Local efficiency corrections to higher order cumulants

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Local Efficiency Corrections to Higher Order Cumulants

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In this brief note we derive and present the formulas necessary to correct measurements of cumulants for detection efficiency. In particular we consider the case where the efficiency may depend on the phase-space, such as transverse momentum, rapidity etc.

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I. INTRODUCTION

The study of the phase structure of the strong interaction has been a central topic in strong interaction physics for many years. Lattice quantum chromodynamics (QCD) calculations have meanwhile established that the transition at vanishing net-baryon density is an analytic cross-over transition [1]. The situation at finite net-baryon density, however, is not yet settled. Indeed many effective models for the strong interaction (see Ref. [2] for an overview) predict a first order phase-coexistence line which ends in a critical point, the location of which being rather model dependent.

In order to explore the QCD phase diagram in experiment, a beam energy scan has been carried out at the Relativistic Heavy Ion Collider (RHIC). By varying the beam energy the temperature and the net-baryon density of the system created in a heavy-ion reaction can be changed, with collisions at lower energies leading to system at higher net-baryon density. Among many observables the cumulants of the proton number distribution has received particular attention, since it can be considered to be a measure for net-baryon number fluctuations [3], which in turn are sensitive to the presence of any structure in the QCD phase diagram at finite net-baryon density [4, 5]. The analysis and interpretation of these cumulants, however, need to be carried out with some care. Effects such as baryon number conservation [6], the non-observation of neutrons [7], and efficiency corrections [8, 9] need to be taken into account before any conclusions on possible phase changes can be drawn from the data.

The purpose of this short note is to revisit the corrections due to finite detection efficiency, ϵ, on particle number cumulants, as discussed in [8], and extend the formalism to include a possible dependence of the efficiency on the phase space, such as transverse momentum, rapidity or azimuthal angle.

Before we start let us recall why the detection efficiency ϵ leads to corrections of any fluctuation observable, such as particle number cumulants of order $n \geq 2$. Suppose in each event we have exactly N particles, i.e., the variance of the number of *produced* particles vanishes. Given a detection efficiency ϵ, the mean number of *observed* particles, n, is then $\langle n \rangle = \epsilon N$. However, this does *not* imply that in each event i the number of observed particles is $n_i = \epsilon N$. Instead, n_i fluctuates around the mean $\langle n \rangle$ so that the distribution of observed particles has a finite variance. More formally, assume that the number of produced particles N is distributed according to $P(N)$ then the number of observed particles follows a distribution $p(n) = \sum_N w(n|N)P(N)$ where $w(n|N)$ denotes the probability to observe n particles given N produced particles. Clearly, in general, the cumulants of $P(N)$ and $p(n)$ are different. Typically $w(n|N)$ is modeled by a binomial distribution, and a recent analysis of net-proton cumulants by the STAR collaboration employed this approach [10].

Efficiency corrections to variances of the net-charge and net-baryon distributions have been studied in [11–13] and have been extended to higher order cumulants in [8, 9]. In all these studies a constant efficiency ϵ has been assumed. In reality, however, the efficiency may very well depend on the kinematics of the particles, such as their transverse momentum, rapidity and azimuthal angle. This will likely result in further corrections especially for higher order cumulants. It is the purpose of this short note to derive the necessary formulae that relate the moments and cumulants of the distribution of *produced* particles to those of the *observed* particles for the situation of phase-space dependent efficiencies. As in Ref. [8] we will assume that the (phase-space dependent) detection probabilities may be modeled by binomial distributions.

