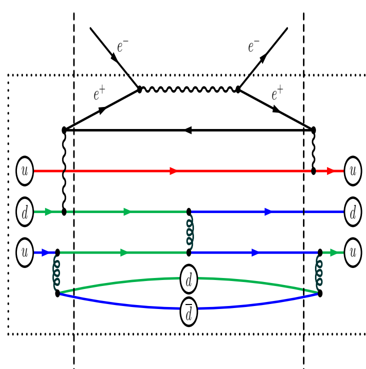

FIG. 1: (color online) Typical Feynman diagram of Ref. [4] illustrating the virtual annihilation of a bound electron with a "light sea lepton" (positron) inside the proton. The up (u) and down (d) quarks, which carry non-integer charge numbers, interact electromagnetically. The vertical dashed lines indicate the lepton-sea component of the proton wave function. The light sea lepton annihilates with the bound electron that enters the diagram from the left.

A natural question to ask is whether or not this diagram is part of a contribution that is already included. In particular, a look at Fig 1 might lead one to conclude that the intermediate baryonic state is that of an excited nucleon. If so, this diagram would be a particular time-ordering of the proton polarizability contribution to the two-photon exchange diagram. However, if the lepton pair is a specific Fock-space component of the complete proton wave function (including QED effects) one may argue [4] that the intermediate state is part of the proton wave function, and therefore not part of the proton polarizability contribution. Here we accept this argument and seek to determine its logical consequences.

This acceptance comes with severe difficulties. If one adopts this Fock-space approach, one must recognize that computing the contribution of a given component may only correspond to computing a particular time ordering of a Feynman diagram, with the consequent obligation to find any remaining time ordering terms of the same order in α. To be specific, the term of Fig. 1 corresponds to a particular time ordering of the graphs that give the radiative corrections to the general two photon exchange diagrams. See for example, Fig. 2. In this figure, the intermediate blob is meant to contain the proton and all of its excited states. Thus this graph contains the time-ordering that is included in Fig. 1. One may immediately estimate the size of the contribution of this term to the energy to be of order α/π times the polarizability correction, and is therefore negligible. The explicit calculations for muonic hydrogen are in accord with this estimate [5]. For electronic hydrogen the contribution is again of order α/π times the polarizability correction, which is relatively much less important for electronic hydrogen than for muonic hydrogen[6]. This means that a correct evaluation of the contribution of the effects of the proton's lepton-pair content must give a negligible result.

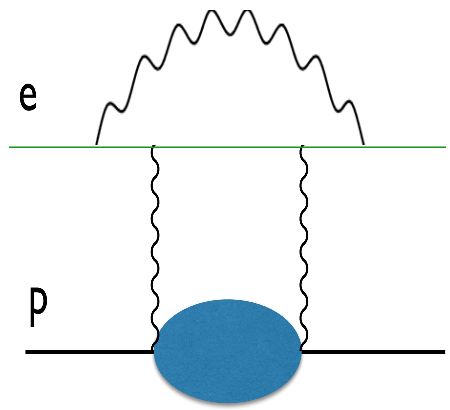

FIG. 2: (color online) Radiative correction to two photon exchange. Including the full gauge invariant set, including crossing the photons arising from the proton (p) and all of the one-loop self-energy terms on the electron(e), is implied.

We pursue the idea of Ref [4] even though the result of any correct calculation will be that the effect is negligible. This is because investigating the lepton-pair content of the nucleon is interesting in its own right and because it useful to see how the correct result arises from the Fock-space idea. The argument of Ref [4] starts by considering the photon annihilation term occurring in positronium. This term leads to the effective interaction [7]:

$$\delta H = \frac{\pi\alpha}{2m_e^2}\,(3+\boldsymbol{\sigma}_+\cdot\boldsymbol{\sigma}_-)\,\delta(\mathbf{r})\,. \tag{1}$$

