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# Nuclear response theory with multiphonon coupling in a covariant framework

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**Background:** Nuclear excited states within a wide range of excitation energies are formally described by the linear response theory. Besides its conventional formulation within the quasiparticle random phase approximation (QRPA) representing excited states as two correlated quasiparticles (2q), there exist extensions for 4q configurations. Such extended approaches are quite successful in the description of gross properties of nuclear spectra, however, accounting for many of their fine features requires further extension of the configuration space.

**Purpose:** This work aims at the development of an approach which is capable of such an extension as well as of reproducing and predicting fine spectral properties, which are of special interest at low energies.

**Method:** The method is based on the covariant density functional theory and time blocking approximation, which is extended for couplings between quasiparticles and multiphonon excitations.

**Results:** The covariant multiphonon response theory is developed and adopted for nuclear structure calculations in medium-mass and heavy nuclei. The equations are formulated in both general and coupled forms in the spherical basis.

**Conclusions:** The developed covariant multiphonon response theory represents a new generation of the approaches to nuclear response, which aims at a unified description of both high-frequency collective states and low-energy spectroscopy in medium-mass and heavy nuclei.

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## I. INTRODUCTION

Linear response theory is a conventional framework adopted for calculations of nuclear spectra in the low-energy regime, i.e. at excitation energies below and around  $\sim 100$  MeV. Such excitations represent the response of nuclear systems to sufficiently weak external fields, so that the induced changes of the nucleonic density are small compared to the ground state density and can be treated in the linear approximation. The self-frequencies of the oscillations of the nucleonic densities can be found as solutions of the secular equation in the limit of the vanishing external field. The simplest non-perturbative approach to the secular equation for strongly interacting Fermions includes scattering process of particle-hole pairs which can be described by ring diagrams summed up to the infinite order. This approach is known as the random phase approximation (RPA) [1] or the quasiparticle random phase approximation (QRPA) [2–4], where the latter is a generalization of the former to the superfluid case and since 1960's has become a standard approach to vibrational spectra of nuclei. Besides its typical diagrammatic structure, the QRPA calculations are based on the concept of nucleon-nucleon (effective) residual interaction which has evolved considerably over the years. The use of relatively simple multipole-multipole forces [4] and Landau-Migdal interaction [5] allowed for reasonable explanation of some experimental findings although the agreement with data could only be achieved after fine tuning of the interaction parameters.

Over the decades, various approaches to the nucleon-nucleon residual interaction, such as G-matrix [6–8],

Skyrme [9, 10], Gogny [11] or Fayans [12] interactions have been developed and successfully tested on nuclear structure calculations. The relativistic approach, based on the Walecka model [13–15] for meson-exchange nucleon-nucleon interaction, has become very successful after the inclusion of non-linear meson coupling [16] or the density dependence of the coupling vertices [17–21], see also review [22] and references therein.

The progress in computer technologies has allowed for fast execution of complex numerical algorithms and for self-consistent QRPA calculations with the above mentioned interactions, in contrast to the earlier ones with simple effective interactions which were disconnected from the underlying mean-field. At the same time, approaches beyond QRPA were developed to account for effects of more complex nature than particle-hole (1p1h) or two-quasiparticle (2q) configurations to overcome the principal limitation of the QRPA in the description of the nuclear response.

Medium-mass and heavy nuclei represent Fermi-systems where single-particle and vibrational degrees of freedom are strongly coupled. Collective vibrations lead to shape oscillations of the mean nuclear potential and, therefore, modify the single-particle motion. To take this effect into account, already in Ref. [23] a general concept for the quasiparticle-vibration (phonon) coupling (QVC) part of the single-nucleon self-energy has been proposed. This concept had various implementations over the years within the Quasiparticle-Phonon Model (QPM) [24–26], Nuclear Field Theory [27–33] and others [34–44]. In particular, the approaches [39, 41, 43, 44] are formulated as a theory for nuclear response function. These studies,

however, are either not self-consistent or do not include pairing correlations of the superfluid type. Recently, a set of self-consistent approaches to QVC in the relativistic framework has become available [45–51], where the latter four account for the superfluid pairing. It has been shown that these models improve considerably the description of the single-particle states around the Fermi surface and explain the strong fragmentation of deep hole states, giant resonances and soft modes quantitatively with a good precision, despite the very limited number of parameters in the underlying Lagrangian.

The present work focuses on the nuclear response theory which includes QVC and superfluid pairing on equal footing. Specifically, the nuclear response in the particle-hole channel, which describes a large variety of typical nuclear excited states, is considered. Based on the relativistic quasiparticle time blocking approximation (RQTBA) developed in Ref. [49] and its two-phonon version [50, 51], it is shown how higher-order QVC effects, or multiphonon coupling, can be included self-consistently. The approach is formulated as a non-perturbative extension of the RQTBA. The convergence of the response function with respect to the number of coupled phonon modes is justified in terms of its multipole expansion in the spherical basis.

It is implied that the response theory with multiphonon coupling presented here is based on the relativistic description of the nuclear uncorrelated ground state known as the covariant density functional theory (CDFT), although the approach can be straightforwardly adopted for calculations with other types of underlying density functionals, such as Skyrme, Gogny, Fayans etc.

## II. RELATIVISTIC QUASIPARTICLE TIME BLOCKING APPROXIMATION: A BRIEF OVERVIEW

This section introduces nuclear response formalism and reviews the relativistic time blocking approximation, which serves as a foundation for the extended approach. To maintain consistency with the previous versions of RQTBA, the notations are kept close to those of Ref. [51].

The response function of a finite Fermi-system with an even particle number describes propagation of two quasiparticles in the medium and quantifies the response of the system to an external perturbation. The exact propagator includes, ideally, all possible kinds of the in-medium interaction between two arbitrary quasiparticles and contains all the information about the Fermi system, which can be, in principle, extracted by a certain experimental probe, if its interaction with the system is represented by a single-quasiparticle operator.

In the case of a weak external field, the response function  $R$  is conventionally described by the Bethe-Salpeter

equation (BSE). The general form of this equation

$$R(14, 23) = G(1, 3)G(4, 2) - i \sum_{5678} G(1, 5)G(6, 2)U(58, 67)R(74, 83), \quad (1)$$

includes the one-nucleon Green function (propagator)  $G(1, 2)$  in the nuclear medium and the effective nucleon-nucleon interaction  $U(14, 23)$  irreducible in the relevant channel. Here and below the particle-hole channel is considered. For the systems with pairing correlations of the superfluid type the conventional degrees of freedom are quasiparticles in Bogoliubov's sense represented by superpositions of particles and holes on top of the Hartree (or Hartree-Fock) Fermi sea. To account for the superfluidity effects, we use the formalism of the extended (doubled) space of quasiparticle states described in Refs. [43, 49]. Thus, the generic number indices 1, 2, ... include all single-quasiparticle variables in an arbitrary representation, components in this doubled space, and time. Respectively, the summation over the number indices implies an integration over the time variables. The amplitude  $U$  is determined as a variational derivative of the nucleonic self-energy  $\Sigma$  with respect to the exact single-quasiparticle Green function  $G$ :

$$U(14, 23) = i \frac{\delta \Sigma(4, 3)}{\delta G(2, 1)}. \quad (2)$$

If the ground state can be with a reasonable accuracy described by a static mean-field, it is convenient to decompose both the single-quasiparticle self-energy  $\Sigma$  and the irreducible effective interaction  $U$  into static  $\tilde{\Sigma}$ ,  $\tilde{V}$  and time-dependent (energy-dependent)  $\Sigma^{(e)}$ ,  $U^{(e)}$  parts as

