Quarkonium above deconfinement as an open quantum system

C. Young and K. Dusling

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Quarkonium above deconfinement as an open quantum system

C. Young*

School of Physics & Astronomy, University of Minnesota, Minneapolis, MN 55455, U.S.A.

K. Dusling†

Department of Physics, North Carolina State University, Raleigh, NC 27695-8202, USA

Abstract

Quarkonium at temperatures above deconfinement is an open quantum system, whose dynamics is determined not just by a potential energy and mass, but also by a drag coefficient which characterizes its interaction with the medium. We develop path-integral Monte Carlo for examining quarkonium at finite-temperature; first, the path integral approach for open quantum systems is developed analytically for imaginary-time, and then the imaginary-time Green function is calculated with a realistic potential, mass, and drag term for quarkonium near deconfinement. We demonstrate that dissipation could affect the Euclidean heavy-heavy correlation functions calculated in lattice simulations at temperatures just above deconfinement.

*Electronic address: young@physics.umn.edu

†Electronic address: kbduslin@ncsu.edu

I. INTRODUCTION

The separation of scales between the masses of the heavy quarks and the QCD energy scale Λ_{QCD} allows heavy-heavy bound states to be treated with first quantization techniques [1–3]. At zero temperature, lattice calculations of static color singlets determine a potential energy which can successfully describe quarkonium spectroscopy. The potential has a non-trivial temperature dependence which which suggests that quarkonium will dissociate above T_c. This effect would result in a modification of the yields of quarkonium in heavy-ion collisions and could be used to measure the temperatures achieved in heavy-ion collisions [4]. Since this seminal work more than twenty years ago, analyses of heavy-ion experiments have shown that some suppression of J/ψ yields at RHIC is anomalous [5]; however, the pattern of suppression does not match what would be expected from these first considerations of changes in the spectrum. It is clear that the dynamics of quarkonium at high temperatures must also be considered carefully.

One step in this direction was made by Shuryak and one of us [6], where charmonium was modeled as an interacting heavy-quark pair undergoing Brownian motion, with the heavy quark spatial diffusion coefficient D_H taken to be quite small (or, equivalently, the heavy quark drag coefficient η taken to be large), as expected from phenomenology of single heavy quarks [7, 8] and by gauge-gravity duality [9–11]. This demonstrated that the survival of J/ψ states above T_c cannot be determined just by examining whether or not the temperature-dependent potentials allow bound states; instead, the dynamics of charm and charmonium and ultimately, their yields are determined by multiple interactions with the medium. The time scales of the heavy-ion collisions cannot be neglected.

Also, lattice calculations have examined quarkonium at finite-temperature more directly [12] by determining Euclidean correlators for local operators related to quarkonium spectroscopy, namely, $G(\tau,T) = \int d^3x \, \langle J_H(\mathbf{x},\tau) J_H(\mathbf{0},0)\rangle$, where $J_H(\mathbf{x},\tau) = \bar{\psi}(\mathbf{x},\tau)\Gamma\psi(\mathbf{x},\tau)$ is a local composite operator related to a given meson. Mocsy and Petreczky compared these correlation functions, at various temperatures and in different channels, with the results of two different potential models [13]. In this work, there is a low-frequency contribution to the spectral densities used to determine the Euclidean correlation functions, which is caused by the diffusion of heavy quarks [14]; however, for $\omega \gg T^2/M$, the diffusion of heavy quarks is assumed not to have any effect on the dynamics of quarkonium. Some effort has also been

made in understanding heavy quark diffusion with lattice simulations [15].

Only a few others have considered medium effects on heavy quarks and quarkonium at finite-temperature. Beraudo *et al.* [16] used the HTL approximation for the heavy quark's interaction with light degrees of freedom that are subsequently integrated out of the path integral. Such considerations should also be taken for quarkonium; also, the medium should be considered in greater generality, away from any assumptions of weak coupling, and using the description most appropriate for heavy particles interacting with a heat bath: open quantum systems. This is the purpose of this paper.

