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We present a parameterization of the statistical rate function, f, for 20 superallowed $0^+ \to 0^+$ nuclear β transitions between $T=1$ analog states, and for 18 superallowed "mirror" transitions between analog $T=1/2$ states. All these transitions are of interest in the determination of V_{ud}. Although most of the transition Q_{EC} values have been measured, their precision will undoubtedly be improved in future. Our parameterization allows a user to easily calculate the corresponding new f value to high precision ($\pm 0.01\%$) without complicated computing.

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I. INTRODUCTION

Precise measurements of nuclear β decay provide a valuable window into the electroweak standard model. In particular, superallowed $0^+ \to 0^+$ transitions between $T=1$ analog states are used to set a limit on the presence of scalar interactions and to determine V_{ud}, the upper left element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and a key contributor to the most demanding available test of the unitarity of that matrix. While these transitions currently lead to the most precise determination of V_{ud}, mirror transitions between $T=1/2$ analog states are becoming of interest as a means of confirming V_{ud} via a different experimental approach. To be useful, not only must the Q_{EC} value for each of these transitions be measured very precisely but the statistical rate function, f, which uses the Q_{EC} as input, must be calculated with equivalent precision.

Because there is no widely available means for calculating f to the required level of precision, we have devised a simple parameterization that reproduces the results of our full code for energies spanning a small range around the currently known Q_{EC} values for both types of superallowed transition. Together, these should provide a convenient resource for experimentalists to use in future to obtain high-precision f values from improved Q_{EC}-value measurements for these transitions.

Our goal in what follows is to parameterize f and present tables of the parameters for the two sets of transitions: 1) the 20 superallowed $0^+ \to 0^+$ nuclear β transitions between $T=1$ analog states, whose properties have been surveyed in Refs. [1, 2]; and 2) the 18 superallowed "mirror" β transitions between the analog $T=1/2$ states surveyed in Ref. [3]. For each transition, we have computed f for 100 values of Q_{EC} taken over a range of ± 60 keV around the transition Q_{EC}-value[1] and fitted these results to determine the coefficients in our parameteriza-

tion. Our aim in fitting these 100 values is to achieve an accuracy of 0.01%, nearly a factor of ten more precise than is currently required.

II. PARAMETERIZATION OF THE STATISTICAL RATE FUNCTION

To achieve 0.01% accuracy, the electron wave function must be determined with great precision. In our detailed evaluation of f [4], we accomplished this by solving the Dirac equation for the emerging electron moving in the Coulomb field of the nuclear charge distribution. The full expression for the computation of f is

$$f = \xi R(W_0) \int_1^{W_0} pW(W_0 - W)^2 F(Z,W) f_1(W) \\ Q(Z,W) r(Z,W) \; dW, \qquad (1)$$

where W is the electron total energy in electron rest-mass units, W_0 is the maximum value of W, $p = (W^2 - 1)^{1/2}$ is the electron momentum, Z is the charge number of the daughter nucleus (positive for electron emission, negative for positron emission), $F(Z,W)$ is the Fermi function and $f_1(W)$ is the shape-correction function as defined by Holstein [5] (but with kinematic recoil corrections omitted).

Further, $Q(Z,W)$ is a screening correction for which we use the analytic prescription of Rose [6] (see Eq. (A44) in Ref. [4]), and $r(Z,W)$ is an atomic overlap correction described in Ref. [1]. The kinematic recoil corrections that Holstein includes in $f_1(W)$ are here written as $R(W_0)$. The expression for $R(W_0)$ is derived in Appendix A, with the result that

$$R(W_0) \simeq 1 - \frac{3W_0}{2M_A}, \qquad (2)$$

where M_A is the average of the initial and final nuclear masses expressed in electron-mass units. Last, for allowed transitions it is customary to remove the leading nuclear matrix element from the definition of f. Thus we have introduced ξ in Eq. (1), where $\xi = 1/|\mathcal{M}_F|^2$ for superallowed Fermi transitions, $\mathcal{M}_F$ being the Fermi matrix element. For mixed Fermi and Gamow-Teller transitions, $\xi = 1/[\mathcal{M}_F^2 + g_A^2 \mathcal{M}_{GT}^2]$ with $\mathcal{M}_{GT}$ being the

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[1] For ^{70}Br the Q_{EC}-value is less precisely known, so the Q_{EC}-value range for fitting was extended to ± 600 keV.