This Hamiltonian gives a nonzero interaction of the bound electron and the light sea positron if their spins add up to one. Ref. [4] assumes that the electron-positron pairs within the proton are not polarized and replaces $\vec{\sigma}_+\cdot\vec{\sigma}_-\to 0$ after averaging over the polarizations of the sea leptons. Further it is argued [4] that for atomic (electronic) hydrogen, the additional interaction of the electron with the proton due to the annihilation channel takes the form

$$H_{\rm ann} = \epsilon_p\,\frac{3\pi\alpha}{2m_e^2}\,\delta(\mathbf{r})\,, \tag{2}$$

where ϵ_p measures the amount of electron-positron pairs within the proton. For muonic hydrogen, the effect is expected to vanish because the dominant contribution to the sea leptons comes from the lightest leptons, namely, electron-positron pairs and thus the annihilation channel is not available. Ref. [4] finds that a value of $\epsilon_p = 2.1\times 10^{-7}$ is sufficient to account for the different values of the extracted radii. Note that the $1/m_e^2$ dependence occurs as the result of assuming that both the electron and positron are at rest.

The hypothesis of sea leptons and the use of Eq. (2) raises a number of interesting questions. Is the quantity ϵ_p well defined? If so, what is its likely range of values? Any positron in the non-perturbative nucleon sea is not likely to be at rest, so one could also ask if the Eq. (2) is applicable. Does the term of Eq. (2) really exist? Our purpose here is to investigate these questions.

Here is an outline of the remainder of this paper. The difficulties in computing and defining ϵ_p are discussed in Sect. II. Observables that depend on lepton-pair content and can be computed in a gauge invariant manner are discussed in Sects.III and IV. The results of these two sections are analyzed and used to understand more detailed loop calculations in Sec. V. The nuclear dependence of all models in which the contribution to the Lamb shift arises from an interaction containing a Dirac delta function in the lepton-nucleon separation is studied in Sec. VI. Some concluding remarks are made in Sect. VII.

II. ATTEMPTING TO COUNT POSITRONS IN THE PROTON

We might proceed to count positrons by defining an anti-lepton number current density operator, $J^\mu_{\bar{l}}$,

$$J^\mu_{\bar{l}} = \bar{\psi}_{\bar{l}}\gamma^\mu\psi_{\bar{l}}, \tag{3}$$

and take its expectation value in the physical proton wave function. The operator $J^\mu_{\bar{l}}$ is defined to act only on anti-leptons ($\bar{l}$), so its 0'th component is the anti-lepton density. We may take the expectation value of this operator in the proton by assuming that the lepton pair arises from the interactions on a single quark, as in Fig. 3a. (Fig.3b is used in Sect. III.) In Fig. 3a j represents the incoming momentum, p and $p' = p + j$ are respectively the incoming and outgoing quark momenta, m and m_q are respectively the lepton mass and the constituent quark mass, q is the momentum flowing into the inner loop, and k is the momentum on one branch of the inner loop.

It is necessary to discuss why we focus on a single quark. In general the momentum in the loop q is very large. Any operator that removes a large momentum from one quark and gives a large momentum to another quark, suffers a vastly reduces matrix element in the proton wave function because each quark can only support a momentum of the order of the inverse radius of the proton, or about 200 MeV/c. There are significant implications to this single-quark dominance see below in Sect. VI.

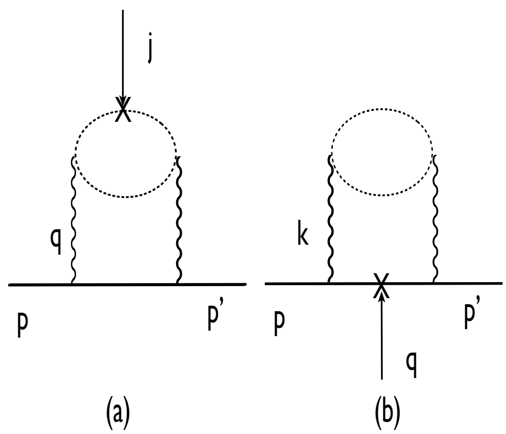

FIG. 3: (color online) The solid line represents a single quark. (a) Diagram for counting anti-leptons. The notation X with the arrow coming into it represents the operator J^μ bringing in a four-momentum j. (b) Lepton-pair contribution to the vertex function.

