Search for time reversal invariance violation in neutron transmission

J. David Bowman and Vladimir Gudkov

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On the Search for Time Reversal Invariance Violation in Neutron Transmission

J. David Bowman*
Oak Ridge National Laboratory, Oak Ridge, TN 37831

Vladimir Gudkov†
Department of Physics and Astronomy,
University of South Carolina, Columbia, SC 29208

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Abstract

Time Reversal Invariant Violating (TRIV) effects in neutron transmission through a nuclear target are discussed. We demonstrate the existence of a class of experiments that are free from false asymmetries. We discuss the enhancement of TRIV effects for neutron energies corresponding to p-wave resonances in the compound nuclear system. We analyze a model experiment and show that such tests can have a discovery potential of $10^2 - 10^4$ compared to current limits.

INTRODUCTION

Time reversal invariance violation (TRIV) in nuclear physics has been studied for several decades. There are a number of TRIV effects in nuclear reactions and nuclear decays, which are sensitive to either CP-odd and P-odd (or T - and P-violating) interactions or T -violating P-conserving (C-odd and P-even) interactions. Here we consider TRIV effects in nuclear reactions which can be measured in the transmission of polarized neutrons through a polarized target[1, 2]. Such reactions can be described within the framework of neutron optics (for a discussion of neutron optics and see for example [3, 4].) The transmitted neutron wave propagates through a medium according to a spin-dependent index of refraction. The index of refraction depends on any applied magnetic field and the polarization of the medium. Because the state of the medium does not change, the polarization of the medium can be treated as a classical field. Because the initial and final propagation vectors of the neutron are the same, the initial and final states of the neutron can be time reversed in an experiment by rotation of the apparatus.

The neutron and nuclei are both composite systems and any measurement of a T-odd process in a particular system may have accidental cancellation of TRIV effects or might be relatively insensitive to one or more of the many possible sources of T-odd amplitudes. A search for TRIV in neutron transmission expands the variety of nuclear systems. This provides assurance that possible "accidental" cancellation of T-violating effects due to unknown structural factors related to the strong interactions in the particular system can be avoided. Taking into account that different models of the CP-violation may contribute differently to a particular T/CP-odd observable, which may have unknown theoretical uncertainties, TRIV nuclear effects could be considered complementary to electric dipole moment (EDM) measurements, whose status as null tests of T invariance is more widely known. Moreover, there is the possibility of an enhancement of T-violating observables by many orders of magnitude in neutron transmission due to the complex nuclear structure (see, i.e. paper [5] and references therein).

For the observation of TRIV and parity violating (PV) effects, we consider neutron optical effects related to the T-odd correlation, $\vec{\sigma}_n \cdot (\vec{k} \times \vec{I})$, where $\vec{\sigma}_n$ is the neutron spin, $\vec{I}$ is the target spin, and $\vec{k}$ is the neutron momentum, which can be observed in the transmission of polarized neutrons through polarized target. This correlation leads to a P-odd and T-odd difference between the total neutron cross sections [2] $\Delta\sigma_{\not{T}\not{P}}$ for $\vec{\sigma}_n$ parallel and anti-parallel to $\vec{k}\times\vec{I}$ and to the neutron spin rotation angle [1] $\phi_{\not{T}\not{P}}$ around the axis $\vec{k}\times\vec{I}$

$$\Delta\sigma_{\not{T}\not{P}} = \frac{4\pi}{k}\mathrm{Im}(f_{\uparrow} - f_{\downarrow}), \qquad \frac{d\phi_{\not{T}\not{P}}}{dz} = -\frac{2\pi N}{k}\mathrm{Re}(f_{\uparrow} - f_{\downarrow}). \tag{1}$$

Here, $f_{\uparrow,\downarrow}$ are the zero-angle scattering amplitudes for neutrons polarized parallel and anti-parallel to the $\vec{k}\times\vec{I}$ axis, respectively; z is the target length and N is the number of target nuclei per unit volume. These TRIV effects can be enhanced [6] by a factor as large as 10^6. Similar enhancement was already observed for PV effects related to $(\vec{\sigma}_n\cdot\vec{k})$ correlation in neutron transmission through nuclear targets. For example, the PV asymmetry in the $0.734\ eV$ p-wave resonance in ^{139}La has been measured to be $(9.56\pm0.35)\cdot10^{-2}$ (see, for example [7] and references therein).

The PV and TRI-conserving difference of total cross sections $\Delta\sigma_{\not{P}}$ in the transmission of polarized neutrons through unpolarized targets which is proportional to the correlation $(\vec{\sigma}\cdot\vec{k})$ can be written in terms of differences of zero angle elastic scattering amplitudes with negative and positive neutron helicities as:

$$\Delta\sigma_{\not{P}} = \frac{4\pi}{k}Im(f_{-} - f_{+}). \tag{2}$$

One can calculate both TRIV and PV amplitudes using distorted wave Born approximation to first order in the parity and time reversal violating interactions (see, for example ref.[6]). Thus, the symmetry violating amplitudes can be written as

$$t^{fi}_{\not{P},\not{P}\not{T}} = <\Psi^{-}_{f}|V_{\not{P},\not{P}\not{T}}|\Psi^{+}_{i}>, \tag{3}$$

where $\Psi^{\pm}_{i,f}$ are the eigenfunctions of the nuclear T-invariant Hamiltonian with the appropriate boundary conditions [8]:

$$\Psi^{\pm}_{i,f} = \sum_{k} a^{\pm}_{k(i,f)}(E)\,\phi_k + \sum_{m}\int b^{\pm}_{m(i,f)}(E,E')\,\chi^{\pm}_{m}(E')\,dE'. \tag{4}$$

Here ϕ_k is the wave function of the k^{th} compound-resonance and $\chi^{\pm}_{m}(E)$ is the potential scattering wave function in the channel m. The coefficient

$$a^{\pm}_{k(i,f)}(E) = \frac{\exp\left(\pm i\delta_{i,f}\right)}{(2\pi)^{\frac{1}{2}}}\frac{(\Gamma^{i,f}_{k})^{\frac{1}{2}}}{E - E_k \pm \frac{i}{2}\Gamma_k} \tag{5}$$

describes compound nuclear resonances reactions and the coefficient $b^{\pm}_{m(i,f)}(E,E')$ describes potential scattering and interactions between the continuous spectrum and compound resonances. (Here E_k, Γ_k, and Γ^{i}_{k} are the energy, the total width, and the partial width in the channel i of the k-th nuclear compound resonance, E is the neutron energy, and δ_i is the potential scattering phase shift in the channel i; $(\Gamma^i_k)^{\frac{1}{2}} = (2\pi)^{\frac{1}{2}} < \chi_i(E)|V|\phi_k >$, where V is a residual interaction operator.)