$$\Sigma = \tilde{\Sigma} + \Sigma^{(e)} \quad (3)$$

$$U = \tilde{V} + U^{(e)}. \quad (4)$$

Accordingly, the uncorrelated response is introduced as  $\tilde{R}^{(0)}(14, 23) = \tilde{G}(1, 3)\tilde{G}(4, 2)$ , where  $\tilde{G}(1, 2)$  are the single-quasiparticle mean-field Green functions in the absence of the term  $\Sigma^{(e)}$  in the self-energy. The Green functions  $G$  and  $\tilde{G}$  are connected by the Dyson equation:

$$G(1, 2) = \tilde{G}(1, 2) + \sum_{34} \tilde{G}(1, 3)\Sigma^{(e)}(3, 4)G(4, 2), \quad (5)$$

so that  $G$  can be eliminated from Eq. (1) and, after some simple algebra, the BSE (1) takes the form:

$$R(14, 23) = \tilde{G}(1, 3)\tilde{G}(4, 2) - i \sum_{5678} \tilde{G}(1, 5)\tilde{G}(6, 2)V(58, 67)R(74, 83), \quad (6)$$

where  $V$  is the new effective interaction amplitude which is specified below. The well-known quasiparticle random phase approximation QRPA including its relativistic version (RQRPA) corresponds to the case of  $V = \tilde{V}$  neglecting the time-dependent term  $U^{(e)}$ . More precisely,

in the (R)QRPA the time-dependent term is included in a static approximation by adjusting the parameters of the effective interaction  $\tilde{V}$  to ground state properties of nuclei such as masses and radii. In the self-consistent (R)QRPA the static effective interaction is the second variational derivative of the covariant energy density functional (CEDF)  $E[\mathcal{R}]$  with respect to the density matrix  $\mathcal{R}$  [22]:

$$\tilde{V}(14, 23) = \frac{2\delta^2 E[\mathcal{R}]}{\delta\mathcal{R}(2, 1)\delta\mathcal{R}(3, 4)}. \quad (7)$$

In the approaches beyond the QRPA both static and time-dependent terms are contained in the residual interactions. In medium-mass and heavy nuclei vibrational and rotational modes are strongly coupled to the single-particle ones. In particular, the coupling to low-lying vibrations is known already for decades [23] as a very important mechanism of the formation of nuclear excited states and serves as a foundation for the so-called (quasi)particle-phonon coupling model. Implementations of this concept on the base of the modern density functionals have been extensively elaborated in non-relativistic [32, 52–57] and relativistic [45–51, 58, 59] frameworks.

Going beyond the Hartree (Hartree-Fock) approach of the CDFT, it is natural to include non-perturbatively bubble and ladder types of nucleon-nucleon correlations associated with multiple meson exchange and re-scattering. This becomes possible because these processes lead to the emergence of collective effects of vibrational character. These vibrations (phonons) manifest themselves as the new degrees of freedom associated with the new order parameter corresponding to the quasiparticle-vibration coupling vertices, which helps to classify and decouple different correlations in the self-consistent non-perturbative treatment. For instance, one-phonon exchange is the leading-order approximation for the time-dependent parts of the effective interaction  $U^{(e)}$  and of the nucleonic self-energy  $\Sigma^{(e)}$ , whose Fourier transform to the energy domain

$$\Sigma^{(e)}(1, 2; \varepsilon) = \sum_{34} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} \Gamma^{(e)}(14, 23; \omega) G(3, 4; \varepsilon + \omega) \quad (8)$$

is formally expressed as a convolution of the exact single-quasiparticle Green function  $G$  and two-quasiparticle scattering amplitude  $\Gamma^{(e)}$ . In the leading-order approximation with respect to the QVC,  $\Gamma^{(e)}$  is obtained as an infinite sum of the ring diagrams with meson-exchange interaction. Thus,  $\Sigma^{(e)}$  is separated from Hartree-Fock contributions which are supposed to be included in the static self-energy  $\tilde{\Sigma}$ . The dynamical part of the self-energy can be represented by the Feynman graph shown in Fig. 1, where in the leading order the straight line stands for the mean-field single-nucleon propagator  $G = \tilde{G}$  and the wiggly line replaces an infinite sum of the ring diagrams.

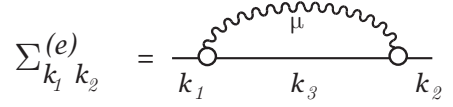


FIG. 1: Top: The skeleton graph representing the single-quasiparticle self-energy  $\Sigma^{(e)}$ . The solid straight line denotes the single-quasiparticle nucleonic propagator, the Latin indices stand for the single-quasiparticle quantum numbers, the wiggly line with Greek index shows the phonon propagator with phonon quantum numbers, empty circles represent QVC vertices.

To account for higher-order correlations, after solution of the Dyson equation (5) the obtained Green function, together with the amplitude  $\Gamma^{(e)}$  calculated from the Eq. (6), for instance, in the approach described below, can be substituted to Eq. (8), thus dressing the skeleton graph shown in Fig. 1.

The consistent response formalism for the BSE (6) based on the QVC self-energy of Eq. (8) has become possible in the time blocking approximation, first proposed in Refs. [43, 44] for superfluid Fermi systems and elaborated in Ref. [49] in detail for the relativistic framework. This approximation allows for exact summation of a selected class of Feynman's diagrams which give the leading contribution of the quasiparticle-phonon coupling effects to the response function. Following [49], it is convenient to write the BSE (6) in the representation in which the mean-field Green function  $\tilde{G}$  is diagonal. This representation is given by the set of the eigenfunctions  $|\psi_k^{(\eta)}\rangle$  of the Relativistic Hartree-Bogoliubov (RHB) Hamiltonian  $\mathcal{H}_{RHB}$  satisfying the equations [60]:

$$\mathcal{H}_{RHB}|\psi_k^{(\eta)}\rangle = \eta E_k |\psi_k^{(\eta)}\rangle, \quad \mathcal{H}_{RHB} = 2 \frac{\delta E[\mathcal{R}]}{\delta \mathcal{R}}, \quad (9)$$

where  $E_k > 0$ , the index  $k$  stands for the set of the single-particle quantum numbers including states in the Dirac sea, and the index  $\eta = \pm 1$  labels positive- and negative-frequency solutions of Eq. (9) in the doubled quasiparticle space. The eigenfunctions  $|\psi_k^{(\eta)}\rangle$  are 8-dimensional Bogoliubov-Dirac spinors:

$$|\psi_k^{(+)}(\mathbf{r})\rangle = \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}, \quad |\psi_k^{(-)}(\mathbf{r})\rangle = \begin{pmatrix} V_k^*(\mathbf{r}) \\ U_k^*(\mathbf{r}) \end{pmatrix}, \quad (10)$$

which form the working basis called Dirac-Hartree-BCS (DHBCS) basis for the subsequent calculations.