One immediate problem with the amplitude of Fig. 3a is that the integrations over the virtual lepton momenta contain an ultraviolet divergence. This means that an unknown counter term is needed to determine a value, and predictive power is lost. Furthermore, the use of of Eq. (3) does not respect current conservation, because $J^\mu_{\bar{l}}$ does not act on all of the charges. Thus any result would be gauge-dependent. These two drawbacks lead us to conclude that the short answer to the question, "is the quantity ϵ_p well defined?", is simply *no*. Thus the validity of Eq. (2), which contains a factor of ϵ_p, is questionable.

However, one may compute matrix elements that depend on the pair content that are free of ultraviolet divergences and are gauge invariant. This is the procedure adopted here.

III. ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON

The goal of this section is to find a simple gauge invariant quantity free of ultraviolet and infrared divergent terms, that depends on the lepton-pair content of the proton and to evaluate that quantity. Computation of the influence of virtual lepton-pairs provides such an example. A leading-order effect of pairs is shown in Fig. 3b, in which the

X represents the usual electromagnetic current operator. In this diagram, the incoming momentum is denoted as q and k is the momentum of the virtual photon. The intermediate electron line exists only if the lepton-pair exists, and computing only the contribution to the magnetic moment insures that gauge invariance is maintained without encountering an ultraviolet or infrared divergence. The effects of this diagram have been evaluated long ago, so our purpose is merely to find an illustrative example.

The effect of pairs is accounted for by dressing the photon propagator using [8]

$$\frac{-ig_{\mu\nu}}{q^2} \to \frac{-ig_{\mu\nu}}{q^2}\frac{1}{1-\hat{\Pi}_2(q^2)} \approx \frac{-ig_{\mu\nu}}{q^2}(1+\hat{\Pi}_2(q^2)), \tag{4}$$

for a virtual photon of four-momemtum q, with

$$\hat{\Pi}_2(q^2) = \frac{-2\alpha}{\pi}\int_0^1 dz\, z(1-z)\log\frac{m_l^2}{m_l^2 - z(1-z)q^2}, \tag{5}$$

where m_l is the lepton mass and the subscript 2 stands for second order in e. It is useful to rewrite this term as

$$\hat{\Pi}_2(q^2) = \frac{-2\alpha}{\pi}\int_0^1 dz\, z(1-z)\, q^2 \int_0^1 \frac{d\lambda}{\lambda}\frac{1}{\hat{m}_l^2 - q^2}, \tag{6}$$

where

$$\hat{m}_l^2 \equiv \frac{m_l^2}{z(1-z)\lambda}. \tag{7}$$

The use of Eq. (6) allows the evaluation of a Feynman diagram involving Π_2 via the usual technique of combining denominators.

The order e^4 correction term to the vertex function, $\delta\Gamma^\mu(q)$ is given by using Eq. (4) in the standard expression [8] for the e^2 term:

$$\delta\Gamma^\mu(q) = 2ie^2\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}(p')\left(\not{k}\gamma^\mu\not{k}' + m^2\gamma^\mu - 2m(k+k')^\mu\right)u(p)}{\left((p-k)^2+i\epsilon)(k'^2-m^2+i\epsilon)(k^2-m^2+i\epsilon)\right)}\hat{\Pi}_2\left((p-k)^2\right), \tag{8}$$

where p is the initial quark momentum, $p' = p+q$, $k' = k+q$ and m is the quark mass, which we take at a constituent value of one-third of the mass of a proton. The use of Eq. (6) in Eq. (8) gives the result:

$$\delta\Gamma^\mu(q) = 2ie^2\frac{2\alpha}{\pi}\int_0^1 dz\, z(1-z)\int_0^1\frac{d\lambda}{\lambda}\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}(p')\left(\not{k}\gamma^\mu\not{k}' + m^2\gamma^\mu - 2m(k+k')^\mu\right)u(p)}{\left((p-k)^2-\hat{m}_l^2+i\epsilon)(k'^2-m^2+i\epsilon)(k^2-m^2+i\epsilon)\right)}. \tag{9}$$