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II. LOCAL EFFICIENCY CORRECTIONS

Let us start by introducing some notation. In general we will use upper case characters to refer to the *produced* particles and lower case for the *observed* particles. Since we are interested in cumulants and moments of the net-baryon number, net-proton number, or net-charge, we will have two types of particles such as baryon/anti-baryon, proton/anti-proton, positive/negative charge. Thus we will denote the number of produced particles and "anti"-particles[1] by N and $\bar{N}$, respectively, and the observed ones by n and $\bar{n}$. Next, we assume that the phase-space will be partitioned into bins of potentially varying size. The location of these bins in rapidity, transverse momentum and azimuthal angle, $[y, p_\perp, \phi]$, will be denoted by x and $\bar{x}$ for particles and anti-particles, respectively. Thus $N(x)$ denotes the number of *produced* particles in the phase-space bin located at x, while $n(x)$ is the number of *observed* particles at x, etc. The event averaged number of produced and observed particles in the phase-space bin located at x are then given by $\langle N(x)\rangle$ and $\langle n(x)\rangle$, respectively. Similarly $\left\langle N(x)\bar{N}(\bar{x})\right\rangle$ is the event average of the product of the number of particles at x and anti-particles at $\bar{x}$. In order to obtain the event average over *all* the particles in the considered phase-space we need to sum over all phase-space bins

$$\langle N\rangle = \sum_x \langle N(x)\rangle \tag{1}$$

$$\left\langle N\bar{N}\right\rangle = \sum_x\sum_{\bar{x}} \left\langle N(x)\bar{N}(\bar{x})\right\rangle \tag{2}$$

$$\left\langle N^i\bar{N}^k\right\rangle = \sum_{x_1,\ldots,x_i}\sum_{\bar{x}_1,\ldots,\bar{x}_k} \left\langle N(x_1)\ldots N(x_i)\bar{N}(\bar{x}_1)\ldots\bar{N}(\bar{x}_k)\right\rangle \tag{3}$$

and similarly for the observed particles.

Next we introduce the probability $w(n(x)|N(x);\epsilon(x))$ to observe $n(x)$ particles in the phase-space bin at x given $N(x)$ produced particles and a phase-space dependent detection efficiency of $\epsilon(x)$. The efficiencies for anti-particles are correspondingly $\bar{\epsilon}(\bar{x})$. As already discussed we will model w as a binomial distribution, where the binomial probability is given by the efficiency $\epsilon(x)$:

$$w(n(x)|N(x);\epsilon(x)) = \frac{N(x)!}{n(x)!\,(N(x)-n(x))!}\,\epsilon(x)^{n(x)}\,(1-\epsilon(x))^{N(x)-n(x)}\,. \tag{4}$$

Given $w(n(x)|N(x);\epsilon(x))$ we can relate the probability to *observe* a given number of particles $n(x_i)$ at the various phase space points x_i to the probability for produced particles at these points,

$$\begin{aligned} p\left(n(x_1),\ldots,n(x_n);\,\bar{n}(\bar{x}_1),\ldots,\bar{n}(\bar{x}_m)\right) = &\sum_{N(x_1)=n(x_1)}^{\infty}\cdots\sum_{\bar{N}(\bar{x}_m)=\bar{n}(\bar{x}_m)}^{\infty} \\ &w\left(n(x_1)|N(x_1);\epsilon(x_1)\right)\ldots w\left(\bar{n}(\bar{x}_m)|\bar{N}(\bar{x}_m);\bar{\epsilon}(\bar{x}_m)\right)\,P\left(N(x_1),\ldots,N(x_n);\,\bar{N}(\bar{x}_1),\ldots,\bar{N}(\bar{x}_m)\right). \end{aligned} \tag{5}$$

Although the above expression looks rather involved, the relations between the various moments of the observed and produced particle distributions are straightforward to write down. The reason is that the binomial distributions for the various bins in phase-space and between particles and anti-particles are independent from each other. For the lowest moments we get

$$\langle n(x)\rangle = \epsilon(x)\,\langle N(x)\rangle \tag{6}$$

$$\langle n(x)\bar{n}(\bar{x})\rangle = \epsilon(x)\bar{\epsilon}(\bar{x})\,\left\langle N(x)\bar{N}(\bar{x})\right\rangle \tag{7}$$

$$\langle n(x_1)n(x_2)\rangle = \epsilon(x_1)\epsilon(x_2)\,\langle N(x_1)N(x_2)\rangle \;\; x_1\neq x_2 \tag{8}$$

$$\langle n(x)(n(x)-1)\rangle = \epsilon(x)^2\,\langle N(x)(N(x)-1)\rangle\,. \tag{9}$$