Within the time blocking approximation and after performing a Fourier transformation to the energy domain, the BSE (6) for the spectral representation of the nuclear response function  $R(\omega)$  in the basis  $\{|\psi_k^{(\eta)}\rangle\}$  reads:

$$R_{k_1 k_4, k_2 k_3}^{\eta \eta'}(\omega) = \tilde{R}_{k_1 k_2}^{(0) \eta}(\omega) \delta_{k_1 k_3} \delta_{k_2 k_4} \delta^{\eta \eta'} + \tilde{R}_{k_1 k_2}^{(0) \eta}(\omega) \sum_{k_5 k_6, k_2 k_5} V_{k_1 k_6, k_2 k_5}^{\eta \eta''}(\omega) R_{k_5 k_4, k_6 k_3}^{\eta'' \eta'}(\omega), \quad (11)$$

being a matrix equation in the DHBCS basis for each external energy variable  $\omega$ . This is the main result of the time-blocking approximation which separates the integrations over the intermediate energy variable in such a way that it is fully integrated out in the interaction amplitude  $V$ . The quantity  $\tilde{R}^{(0)}$

$$\tilde{R}_{k_1 k_2}^{(0)\eta}(\omega) = \frac{1}{\eta\omega - E_{k_1} - E_{k_2}} \quad (12)$$

describes the free propagation of two quasiparticles with their Bogoliubov energies  $E_{k_1}$  and  $E_{k_2}$  in the relativistic mean field. The interaction amplitude of Eq. (11) contains both static  $\tilde{V}$  and dynamical (frequency-dependent)  $W(\omega)$  parts as follows:

$$\begin{aligned} V_{k_1 k_4, k_2 k_3}^{\eta\eta'}(\omega) &= \tilde{V}_{k_1 k_4, k_2 k_3}^{\eta\eta'} + W_{k_1 k_4, k_2 k_3}^{\eta\eta'}(\omega), \\ W_{k_1 k_4, k_2 k_3}^{\eta\eta'}(\omega) &= \left[ \Phi_{k_1 k_4, k_2 k_3}^{\eta}(\omega) - \Phi_{k_1 k_4, k_2 k_3}^{\eta}(0) \right] \delta^{\eta\eta'} \end{aligned} \quad (13)$$

The diagrammatic representation of the Eq. (11) with the interaction of Eq. (13) is given in Fig. 2. The black circle in the second term on the right hand side of the top line corresponds to the static effective interaction denoted by  $\tilde{V}$  and, in the absence of the third term containing the phonon coupling effects via the amplitude  $W$ , one would have the QRPA equation. The energy-dependent resonant part of the two-quasiparticle amplitude  $\Phi(\omega)$  can be factorized [43] and takes the following form:

$$\Phi_{k_1 k_4, k_2 k_3}^{\eta}(\omega) = \sum_{k_5 k_6, \mu} \zeta_{k_1 k_2; k_5 k_6}^{\mu\eta} \tilde{R}_{k_5 k_6}^{(0)\eta}(\omega - \eta\Omega_{\mu}) \zeta_{k_3 k_4; k_5 k_6}^{\mu\eta*}, \quad (14)$$

so that  $\tilde{R}_{k_5 k_6}^{(0)\eta}(\omega - \eta\Omega_{\mu})$  are the matrix elements of the two-quasiparticle propagator in the mean field with the frequency shifted forward or backward by the phonon energy  $\Omega_{\mu}$ . The quantities  $\zeta$  are the generalized phonon vertices:

$$\begin{aligned} \zeta_{k_1 k_2; k_5 k_6}^{\mu(+)} &= \delta_{k_1 k_5} \gamma_{\mu; k_6 k_2}^{(-)} - \gamma_{\mu; k_1 k_5}^{(+)} \delta_{k_6 k_2}, \\ \zeta_{k_1 k_2; k_5 k_6}^{\mu(-)} &= \delta_{k_5 k_1} \gamma_{\mu; k_2 k_6}^{(+)*} - \gamma_{\mu; k_5 k_1}^{(-)*} \delta_{k_2 k_6}, \end{aligned} \quad (15)$$

revealing the four terms in Eq. (14) which correspond to those in the diagrammatic representation of the amplitude  $\Phi(\omega)$  in the bottom line of Fig. 2.

The shorthand notation for the phonon emission (absorption) amplitudes imply:

$$\gamma_{\mu; k_1 k_2}^{\eta} = \gamma_{\mu; k_1 k_2}^{\eta_1 \eta_2} \delta_{\eta_1 \eta} \delta_{\eta_2 \eta}, \quad \eta = (\pm), \quad (16)$$

where  $\gamma_{\mu; k_1 k_2}^{\eta_1 \eta_2}$  are the matrix elements of these amplitudes in the doubled quasiparticle space. They determine the probability of the coupling of a quasiparticle pair in the states  $\{k_1 \eta_1\}, \{k_2 \eta_2\}$  to the collective vibrational state (phonon) with quantum numbers  $\mu = \{\Omega_{\mu}, J_{\mu}, M_{\mu}, \pi_{\mu}\}$ . In the RQTBA these vertices are derived from the corresponding RQRPA transition densities  $\mathcal{R}_{\mu}$  and the static effective interaction as

$$\gamma_{\mu; k_1 k_2}^{\eta_1 \eta_2} = \sum_{k_3 k_4} \sum_{\eta} \tilde{V}_{k_1 k_4, k_2 k_3}^{\eta_1, -\eta, \eta_2, \eta} \mathcal{R}_{\mu; k_3 k_4}^{\eta}, \quad (17)$$

where  $\tilde{V}_{k_1 k_4, k_2 k_3}^{\eta_1 \eta_4, \eta_2 \eta_3}$  is the matrix element of the amplitude  $\tilde{V}$  of Eq. (7) in the basis  $\{|\psi_k^{(\eta)}\rangle\}$ . The matrix elements of the phonon transition densities are calculated, in first approximation, within the relativistic quasiparticle random phase approximation [21]. In the Dirac-Hartree-BCS basis  $\{|\psi_k^{(\eta)}\rangle\}$  it has the following form:

$$\mathcal{R}_{\mu; k_1 k_2}^{\eta} = \tilde{R}_{k_1 k_2}^{(0)\eta}(\Omega_{\mu}) \sum_{k_3 k_4} \sum_{\eta'} \tilde{V}_{k_1 k_4, k_2 k_3}^{\eta\eta'} \mathcal{R}_{\mu; k_3 k_4}^{\eta'}, \quad (18)$$

where  $\tilde{V}_{k_1 k_4, k_2 k_3}^{\eta\eta'} = \tilde{V}_{k_1 k_4, k_2 k_3}^{\eta, -\eta', -\eta, \eta'}$ , since we cut out the particle-hole components of the tensors in the quasiparticle space.

In the diagrammatic expression of the amplitude (14) in the upper line of the Fig. 2 the uncorrelated propagator  $\tilde{R}_{k_1 k_2}^{(0)\eta}$  is represented by the two straight nucleonic lines between the circles denoting emission and absorption of a phonon by a single quasiparticle with the amplitude  $\gamma_{\mu; k_1 k_2}^{\eta_1 \eta_2}$ . The approach to the amplitude  $\Phi(\omega)$  expressed by Eq. (14) represents a version of first-order perturbation theory compared to RQRPA and the amplitude  $W(\omega)$  of Eq. (13) is the first-order correction to the effective interaction  $\tilde{V}$ , because the dimensionless matrix elements of the phonon vertices are such that  $\gamma_{\mu; k_1 k_2}^{\eta_1 \eta_2} / \Omega_{\mu} \ll 1$  in most physical cases. The phonon-coupling term  $\Phi$  generates fragmentation of the excitation modes obtained in QRPA. In particular, the high-frequency oscillations known as giant resonances acquire their spreading width due to the term  $\Phi$ . In the low-energy region below the neutron threshold of medium-mass even-even nuclei this term is responsible solely for the appearing strength. In the relativistic framework, the latter was confirmed and extensively studied [49, 61], and verified by comparison to experimental data [50, 51, 62–64]. However, a comparison with high-resolution experiments on the dipole strength below the neutron threshold has revealed that, although the total strength and some gross features of the strength are reproduced well, the fine features are sensitive to truncation of the configuration space by  $2q \otimes \text{phonon}$  configurations and further extensions of the method are needed. Such an extension forms the content of the subsequent sections.