The above correction term contributes to both the Dirac and Pauli form factors of the quark. Evaluating the Dirac form factor requires treatments of infrared and ultraviolet divergences that are not present in the Pauli form factor. We therefore compute only the contribution to the Pauli form factor. Then we proceed by combining denominators, and integrating over the four-momentum and two of the Feynman parameters and λ.

The relevant momentum transfer for atomic physics is the inverse of the Bohr radius, this is much, much smaller than the quark mass. Therefore we need only evaluate $\delta\Gamma^\mu(q)$ at $q^2 = 0$. We find the following result

$$\delta\Gamma^\mu(q^2=0) = 2(\tfrac{\alpha}{\pi})^2\,\bar{u}(p')\tfrac{i\sigma^{\mu\nu}q_\nu}{2m}u(p)I(m^2/m_l^2) \tag{10}$$

$$I(m^2/m_l^2) = \textstyle\int_0^1 dz\, z(1-z)\int_0^1 dz'\,\log\left(1+\frac{(1-z')^2 z(1-z)}{z'}\frac{m^2}{m_l^2}\right) \tag{11}$$

Numerical evaluation leads to the result

$$I(\text{electron}) = 1.7, \quad I(\text{muon}) = 0.17, \tag{12}$$

and the virtual e^+e^- pair contribution to the magnetic moment of the quark is given by $3.4\,(\frac{\alpha}{\pi})^2$ or 1.8×10^{-5}. This number is about 100 times larger than the value of ϵ_p used to account for the proton radius puzzle. Moreover, if the value of the lepton mass in the loop were to dominate the value we would expect a muon to electron ratio of about 1/200 instead of the ratio $\sim 1/10$ of the values of Eq. (20).

It is useful to attempt to relate the results of Eq. (20) to access the value of ϵ_p, even though this can only be done heuristically. In non-relativistic quantum mechanics the contribution of a component n to the expectation value of an operator $\mathcal{O}$ is written as $\langle\mathcal{O}\rangle = \int d^3r\psi_n^*(\mathbf{r})\mathcal{O}\psi_n(\mathbf{r})$. This can be interpreted as $\int d^3r|\psi_n(\mathbf{r})|^2 \times \bar{\mathcal{O}} = P_n\bar{\mathcal{O}}$. where $\bar{\mathcal{O}}$ is the average of $\mathcal{O}$ in the component n. If this operator is written in natural dimensionless units, such as a coefficient multiplying the factor $\bar{u}(p')\frac{i\sigma^{\mu\nu}q_\nu}{2m}u(p)$, then $\bar{\mathcal{O}}$ can be expected to be of the order of unity. In that case the numerical coefficient, here 1.8×10^{-5} can be expected to be of the order of the probability for the electron-postiron pair to exist. Thus, we state, very roughly, that

$$\epsilon_p \sim 1.8 \times 10^{-5}. \tag{13}$$

Note that the dimensionless factor, I, multiplying $\bar{u}(p')\frac{i\sigma^{\mu\nu}q_\nu}{2m}u(p)$ depends on the ratio of the quark to lepton masses. This is not surprising-at $q^2 = 0$ the only parameters with dimension are m, m_l and the only way to make a dimensionless number is a dependence on the ratio. The consequences of this are discussed below in Sect. V.