The last two equations, Eqs. (8),(9), can be conveniently written as

$$\langle n(x_1)(n(x_2)-\delta_{x_1,x_2})\rangle = \epsilon(x_1)\epsilon(x_2)\,\langle N(x_1)(N(x_2)-\delta_{x_1,x_2})\rangle \tag{10}$$

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[1] We use the term anti-particles quite generally. For example, in case of net-charge, "particles" refer to positively charged particles and "anti-particles" refer to negatively charged particles.

where $\delta_{x_1,x_2} = 1$ if $x_1 = x_2$ and zero otherwise. In order to proceed and to arrive at a general relation between the cumulants of the observed and produced particle distributions we follow the same strategy as in our previous work [8]. There we expressed the cumulants in terms of factorial moments and used a general relation between the factorial moments of the distribution of observed and produced particles. The factorial moments are given by

$$F_{i,k} \equiv \left\langle \frac{N!}{(N-i)!} \frac{\bar{N}!}{(\bar{N}-k)!} \right\rangle \tag{11}$$

$$f_{i,k} \equiv \left\langle \frac{n!}{(n-i)!} \frac{\bar{n}!}{(\bar{n}-k)!} \right\rangle \tag{12}$$

for the distribution of produced and observed particles, respectively. For constant, phase-space independent, efficiency correction [8]

$$f_{i,k} = \epsilon^i \bar{\epsilon}^k F_{i,k}. \tag{13}$$

In order to allow for phase space depended efficiency corrections we introduce the "local" factorial moments

$$\begin{aligned} A_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k) &= \langle N(x_1)[N(x_2) - \delta_{x_1,x_2}] \ldots [N(x_i) - \delta_{x_1,x_i} - \ldots - \delta_{x_{i-1},x_i}] \\ &\qquad \bar{N}(\bar{x}_1)[\bar{N}(\bar{x}_2) - \delta_{\bar{x}_1,\bar{x}_2}] \ldots [\bar{N}(\bar{x}_k) - \delta_{\bar{x}_1,\bar{x}_k} - \ldots - \delta_{\bar{x}_{k-1},\bar{x}_k}] \rangle \end{aligned} \tag{14}$$

$$\begin{aligned} a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k) &= \langle n(x_1)[n(x_2) - \delta_{x_1,x_2}] \ldots [n(x_i) - \delta_{x_1,x_i} - \ldots - \delta_{x_{i-1},x_i}] \\ &\qquad \bar{n}(\bar{x}_1)[\bar{n}(\bar{x}_2) - \delta_{\bar{x}_1,\bar{x}_2}] \ldots [\bar{n}(\bar{x}_k) - \delta_{\bar{x}_1,\bar{x}_k} - \ldots - \delta_{\bar{x}_{k-1},\bar{x}_k}] \rangle . \end{aligned} \tag{15}$$

Using Eq. (3), it is straightforward to show that the "local" factorial moments, $A_{i,k}$ and $a_{i,k}$, are related to the factorial moments $F_{i,k}$ and $f_{i,k}$ by summation over the phase-space bins

$$F_{i,k} = \sum_{x_1,\ldots,x_i} \sum_{\bar{x}_1,\ldots,\bar{x}_k} A_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k) \tag{16}$$

$$f_{i,k} = \sum_{x_1,\ldots,x_i} \sum_{\bar{x}_1,\ldots,\bar{x}_k} a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k) \tag{17}$$

Analogous to Eq. (13) the local factorial moments of the observed particle distribution are related to that of the produced particles by

$$a_{i,k} = \epsilon(x_1) \ldots \epsilon(x_i) \bar{\epsilon}(\bar{x}_1) \ldots \bar{\epsilon}(\bar{x}_k) A_{i,k}. \tag{18}$$

This relation follows from Eq. (13) and the fact, that the binomial efficiency corrections for different phase-space bins are independent from each other. Clearly our generalized relation, Eq. (18), gives the correct results for the second order moments, Eqs. (6-7).