Before proceeding further, let us notice that the diagrammatic equation of Fig. 2 written, as in Ref. [49], for the system with pairing correlations has the same form as that for the normal (non-superfluid) system. The formal similarity of the equations for normal and superfluid systems is achieved by the use of the representation of the basis functions  $|\psi_k^{(\eta)}\rangle$  satisfying Eq. (9). This basis is a counterpart of the particle-hole basis of the conventional RPA in which the (Q)RPA equations have the most simple and compact form. In the representation of the functions  $|\psi_k^{(\eta)}\rangle$  the generalized superfluid mean-field Green function  $\tilde{G}$  (often called Gor'kov-Green's function) has a diagonal form and describes the propagation of the quasiparticle with fixed energy. In this diagonal representation the directions of the fermion lines of the diagrams

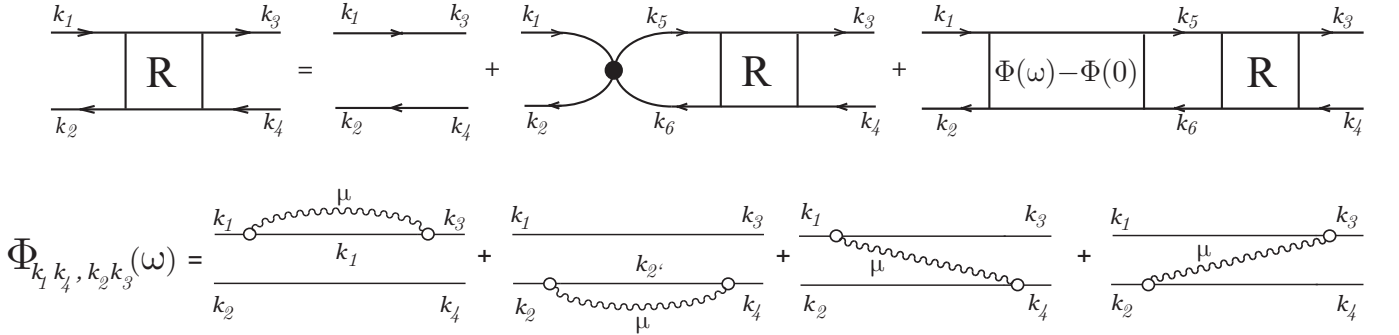


FIG. 2: Top: Bethe-Salpeter equation for the response function  $R$  in the ph-channel in diagrammatic representation. The solid lines denote single-quasiparticle mean-field propagators. The integral part is divided into two terms; the small black circle represents the static effective interaction  $\tilde{V}$  and the energy-dependent block  $\Phi(\omega) - \Phi(0)$  contains the dynamic contributions. Bottom: The dynamical part of the effective interaction in the quasiparticle-vibration coupling (QVC) model in the leading order on QVC.

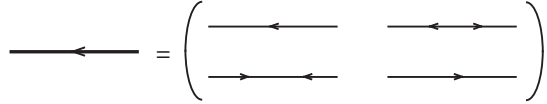


FIG. 3: The 4-component Green's function in the diagrammatic representation.

(of the type shown in Fig. 2) denote the positive- or the negative-frequency components of the functions  $\tilde{G}$ . The so-called backward-going diagrams, corresponding to the ground-state correlations in the RQRPA, are not marked out in Fig. 2 though they are included in Eq. (11). In the coordinate representation, the non-diagonal Green function  $\tilde{G}$  for the quasiparticle has no definite energy. This Green function can be represented by the  $2 \times 2$  block matrix shown in Fig. 3, see an extended discussion in Ref. [51].

In RQTBA based on the CDFT, the elimination of double counting effects of the phonon coupling is performed by the subtraction of the static contribution of the amplitude  $\Phi$  from the residual interaction in Eq. (13), since the parameters of the underlying functional have been adjusted to experimental data for ground states and include, thereby, the phonon coupling contributions to the ground state. The subtraction of the phonon coupling amplitude at zero frequency  $\Phi(0)$  in Eq. (13) acquires another important role for the excitations which have an isoscalar dipole component, for example, the electromagnetic dipole response. On the RQRPA level the elimination of the  $1^-$  spurious state is achieved by the use of a sufficiently large  $2q$  configuration space within a fully self-consistent approach [21]. In the extended theories based on the self-consistent RQRPA the translational invariance can be restored by the subtraction of the energy dependent interaction amplitude at zero frequency. In the numerical implementation, due to numerical inaccuracies, this state appears at a finite energy below 1 MeV already in RQRPA, but due to the subtraction procedure, in extended theories such as RQTBA and RQTBA-2 of

Refs. [50, 51] the accuracy of elimination of the spurious state is preserved. A detailed description of the subtraction procedure which, in addition, guarantees stability of solutions of the extended RPA theories, is presented in Refs. [66]. A similar procedure is proposed in the higher-order RQTBA described in Section IV.

In practice, calculations for the response function (11) are divided into two major steps. First, the BSE for the correlated propagator  $R^{(e)}(\omega)$

$$R_{k_1 k_4, k_2 k_3}^{(e)\eta}(\omega) = \tilde{R}_{k_1 k_2}^{(0)\eta}(\omega) \delta_{k_1 k_3} \delta_{k_2 k_4} + \tilde{R}_{k_1 k_2}^{(0)\eta}(\omega) \times \sum_{k_5 k_6} [\Phi_{k_1 k_6, k_2 k_5}^{\eta}(\omega) - \Phi_{k_1 k_6, k_2 k_5}^{\eta}(0)] R_{k_5 k_4, k_6 k_3}^{(e)\eta}(\omega) \quad (19)$$

is solved in the Dirac-Hartree-BCS basis. Second, the BSE for the full response function  $R(\omega)$

$$R_{k_1 k_4, k_2 k_3}^{\eta\eta'}(\omega) = R_{k_1 k_4, k_2 k_3}^{(e)\eta}(\omega) \delta^{\eta\eta'} + \sum_{k_5 k_6 k_7 k_8} R_{k_1 k_6, k_2 k_5}^{(e)\eta}(\omega) \sum_{\eta''} \tilde{V}_{k_5 k_8, k_6 k_7}^{\eta\eta''} R_{k_7 k_4, k_8 k_3}^{\eta''\eta'}(\omega), \quad (20)$$

where

$$R_{k_1 k_4, k_2 k_3}^{\eta\eta'}(\omega) = R_{k_1 k_4, k_2 k_3}^{\eta, -\eta', -\eta, \eta'}(\omega), \quad (21)$$

is solved either in the DHBCS or in the momentum-channel representation which is especially convenient because of the structure of the one-boson exchange interaction. The details are given in Appendix C of Ref. [49].