Gamow-Teller matrix element and g_A the axial-vector coupling constant.

In order to parameterize f, it is convenient to factor it into two contributions:

$$f = f_0(1 + \delta_S), \qquad (3)$$

$$f_0 = \int_1^{W_0} pW(W_0 - W)^2 F(Z,W) Q(Z,W) r(Z,W) \, dW, \qquad (4)$$

$$\delta_S = (f - f_0)/f_0. \qquad (5)$$

The purpose of this factorization is to place the role of the shape-correction function $f_1(W)$ entirely within the correction term δ_S, which is typically of the order of a few percent. The shape-correction function depends on nuclear matrix elements and differs for Fermi and Gamow-Teller transitions. This piece of the calculation is somewhat less certain since it is nuclear-structure dependent; however, being small, its accuracy is also less critical.

In the limit that $F(Z,W) Q(Z,W) r(Z,W) \to 1$, which occurs when $Z = 0$, the integral f_0 has an analytic value:

$$f_0(Z=0) = \tfrac{1}{30} W_0^4 p_0 - \tfrac{3}{20} W_0^2 p_0 - \tfrac{2}{15} p_0 + \tfrac{1}{4} W_0 \ln(W_0 + p_0), \qquad (6)$$

with $p_0 = (W_0^2 - 1)^{1/2}$. This suggests a fitting function of the form

$$f_0 = a_0 W_0^4 p_0 + a_1 W_0^2 p_0 + a_2 p_0 + a_3 W_0 \ln(W_0 + p_0). \qquad (7)$$

In fitting 100 values of f_0, we found that the four parameters a_0, a_1, a_2 and a_3 could not be uniquely determined with precision. Thus it was decided to fix a_2 and a_3 to their $Z = 0$ values, namely $a_2 = -2/15$ and $a_3 = 1/4$, and use the fitting process to determine a_0 and a_1. This procedure yielded the required precision for f_0. The resultant values of a_0 and a_1 are given in Table I for the $0^+ \to 0^+$ transitions, and in Table II for the $T=1/2$ mirror transitions.

We note that in Eq. (7) each successive term gives a smaller and smaller contribution to the total. Thus in deciding to fix a_2 and a_3 to their $Z = 0$ values, we have fixed the coefficients for the two smallest terms in the expression for f_0. This parameterization is not unique however. We could equally well have chosen to hold two different coefficients to their $Z = 0$ values and consequently have obtained another parameterization that would also produce f_0 values accurate to within 0.01%.

For the correction δ_S we start with the approximate expression

$$f_0 \delta_S \simeq \int_1^{W_0} pW(W_0 - W)^2 F(Z,W)$$
$$\left[\xi f_1(W) - 1 - \frac{3W_0}{2M_A} \right] \, dW \qquad (8)$$

and write

$$\xi f_1(W) - 1 = B_0 + B_1 W + B_2/W + B_3 W^2. \qquad (9)$$

The coefficients B_0, B_1, B_2 and B_3 are different for Fermi and Gamow-Teller transitions. This choice of parameterization is guided by the early work of Schopper [7] who used such a parameterization for the shape-correction function.

A. Superallowed $0^+ \to 0^+$ Fermi transitions

For Fermi (vector) transitions,

$$B_0^F = -\tfrac{1}{5}(W_0 R)^2 + \tfrac{1}{15} R^2 - \tfrac{6}{35}(\alpha Z)(W_0 R) + \tfrac{61}{630}(\alpha Z)^2,$$
$$B_1^F = \tfrac{4}{15}(W_0 R) R - \tfrac{48}{35}(\alpha Z) R,$$
$$B_2^F = \tfrac{2}{15}(W_0 R) R - \tfrac{18}{35}(\alpha Z) R,$$
$$B_3^F = -\tfrac{4}{15} R^2, \qquad (10)$$

where R is the radius of the nuclear charge distribution expressed in electron Compton wavelength units. We derived these equations from the work of Behrens and Bühring [8] who give algebraic expressions for the shape-correction function as expansions in the small quantities R and (αZ). Our Eqs. (10) and (14) below are correct to second order in these quantities, namely to order R^2, $(\alpha Z)^2$ and $(\alpha Z)R$. Inserting Eqs. (10) and (9) into Eq. (8) we obtain

$$\delta_S \simeq B_0 + B_1 \langle W \rangle + B_2 \langle 1/W \rangle + B_3 \langle W^2 \rangle - \frac{3W_0}{2M_A}, \qquad (11)$$

where $\langle W^n \rangle$ is the value of W^n averaged over the electron spectrum. Estimates of these quantities are: $\langle W \rangle = W_0/2$, $\langle W^{-1} \rangle = 5 W_0^{-1}/2$ and $\langle W^2 \rangle = 2 W_0^2/7$.