IV. AXIAL COUPLING

We seek another example of a divergent-free matrix element that depends on the lepton-pair content of the proton. Consider the axial coupling $\gamma^\mu\gamma^5$ operator as an insertion (X) in Fig. 3b. The resulting term is defined as δA^μ, with

$$\delta A^\mu = ie^2\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}(p')\gamma_\rho\left(\not{k}'+m)\gamma^\mu\gamma^5(\not{k}+m)\right)\gamma^\rho u(p)}{\left((p-k)^2+i\epsilon)(k'^2-m^2+i\epsilon)(k^2-m^2+i\epsilon)\right)}\hat{\Pi}_2((p-k)^2), \tag{14}$$

Using parity conservation and time reversal invariance tells us that δA^μ takes the form

$$\delta A^\mu = \bar{u}(p')\left[G_A(q^2)\gamma^\mu\gamma^5 + i\frac{G_T(q^2)}{2m}\sigma^{\mu\nu}q_\nu\gamma^5 + \frac{G_P(q^2)}{2m}q^\mu\gamma^5\right]u(p). \tag{15}$$

The term G_A, as computed from Eq. (14) will contain an ultraviolet divergence, and is determined after a renormalization procedure. The terms G_T, G_P are free of such problems, so we will examine only those terms. We again take $q^2 = 0$.

We proceed by using Eq. (6) in Eq. (14) to find

$$\delta A^\mu = ie^2\frac{2\alpha}{\pi}\int_0^1 dz\,z(1-z)\int_0^1\frac{d\lambda}{\lambda}\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}(p')\gamma_\rho\left(\not{k}'+m)\gamma^\mu\gamma^5(\not{k}+m)\right)\gamma^\rho u(p)}{\left((p-k)^2-\hat{m}_l^2+i\epsilon)(k'^2-m^2+i\epsilon)(k^2-m^2+i\epsilon)\right)}. \tag{16}$$

We proceed by combining the propagators, shifting the origin of integration, integrating over the shifted four-momentum, integrating over one of the Feynman parameters, taking $q^2 = 0$ and keeping only the contribution to G_T, G_P. The result is

$$\delta A^\mu_{T,P}(q^2=0) = \tfrac{1}{2}(\tfrac{\alpha}{\pi})^2\,m\int_0^1 dz\,z(1-z)\int_0^1\tfrac{d\lambda}{\lambda}\int_0^1 dz'\int_0^{1-z'}dy\left[q^\mu\gamma^5(2+z'(1+z'))+i\sigma^{\mu\nu}q_\nu\gamma^5(-2yz'+z'(1-z')\right]\times \frac{1}{(1-z')^2m^2+z'\hat{m}_l^2+i\epsilon}. \tag{17}$$

The integration over y gives a factor of $1-z'$ to the term proportional to $q^\mu\gamma^5$, but leads to a cancellation in the term proportional to $i\sigma^{\mu\nu}q_\nu\gamma^5$. Thus the G_T term vanishes at $q^2 = 0$. This cancellation does not occur for other values of q^2. The integration over λ is performed to give the result:

$$\delta A^\mu_P(q^2=0) = \tfrac{1}{2}\left(\tfrac{\alpha}{\pi}\right)^2\tfrac{q^\mu\gamma^5}{m}J(m^2/m_l^2) \tag{18}$$

$$J(m^2/m_l^2) \equiv \int_0^1 dz\,z(1-z)\int_0^1 dz'\tfrac{((2+z'(1+z'))}{1-z'}\log[1+\tfrac{(1-z')^2z(1-z)}{z'}\tfrac{m^2}{m_l^2}] \tag{19}$$

The remaining integrals are handled numerically. The result is that

$$J(q^2=0,\text{electron}) = 18.3, \quad J(q^2=0,\text{muon}) = 0.593, \tag{20}$$

We again see that the effect of e^+e^- pairs is much larger than the 2 $\times10^{-7}$ that is supposed to enter in the proton radius puzzle. Using the logic of the previous section leads to the approximate relation

$$\epsilon_p \sim 5\times10^{-5}. \tag{21}$$

This has about 250 times larger than the value used in Ref. [4]. The numbers displayed in Eq. (13) and Eq. (21) are of the same order of magnitude, which is all that can be expected from the quantitative approach used here.