In this paper, we will outline how the imaginary-time propagator with periodicity $\beta = 1/T$ can be determined for quarkonium interacting with a heat bath. In Section II, we review the reduced density matrix and apply the model of Caldeira and Leggett for quantum Brownian motion in imaginary time, determining an analytic expression for the reduced density matrix. In Section III, we find an expression for the periodic imaginary-time Green function commensurate to the Euclidean quarkonium correlation functions calculated in lattice QCD. Using path-integral Monte Carlo, the Green function is computed and the effect of diffusion on Euclidean correlation functions is demonstrated.

II. THE REDUCED DENSITY MATRIX OF A DISSIPATIVE SYSTEM

This section follows the approach of Caldeira and Leggett and uses path integrals to describe quantum Brownian motion; specifically examining quantities that can be calculated in imaginary-time. First, an expression for the reduced density matrix is determined for a heavy particle undergoing interactions with a heat bath. As an introductory example, we find an analytic expression for the reduced density matrix of an "otherwise free" particle.

For an excellent review of the functional integral approach to quantum Brownian motion, in both imaginary and real time, see [17]. These results are discussed in generality, and shown to arise from the Schwinger-Keldysh contour integral for a heavy quark's real-time partition function, by Son and Teaney [18]. These systems can be studied with approaches besides the path-integral formulation, and the study of quantum Brownian motion, in terms of a partial differential equation for the density matrix, has been studied by Hu, Paz, and Zhang [19, 20].

A. The reduced density matrix

Consider a system consisting of a heavy particle of mass M minimally coupled to a harmonic oscillator of mass m.

$$\begin{aligned} L &= L_S + L_I; \\ L_S &= \frac{1}{2}M\dot{x}^2 - V(x), \\ L_I &= \frac{1}{2}m\dot{r}^2 - \frac{1}{2}m\omega^2 r^2 - Cxr. \end{aligned} \tag{1}$$

In the above expression L_S is Lagrangian for the heavy particle (which we call the system) interacting with potential $V(x)$. The Lagrangian L_I includes the kinetic energy of the light particle (which will make up the heat bath) as well as its interaction with the heavy particle. Without any loss of generality the above Lagrangian describes a pair of mutually interacting heavy particles as long as M is treated as the reduced mass of the system and $V(x)$ is the potential of the two heavy particles in terms of their relative coordinate x. The above Lagrangian analytically continued to imaginary time $\tau = it$ is

$$\begin{aligned} S_S^E[x] &= \int_0^\beta \left[\frac{1}{2}M\dot{x}^2 + V(x)\right] d\tau, \\ S_I^E[x,r] &= \int_0^\beta \left[\frac{1}{2}m\dot{r}^2 + \frac{1}{2}m\omega^2 r^2 + Cxr\right] d\tau\,. \end{aligned} \tag{2}$$

We can simplify this Lagrangian by a change of variables. We subtract the particular solution to the classical equations of motion as determined by Eq. 2:

$$\begin{aligned} r(\tau) &\equiv r'(\tau) + \frac{C}{m\omega}\int_0^\tau d\tau' x(\tau') \sinh\left[\omega(\tau - \tau')\right] \\ &\equiv r'(\tau) + A[x,\tau] \end{aligned} \tag{3}$$

In terms of the shifted coordinate r' the Euclidean action becomes that of a simple harmonic oscillator:

$$\begin{aligned} S_I^E[x,r] &= \int_0^\beta \left[\frac{1}{2}m\dot{r}'^2 + \frac{1}{2}m\omega^2 r'^2 + \frac{1}{2}Cx(\tau)A[x,\tau]\right] d\tau \\ &\quad + m\dot{A}[x,\beta]\left(r'(\beta) + \frac{1}{2}A[x,\beta]\right) \\ &\equiv S_I'^E[x,r']. \end{aligned} \tag{4}$$

As always, the propagator of a system for imaginary time $\beta = 1/T$ gives matrix elements of the thermal density operator. In our example, the density matrix has 4 indices; two for

the heavy particle denoted $x_{i,f}$ and two for the light particle denoted by $r_{i,f}$ where $i(f)$ is shorthand for the initial (final) position. In the example given above the density matrix is given as

$$\rho(x_i, r_i; x_f, r_f; \beta) == \int_{x(0)=x_i}^{x(\beta)=x_f} \mathcal{D}x \int_{r'(0)=r_i}^{r'(\beta)=r_f-\frac{1}{2}A[x,\beta]} \mathcal{D}r' \exp\left(-S_S^E[x] - S_I'^E[x, r']\right). \quad (5)$$