Since it is already known that the dominant mechanism of symmetry violation in heavy nuclei is the mechanism of symmetry mixing in the compound nuclear resonances [6], only first term in Eq. (4) is important to include for our estimates. For sake of simplicity we consider the case of a two resonance approximation, which is reasonably good for many heavy nuclei in the low neutron energy region $E \sim 1eV - 100eV$. Then, symmetry violating amplitudes due to mixing of nearby s-wave and p-wave resonances can be written as:

$$< p|t|s >= -\frac{1}{2\pi}\frac{(v+iw)(\Gamma^n_s\Gamma^f_p)^{\frac{1}{2}}}{(E-E_s+i\Gamma_s/2)(E-E_p+i\Gamma_p/2)}e^{i(\delta^n_s+\delta^n_p)}, \tag{6}$$

and

$$< s|t|p >= -\frac{1}{2\pi}\frac{(v-iw)(\Gamma^n_p\Gamma^n_s)^{\frac{1}{2}}}{(E-E_s+i\Gamma_s/2)(E-E_p+i\Gamma_p/2)}e^{i(\delta^n_p+\delta^n_s)}, \tag{7}$$

where v and w are and imaginary parts of the real and imaginary parts of the matrix elements for PV and TRIV mixing between s- and p-wave compound resonances

$$v + iw =< \phi_p|V_{\not P} + V_{\not P \not T}|\phi_s > \tag{8}$$

due to $V_{\not P}$ (PV) and $V_{\not P \not T}$ (TRIV) interactions. One can see that PV and TRIV matrix elements are real and imaginary parts of the same matrix element calculated with exactly the same wave functions. Also, the difference of amplitudes $(f_- - f_+)$ for the PV effect in Eq. (2) is proportional to the sum of the symmetry violating amplitudes (Eq. (6) and Eq. (7)) but the difference of amplitudes $(f_\uparrow - f_\downarrow)$ for the PT-violating effect in Eq. (1) is proportional to the difference of the same amplitudes (Eq. (6) and Eq. (7)). This results in the same energy dependencies for both PV and TRIV effects. Indeed, taking into account all numerical factors one gets:

$$\Delta\sigma_{\not P \not T} = -\frac{2\pi G^T_J}{k^2}\frac{w(\Gamma^n_s\Gamma^n_p(S))^{\frac{1}{2}}}{[s][p]}[(E-E_s)\Gamma_p + (E-E_p)\Gamma_s], \tag{9}$$

and

$$\Delta\sigma_{\not P} = \frac{2\pi G^P_J}{k^2}\frac{v(\Gamma^n_s\Gamma^n_p)^{\frac{1}{2}}}{[s][p]}[(E-E_s)\Gamma_p + (E-E_p)\Gamma_s], \tag{10}$$

where $[s,p] = (E - E_{s,p})^2 + \Gamma^2_{s,p}/4$, G^T_J and G^P_J are spin factors, and J is the spin of compound nucleus (see details in ref.[5, 6, 9]). One can see that due to the similarity of these two equations, the TRIV effect has the same enhancement on resonance as the PV one.

Now one can find the relation between the values of the PV and TRIV effects as

$$\Delta\sigma_{\not{T}\not{P}} = \kappa(J)\frac{w}{v}\Delta\sigma_{\not{P}}, \tag{11}$$

where

$$\kappa(I+1/2) = -\frac{3}{2^{3/2}}\left(\frac{2I+1}{2I+3}\right)^{3/2}\left(\frac{3}{\sqrt{2I+3}}\gamma - \sqrt{I}\right)^{-1},$$
$$\kappa(I-1/2) = -\frac{3}{2^{3/2}}\left(\frac{2I+1}{2I-1}\right)\left(\frac{I}{I+1}\right)^{1/2}\left(-\frac{I-1}{\sqrt{2I-1}}\frac{1}{\gamma} + \sqrt{I+1}\right)^{-1}. \tag{12}$$

Here $\gamma = [\Gamma_p^n(I+1/2)/\Gamma_p^n(I-1/2)]^{1/2}$ is the ratio of the neutron width amplitudes for the different channel spins. In general, the parameter γ may be obtained from gamma-ray angular correlation measurements in neutron capture reactions on resonance [6, 10]. Using standard unitary transformations one can rewrite the parameter γ in the neutron spin ($j = l \pm 1/2$) representation scheme $\Gamma_p^n(j)^{1/2}$ as

$$\gamma = \frac{-\sqrt{2}\Gamma_p^n(1/2)^{1/2} + \Gamma_p^n(3/2)^{1/2}}{\Gamma_p^n(1/2)^{1/2} + \sqrt{2}\Gamma_p^n(3/2)^{1/2}}. \tag{13}$$

One can see from eq.(11), that larger values of the parameter $\kappa(J)$ increase the sensitivity of the TRIV difference of total cross sections compared to the PV. One can therefore enhance the sensitivity of TRIV experiments in polarized neutron transmission by choosing a p-wave resonance in a nucleus with favorable properties.

ENHANCEMENT FACTORS

Let us recall the main features of the enhancement factors for TRIV and PV effects using as an example the P-odd difference $\Delta\sigma_{\not{P}}$ of total cross sections. The quantity $\Delta\sigma_{\not{P}}$ displays resonance peaks near both s- and p-wave resonances, increasing its value by a factor of $(D/\Gamma)^2$ with respect to an energy between the resonances ($D = \mid E_s - E_p \mid$). These peaks are caused by the resonant enhancement of the wave function amplitude in the region of the interaction. The physical meaning of the resonance enhancement is similar to the estimates of the lifetime of the compound nucleus. This lifetime τ can be understood as the additional time, that the neutron spends in the range of the nuclear interaction due to the resonant component of the neutron-nucleus interaction. In terms of the neutron scattering phase shift $\delta(E)$, one can write

$$\tau = 2\frac{d\delta(E)}{dE}, \tag{14}$$

where the resonant part of the phase shift for the i-th resonance is $\delta(E) \simeq -\arctan\left((\Gamma_i/2)/(E-E_i)\right)$ near the resonance energy. In the resonance state, the particle remains within the nucleus for a longer time of the order of the resonance lifetime $\sim (1/\Gamma)$. Therefore, it is natural to expect an enhancement of symmetry violation proportional to the ratio of the resonance lifetime $(1/\Gamma)$ to the lifetime of compound- nucleus away from the resonance (Γ/D^2), that is to $(D/\Gamma)^2$.

Let us consider the ratio $P = \Delta\sigma_{\not P}/(2\sigma_{tot})$, where σ_{tot} is the total cross section and consists of the s-resonance , p-resonance and the potential scattering contributions. The quantity σ_{tot} also displays a marked resonance peak in the vicinity of s -wave resonance, which compensates completely for the corresponding peak of the numerator P. Therefore, the quantity P is not enhanced in the vicinity of the s-wave resonance and remains approximately on the same level as the value between the resonances. In general, σ_{tot} is dominated by the smooth background of the s-wave resonance and potential scattering cross section in the vicinity of the p-wave resonance, since for the neutron energies under consideration here $(kR) \ll 1$ (R is the nuclear radius). Therefore, the resonance peak of $\Delta\sigma_{\not P}$ near the p-resonance is retained in the quantity P, which is enhanced here by a factor of $(D/\Gamma)^2$

$$P(E_p) \sim 8\frac{v}{D}\sqrt{\frac{\Gamma^n_p}{\Gamma^n_s}\frac{D^2}{\Gamma_s\Gamma_p}}\left[1+\frac{\sigma_p+\sigma_{pot}}{\sigma_s}\right]^{-1}. \tag{15}$$

The presence of the "penetration factor" $\sqrt{\Gamma^n_p/\Gamma^n_s} \sim (kR)$ in eq.(15) is characteristic of all correlations observed in low energy nuclear reactions which arise due to initial state interference and, consequently, are proportional to the neutron momentum in the correlation $(\vec{\sigma}\cdot\vec{k})$. It should be noted that P might have the maximal magnitude

$$P_{max} \simeq \frac{v}{D}\frac{D}{\Gamma} = \frac{v}{\Gamma}, \tag{16}$$

when the total cross section contributions from the s- and p-resonances have similar magnitudes in the vicinity of the p-wave resonance.