This leads to our final choice of parameterization for the correction δ_S:

$$\delta_S = b_0 + b_1 W_0 + b_2/W_0 + b_3 W_0^2, \qquad (12)$$

where approximate values of the coefficients are

$$b_0^F \simeq \tfrac{2}{5} R^2 + \tfrac{61}{630}(\alpha Z)^2,$$
$$b_1^F \simeq -\tfrac{6}{7}(\alpha Z) R - \tfrac{3}{2M_A},$$
$$b_2^F \simeq -\tfrac{9}{7}(\alpha Z) R,$$
$$b_3^F \simeq -\tfrac{1}{7} R^2. \qquad (13)$$

We fitted the expression in Eq. (12) to the exactly computed value of δ_S from Eq. (5) to obtain the parameters b_0, b_1, b_2 and b_3. Again, it was found that all four parameters could not be uniquely determined with precision, so the coefficients b_2 and b_3 were fixed at the values given in Eq. (13) for b_2^F and b_3^F, and the fitting process was used to determine b_0 and b_1. Table I gives the values of the parameters b_0, b_1, b_2 and b_3 for the superallowed $0^+ \to 0^+$ Fermi transitions.

B. Mirror $T=1/2$ transitions

For pure Gamow-Teller (axial-vector) transitions, coefficients in the expression for the shape-correction func-

TABLE I: Values of the coefficients a_0 and a_1 that yield the statistical rate function f_0 from Eq. (7), and coefficients b_0, b_1, b_2 and b_3 that yield the correction δ_S from Eq. (12). Coefficients a_2 and a_3 are held fixed at the values: $a_2 = -2/15$ and $a_3 = 1/4$. The cases shown are the superallowed Fermi transitions between $T=1$, $J^\pi = 0^+$ analog states surveyed in Ref. [1, 2].

Parent nucleus	a_0	a_1	$b_0(\%)$	$b_1(\%)$	$b_2(\%)$	$b_3(\%)$
^{10}C	0.0297225	−0.1431540	0.01178	0.02006	0.05203	−0.00096
^{14}O	0.0285463	−0.1417222	0.03176	0.03123	0.06506	−0.00101
^{18}Ne	0.0274005	−0.1398743	0.04750	0.04995	0.08945	−0.00134
^{22}Mg	0.0263237	−0.1374785	0.07036	0.06393	0.10796	−0.00139
^{26}Si	0.0253385	−0.1369946	0.10306	0.07827	0.12856	−0.00146
^{30}S	0.0242904	−0.1271838	0.14905	0.09529	0.15413	−0.00161
^{34}Ar	0.0233252	−0.1182701	0.15336	0.11896	0.18478	−0.00184
^{38}Ca	0.0223867	−0.1018182	0.17301	0.13558	0.20674	−0.00186
^{42}Ti	0.0216593	−0.1105386	0.15625	0.15293	0.23380	−0.00196
^{26m}Al	0.0257927	−0.1355697	0.09813	0.07208	0.12037	−0.00149
^{34}Cl	0.0238533	−0.1281700	0.16759	0.10598	0.16911	−0.00173
^{38m}K	0.0228360	−0.1090747	0.17630	0.12480	0.19325	−0.00180
^{42}Sc	0.0220302	−0.1082192	0.17335	0.14265	0.22147	−0.00193
^{46}V	0.0211437	−0.0894977	0.20200	0.16556	0.25213	−0.00208
^{50}Mn	0.0202722	−0.0597791	0.25281	0.18330	0.27834	−0.00214
^{54}Co	0.0195698	−0.0524836	0.32812	0.19757	0.30337	−0.00217
^{62}Ga	0.0181322	−0.0141676	0.49342	0.24418	0.36915	−0.00243
^{66}As	0.0173202	0.0473840	0.57218	0.27260	0.40770	−0.00261
^{70}Br	0.0167829	0.0417070	0.56671	0.29670	0.43602	−0.00265
^{74}Rb	0.0162385	0.0446304	0.44643	0.32180	0.46490	−0.00269