### III. EXTENDED RQTBA: THE NEXT ORDER

The first extension of the RQTBA described above used the idea proposed in Ref. [43] and is based on the factorization of Eq. (14): the uncorrelated propagator  $\tilde{R}^{(0)\eta}$  in Eq. (14) is replaced by the positive- ( $\eta = +1$ )

or the negative- ( $\eta = -1$ ) frequency part of a correlated one. The first order approximation to a correlated propagator is the RQRPA, in which a two-quasiparticle pair scatters via a quasi-bound phonon configuration. As a result, two-phonon configurations appear in the amplitude  $\Phi(\omega)$ , as it is described in detail in Ref. [51]. This two-phonon version of the RQTBA, RQTBA-2, contains, by definition, more correlations than the original RQTBA truncated by  $2q \otimes$  phonon configurations, but it is still on the same two-particle-two-hole (2p2h) level of configuration complexity.

Here we make another step forward with introducing correlations inside the amplitude  $\Phi(\omega)$ . But now we go beyond the 2p2h configurations. In order to keep the notations consistent with those used before, let us define:

$$\begin{aligned}
\Phi_{k_1 k_4, k_2 k_3}^{(1)\eta}(\omega) &= 0 \\
\Phi_{k_1 k_4, k_2 k_3}^{(2)\eta}(\omega) &= \Phi_{k_1 k_4, k_2 k_3}^{\eta}(\omega) \\
R_{k_1 k_4, k_2 k_3}^{e(1)\eta}(\omega) &= \tilde{R}_{k_1 k_2}^{(0)\eta}(\omega) \delta_{k_1 k_3} \delta_{k_2 k_4} \\
R_{k_1 k_4, k_2 k_3}^{e(2)\eta}(\omega) &= R_{k_1 k_4, k_2 k_3}^{(e)\eta}(\omega) \\
R_{k_1 k_4, k_2 k_3}^{(1)\eta\eta'}(\omega) &= \tilde{R}_{k_1 k_2}^{(0)\eta}(\omega) \delta_{k_1 k_3} \delta_{k_2 k_4} \delta^{\eta\eta'} \\
R_{k_1 k_4, k_2 k_3}^{(2)\eta\eta'}(\omega) &= R_{k_1 k_4, k_2 k_3}^{\eta\eta'}(\omega). \quad (22)
\end{aligned}$$

Now the response function  $R^{(2)}$  of the conventional RQTBA substitutes the uncorrelated intermediate propagator and, instead of the amplitude  $\Phi$  of Eq. (14), we have the new amplitude  $\Phi^{(3)}$ :

$$\begin{aligned}
\Phi_{k_1 k_4, k_2 k_3}^{(3)\eta}(\omega) &= \\
&= \sum_{k_5 k_6, k_5' k_6' \mu} \zeta_{k_1 k_2; k_5 k_6}^{\mu\eta} R_{k_5 k_6', k_6 k_5'}^{(2)\eta}(\omega - \eta \Omega_\mu) \times \\
&\quad \times \zeta_{k_3 k_4; k_5' k_6'}^{\mu\eta*}, \quad (23)
\end{aligned}$$

where the response function  $R_{k_1 k_4, k_2 k_3}^{(2)\eta}$  is the solution of the equation:

$$\begin{aligned}
R_{k_1 k_4, k_2 k_3}^{(2)\eta}(\omega) &= R_{k_1 k_4, k_2 k_3}^{(e)\eta}(\omega) + \\
&+ \sum_{k_5 k_6 k_7 k_8} R_{k_1 k_6, k_2 k_5}^{(e)\eta}(\omega) \tilde{V}_{k_5 k_8, k_6 k_7}^{\eta\eta'} R_{k_7 k_4, k_8 k_3}^{\eta}(\omega), \quad (24)
\end{aligned}$$

which is an analog of Eq. (20), but does not contain ground state correlations. This simplification is made to exclude 'zigzag' diagrams from the amplitude  $\Phi^{(3)}$  [65]. By this substitution, we introduce  $2q \otimes$  phonon correlations into the intermediate two-quasiparticle propagators, i.e., in diagrammatic language, we perform the operation shown in Fig. 4. The analytic expression for the new QVC amplitude in terms of the phonon vertices

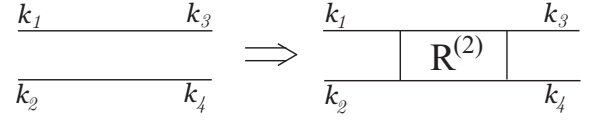


FIG. 4: Replacement of the uncorrelated two-nucleon propagator by the correlated one.

(16) reads:

$$\begin{aligned}
\Phi_{k_1 k_4, k_2 k_3}^{(3)\eta}(\omega) &= \sum_{k_1', k_3'; \mu} \gamma_{\mu; k_1 k_1'}^{\eta} R_{k_1' k_2', k_3' k_4}^{(2)\eta} \gamma_{\mu; k_3 k_3'}^{\eta*} + \\
&+ \sum_{k_2', k_4'; \mu} \gamma_{\mu; k_2' k_2}^{\eta} R_{k_1 k_2', k_3 k_4'}^{(2)\eta} \gamma_{\mu; k_4' k_4}^{\eta*} - \\
&- \sum_{k_1', k_4'; \mu} \gamma_{\mu; k_1 k_1'}^{\eta} R_{k_1' k_2, k_3 k_4'}^{(2)\eta} \gamma_{\mu; k_4' k_4}^{\eta*} - \\
&- \sum_{k_2', k_3'; \mu} \gamma_{\mu; k_2' k_2}^{\eta} R_{k_1 k_2, k_3' k_4}^{(2)\eta} \gamma_{\mu; k_3' k_3}^{\eta*}, \quad (25)
\end{aligned}$$

where the four terms correspond to the four diagrams in the bottom line of Fig. 5. This Figure also illustrates the relation between the QVC amplitudes in the conventional RQTBA, RQTBA-2 and in the next-order approach.

In fact, the amplitude  $\Phi^{(3)}$  contains the contributions of the graphs shown in Fig. 6. However, the substitution shown in Fig. 4 allows calculation of their contribution without explicit calculations of the diagrams of Fig. 6. It is easy to see that these terms contain  $2q \otimes 2$ phonon configurations and thereby represent the next, three-particle-three-hole (3p3h), level of configuration complexity, as compared to the RQTBA and RQTBA-2. The two-phonon amplitude  $\Phi^{(2)}$  is also contained in the amplitude  $\Phi^{(3)}$ , although approximately, since the ground state correlations are neglected in Eq. (24) [65].