In addition to the resonance enhancement factor, there is also the so called "dynamic" enhancement factor, which is connected with the ratio v/D. For a crude estimate of this ratio, one can expand the compound resonance wave function ϕ in terms of simple-configuration wave functions (e.g., one-particle wave functions) ψ_i which are admixed to compound resonances by strong interactions:

$$\phi = \sum_{i=1}^{N} c_i \psi_i. \tag{17}$$

Using the normalization condition for the coefficients c_i and the statistical random-phase hypothesis for matrix elements $< \psi_i|W|\psi_k >$ we obtain

$$v =< \phi_s|W|\phi_p >=< \psi_i|W|\psi_k >_{RMS} N^{-1/2}. \tag{18}$$

Here $< \psi_i|W|\psi_k >_{RMS}$ is the root mean square value of the matrix elements between simple configurations. In the black-nucleus statistical model, the number of components N is estimated in terms of the average spacing $\overline{D}$ of compound resonances and the average spacing $\overline{D}_0$ of single-particle states:

$$N \approx \overline{D}_0/\overline{D}. \tag{19}$$

One can estimate N from the experimental data on neutron strength functions since, in the statistical model of heavy nuclei, the neutron strength function is proportional to N^{-1} (see, e.g., [11]). The value of N is about 10^6. Hence

$$\frac{v}{D} \simeq \frac{< \psi_i|W|\psi_k >_{RMS}}{\overline{D}_0}\sqrt{N}, \tag{20}$$

where the ratio of the single-particle weak matrix element to the single particle level distance is about 10^{-7} (or the usual scale of the nucleon-nucleon weak interaction). The enhancement factor $\sqrt{N}$ occurs as a result of the small level distance between compound nuclear resonances ($D^{-1} \sim N$) and the random-phase averaging procedure ($\sim N^{-1/2}$).

Using the one particle formula (18) for the weak matrix element:

$$v \simeq 2 \cdot 10^{-4}\sqrt{\overline{D}(eV)}, \tag{21}$$

one can see that the maximal possible P-odd effect is estimated to be

$$P_{max} \sim 10^{-4}\sqrt{\overline{D}(eV)}/\Gamma \leq 10\% \tag{22}$$

for the case of medium and heavy nuclei, which have typical values of the parameters $\overline{D} \in (1 - 10^3)eV$, $\Gamma \in (0.05 - 0.2)eV$.

Using one particle PV and TRIV potentials

$$V_P = \frac{G}{8^{1/2}M}\{(\vec{\sigma}\cdot\vec{p}), \rho(\vec{r})\}_+, \tag{23}$$

$$V_{PT} = \frac{iG\lambda}{8^{1/2}M}\{(\vec{\sigma}\cdot\vec{p}), \rho(\vec{r})\}_-, \tag{24}$$

where G is the weak interaction Fermi constant, M is the proton mass, $\rho(\vec{r})$ is the nucleon density, $\vec{p}$ is the momentum of the valence nucleon, one can get a relation between the ratio of matrix elements $\overline{\lambda} = w/v$ and the ratio of nucleon coupling constants $\lambda = g_{\not{P}\not{T}}/g_{\not{P}}$:

$$\overline{\lambda} = \frac{\lambda}{1+2\xi}. \tag{25}$$

Here $\xi \sim (1-7)$ (for detailed discussions see papers[12–15]). with

$$\xi = \frac{<\phi_p|\rho(\vec{\sigma}\vec{p})|\phi_s>}{<\phi_p|(\vec{\sigma}\vec{p})\rho|\phi_s>}. \tag{26}$$

$\phi_{s,p}$ are the $s, p-$resonance wave functions of the compound nucleus. The value of the matrix element in numerator can be estimated [12] using the operator identity $2\vec{p} = iM[H_{sp}, \vec{r}]$ as

$$<\phi_p|\rho(\vec{\sigma}\vec{p})|\phi_s> \simeq \frac{i\overline{\rho}M}{2}D_{sp}<\phi_p|(\vec{\sigma}\vec{p})|\phi_s>. \tag{27}$$

Here H_{sp} is the single particle nuclear Hamiltonian, D_{sp} is the average single particle level spacing, and $\overline{\rho}$ is the average value of the nuclear density. For the denominator of eq.(26) one can show

$$<\phi_p|(\vec{\sigma}\vec{p})\rho|\phi_s> = -<\phi_p|(\vec{\sigma}\vec{r})\frac{1}{r}\frac{\partial\rho}{\partial r}|\phi_s> = \frac{2i\overline{\rho}}{R^2}<\phi_p|(\vec{\sigma}\vec{r})|\phi_s>, \tag{28}$$

where R is the nuclear radius. Then, we obtain

$$\xi = \frac{1}{4}MD_{sp}R^2 = \frac{1}{4}\pi(KR), \tag{29}$$

where

$$D_{sp} = \frac{1}{MR^2}\pi KR, \tag{30}$$

for square-well potential model [11], with K the nucleon momentum in the nucleus. This leads to a value of $\xi \simeq 1$. Taking into account that theoretical predictions for λ vary from 10^{-2} to 10^{-10} for different models of CP violation (see, for example, [16] and references therein), one can estimate a range of possible values of the TRIV observable and relate a particular mechanism of the CP-violation to their values.

ABSENCE OF FINAL STATE INTERACTIONS IN FORWARD SCATTERING

The unique feature of the TRIV neutron optical effects in forward neutron-nucleus elastic scattering (as well as the similar effects related to the TRIV and parity conserving correlation $\vec{\sigma}_n \cdot (\vec{k} \times \vec{I}) \cdot (\vec{k} \cdot \vec{I})$) is *the absence of false TRIV effects due to the final state interactions (FSI)* (see, for example [5] and references therein). The possibility to construct a null test of T invariance in this case is related to the fact that neutron optical effects involve elastic scattering at zero angle. The general theorem about the absence of FSI for TRIV effects in elastic scattering has been proved first by R. M. Ryndin [17] (see, also [5, 18–20]). Since this theorem is very important, we give a brief sketch of the proof for the case of the zero angle elastic scattering following [5, 17].