tion in Eq. (9) are:

$$
\begin{aligned}
B_0^{GT} &\simeq -\tfrac{1}{5}(W_0 R)^2 + \tfrac{11}{45}R^2\left(1 - \tfrac{2}{11}x\right) \\
&\quad + \tfrac{2}{35}(\alpha Z)(W_0 R)(1-x) + \tfrac{1}{3}(W_0 R)\left[\mp 2\overline{b} + \overline{d}\right] \\
&\quad + \tfrac{1}{3}\beta(\alpha Z)\left[\pm 2\overline{b} + \overline{d}\right] + \tfrac{61}{630}(\alpha Z)^2, \\
B_1^{GT} &\simeq \tfrac{4}{9}(W_0 R)R\left(1 - \tfrac{1}{10}x\right) - \tfrac{8}{5}(\alpha Z)R\left(1 - \tfrac{1}{28}x\right) \\
&\quad \pm \tfrac{4}{3}R\overline{b}, \\
B_2^{GT} &\simeq -\tfrac{2}{45}(W_0 R)R(1-x) - \tfrac{18}{35}(\alpha Z)R \\
&\quad - \tfrac{1}{3}R\left[\pm 2\overline{b} + \overline{d}\right], \\
B_3^{GT} &\simeq -\tfrac{4}{9}R^2\left(1 - \tfrac{1}{10}x\right), \quad (14)
\end{aligned}
$$

where

$$ x = -\sqrt{10}\,\mathcal{M}_{1y}/\mathcal{M}_{\sigma r^2}, \quad (15) $$

$$ \overline{b} = \frac{1}{MR}\left[\frac{g_M}{g_A} + \frac{\mathcal{M}_L}{\mathcal{M}_{GT}}\right], \quad (16) $$

$$ \overline{d} = \frac{1}{MR}\frac{\mathcal{M}_{\sigma L}}{\mathcal{M}_{GT}}, \quad (17) $$

and also $\beta \simeq 6/5$, $g_M = 4.706$ and M is the nucleon mass in electron rest-mass units. Where there is a $\pm$ symbol, the upper sign is used for electron emission beta decays, the lower sign for positron emitters. All the transitions discussed in this work are positron emitters, so the

lower sign is consistently used. The nuclear matrix elements are defined in Eq. (68) of Ref. [5]. Schematically, they are written: $\mathcal{M}_{GT} = \langle\sigma\rangle$, $\mathcal{M}_{\sigma r^2} = \langle r^2\sigma\rangle$, $\mathcal{M}_{1y} = (16\pi/5)^{1/2}\langle r^2\left[Y_2 \times \sigma\right]\rangle$, $\mathcal{M}_L = \langle L\rangle$ and $\mathcal{M}_{\sigma L} = \langle\sigma \times L\rangle$. Note that the matrix element $\mathcal{M}_{\sigma L}$, and hence $\overline{d}$, vanishes in diagonal matrix elements, as would occur in a mirror transition between isobaric analogue states.

The correction δ_S is again parameterized as in Eq. (12) with approximate expressions for the coefficients derived from Eq. (14). For pure Gamow-Teller transitions they yield

$$
\begin{aligned}
b_0^{GT} &\simeq \tfrac{2}{15}R^2 + \tfrac{1}{15}R^2 x + \tfrac{1}{3}\beta(\alpha Z)\left[\pm 2\overline{b} + \overline{d}\right] + \tfrac{61}{630}(\alpha Z)^2, \\
b_1^{GT} &\simeq -\tfrac{26}{35}(\alpha Z)R - \tfrac{1}{35}(\alpha Z)Rx + \tfrac{1}{3}R\overline{d} - \tfrac{3}{2M_A}, \\
b_2^{GT} &\simeq -\tfrac{9}{7}(\alpha Z)R - \tfrac{5}{6}R\left[\pm 2\overline{b} + \overline{d}\right], \\
b_3^{GT} &\simeq -\tfrac{11}{105}R^2\left(1 + \tfrac{1}{11}x\right). \quad (18)
\end{aligned}
$$

Again in fitting the exact values of δ_S from Eq. (5) with the expression in Eq. (12) we held the parameters b_2 and b_3 fixed at the values given for b_2^{GT} and b_3^{GT} in Eq. (18) and then obtained the parameters b_0 and b_1 from the fit.