By virtue of Eqs. (16) and (18), the factorial moments of the produced particle distribution can be extracted from the measured local particle distribution via

$$F_{i,k} = \sum_{x_1,\ldots,x_i} \sum_{\bar{x}_1,\ldots,\bar{x}_k} \frac{a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k)}{\epsilon(x_1) \ldots \epsilon(x_i) \bar{\epsilon}(\bar{x}_1) \ldots \bar{\epsilon}(\bar{x}_k)}. \tag{19}$$

For example for $F_{2,2}$ we obtain

$$\begin{aligned} F_{2,2} &= \sum_{x_1,x_2,\bar{x}_1,\bar{x}_2} \frac{\langle n(x_1)[n(x_2) - \delta_{x_1,x_2}]\, \bar{n}(\bar{x}_1)[\bar{n}(\bar{x}_2) - \delta_{\bar{x}_1,\bar{x}_2}] \rangle}{\epsilon(x_1)\epsilon(x_2)\bar{\epsilon}(\bar{x}_1)\bar{\epsilon}(\bar{x}_2)} \\ &= \sum_{x_1,x_2,\bar{x}_1,\bar{x}_2} \frac{\langle n(x_1)n(x_2)\bar{n}(\bar{x}_1)\bar{n}(\bar{x}_2) \rangle}{\epsilon(x_1)\epsilon(x_2)\bar{\epsilon}(\bar{x}_1)\bar{\epsilon}(\bar{x}_2)} - \sum_{x_1,x_2,\bar{x}_1} \frac{\langle n(x_1)n(x_2)\bar{n}(\bar{x}_1) \rangle}{\epsilon(x_1)\epsilon(x_2)\bar{\epsilon}(\bar{x}_1)^2} \\ &\quad - \sum_{x_1,\bar{x}_1,\bar{x}_2} \frac{\langle n(x_1)\bar{n}(\bar{x}_1)\bar{n}(\bar{x}_2) \rangle}{\epsilon(x_1)^2\bar{\epsilon}(\bar{x}_1)\bar{\epsilon}(\bar{x}_2)} + \sum_{x_1,\bar{x}_1} \frac{\langle n(x_1)\bar{n}(\bar{x}_1) \rangle}{\epsilon(x_1)^2\bar{\epsilon}(\bar{x}_1)^2}. \end{aligned} \tag{20}$$

The relation, Eq. (19), between the factorial moments of the distribution of produced particles, $F_{i,k}$, and the local factorial moments of the observed particles, $a_{i,k}$, is the main result of this paper. To extract cumulants from the factorial moments $F_{i,k}$ is straightforward, and it has been discussed in [8], where the relevant formulas for cumulants up to the sixth order are provided.

Finally, it is worth noticing that the number of terms in Eq. (19) is m^{i+k}, where m is the number of bins.[2] For example, for the fourth order cumulant $i+k=4$ leading to m^4 terms.