It is well known that the T-odd angular correlations in scattering and in particle decays are not sufficient to establish TRIV, i.e. they can have non-zero values in any process with strong, electromagnetic, and weak interactions. For example, the parity-conserving analyzing power in the scattering of polarized particles $\vec{\sigma} \cdot (\vec{k}_i \times \vec{k}_f)$ is formally odd under time reversal judged superficially according to the change in signs of the vectors under a T transformation, and is known to be $O(1)$ for many systems. This is because TRI, unlike parity conservation, does not provide a constraint on a single amplitude for any process, but rather relates the amplitudes for two different processes: for example, direct and inverse channels of reactions. We can relate T-odd correlations to TRIV interactions in such processes only in the first order Born approximation to the scattering amplitude: higher order processes can be sources of "final state effects" which introduce (formally) T-odd correlations from T-invariant interactions. Indeed, the unitarity condition for the scattering matrix in terms of the reaction matrix T, which is proportional to the scattering amplitude, can be written as [21]

$$T^\dagger - T = iTT^\dagger \tag{31}$$

The first Born approximation can be used when the right side of the unitarity equation is much smaller than the left side, and results in a hermitian T-matrix

$$< i|T|f >=< i|T^*|f >, \tag{32}$$

which with TRI condition

$$< f|T|i >=< -i|T| - f >^* \tag{33}$$

leads to the constraint on the T-matrix as

$$< f|T|i >=< -f|T| - i >^* . \quad (34)$$

This condition forbids T-odd angular correlations, as is the case with the P-odd correlations when parity is conserved. (Here the minus signs in matrix elements mean the opposite signs for particle spins and momenta in the corresponding states.) In the case of forward scattering relevant for neutron optics, which corresponds to zero angle elastic scattering, the initial and final states coincide ($i = f$). Combined with the TRI condition (33), this condition gives Eq.(34) without the violation of unitarity (32). Therefore, in this case, FSI cannot mimic T-odd correlations originated from TRIV interactions. Therefore, an observation of a non-zero value of TRIV effects in neutron transmission directly indicates TRIV, exactly like in the case of neutron EDM [22].

To measure TRIV effects for neutron propagation with the simple changing of neutron and/or nucleus polarizations is unpractical since it requires unobtainably precise control for many parameters which can contribute to systematic errors (see, for example, [23, 24]. The approach to eliminate this difficulties was suggested in [20] (see also [25–27]), which will be implemented and discussed later in this paper.

TRIV TRANSMISSION THEOREM

We have shown in the previous section that a null test of T invariance can in principle be constructed from transmission differences involving the forward elastic amplitudes in neutron optics. How best to conduct a practical experiment that makes use of the full potential of this null test for T invariance is a separate question. We now start to address this question in the rest of the paper.

Many authors have considered this question and have outlined various experimental strategies in the literature. The first impulse one might have when presented with the triple correlation of vectors of interest in the forward scattering amplitude is that one can simply reverse the sign of whatever vectors are most convenient experimentally and measure the resulting cross section difference. Since it is typically much easier to flip a spin without changing other aspects of the apparatus, the great majority of these papers have analyzed procedures in which either the neutron spin or the target spin are reversed. Unfortunately detailed considerations of these schemes have shown that this approach is very sensitive to the alignment of the relevant vectors which are very difficult to control to the required precision.

In this paper we advocate an experimental approach whose essential reversal involves not only a spin flip but also a rotation of the apparatus. This approach to the realization of the experiment, which we advocate below, has also been suggested before in the literature [25, 26]. To clarify why we believe that this approach can be superior to many of the previous schemes proposed in the literature, we first prove a theorem for polarized neutron motion in a medium in the presence of any external fields (neutron optical, magnetic,) whose interaction with the neutrons can be treated in the classical limit.

Systematic errors in a transmission test of T invariance can arise from one or more of the following sources: imperfect alignment of polarizer, target and analyzer, differences in the polarizer and analyzer, inhomogeneity of the target medium, rotations of the neutron spin due to the holding field of a polarized target, and the interaction of the neutron spin with the target spin from the spin dependence of the strong interaction (sometimes referred to in the literature as nuclear pseudomagnetism) [28, 29]. Masuda [26, 30, 31], and Serebrov [32] have proposed experiments that involve adding additional spin flips to the basic polarizer and polarized target apparatus. The difficulty in these approaches is that each added spin flip increases the number of parameters needed to characterize the apparatus by three: two alignment angles and an analyzing power. Lamoreaux and Golub [23] argue that, "…it will be necessary to develop new methods to make very precise absolute measurements of the neutron-spin direction. It seems hopeless to devise a experiment that would convincingly measure TRIV in the presence of such a wide variety of potential sources of false effects."

To resolve this problem we consider a configuration of the apparatus related to the approach originally proposed by Kabir [20, 33], which is shown in Fig.(1), where the polarizer and analyzer prepare and select spin perpendicular to neutron momentum $\vec{k}$. The target is polarized perpendicular to both $\vec{k}$ and the polarizer direction.

To describe the transmission difference between these two configurations with both the polarizer and analyzer reversed, we can use the equation of motion for the neutron spin as the neutron propagates through a medium and an external magnetic field, $\vec{B}$, given by