If we were never interested in measurements of the degree of freedom r, we could take the trace over the indices corresponding to this degree of freedom, and work with a density operator with only two indices. With this in mind, we define the *reduced density matrix* as

$$\rho_{red}(x_i, x_f, \beta) \equiv \int dr \rho(x_i, r; x_f, r; \beta). \quad (6)$$

This is the only operator practical for calculating thermal averages. For the system defined by Eq. 1, we can write a path-integral description for the reduced density matrix,

$$\begin{aligned} \rho_{red}(x_i, x_f, \beta) &= \int dr \int \mathcal{D}x\, \mathcal{D}r' \, \exp\left(-S_S^E[x] - S_I^E[x, r']\right) \\ &= \int \mathcal{D}x \exp\left(-S_x^E[x]\right) \int dr \int \mathcal{D}r' \exp\left(-S_I'^E[x, r']\right), \end{aligned} \quad (7)$$

where we integrate over paths with endpoints $x(0) = x_i$, $x(\beta) = x_f$, $r'(0) = r$, and $r'(\beta) = r - A[x, \beta]$. Thanks to the change in variables enacted in Eq. 3, the integral over the paths $r'(\tau)$ is Gaussian and can be done easily with the result

$$\begin{aligned} \rho_{red}(x_i, x_f, \beta) &= \int_{x(0)=x_i}^{x(\beta)=x_f} \mathcal{D}x \\ \times\ \exp&\left(-S_S^E[x] + \sum_k \frac{C_k^2}{2m\omega_k \sinh(\frac{\omega_k\beta}{2})} \int_0^\beta d\tau \int_0^\tau ds\, x(\tau)x(s) \cosh\left[\omega_k\left(\tau - s - \beta/2\right)\right]\right) \end{aligned} \quad (8)$$

The summation over k has been introduced in order to generalize the result to a system where a heavy particle interacts with a bath of independent simple harmonic oscillators each having coupling C_k and frequency ω_k. Eq. 8 is the path integral form for the reduced density matrix.

In Eq. 8, the degrees of freedom of the heat bath are arbitrary; the values of C_k and ω_k can take any set of values as long as the integral remains convergent. For a larger number of light particles, the heat bath can be represented by a density of states $\rho(\omega)$ with $\sum_k \to \int d\omega$.

B. Making a dissipative system

Any finite quantum-mechanical system is reversible and therefore inappropriate for describing Brownian motion. One might find it intuitive that if the bath of harmonic oscillators were taken to an infinite limit, it would be "large enough" so that energy from the heavy particle could dissipate into the system and never return. This intuition was proven to be true by the authors of [22], who considered the real-time evolution of the density matrix for our system and showed that when the bath of harmonic oscillators is characterized by the continuous density of states

$$C^2(\omega)\rho_D(\omega) = \begin{cases} \frac{2m\eta\omega^2}{\pi} & \text{if } \omega < \Omega \\ 0 & \text{if } \omega > \Omega \end{cases} \tag{9}$$

the force autocorrelator for the heavy particle is proportional to $\delta(t-t^{'})$ at high temperatures. In this "white noise" limit, the density matrix describes an ensemble of particles interacting according to the Langevin equation, which has been used to describe Brownian motion for over a century. It is a stochastic differential equation, is irreversible, and evolves any ensemble towards the thermal phase space distribution. Finally, the authors of [22] showed that η in Eq. 9 corresponds exactly to the drag coefficient η in the Langevin equation; knowing a transport coefficient, for example the heavy quark drag coefficient, allows this path integral to be matched with a system whose classical behavior is known.