III. DISCUSSION

It would be interesting to get an idea about the magnitude of the corrections due to the local efficiency corrections. A precise determination is very difficult, since the correction will depend on the true multiplicity distribution and on the specific distribution of the particles in phase-space. All we can attempt here is a rough estimate using certain, simplifying, assumptions. In order to keep the formalism manageable let us consider the variance of the distribution of positively charged particles only. The extension to net-charge distribution is straightforward. The variance σ^2 is given in terms of the produced particles N by

$$\sigma^2 \equiv \left\langle N^2 \right\rangle - \left\langle N \right\rangle^2 = \sum_{x_1,x_2} \left[\left\langle N\left(x_1\right)\left(N\left(x_2\right) - \delta_{x_1,x_2}\right)\right\rangle - \left\langle N\left(x_1\right)\right\rangle \left\langle N\left(x_2\right)\right\rangle \right] + \sum_{x} \left\langle N\left(x_1\right)\right\rangle \tag{21}$$

Next, we introduce a correlation function $C\left(x_1,x_2\right)$ such that

$$\left\langle N\left(x_1\right)\left(N\left(x_2\right) - \delta_{x_1,x_2}\right)\right\rangle = \left\langle N\left(x_1\right)\right\rangle \left\langle N\left(x_2\right)\right\rangle \left(1 + C\left(x_1,x_2\right)\right). \tag{22}$$

The correlation function $C\left(x_1,x_2\right)$ controls the correlations of the particles in phase-space. In its absence, $C=0$, particles are distributed according to a Poisson distribution in each bin, and there are no bin-to-bin correlations. In this case, as we shall see, there is no difference between local and global efficiency corrections. Given the correlation function $C\left(x_1,x_2\right)$ the variance can be expressed as

$$\sigma^2 \;=\; \sum_{x_1,x_2} \left\langle N\left(x_1\right)\right\rangle \left\langle N\left(x_2\right)\right\rangle C\left(x_1,x_2\right) + \left\langle N\right\rangle = \left\langle N\right\rangle + \delta \tag{23}$$

where δ denotes the deviation from Poisson behavior.

In order to see the difference between local and global efficiency corrections, let us suppose we measure the variance but only correct for the global or rather mean efficiency, $\bar{\epsilon}$, which is given by

$$\bar{\epsilon} = \frac{\sum_x \epsilon(x)\left\langle N(x)\right\rangle}{\sum_x \left\langle N(x)\right\rangle} = \frac{\sum_x \epsilon(x)\left\langle N(x)\right\rangle}{\left\langle N\right\rangle} \tag{24}$$

Using the expression derived in [8] we would extract the following for the variance

$$\bar{\sigma}^2 = \frac{1}{\bar{\epsilon}^2}\left\langle n\left(n-1\right)\right\rangle - \frac{1}{\bar{\epsilon}^2}\left\langle n\right\rangle^2 + \frac{1}{\bar{\epsilon}}\left\langle n\right\rangle = \frac{1}{\bar{\epsilon}^2}\sum_{x_1,x_2}\left[\left\langle n\left(x_1\right)\left(n\left(x_2\right)-\delta_{x_1,x_2}\right)\right\rangle - \left\langle n\left(x_1\right)\right\rangle\left\langle n\left(x_2\right)\right\rangle\right] + \frac{1}{\bar{\epsilon}}\sum_{x_1}\left\langle n\left(x_1\right)\right\rangle. \tag{25}$$

The difference between the true variance, σ^2, which we would recover by applying local efficiency corrections, and that extracted by correcting only for the average efficiency, $\bar{\sigma^2}$, will be a measure for the importance of local efficiency corrections. In order to proceed, we express observed local moments in Eq. (25) by the true moments following the relations derived above, Eqs. (6–9)

$$\bar{\sigma}^2 = \left\langle N\right\rangle^2 \frac{\sum_{x_1,x_2}\epsilon\left(x_1\right)\epsilon\left(x_2\right)\left\langle N\left(x_1\right)\right\rangle\left\langle N\left(x_2\right)\right\rangle C\left(x_1,x_2\right)}{\left[\sum_{x_1}\epsilon\left(x_1\right)\left\langle N\left(x_1\right)\right\rangle\right]^2} + \left\langle N\right\rangle = \bar{\delta} + \left\langle N\right\rangle, \tag{26}$$

where $\bar{\delta}$ denotes again the deviation from Poisson. We note, without correlations, $\delta = \bar{\delta} = 0$ and there is no difference between local and global efficiency corrections.