The amplitude  $\Phi^{(3)}$  forms the kernel of the BSE for the correlated propagator  $R^{e(3)}$  taking into account 3p3h correlations (to be compared to  $R^{e(2)} = R^{(e)}$  which includes 2p2h ones):

$$\begin{aligned}
R_{k_1 k_4, k_2 k_3}^{e(3)\eta}(\omega) &= \tilde{R}_{k_1 k_2}^{(0)\eta}(\omega) \delta_{k_1 k_3} \delta_{k_2 k_4} + \tilde{R}_{k_1 k_2}^{(0)\eta}(\omega) \times \\
&\times \sum_{k_5 k_6} \left[ \Phi_{k_1 k_6, k_2 k_5}^{(3)\eta}(\omega) - \Phi_{k_1 k_6, k_2 k_5}^{(3)\eta}(0) \right] R_{k_5 k_4, k_6 k_3}^{e(3)\eta}(\omega). \quad (26)
\end{aligned}$$

Analogously to the conventional RQTBA (20), the equation for the full response function is formulated in terms of the correlated propagator  $R^{e(3)}$  as a free term and the static effective interaction as a kernel:

$$\begin{aligned}
R_{k_1 k_4, k_2 k_3}^{(3)\eta\eta'}(\omega) &= R_{k_1 k_4, k_2 k_3}^{e(3)\eta}(\omega) \delta^{\eta\eta'} + \\
&+ \sum_{k_5 k_6} R_{k_1 k_6, k_2 k_5}^{e(3)\eta}(\omega) \sum_{k_7 k_8 \eta''} \tilde{V}_{k_5 k_8, k_6 k_7}^{\eta\eta''} R_{k_7 k_4, k_8 k_3}^{(3)\eta''\eta'}(\omega), \quad (27)
\end{aligned}$$



$$\Phi_{k_1 k_4, k_2 k_3}^{(2)} = \frac{k_1 \text{---} \mu \text{---} k_3}{k_2 \text{---} k_4} + \frac{k_1 \text{---} k_3}{k_2 \text{---} \mu \text{---} k_4} + \frac{k_1 \text{---} \mu \text{---} k_3}{k_2 \text{---} k_4} + \frac{k_1 \text{---} \mu \text{---} k_3}{k_2 \text{---} k_4}$$

$$\bar{\Phi}_{k_1 k_4, k_2 k_3}^{(2)} = \frac{k_1 \text{---} \mu \text{---} k_3}{k_2 \text{---} \nu \text{---} k_4} + \frac{k_1 \text{---} \nu \text{---} k_3}{k_2 \text{---} \mu \text{---} k_4} + \frac{k_1 \text{---} \mu \text{---} k_3}{k_2 \text{---} \nu \text{---} k_4} + \frac{k_1 \text{---} \nu \text{---} k_3}{k_2 \text{---} \mu \text{---} k_4}$$

$$\Phi_{k_1 k_4, k_2 k_3}^{(3)} = \frac{k_1 \text{---} \mu \text{---} k_3}{k_2 \text{---} R^{(2)} \text{---} k_4} + \frac{k_1 \text{---} k_3}{k_2 \text{---} R^{(2)} \text{---} k_4} + \frac{k_1 \text{---} k_1' \text{---} k_3}{k_2 \text{---} R^{(2)} \text{---} k_4} + \frac{k_1 \text{---} \mu \text{---} k_3}{k_2 \text{---} R^{(2)} \text{---} k_4}$$

FIG. 5: Hierarchy of the quasiparticle-phonon coupling amplitudes: 2q⊗phonon amplitude  $\Phi = \Phi^{(2)}$  of the leading-order QVC (compare to the bottom line of Fig. 2), the 2-phonon amplitude  $\bar{\Phi}^{(2)}$  and the 2q⊗2phonon amplitude  $\Phi^{(3)}$  with the correlated intermediate two-quasiparticle propagator  $R^{(2)}$ .

$$\Phi^{(3)} = \bar{\Phi}^{(2)} +$$

FIG. 6: The time-ordered diagrams taken into account in the extended RQTBA in the second order of the quasiparticle-vibration coupling.

where the superscript '(3)' indicates that this response function takes into account 3p3h configurations. Analogously to the 2q⊗phonon RQTBA, the subtraction of the amplitude  $\Phi^{(3)}$  at zero frequency from the effective interaction in Eq. (27) eliminates double counting of the static contribution of phonon coupling effects.

#### IV. HIGHER-ORDER CORRELATIONS: MULTIPHONON COUPLING

In principle, the procedure shown in Fig. 4 can be repeated with the replacement  $R^{(2)} \Rightarrow R^{(3)}$  to take into account the next-order effects, and it can be continued until convergence. Each iteration in this procedure will

add another correlated two-quasiparticle pair into the phonon coupling amplitude  $\Phi^{(n)}(\omega)$ , resolving finer and finer features of the response function. The latter means that in the model-independent spectral expansion of the response function

$$R_{k_1 k_4, k_2 k_3}^{(n)\eta\eta'}(\omega) = \sum_{\nu} \left[ \frac{\mathcal{R}_{\nu; k_1 k_2}^{(n)\eta*} \mathcal{R}_{\nu; k_3 k_4}^{(n)\eta'}}{\omega - \omega_{\nu} + i\delta} - \frac{\mathcal{R}_{\nu; k_2 k_1}^{(n)-\eta} \mathcal{R}_{\nu; k_4 k_3}^{(n)-\eta'*}}{\omega + \omega_{\nu} - i\delta} \right] \quad (28)$$

more and more terms numbered by the index  $\nu$  will appear with the increase of  $n$ , so that the spectrum will become more and more fragmented. In this way, the parameter  $n$  establishes a hierarchy of the excited states: larger  $n$  numbers correspond to fine structure while small  $n$ 's are responsible for gross structure of the spectra. As it is shown below in Section V by the multipole expansion of  $R^{(n)}$ , each iteration introduces such a geometrical factor into the kernel of the Bethe-Salpeter equation for  $R^{(n)}$ , that contains some smallness providing a condition for the convergence of the iterative procedure.

The chain of operator equations for the correlated propagator  $R^{(n)}$ , phonon coupling amplitude  $\Phi^{(n)}$  and response function  $R^{(n)}$  looks as follows:

$$\begin{cases} R^{(1)}(\omega) = R^{e(1)}(\omega) = \tilde{R}^0(\omega) \\ R^{e(n)}(\omega) = \tilde{R}^0(\omega) + \\ + \tilde{R}^0(\omega) \left[ \Phi[R^{(n-1)}(\omega)] - \Phi[R^{(n-1)}(0)] \right] R^{e(n)}(\omega) \\ R^{(n)}(\omega) = R^{e(n)}(\omega) + R^{e(n)}(\omega) \tilde{V} R^{(n)}(\omega), \end{cases} \quad (29)$$

where  $n > 1$  and the matrix elements of the amplitude



$\Phi[R^{(n-1)}(\omega)] = \Phi^{(n)}(\omega)$  are defined as:

$$\begin{aligned} & \Phi_{k_1 k_4, k_2 k_3}^{(n)\eta}(\omega) = \\ & = \sum_{k_5 k_6, k_5', k_6' \mu} \zeta_{k_1 k_2; k_5 k_6}^{\mu\eta} R_{k_5 k_6', k_6 k_5'}^{(n-1)\eta}(\omega - \eta\Omega_\mu) \times \\ & \quad \times \zeta_{k_3 k_4; k_5' k_6'}^{\mu\eta*}. \end{aligned} \quad (30)$$

In this context, the model, which was previously called RQTBA ('conventional' RQTBA), represents the second-order approach: if the procedure (29) is truncated at  $n = 2$ , one would obtain the conventional RQTBA of Ref. [49]. The approach (29) of the  $n$ -th order describes the nuclear response function which includes couplings of two quasiparticles to up to  $(n-1)$  phonons, or np-nh ( $2n$  quasiparticles,  $2nq$ ) configurations. These effects are included in the time blocking approximation which is, thereby, generalized to multiphonon coupling. Here only the resonant part of the phonon coupling is taken into account, and the so-called associated components introduced in Ref. [41, 43] are neglected. Their quantitative role is known to be minor for the spectral gross features, however, they represent the ground state correlations caused by phonon dynamics and may affect the fine structure of low-lying states [41].