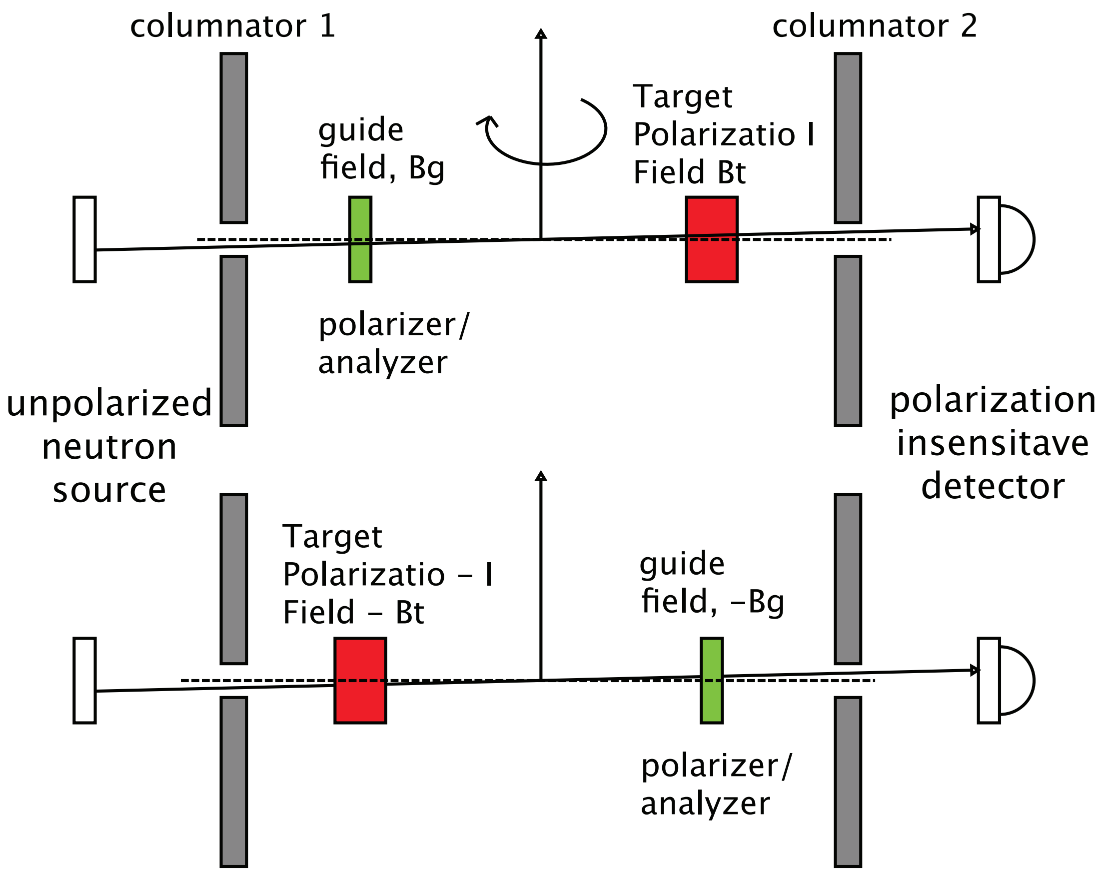

FIG. 1. (Color online) Apparatus to search for time reversal symmetry violation. The collimators, polarizer/analyzer and polarized target are mounted on a turn table that rotates about a vertical axis. In the forward configuration, the neutron first pass through the polarizer/analyzer, then through the target, and are detected. In the reversed configuration the neutrons pass through the polarized target, then through the polarizer/analyzer and are detected. The dashed line is the horizontal axis of symmetry of the apparatus. The solid line is a neutron trajectory. The collimators select the same bundles of trajectories in the two configurations. The signs of the magnetic fields, the target polarization, and the polarizer/analyzer direction are all opposite in the two configurations.

Schrödinger's equation with the effective Hamiltonian (Fermi potential):

$$H = \frac{2\pi\hbar^2}{m_n} N f - \frac{\mu}{2}(\vec{\sigma} \cdot \vec{B}) \tag{35}$$

where m_n is the neutron mass, N is the number of scattering centers per unit volume, f is the forward elastic scattering amplitude, and $\vec{\sigma}$ are the Pauli spin matrices. (For discussion of the conditions under which equation (35) applies, see [23] and references therein.) We can write f as the sum of four terms:

$$f = a_0 + b_0(\vec{\sigma} \cdot \vec{I}) + c_0(\vec{\sigma} \cdot \vec{k}) + d_0(\vec{\sigma} \cdot [\vec{k} \times \vec{I}]), \tag{36}$$

where I is the polarization of the target medium, and quantities other than the neutron spin $\vec{\sigma}$ are treated as classical fields. Neutron spin-optics tests of TRIV have the goal of measuring d, which is the only term that originates from a TRIV interaction. Terms a and b, give the strengths of the spin-independent and strong spin-spin (pseudo magnetic) interactions, while terms c and d give the degree of PV and TRIV arising from symmetry mixing in the neutron resonances in the target medium.

We now show that if $\vec{B}$ and $\vec{I}$ are reversed, the forward and reversed transmissions for the apparatus configuration presented in Fig. 1 are equal if $d = 0$. Note carefully that, in this proposed approach, the magnetic field $\vec{B}$ is reversed, but the orientation of the target polarization $\vec{I}$ with respect to $\vec{B}$ is unchanged. Therefore, one can re-write Hamiltonian (35) as

$$H = a + b(\vec{\sigma} \cdot \vec{I}) + c(\vec{\sigma} \cdot \vec{k}) + d(\vec{\sigma} \cdot [\vec{k} \times \vec{I}]), \tag{37}$$

where $a = \frac{2\pi\hbar^2}{m_n} N a_0$, $b = \frac{2\pi\hbar^2}{m_n} N b_0 - (\mu B)/2$, $c = \frac{2\pi\hbar^2}{m_n} N c_0$, and $d = \frac{2\pi\hbar^2}{m_n} N d_0$. The neutron beam phase space acceptance of the apparatus is defined by a pair of collimators mounted on a rigid rotatable platform with the polarizers (analyzer) and target as shown in Fig.1. Rotating the apparatus by an angle π about an axis perpendicular to the symmetry axis of the collimators reverses the sign of $\vec{k}$ for neutrons. We assume that the product of the neutron source strength and neutron detector efficiency is symmetric with respect to the plane of the symmetry axis and the rotation axis. Then, the time evolution operator for the forward neutron transmission, U_F, gives the relationship between the initial and final spin wave functions for a neutrons that propagate from the source through the apparatus, and ends on the detector.

Let us consider the case when we have only TRI interactions. Then we divide the apparatus into m slabs and write the time evolution operator U_F as a time ordered product of the evolution operators for each of the slabs:

$$U_F = \prod_{j=1}^{m} \exp\left(-i\frac{\Delta t_j}{\hbar} H_j^F\right) = \alpha + (\vec{\beta} \cdot \vec{\sigma}). \tag{38}$$

Here H_j^F is the Hamiltonian from equation (37) evaluated at slab j, and α and $\vec{\beta}$ contain only TRI terms, since we temporarily assume that the TRIV parameter $d = 0$. In the expression for the reverse evolution operator, U_R, the time ordering of the product and the signs of the spin-dependent terms in H_j^R are reversed from those in H_j^F. Then, the reverse evolution operator is

$$U_R = \prod_{j=m}^{1} \exp{(-i\frac{\Delta t_j}{\hbar} H_j^R)} = \alpha - (\vec{\beta} \cdot \vec{\sigma}). \tag{39}$$

The fact that the signs of the spin-dependent terms in the reverse evolution operator are changed eliminates potential systematic effects which may mimic TRIV effects in scattering experiments. This analysis agrees with Kabir's result about the possibility to unambiguously [20] measure TRIV effects in neutron scattering. Since the relation asserted in equation (39) between forward and reverse evolution operators is very important for further consideration and not obvious, we will prove it here.

First, let us consider two-slab medium. The forward and reverse evolution operators are

$$\begin{aligned} U_F &= U_F^{(1)} U_F^{(2)} = \exp{(-i\frac{\Delta t_1}{\hbar} H_1^F)} \exp{(-i\frac{\Delta t_2}{\hbar} H_2^F)}, \\ U_R &= U_R^{(2)} U_R^{(1)} = \exp{(-i\frac{\Delta t_2}{\hbar} H_2^R)} \exp{(-i\frac{\Delta t_1}{\hbar} H_1^R)}. \end{aligned} \tag{40}$$

For the case of infinitesimally small widths of the slabs, each exponential operator in the above equations can be written as

$$\begin{aligned} U_F^{(j)} &= (1 - i\frac{\Delta t_j}{\hbar} H_j^F) = F^{(j)} + (\vec{A}^{(j)} \cdot \vec{\sigma}), \\ U_R^{(j)} &= (1 - i\frac{\Delta t_j}{\hbar} H_j^R) = F^{(j)} - (\vec{A}^{(j)} \cdot \vec{\sigma}), \end{aligned} \tag{41}$$

correspondingly, where

$$\begin{aligned} F^{(j)} &= 1 - i\frac{\Delta t_j}{\hbar} a^{(j)}, \\ \vec{A}^{(j)} &= \frac{-i\Delta t_j}{\hbar}(b^{(j)}\vec{I} + c^{(j)}\vec{k}). \end{aligned} \tag{42}$$