Using the above density of states the reduced density matrix becomes

$$\begin{aligned} \rho_{red}(x_i, x_f, \beta) &= \int_{x(0)=x_i}^{x(\beta)=x_f} \mathcal{D}x \\ \times\ \exp&\left(-S_S^E[x] + \frac{\eta}{\pi}\int_0^\Omega d\omega \int_0^\beta d\tau \int_0^\tau ds\ x(\tau)x(s)\frac{\omega\cosh\left[\omega(\tau - s - \beta/2)\right]}{\sinh(\frac{\omega\beta}{2})} \right). \end{aligned} \tag{10}$$

The divergences of this action can be isolated by integrating by parts twice

$$\begin{aligned}
&\int_0^\Omega d\omega \int_0^\beta d\tau \int_0^\tau ds\; x(\tau)x(s)\frac{\omega\cosh\left[\omega(\tau-s-\beta/2)\right]}{\sinh(\frac{\omega\beta}{2})} \\
&= \Omega\int_0^\beta d\tau\;(x(\tau))^2 - \frac{1}{2}(x_i-x_f)^2\ln(M\Omega/\eta)\frac{\cosh(\Omega\beta/2)-1}{\sinh(\Omega\beta/2)} \\
&\quad -\frac{1}{2}(x_i-x_f)^2\left[\gamma_E+\ln\left(\frac{\eta\beta}{\pi M}\right)\right] \\
&\quad +(x_i-x_f)\int_0^\beta d\tau\;\dot{x}(\tau)\;\mathrm{lnsin}\left(\frac{\pi\tau}{\beta}\right) \\
&\quad +\int_0^\beta d\tau\int_0^\tau ds\;\dot{x}(\tau)\dot{x}(s)\;\mathrm{lnsin}\left(\frac{\pi(\tau-s)}{\beta}\right), \qquad (11)
\end{aligned}$$

where $\mathrm{lnsin}(x)\equiv\ln\left[\sin\left(x\right)\right]$ and γ_E is the Euler-Mascheroni constant. The first two terms on the right-hand side correspond to a renormalization of the potential for the heavy particle, always necessary when considering the interaction of a particle with infinitely many additional degrees of freedom. They are temperature-independent in the limit of large Ω, and may be renormalized into temperature-independent counter terms in the free Lagrangian. The final three terms are finite and can be readily evaluated in the $\Omega\to\infty$ limit,

$$\begin{aligned}
\rho_{red}(x_i,x_f,\beta) = \int_{x(0)=x_i}^{x(\beta)=x_f}\mathcal{D}x\;\exp\Bigg\{ &-\;S_S^E[x]-\frac{\eta}{2\pi}(x_i-x_f)^2\left[\gamma_E+\ln\left(\frac{\eta\beta}{\pi M}\right)\right] \\
&+\;\frac{\eta}{\pi}\left(x_i-x_f\right)\int_0^\beta d\tau\;\dot{x}(\tau)\;\mathrm{lnsin}\left(\frac{\pi\tau}{\beta}\right) \\
&+\;\frac{\eta}{\pi}\int_0^\beta d\tau\int_0^\tau ds\;\dot{x}(\tau)\dot{x}(s)\;\mathrm{lnsin}\left(\frac{\pi(\tau-s)}{\beta}\right)\Bigg\} \qquad (12)
\end{aligned}$$

In summary, we have determined an expression for the reduced density matrix of a system (consisting of a massive particle in a potential or equivalently a pair of mutually interacting particles) coupled to a heat bath of oscillators. The coupling to the bath is chosen to reproduce the results of classical Brownian motion in the high temperature limit. We have taken care to make the term in the exponential finite, by isolating the divergences through integration by parts. This is important for path integral Monte Carlo simulation to be possible for this functional integral.

C. Example: The otherwise free particle

The reduced density matrix for a particle interacting with such a bath can be determined analytically for the otherwise free particle ($V_R(x)=0$ where V_R is the renormalized poten-

tial). In order to arrive at an analytic result start by writing an arbitrary path in Eq. 12 as an expansion around the classical solution

$$\begin{aligned} x(\tau) &= x_{cl}(\tau) + \xi(\tau); \\ x_{cl}(\tau) &\equiv x_i + (x_f - x_i)\tau/\beta, \\ \xi(\tau) &\equiv \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi\tau}{\beta}\right). \end{aligned} \tag{13}$$