V. ASSESSMENT

We have provided arguments that the lepton pair content is an order of magnitude larger than the value of ϵ_p needed to account for the proton radius puzzle. One might expect that this strengthens the case for the explanation of Ref. [4].

That this is not so can be understood by examining the validity of Eq. (2), derived by assuming that the electron and positron annihilate when both are at rest. Firstly, if this equation is correct, and the lepton-pair probabilities are as large as obtained in this paper, then the computed effect would be between 100 and 250 times times too big., causing a disagreement with experiment that would rule out the explanation of Ref. [4]. But it is necessary to assess the lepton mass dependence that arises from evaluating the loop diagrams. This is in both examples approximately a logarithm of the ratio m^2/m_l^2, as shown in Fig. 4. This means that a dependence as the inverse square of the lepton mass will not result from evaluating any loop diagram.

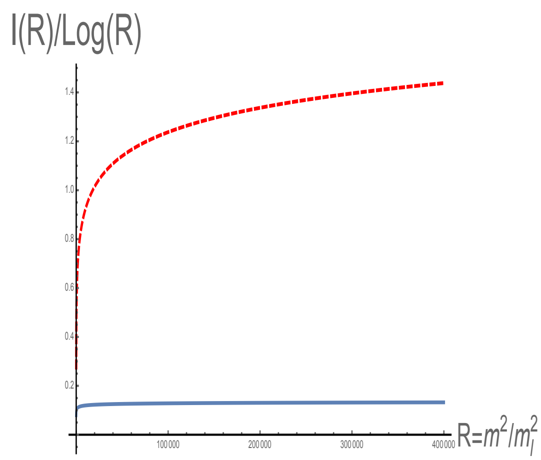

FIG. 4: (color online) Dependence on the ratio m^2/m_l^2. The functions $I(m^2/m_l^2)/\log(m^2/m_l^2)$,Eq. (11) (blue, solid) and $J(m^2/m_l^2)/\log(m^2/m_l^2)$, Eq. (19) (red, dashed) are displayed.

Moreover, we may use dimensional analysis to argue that a complete evaluation of the effect depicted in Fig. 1 would lead to a $1/m^2$ dependence. This is because the inverse factor of $4m_e^2$ appearing in Eq. (1) arises from the propagator of the virtual photon that is produced by the annihilation. In the diagram this factor would be $(p_e^- + p_e^+)^2$. The momentum p_e^- of the external electron is given almost entirely by the electron mass, but the four-momentum of the space-like virtual positron, p_e^+, is dominated by its three-momentum. This in turn is governed in loop integrals by the largest mass, which is that of the quark, m. In particular, a correct and complete evaluation would lead one to obtain

$$\delta H_{\rm ann} \propto \frac{\pi\alpha}{2m^2}\,(3+\boldsymbol{\sigma}_+\cdot\boldsymbol{\sigma}_-)\,\delta(\mathbf{r}) \tag{22}$$

instead of Eq. (1). Since $m \approx 600\, m_e$ the effect is about four hundred thousand times smaller than what is needed and therefore can be said to negligible.

We have used the term "a full and complete evaluation". What would be needed for that? The minimum requirement is gauge invariance. That the diagram of Fig. 1 cannot satisfy gauge invariance by itself is apparent because there are many other diagrams of the same order. In particular, there is a diagram corresponding to crossing of the photons emitted by the u and d quarks of Fig 1. Terms with both crossed and uncrossed photons are needed to satisfy gauge invariance in Compton scattering. This means that the lepton pair content can not be evaluated without also including the effects of the $2\gamma\, e^-\, e^+$ component. One simply needs to compute the complete gauge invariant set of diagrams corresponding to those implied in Fig 2. This has been done for the case of muonic hydrogen. Given the insensitivity of the results to the value of the lepton mass, we expect that for electronic hydrogen the lepton-pair effect will be of order of α/π of the proton polarizability correction and therefore negligible.