These one slab evolution operators have exactly the same structure as the operators in eqs. (38) and (39), provided $F^{(j)} \to \alpha^{(j)}$ and $\vec{A}^{(j)} \to \vec{\beta^{(j)}}$. Substitution into eq.(40) lead to exactly the same form as for eqs. (38) and (39), again, with

$$\begin{aligned} \alpha &= \alpha^{(1)}\alpha^{(2)} + (\vec{\beta}^{(1)} \cdot \vec{\beta}^{(2)}), \\ \vec{\beta} &= \alpha^{(1)}\vec{\beta}^{(2)} + \alpha^{(2)}\vec{\beta}^{(1)} - [\vec{\beta}^{(1)} \times \vec{\beta}^{(2)}]. \end{aligned} \tag{43}$$

Then, applying mathematical induction, one can prove the proposition in general (multi-slab) case as is given in eqs. (38) and (39). Applying this result for the calculations of the forward and reverse transmissions, T_F and T_R , for our experimental setup we obtain the relation

$$T_F = \frac{1}{2} Tr(U_F^\dagger U_F) = \alpha^* \alpha + (\vec{\beta}^* \vec{\beta}) = \frac{1}{2} Tr(U_R^\dagger U_R) = T_R, \tag{44}$$

which we call TRIV transmission theorem. This theorem shows that if $d = 0$ and whole apparatus is rotated with $\vec{B}$ and $\vec{I}$ being reversed, then the transmissions of (un-polarized) neutrons through the apparatus in opposite directions are equal. The proof of TRIV theorem makes no assumption concerning the geometrical symmetry of the classical fields and materials of the apparatus. Therefore, any deviation from the equality of the forward and reversed transmissions in eq.(44) is a clear manifestation of the existence of TRIV interactions (non-zero d coefficient in eq.(37)). It should be noted that for non-zero d coefficient the difference between T_F and T_R transmissions arises from both spin dependent and spin independent parts of the evolution operators, which is in agreement with Kabir's [20, 33] conclusion about the existence of a number of possible unambiguous tests.

EVALUATION OF A MODEL EXPERIMENT

No TRIV experiment in neutron optics has been done to date: polarized targets of materials that have compound nuclear resonances that exhibit large PV asymmetries are not easy to construct. It has proved difficult to devise an experiment that would eliminate false effects that arise from combinations of instrumental imperfections and TRI interactions of the neutron spin with materials and external fields. We believe that we have made progress on the second issue with our TRIV transmission theorem. Considerable progress has also been made on the first problem: groups at the KEK national laboratory in Japan [34, 35], at Kyoto University [36], and at PSI in Switzerland [37] have achieved substantial polarizations of ^{139}La nuclei in Lanthanum Aluminate crystals as large as 10 cc. Thus the 0.734 eV p-wave resonance in ^{139}La, which has a parity-odd longitudinal transmission asymmetry of 9.5% [38], is a good candidate for TRIV studies.

To polarize the epithermal neutron beam for the proposed experiment based on the TRIV transmission theorem, we can use cells of polarized ^3He as neutron polarizers and analyzers. The direction of the ^3He polarization in these polarizers, based on spin-exchange optical pumping, is parallel to the external magnetic field and will reverse direction when the field direction is reversed adiabatically. Note that polarizers and analyzers based on ferromagnetic materials can be difficult to use in this experiment because hysteresis effects prevent their precise reversal. Also, since the earth's magnetic field cannot be reversed, it must be compensated or shielded in this experimental approach. It is also essential that the values of the classical fields be stable in time. Magnetic field strengths and the polarizations of ^3He and the target medium can be accurately monitored using nuclear-magnetic-resonance techniques.

For the target we use ^{139}La nuclei in Lanthanum Aluminate crystals which has a very large PV effect in the vicinity of 0.734 eV resonance. Using the experimentally achieved value of ^{139}La polarization of 47.5% combined with the existing knowledge of the spin dependence of the polarized neutron scattering amplitudes on polarized ^{139}La nuclei in the $J + 1/2$ and $J - 1/2$ spin channels, we can estimate [28, 39] the size of the pseudo-magnetic field inside the crystal as a function of neutron energy (see Fig. 2), the pseudo-magnetic field is opposite the applied field. This gives an advantage for using Lanthanum Aluminate crystals, since values of TRIV effects in neutron optics, in general, are inverse proportional to the sum of magnetic and pseudo magnetic fields [39, 40].

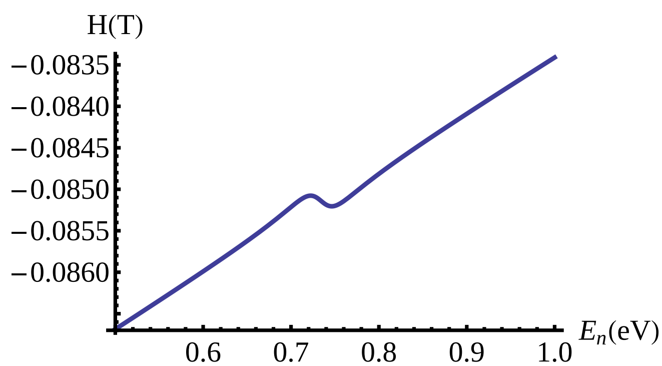

FIG. 2. (Color online) Pseudo-magnetic field in Lanthanum Aluminate crystals.

As an example of the statistical accuracy that can be achieved with present spallation neutron sources, we make a rough estimate of the statistical uncertainty in the T-odd cross section that could be achieved in 10^7 seconds of data collection on the water moderator of Flight Path 16A at the Spallation Neutron Source at Oak Ridge National Laboratory. At the present time this beamline has not been instrumented. We assume a proton current of 1.4 mA at 1 GeV proton energy. We carry out the estimate for the 0.734 eV resonance in ^{139}La. We assume that the target consists of one neutron interaction length of dynamically polarized Lanthanum Aluminate and that the neutron beam is polarized by a one-interaction-length 70% polarized ^3He spin filter.

We were unable to find a calculation or measurement of the neutron flux for FP16A. We estimated the neutron flux using the measurement of the flux from the water moderator of Flight Path 2 at the Los Alamos Neutron Scattering Center at the Los Alamos National Laboratory. Roberson et al. [41] found that the moderator brightness was well described by the expression

$$\frac{d^3N}{dAdtd\Omega} = k\frac{\Delta E}{E}\left(\frac{E}{1eV}\right)^{\gamma}\left(\frac{i}{e}\right)(neutrons\ cm^{-2}sec^{-1}sr^{-1}), \tag{45}$$

with $k = 5.8 \cdot 10^{-3}$ and $\gamma = 0.1$. E is the neutron energy, i is the proton current, e is the charge quantum, A is the area of the moderator that is viewed, ΔE is the range of neutron energies accepted, and Ω is the solid angle acceptance of the apparatus. We assume that the neutron production rate is proportional to the proton energy and increase k by 1000/800, the ratio of proton energies. We assume that SNS will operate at 1.4 MW and $i = 1.4\ mA$.