[2] To clarify the notation all x_i and $\bar{x}_k$ in Eq. (19) are summed from the first bin to the m-th bin.

In order to estimate the difference between δ and $\bar{\delta}$ we further assume that the true particles distribution, $\langle N(x)\rangle$, the efficiency, $\epsilon(x)$ and the correlation function, $C(x_1,x_2)$ are all given by Gaussians

$$\begin{aligned}\langle N(x)\rangle &= \frac{N_{\rm tot}}{\sqrt{2\pi}\sigma_N}\exp\left(-\frac{x^2}{2\sigma_N^2}\right) && (27)\\ \epsilon(x) &= \epsilon_0\exp\left(-\frac{x^2}{2\sigma_\epsilon^2}\right) && (28)\\ C(x_1,x_2) &= C_0\exp\left(-\frac{(x_1-x_2)^2}{2\sigma_c^2}\right), && (29)\end{aligned}$$

and that the bins are sufficiently small so that we may replace the sums by integrals.

Typically the range of the correlation functions is expected to be shorter than that of the particles distribution, $r_c \equiv \sigma_c/\sigma_N < 1$. On the other hand $r_\epsilon \equiv \sigma_\epsilon/\sigma_N$ depends on the specific detector system. In addition to the various Gaussians, the overall acceptance of the detector needs to be taken into account as well, and we denote the interval in phase space where particles are actually measured by $x \in (-\Delta,\Delta)$. This will determine the range of integration (summation) for the above expressions. If the acceptance Δ is comparable to the range of the particle distribution we may integrate over the full phase-space, $x \in (-\infty,\infty)$ and we get

$$R \equiv \frac{\bar{\delta}}{\delta} = \sqrt{\frac{(2+r_c^2)}{r_c^2 + \frac{2r_\epsilon^2}{(1+r_\epsilon^2)}}}. \qquad (30)$$

If the efficiency changes only little over the range of the particle distribution than the effect of local efficiency corrections is negligible, and in the limit of $r_\epsilon \to \infty$ we recover the result for constant efficiency, i.e. $R = 1$. If, on the other hand, the range of the efficiency corrections are smaller than or comparable with that of the particle distribution, $r_\epsilon \lessapprox 1$, the correction become significant, $R \gtrapprox \sqrt{2}$. However, in this is case a more quantitative estimate require to account for the overall acceptance, Δ, since in this limit we will have particles in the region with vanishingly small efficiency, i.e. no acceptance.

If the measurement is performed in the region smaller than the range of the particle distribution we may only integrate over $x \in (-\Delta,\Delta)$. In this case no simple analytical expression can be obtained for R. Numerical studies show that for example $R = \bar{\delta}/\delta \approx 1.25$ if we assume that the efficiency is 20% at the boundary of the acceptance, $\epsilon(\Delta) = 0.2$. In other words, global efficiency correction leads to 25% larger deviations from the Poisson limit than the local efficiency correction. Although, one would expect the effect to increase for higher order cumulants, it is difficult to imagine factors of 2 or more due to the neglect of local efficiency corrections. We note, that the signal for a potential phase structure is the deviation from Poisson behavior and, therefore, the above corrections, while not tremendous, are still significant and need to be properly accounted for.

IV. CONCLUDING REMARKS

Let us conclude with a few remarks.

1. We note that the above result, Eq. (19), does not require the phase-space bins to be of equal size. Thus Eq. (19) applies to any choice of binning most suitable for a given experiment.

2. In the limit of constant efficiency over the entire phase-space under consideration, our result, Eq. (19), reduces to Eq. (13), the consequences of which were subject of our previous paper [8].

3. As already pointed out at the beginning of the paper, the above results assume a binomial distribution as a model for particle detection efficiencies. Thus the above expressions need to be suitably modified if this is not the case in a given experiment.

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