In the diagrammatic language, the proposed solution (29) for the multiphonon response function is obtained by iteration of the intermediate double-line (two-quasiparticle propagator) in the QVC amplitude  $\Phi$  of Fig. 2. This procedure is similar to the method applied to the solution of the Dyson equation beyond the leading order of QVC, when the single-quasiparticle Green function entering the self-energy (8) is iterated.

The contribution of the terms with vertex corrections are known to contain smallness, as compared to the line corrections, in analogy to Migdal's theorem for electron-phonon systems [67]. In particular, in spherical nuclei all phonon-exchange terms which represent vertex corrections (such as the last two terms in the bottom line of Fig. 2), in their coupled form contain 6j-symbols which make these terms smaller than those associated with self-energy insertions (first two terms in Fig. 2), see Eq. (C4) of Ref. [49]. Numerical calculations within the RQTBA [49, 61] have confirmed that the corrections to the phonon vertices and frequencies beyond the RQRPA can be neglected for the most important phonon modes. However, these corrections can be, in principle, included into the iterative scheme (29) by extraction of the phonon vertices from the  $n$ -th order RQTBA response function on each iteration.

In the next section the equations (29) are formulated in the coupled form in the spherical DHBCS basis, which allows one to adopt the approach for numerical calculations for finite nuclei.

## V. THE MULTIPHONON RESPONSE FUNCTION IN THE COUPLED FORM

For practical calculations for finite nuclei, it is convenient to formulate the equations (25) - (30) in terms of the reduced matrix elements with the transferred angular momentum  $J$ , i.e. in the so-called coupled form. The reduced matrix elements of the phonon-coupling amplitude  $\Phi^{(n)}$  read:

$$\begin{aligned} \Phi_{(k_1 k_4, k_2 k_3)}^{(n)J, \eta}(\omega) &= \frac{(-1)^{j_1 + j_2 + j_3 + j_4}}{2J + 1} \sum_{(\mu) J_e} \times \\ & \left[ \sum_{(k_1', k_3')} \gamma_{(\mu; k_1 k_1')}^\eta R_{(k_1', k_2, k_3', k_4)}^{(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_3 k_3')}^{\eta*} \times \right. \\ & \quad \times \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{1'} & j_2 & j_1 \end{matrix} \right\} \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{3'} & j_4 & j_3 \end{matrix} \right\} + \\ & + \sum_{(k_2', k_4')} \gamma_{(\mu; k_2' k_2)}^\eta R_{(k_1 k_2', k_3 k_4')}^{(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_4' k_4)}^{\eta*} \times \\ & \quad \times \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{2'} & j_1 & j_2 \end{matrix} \right\} \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{4'} & j_3 & j_4 \end{matrix} \right\} - \\ & - \sum_{(k_1', k_4')} \gamma_{(\mu; k_1 k_1')}^\eta R_{(k_1', k_2, k_3 k_4')}^{(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_4' k_4)}^{\eta*} \times \\ & \quad \times \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{1'} & j_2 & j_1 \end{matrix} \right\} \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{4'} & j_3 & j_4 \end{matrix} \right\} - \\ & - \sum_{(k_2', k_3')} \gamma_{(\mu; k_2' k_2)}^\eta R_{(k_1 k_2', k_3', k_4)}^{(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_3 k_3')}^{\eta*} \times \\ & \quad \times \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{2'} & j_1 & j_2 \end{matrix} \right\} \left\{ \begin{matrix} J & J_\mu & J_e \\ j_{3'} & j_4 & j_3 \end{matrix} \right\} \left. \right]. \end{aligned} \quad (31)$$

Here the indices in the brackets denote full sets of single-particle quantum numbers with the excluded magnetic quantum numbers (total angular momentum projections:  $k_1 = \{(k_1), m_1\}$ ). The correlated propagator  $R^{e(n)}$  is calculated in the symmetrized form:

$$\begin{aligned} R_{s(k_1 k_4, k_2 k_3)}^{e(n)J, \eta}(\omega) &= \tilde{R}_{s(k_1 k_4, k_2 k_3)}^{(0)J, \eta}(\omega) + \\ & + \tilde{R}_{(k_1 k_2)}^{(0)\eta}(\omega) \sum_{(k_6 \leq k_5)} \left[ \Phi_{s(k_1 k_6, k_2 k_5)}^{(n)J, \eta}(\omega) - \Phi_{s(k_1 k_6, k_2 k_5)}^{(n)J, \eta}(0) \right] \times \\ & \quad \times R_{s(k_5 k_4, k_6 k_3)}^{e(n)J, \eta}(\omega), \end{aligned} \quad (32)$$

where the matrix elements with the subscript "s" are symmetrized with respect to one non-conjugated and one conjugated quasiparticle index. Such a symmetrization allows a shortened summation in the integral part of the Eq. (32) and simplifies, to some extent, the numerical calculations. The symmetrized matrix elements of the mean field propagator  $\tilde{R}_s^{(0)}$  and of the phonon coupling

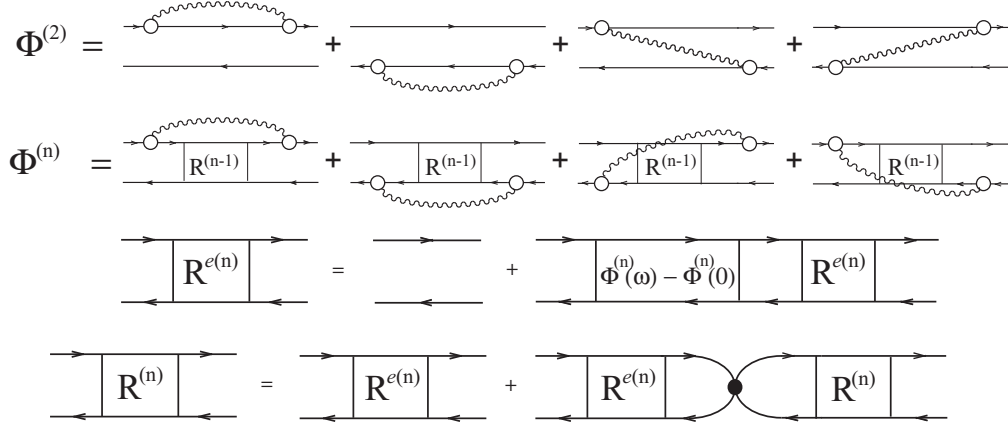


FIG. 7: The diagrammatic representation of the iterative series for the QVC amplitude and response function with multiphonon couplings: The lowest order  $2q \otimes \text{phonon}$  amplitude  $\Phi^{(2)}$  of the conventional phonon coupling model, the higher-order  $2q \otimes (n-1)\text{phonon}$  amplitude  $\Phi^{(n)}$  for  $n > 2$ ; Bethe-Salpeter equations in the ph-channel for the correlated propagator  $R^{e(n)}$  and for the response function  $R^{(n)}$ .