We assume that $A = 100cm^2$ and that the acceptance of the apparatus is defined by a 10 cm diameter polarized target located 15 meters from the moderator: $\Omega = 3.5 \cdot 10^{-5}\ sr$. We set $\Delta E = .045\ eV$ to cover the total width of the resonance. The neutron flux within the ^{139}La p-wave resonance width is $dN/dt = 7.8 \cdot 10^7\ neutrons/sec$.

In order to determine the uncertainty in the TRIV asymmetry we must make some assumptions concerning running time, source, polarizer, polarized target, detector, and cross sections. We assume a running time of $10^7\ sec$. We use the peak value of the resonance cross section is 2.9 barns, the potential scattering cross section is 3.1 barns, and the capture cross section at the resonance energy is 1.6 barns. We use the cross sections of aluminum and oxygen are 3.8 barns and 1.4 barns [42]. We calculate that the neutron polarization is 46% and the transmission of the polarizer is 46%. We assume a one-interaction-length $LaAlO_3$ target. We further reduce the transmission by a factor of 2 to account for various windows. The transmission of the apparatus for 0.7 eV neutrons is then estimated to be 11%. The transmitted beam intensity in ΔE is $Flux = .86 \cdot 10^7\ neutrons/sec$. The fractional uncertainty in TRIV cross section is given by

$$\frac{\delta\sigma}{\sigma} = \frac{1}{\sqrt{Flux \cdot Time}}\frac{\sum \sigma_k}{\sigma_p}. \tag{46}$$

(The sum runs over all the cross sections given above.) If we adopt the fractional parity-violating asymmetry for the resonance to be 9.5% [38], we obtain an uncertainty in λ , the ratio of the TRIV to PV asymmetries of $6.0 \cdot 10^{-6}$.

DISCOVERY POTENTIAL

As noted above in the Introduction, the question of how sensitive any T-odd observable is to a particular source of T violation in the nucleon system is theoretically nontrivial, due in part to our lack of quantitative understanding of many of the relevant aspects of the strong interaction. As an example to set the scale for the potential sensitivity of a TRIV search in neutron transmission, we start first with a case in which a quantitative analysis is possible and has already been performed: the neutron-deuteron system. Using the results of the recent calculations of PV and TRIV effects in neutron deuteron scattering [43, 44], one can calculate the parameter $\overline{\lambda}$ for this reaction and compare it to the case of the complex nuclei. Let us consider the ratio of the TRIV difference of total cross sections in neutron deuteron scattering given in [44]

$$P_{\not{T}\not{P}} = \frac{\Delta\sigma_{\not{P}\not{T}}}{2\sigma_{tot}} = \frac{(-0.185\ \mathrm{b})}{2\sigma_{tot}}[\bar{g}_{\pi}^{(0)} + 0.26\bar{g}_{\pi}^{(1)} - 0.0012\bar{g}_{\eta}^{(0)} + 0.0034\bar{g}_{\eta}^{(1)} - 0.0071\bar{g}_{\rho}^{(0)} + 0.0035\bar{g}_{\rho}^{(1)} + 0.0019\bar{g}_{\omega}^{(0)} - 0.00063\bar{g}_{\omega}^{(1)}] \tag{47}$$

to the corresponding PV difference [43]

$$P_{\not{P}} = \frac{\Delta\sigma_{\not{P}}}{2\sigma_{tot}} = \frac{(0.395\ \mathrm{b})}{2\sigma_{tot}}[h_{\pi}^{1} + h_{\rho}^{0}(0.021) + h_{\rho}^{1}(0.0027) + h_{\omega}^{0}(0.022) + h_{\omega}^{1}(-0.043) + h_{\rho}^{\prime 1}(-0.012)]. \tag{48}$$

Here, we use the one meson exchange model, known as the DDH model for PV nucleon-nucleon interactions, to calculate both effects; in the above expressions, $\bar{g}$ and h are meson-nucleon TRIV and PV coupling constants, correspondingly (see for details [43, 44]). The dimensionless numerical constants multiplying these couplings come from the detailed evaluation of n-D scattering given the measured properties of the strong NN interactions. These factors naturally become progressively more difficult to calculate for heavier nuclei. From these expressions, one can see that in this case contributions from pion exchange are dominant for both TRIV and PV parameters. Taking into account only the dominant pion contributions, one can estimate $\overline{\lambda}$ as

$$\overline{\lambda} = \frac{\Delta\sigma_{\not{T}\not{P}}}{\Delta\sigma_{\not{P}}} \simeq (-0.47)\left(\frac{\bar{g}_{\pi}^{(0)}}{h_{\pi}^{1}} + (0.26)\frac{\bar{g}_{\pi}^{(1)}}{h_{\pi}^{1}}\right). \tag{49}$$

This result is in reasonable agreement with an estimate for complex nuclei [12].

We can attempt to relate the parameter $\overline{\lambda}$ to the existing experimental constrains obtained from EDM measurements, with the understanding that even such a relative comparison is highly model dependent. The CP-odd coupling constant $\bar{g}_{\pi}^{(0)}$ can be related to the value of the neutron EDM d_n generated via a π-loop in the chiral limit [45]. Using the experimental limit [46] on d_n, one can estimate $\bar{g}_{\pi}^{(0)} < 2.5 \times 10^{-10}$. The constant $\bar{g}_{\pi}^{(1)}$ can be bounded using the constraint [47] on the ^{199}Hg atomic EDM as $\bar{g}_{\pi}^{(1)} < 0.5 \times 10^{-11}$ [48].

The comparison of the $\overline{\lambda}$ parameter with the constrains on the coupling constants from the EDM experiments gives us the opportunity to estimate the possible sensitivity of TRIV effects to the value of TRIV nucleon coupling constant, which we call a "*discovery potential*" for neutron scattering experiments [49, 50], since it shows a possible factor for improving the current limits of the EDM experiments. Taking the DDH "best value" of $h_{\pi}^{1} \sim 4.6 \cdot 10^{-7}$, the nuclear enhancement factors estimated above, and assuming that the parameter $\overline{\lambda}$ could be measured with an accuracy of 10^{-5} on complex nuclei, one can see from Eq.(49) that the existing limits on the TRIV coupling constants could be improved in neutron optics transmission measurements using existing neutron sources by two orders of magnitude. To obtain Eq.(49), we assumed that the π exchange contribution dominates the PV effects. However, there is an indication [51–53] that the PV coupling constant h_{π}^{1} could well be much smaller than the "best value" of the DDH. Should this hint be confirmed by the $\vec{n} + p \to d + \gamma$ experiment, the estimate for the sensitivity of $\overline{\lambda}$ to the TRIV coupling constant would be increased, as can be seen from Eqs.(47-49), since in most theoretical estimates the parameter $\overline{\lambda}$ is a ratio of TRIV to PV pion coupling constants (λ). (Note that there is absolutely no fundamental reason to our knowledge why the an effective TRIV pion coupling should be suppressed if the PV pion coupling is suppressed: Barton's theorem, for example, suppresses neutral pion exchange in PV meson-nucleon interactions but not in TRIV interactions.) This increased sensitivity combined with a possible choice of the target with large spin factor (13) might increase the relative values of TRIV effects by two orders of magnitude, and as a consequence, the discovery potential of the TRIV experiments could be about 10^{4}.