The $T=1/2$ mirror transitions are mixed transitions, with both Fermi and Gamow-Teller components. The fitted b coefficients for both the Fermi and Gamow-Teller components are given in Table II along with the a_0 and a_1 coefficients. In such mixed transitions the inverse of the partial lifetime is proportional to

$$ t^{-1} \propto f_V\left[|\mathcal{M}_F|^2 + \frac{f_A}{f_V}|g_A\mathcal{M}_{GT}|^2\right], \quad (19) $$

TABLE II: Values of the coefficients, a_0 and a_1, that yield the statistical rate function f_0 from Eq. (7), and coefficients, b_0, b_1, b_2 and b_3, that yield the correction, δ_S, from Eq. (12). For each case, the b coefficients in the first row correspond to the Fermi component, b_0^F etc., and those in the second row correspond to the Gamow-Teller component, b_0^{GT} etc. Coefficients a_2 and a_3 are held fixed at values: $a_2 = -2/15$ and $a_3 = 1/4$. The cases shown are the mixed Fermi and Gamow-Teller transitions between mirror $T = 1/2$ states in odd-mass nuclei surveyed in Ref. [3].

Parent nucleus	a_0	a_1	$b_0(\%)$	$b_1(\%)$	$b_2(\%)$	$b_3(\%)$
^{11}C	0.0297280	−0.1431964	0.01227	0.02003	0.05030	−0.00093
			0.71578	0.07695	0.56495	−0.00055
^{13}N	0.0291054	−0.1420911	0.02166	0.02566	0.05640	−0.00095
			0.31915	0.04052	0.36491	−0.00026
^{15}N	0.0285370	−0.1416159	0.04552	0.03330	0.06846	−0.00117
			0.25027	0.02949	0.31261	−0.00024
^{17}F	0.0279301	−0.1400527	0.02528	0.04132	0.07482	−0.00113
			1.47963	0.06556	0.67635	−0.00074
^{19}Ne	0.0273984	−0.1398186	0.05521	0.04998	0.08914	−0.00135
			1.34270	0.05876	0.55039	−0.00099
^{21}Na	0.0268709	−0.1394429	0.05922	0.05859	0.09992	−0.00142
			1.73365	0.06313	0.64005	−0.00101
^{23}Mg	0.0263324	−0.1378844	0.06375	0.06436	0.10731	−0.00138
			1.91334	0.06348	0.64697	−0.00093
^{25}Al	0.0258123	−0.1364357	0.08154	0.07160	0.11849	−0.00144
			2.34766	0.07237	0.73737	−0.00090
^{27}Si	0.0252815	−0.1330446	0.09398	0.07980	0.12945	−0.00149
			2.75164	0.07192	0.79082	−0.00086
^{29}P	0.0247788	−0.1301483	0.11110	0.08717	0.13982	−0.00151
			2.41055	0.06857	0.66134	−0.00086
^{31}S	0.0243506	−0.1328386	0.14554	0.09612	0.15460	−0.00163
			2.17479	0.07799	0.60883	−0.00092
^{33}Cl	0.0239077	−0.1333333	0.15209	0.10627	0.16772	−0.00170
			−0.78012	0.06397	0.14681	−0.00050
^{35}Ar	0.0233631	−0.1226819	0.19658	0.11636	0.18453	−0.00183
			−0.53603	0.07047	0.19515	−0.00055
^{37}K	0.0229710	−0.1251564	0.18369	0.12491	0.19342	−0.00180
			0.89286	0.09010	0.41192	−0.00084
^{39}Ca	0.0224606	−0.1123165	0.21779	0.13259	0.20653	−0.00185
			0.58246	0.09757	0.37316	−0.00087
^{41}Sc	0.0220044	−0.1046436	0.20989	0.14001	0.22165	−0.00193
			3.94590	0.12635	0.91561	−0.00124
^{43}Ti	0.0216749	−0.1134605	0.15801	0.15650	0.23644	−0.00200
			3.54635	0.13277	0.82630	−0.00135
^{45}V	0.0211420	−0.0910683	0.31418	0.16234	0.25779	−0.00218
			4.54443	0.14448	0.99222	−0.00134

where

$$f_V = f_0 \left(1 + \delta_S^F\right),$$
$$f_A = f_0 \left(1 + \delta_S^{GT}\right). \quad (20)$$

The statistical rate functions f_V and f_A are easily obtained from the parameters listed in Table II.