amplitude  $\Phi_s^{(n)}$  have the following form:

$$\tilde{R}_{s(k_1 k_4, k_2 k_3)}^{(0)J, \eta}(\omega) = \tilde{R}_{(k_1 k_2)}^{(0)\eta}(\omega) \times [\delta_{(k_1 k_3)} \delta_{(k_2 k_4)} + (-)^{\phi_{12}} \delta_{(k_1 k_4)} \delta_{(k_2 k_3)}], \quad (33)$$

$$\Phi_{s(k_1 k_4, k_2 k_3)}^{(n)J, \eta}(\omega) = \frac{1}{1 + \delta_{(k_3 k_4)}} \times [\Phi_{(k_1 k_4, k_2 k_3)}^{(n)J, \eta}(\omega) + (-)^{\phi_{12}} \Phi_{(k_2 k_4, k_1 k_3)}^{(n)J, \eta}(\omega)], \quad (34)$$

with  $\phi_{12} = J + l_1 - l_2 + j_1 - j_2$ . The BSE for the full response function  $R^{(n)}(\omega)$

$$R_{(k_1 k_4, k_2 k_3)}^{(n)J, \eta \eta'}(\omega) = R_{s(k_1 k_4, k_2 k_3)}^{e(n)J, \eta}(\omega) \delta^{\eta \eta'} + \sum_{(k_6 \leq k_5)} \times \sum_{(k_8 \leq k_7) \eta''} R_{s(k_1 k_6, k_2 k_5)}^{e(n)J, \eta}(\omega) \tilde{V}_{(k_5 k_8, k_7 k_6)}^{J, \eta \eta''} R_{(k_7 k_4, k_8 k_3)}^{(n)J, \eta'' \eta'}(\omega) \quad (35)$$

is solved either in Dirac-Hartree-BCS or in momentum-channel representations, see Appendix C of Ref. [49].

Similar to the conventional RQTBA, the subtraction of the static contribution of the phonon coupling amplitude  $\Phi(\omega = 0)$  from the effective interaction should be performed to avoid double counting effects of the quasiparticle-vibration coupling [66]. The subtraction can be done in the integral part either of the equation for the correlated propagator  $R_s^{e(n)}$  or of the equation for the response function  $R^{(n)}$ . In the latter case the subtraction acquires the meaning of renormalization of the static effective interaction  $\tilde{V}$ . In this section, giving the coupled-form expressions for these quantities, the subtraction is performed in Eq. (32) for the symmetrized correlated propagator  $R_s^{e(n)}$ . This is more convenient technically because the subtracted term has the same form as its energy-dependent counterpart  $\Phi(\omega)$ .

After finding the response function  $R^{(n)}$  of Eq. (35) it can be substituted to the Eq. (31) for the next-order

QVC amplitude. In principle, the iterations can be continued until the desired accuracy is reached. The closed system of equations for the nuclear response function in the coupled form presented in this section can be directly implemented for numerical calculations. The case of  $n = 3$  is the first step beyond the conventional RQTBA and includes  $2q \otimes 2\text{phonon}$  configurations. It is clear from Eq. (31) that on the large scale the  $2q \otimes 2\text{phonon}$  effects play a smaller role compared to the  $2q \otimes \text{phonon}$  ones of the RQTBA because of the products of the two 6j-symbols in each term on the right hand side of the Eq. (31), which are of geometrical nature. Every next iteration contains an additional smallness of this origin, thus providing conditions for convergence of the whole procedure (29). The convergence will be examined in more detail and verified by numerical implementation of the approach in the future work.

The resulting linear response function  $R^{(n)}(\omega)$  contains all the information on the nuclear response to external one-body operators. The observed spectrum of a nucleus excited by a sufficiently weak external field  $P$  as, for instance, an electromagnetic field or a weak current, is described by the nuclear polarizability which is a double convolution of the response function with this field operator. The reduced matrix elements of the external field operator have the following general coupled form:

$$P_{(k_1 k_2)}^{(p)J, \eta} = \sum_{LS} \frac{\delta_{\eta, 1} + (-1)^S \delta_{\eta, -1}}{\sqrt{1 + \delta_{(k_1 k_2)}}} \eta_{(k_1 k_2)}^S \times \langle (k_1) \| P_{LS}^{(p)J} \| (k_2) \rangle, \quad (36)$$

where the index  $(p)$  contains all possible quantum numbers, other than those concretized here. The factors  $\eta_{(k_1 k_2)}^S$  are determined by combinations of the quasiparticle occupation numbers  $u_k, v_k$  [68]:

$$\eta_{(k_1 k_2)}^S = \frac{1}{\sqrt{1 + \delta_{(k_1 k_2)}}} (u_{k_1} v_{k_2} + (-1)^S v_{k_1} u_{k_2}), \quad (37)$$

obtained as a solution of Eq. (9). These combinations reflect symmetrization in the integral part of Eq. (35), which enables one to take each  $2q$ -pair into account only once because of the symmetry properties of the reduced matrix elements  $\tilde{V}_{(k_5 k_8, k_7 k_6)}^{J, \eta \eta''}$ .

Nuclear polarizability in np-nh time blocking approximation reads:

$$\begin{aligned} \Pi_P^{(n)}(\omega) = & \sum_{(k_2 \leq k_1) \eta} \sum_{(k_4 \leq k_3) \eta'} P_{(k_1 k_2)}^{(p) J, \eta*} \times \\ & \times R_{(k_1 k_4, k_2 k_3)}^{(n) J, \eta \eta'}(\omega) P_{(k_3 k_4)}^{(p) J, \eta'}, \end{aligned} \quad (38)$$

and determines the microscopic strength function  $S(E)$  as:

$$S(E) = -\frac{1}{\pi} \lim_{\Delta \rightarrow +0} \text{Im} \Pi_P^{(n)}(E + i\Delta), \quad (39)$$

where a finite imaginary part  $\Delta$  of the energy variable is introduced in order to obtain a smoothed envelope of the spectrum, if needed, which is often the case for the correct comparison to experimental data with limited resolution. Thus, Eqs. (38), (39) relate the obtained response function to experimental observations.

## VI. SUMMARY

In this work the nuclear response theory is advanced beyond the existing approaches in order to include the effects of multiphonon coupling. The theory is formulated consistently for the covariant framework based on meson-exchange nuclear forces, i.e. on the effective quantum hadrodynamics as the underlying concept, although it can be adopted for a non-relativistic framework based on one of the modern density functionals.

While quantum hadrodynamics provides a fundamental description of nuclear processes of short and medium range, there are long-range correlations with ranges of the order of nuclear size, which can not be described by an exchange of heavy and intermediate-mass mesons in

perturbative methods. In medium-mass and heavy nuclei, the collective effects, such as low-lying vibrational modes, emerge as 'effective' degrees of freedom, which are in immediate relevance to the energy scale of nuclear structure. An order parameter associated with these degrees of freedom appears naturally in the covariant response theory already on the RQRPA level, which helps to treat them as effective quasi-bosonic fields responsible for the long-range correlations. Their characteristics are computed consistently from the meson-exchange interaction using, in the leading-order approximation, RQRPA or equivalent techniques. Thereby, the link between the short-range, medium-range, and long-range correlations is established, that forms an essential part of the covariant response theory.

The mathematical structure of the presented extension of nuclear response theory is based on the idea of time-blocking which is another key ingredient for this work. The time-blocking approximation makes possible the selection of the most important Feynman graphs containing quasiparticle-vibration coupling and their subsequent non-perturbative treatment in a controlled way. The developed method generalizes the time-blocking approximation to multiphonon coupling and, thus, is capable of resolving fine details of nuclear excitation spectra, which was quite limited in the previous versions of the RQTBA. The generalized response theory presented here is, thereby, a step forward to a more precise solution of the nuclear many-body problem, which aims at a unified description of both high-frequency collective states and low-energy spectroscopy. The proposed approach is of a rather general character and can be applied for other many-body Fermi systems with collective degrees of freedom.

## VII. ACKNOWLEDGEMENT

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