The TRIV effects in neutron transmission through a nuclei target are unique TRIV observables being free from FSI, and constitute null tests for time reversal invariance as do EDM experiments. These TRIV effects can be enhanced on certain p-wave epithermal neutron resonances by about a factor of 10^6 due to the nuclear enhancement well-understood mechanisms discussed above. In addition to this resonant enhancement in complex nuclei, the sensitivity to TRIV interactions in these effects might be structurally enhanced by about 10^2 if PV π-nucleon coupling constant is less than the "best value" DDH estimate. Therefore, these types of experiments with high intensity neutron sources have a discovery potential of about $10^2 - 10^4$ for the improvement of the current limits on the TRIV interaction obtained from the EDM experiments.

Another important feature of these experiments is the complementarity to other searchers for TRIV. To illustrate this we use results of the calculations of neutron and proton EDMs [54] and EDMs of few body nuclei [55] presented in terms of TRIV meson-nucleon coupling constants. Then, assuming that TRIV pion, rho, eta, and omega meson coupling constants have about the same order of magnitude, we can write the main contributions to these EDMs in $e \cdot fm$ units as

$$d_n \simeq 0.14(\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)}), \tag{50}$$

$$d_p \simeq 0.14\bar{g}_\pi^{(2)}, \tag{51}$$

$$d_D \simeq 0.22\bar{g}_\pi^{(1)}, \tag{52}$$

$$d_{^3He} \simeq 0.2\bar{g}_\pi^{(0)} + 0.14\bar{g}_\pi^{(1)}, \tag{53}$$

$$d_{^3H} \simeq 0.22\bar{g}_\pi^{(0)} - 0.14\bar{g}_\pi^{(1)}, \tag{54}$$

where $\bar{g}_\pi^{(T)}$ is pion-nucleon TRIV coupling constant with isospin T. The comparison these results with eq.(47) shows that all these observable have different sensitivities to the models of TRIV. This becomes even more pronounced if we relax the assumption about values of TRIV coupling constants. These sensitivities of TRIV neutron scattering effect and neutron and light nuclei to TRIV π-mesons coupling constants are shown figures 3 and 4. Therefore, one can see that even for the simplest case with the dominance of TRIV pion-nucleon coupling constants, it is necessary to measure at least three independent TRIV effects to constrain the source of CP-violation.

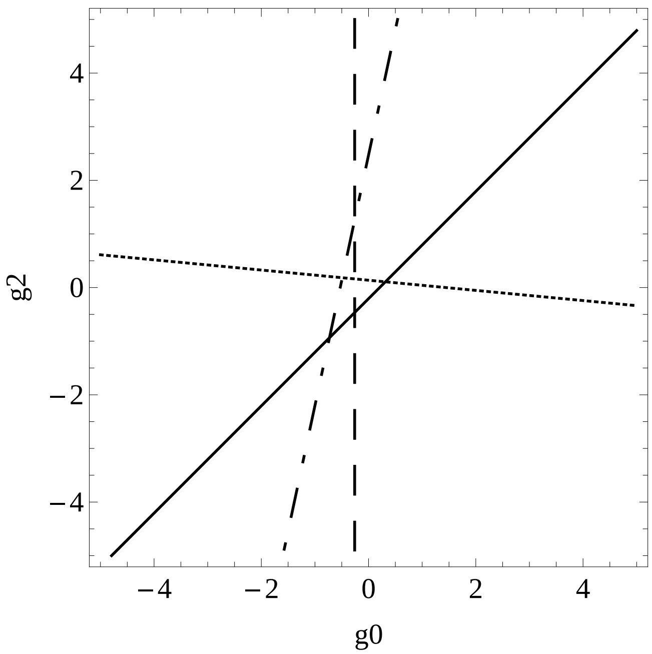

FIG. 3. The dependence of neutron EDM (solid), ^3He EDM (dotted-dashed), ^3H EDM (dotted) and parameter λ on TRIV π-mesons iso-scalar and iso-tensor coupling constants.

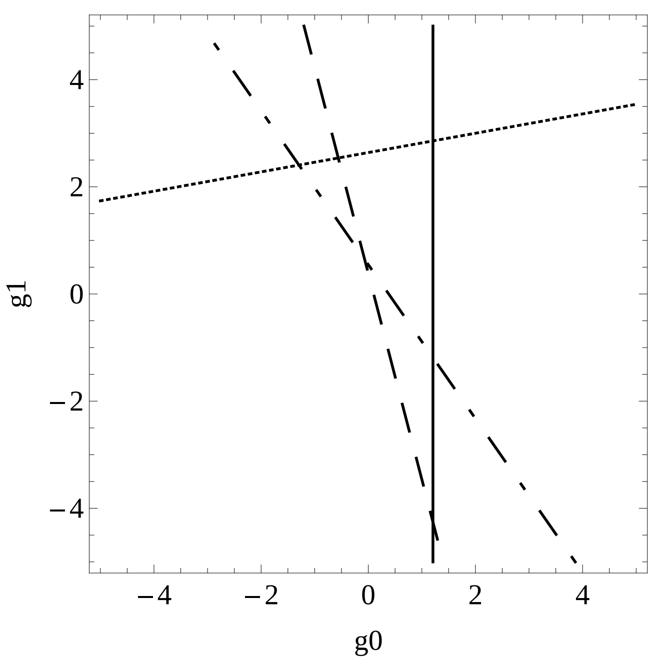

FIG. 4. The dependence of neutron EDM (solid), ^3He EDM (doted-dashed), ^3H EDM (doted) and parameter λ on TRIV π-mesons iso-scalar and iso-vector coupling constants.

CONCLUSIONS

We presented the summary of theoretical description of the TRIV effects in neutron transmission through a nuclei target and demonstrated that these TRIV observables are free from FSI, and, as a consequence, are of the same quality as the EDM experiments. The neutron transmission effects can be enhanced by about 10^6 due to the nuclear enhancement factor. In addition to this enhancement, the sensitivity to TRIV interactions in these effects compared to observed PV effects might be enhanced by about 10^2 if PV π-nucleon coupling constant is less than the "best value" DDH estimate, and by choosing a target with large partial neutron width related to TRIV observables.

The main result of this paper is the proof of the TRIV transmission theorem showing that the transmission of neutrons through an apparatus with arbitrary spin-dependent interactions that arise from time-reversal-invariant interactions is unchanged when the signs of all classical fields that interact with the neutron spin are reversed. We have used this result to propose a specific experimental procedure to test time-reversal invariance which is in principle free of false asymmetries arising from combinations of time-reversal-invariant interactions and asymmetries in the apparatus. These types of experiments with high intensity neutron sources have a discovery potential of about $10^2 - 10^4$ for the improvement of the current limits on the TRIV interaction obtained from the EDM experiments.

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* bowmanjd@ornl.gov

† gudkov@sc.edu

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