VI. NUCLEAR DEPENDENCE

Models such as the contact interactions of Eq. (2) which behave as a delta function in the separation between the lepton and nucleon contain very specific predictions for the nuclear dependence of the Lamb shift.

The first example we consider is the model of Ref. [4]. To make a prediction for the nucleus, one needs to know the contribution of the neutrons. Using the single-quark dominance idea of Sect II gives a specific result obtained by considering the factors of the square of the quark charge that would appear in the calcuation. A proton contains two up quarks and one down quark, with a resulting quark charge squared factor of $2(2/3)^2e^2 + 1(1/3)^2e^2 = e^2$. For a neutron one would have $1(2/3)^2e^2 + 2(1/3)^2e^2 = 2/3e^2$. Thus the neutron contribution would be 2/3 that of the proton. As result, if one considers the Lamb shift in the electron-deuteron atom, the effect would 5/3 as large as for a proton. Such an effect would contradict the existing good agreement between theory and experiment [9].

More generally, suppose the contribution of a proton to the Lamb shift is E_p (0.3meV) to resolve the proton radius puzzle) and that of the neutron is E_n. Then for a nucleus with A nucleons and Z protons, we find

$$E_A = \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p + NE_n)\left(1 - \mathcal{O}(\frac{R_A^2}{a_\mu^2})\right) \approx \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p + NE_n), \tag{23}$$

where a_μ is the muon Bohr radius (>100 times larger than nuclear radius, R_A). The meaning of Eq. (23) is that the contributions of such contact interactions increase very rapidly with atomic number.

In particular, the prediction of Ref. [4](with $E_n = 2/3E_p$) for ^{4}He is a Lamb shift that is (1.27) 8 (2)5/3≈ 10 meV, a huge number. The expression Eq. (23) applies to all models in which the contribution to the Lamb shift enters as a delta function (or of very short range) in the lepton-nucleon coordinate, including [10, 11]. In Ref. [10], which concerns polarizability corrections, the neutron contribution, E_n could vanish, so that the contribution for ^4He would be 20 E_p=6 meV. In Ref. [11], which concerns a gravitational effect, $E_n = E_p$, so the prediction for ^4He would be $40E_p$=12 meV. These various predictions will be tested in an upcoming experiment [12]. It is also worth mentioning the MUon proton Scattering Experiment (MUSE) [13], a simultaneous measurement of $\mu^+ p$ and $e^+ p$ scattering and also a simultaneous measurement of $\mu^- p$ and $e^- p$ scattering that is particularly sensitive to the presence of contact interactions [10] .

VII. SUMMARY

The work presented here supports the idea of the existence of a non-perturbative lepton-pair content of the proton. Such components are not forbidden by any symmetry principle and therefore can appear. However, the presented calculations show that such a content is not a candidate to explain the proton radius puzzle. This is because computation of the necessary loop effects is cannot yield an effective Hamiltonian of the strength and form of Eq. (2). The $1/m_l^2$ behavior of that equation in Ref. [4] is necessary to obtain the needed magnitude of the separate electron and muon Lamb shifts. Instead, loop calculations are expected to lead to a a dependence of $1/m^2$, with m the constituent quark mass. This means that the effect of Ref. [4] is expected to be entirely negligible. This is in accord with the estimate: (α/π) times the polarizability correction that is obtained from evaluating the relevant Feynman diagrams.

More generally: it can be said that the proton may be considered to have Fock -space components containing lepton pairs. However, the simplest and most reliable method of treating such pairs is to compute gauge-invariant sets of Feynman diagrams using QED perturbation theory.

VIII. ACKNOWLEDGEMENTS

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