III. CONCLUSIONS

We have provided simple parameterizations of the statistical rate functions, f, for nuclear β transitions of current interest in determining V_{ud} and testing CKM unitarity. In most but not all cases, the transition Q_{EC} values have already been measured with $\sim$1-keV precision or better. In a few cases they are much less well known. In all cases, the Q_{EC} values will undoubtedly be remeasured, leading possibly to different values and certainly to reduced uncertainties. When this happens, experimenters will need f values of equivalent precision, and the parameterizations presented here will satisfy that need without complicated computing.

It is important to note that our parameterization is only valid for the transitions identified and only for a limited range of energies (± 60 keV for all cases except for the decay of ^{70}Br which covers ± 600 keV) around the currently accepted Q_{EC} values for those transitions

[2, 3]. The coefficients of our parameterization should not be applied outside the range of energies specified or to any other transitions.

Appendix A: Kinematic recoil corrections

Let M_A be the average mass of the initial and final nuclei. Then the kinematic recoil corrections are of order W_0/M_A and, in all but the most precise work, they can generally be ignored. The recoil correction enters the calculation in two places: firstly, the end-point energy is slightly modified, a correction we denote Δf^a; and secondly, additional terms are added to the shape-correction function $f_1(W)$, providing a correction we call Δf^b.

For the first correction: If W_0 is the end-point energy without consideration of recoil and W_0^{corr} is the corrected value, then from Eq.(3) of Holstein [5] we get

$$
\begin{aligned}
W_0^{\mathrm{corr}} &= W_0 \left(1 + \frac{1}{2W_0 M_A}\right)\left(1 + \frac{W_0}{2M_A}\right)^{-1} \\
&\simeq W_0 \left(1 - \frac{W_0}{2M_A} + \frac{1}{2W_0 M_A}\right). \quad \text{(A1)}
\end{aligned}
$$

So, since the statistical rate function is approximately proportional to W_0^5, the correction to f must be of order

$$
\frac{\Delta f^a}{f} \simeq 1 - \frac{5}{2}\frac{W_0}{M_A} + \frac{5}{2}\frac{1}{W_0 M_A}. \quad \text{(A2)}
$$

Unlike Δf^a, the recoil correction to the shape-correction function, Δf^b, is different for Fermi and Gamow-Teller transitions. The modifications are

$$
\begin{aligned}
f_1^{F,\mathrm{corr}}(W) &= f_1^F(W)\left(1 + 2\frac{W}{M_A}\right), \\
f_1^{GT,\mathrm{corr}}(W) &= f_1^{GT}(W)\left(1 - \frac{2}{3}\frac{W_0}{M_A} + \frac{10}{3}\frac{W}{M_A} \right. \\
&\qquad\qquad \left. - \frac{2}{3}\frac{1}{M_A W}\right). \quad \text{(A3)}
\end{aligned}
$$

If these corrections are integrated over the electron spectrum, they yield corrections to the statistical rate function of

$$
\begin{aligned}
\frac{\Delta f^{b,F}}{f} &\simeq 1 + \frac{W_0}{M_A}, \\
\frac{\Delta f^{b,GT}}{f} &\simeq 1 - \frac{2}{3}\frac{W_0}{M_A} + \frac{5}{3}\frac{W_0}{M_A} - \frac{5}{3}\frac{1}{M_A W_0}. \quad \text{(A4)}
\end{aligned}
$$

Finally, combining corrections Δf^a and Δf^b, we obtain the final recoil correction to the statistical rate function

$$
\begin{aligned}
\frac{\Delta f^F}{f} &= 1 - \frac{3}{2}\frac{W_0}{M_A} + \frac{5}{2}\frac{1}{W_0 M_A} \\
&\simeq 1 - \frac{3}{2}\frac{W_0}{M_A}, \\
\frac{\Delta f^{GT}}{f} &= 1 - \frac{3}{2}\frac{W_0}{M_A} + \frac{5}{6}\frac{1}{W_0 M_A} \\
&\simeq 1 - \frac{3}{2}\frac{W_0}{M_A}. \quad \text{(A5)}
\end{aligned}
$$

Thus, Fermi and Gamow-Teller transitions are subject to essentially the same correction and it is this correction that we have recorded in Eq. (2) and used in our fitting algorithms. Of course, the exactly computed f values, to which our parameterizations are fitted, include the complete kinematic recoil treatment.

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