After evaluation of the integrals using contour integration along with a change in variables for the integration over the even Fourier coefficients, we find the reduced density matrix

$$\begin{aligned} \rho_{red}(x_i, x_f, \beta) &= \sqrt{\frac{M}{2\pi\beta} + \frac{\eta}{2\pi^2}\left[\log(2) + \gamma_E + \Psi\left(1 + \frac{\eta\beta}{2\pi M}\right)\right]} \\ &\times \exp\left\{-\left[\frac{M}{2\beta} + \frac{\eta}{2\pi}\left[\log\left(\frac{\eta\beta}{2\pi M}\right) - \Psi\left(1 + \frac{\eta\beta}{2\pi M}\right)\right]\right](x_i - x_f)^2\right\}, \end{aligned} \tag{14}$$

where $\Psi(x)$ is the digamma function and the overall normalization is determined by analytic continuation of β and requiring the propagator to conserve probability for purely imaginary β. A Fourier transform of this density matrix determines an effective mass for a the heavy particle:

$$\left\langle p^2 \right\rangle = \frac{M_{eff}}{\beta}, \quad M_{eff} = M + \frac{\eta\beta}{\pi}\left[\log\left(\frac{\eta\beta}{2\pi M}\right) - \Psi\left(1 + \frac{\eta\beta}{2\pi M}\right)\right]. \tag{15}$$

D. PIMC for the reduced density matrix

Obtaining the analytic result for our reduced density matrix was reasonable for the otherwise free particle. For the simple harmonic oscillator, the analytic result exists but has a rather complicated expression. For the potentials which describe quarkonium spectroscopy with some precision, analytic work becomes entirely impractical. We would like to use numerical simulation to obtain reliable estimates of reduced density matrices and Euclidean correlation functions.

The most natural numerical approach for the formalism is path-integral Monte Carlo (PIMC). For an excellent review of the technique, see D. M. Ceperley [23]. Paths are either sampled according to a Metropolis algorithm determined by the action of interest, or sampled from some convenient distribution which samples the entire space of paths with some weight. For our work with a single degree of freedom, we found that sampling a

convenient distribution (in our case, the distribution of paths determined by $\exp(-S_{free}[x])$) to be sufficient, which is easily sampled for any discretization of the path with a bisection method. When sampling the space of paths, the next step is to determine an estimate for the action of each path. For our case, the primitive action with the simplest integration of the new dissipative terms is sufficient. Once this is determined, any correlation function can be calculated by sampling paths and making weighted averages.

III. IMAGINARY-TIME GREEN FUNCTIONS FOR QUARKONIUM: NUMERICAL RESULTS

In order to generate results relevant to the lattice, a different imaginary-time path integral must be discussed. We will now be interested in the finite-temperature imaginary-time Green function

$$G(\tau) = \sum_{n=-\infty}^{\infty} \langle \mathbf{x} = 0; \tau + n\beta | \mathbf{x} = 0; 0 \rangle \,, \tag{16}$$

as it is directly related to the S-channel quarkonium correlation functions calculated on the lattice [13]. In this section, path-integral Monte Carlo is developed for determining this Green function for *any* potential or diffusion coefficient, and numerical results for the $n = 0$ term in this sum are shown.

For a given channel, the two-point Euclidean correlator for a composite mesonic operator is given by

$$\begin{aligned} G(\tau, T) &= \int d^3x \, \langle J_\Omega(\mathbf{x}, \tau) J_\Omega(\mathbf{0}, 0) \rangle_\beta \,, \\ J_\Omega(\mathbf{x}, \tau) &= \bar{\psi}(\mathbf{x}, \tau) \Omega \psi(\mathbf{x}, \tau), \end{aligned} \tag{17}$$

where $\Omega = 1$, γ^0, γ^μ, $\gamma^0\gamma^\mu$ determine the mesonic channel to be scalar, pseudo-scalar, vector, or pseudo-vector, respectively.

For now, consider only the vector channel (the following arguments must be modified for the scalar and pseudo-vector channels). Think of this correlator as being the sum of the expectation values of an operator over all of the states in the Fock space of N-particle mesonic systems, with each expectation value entering the sum weighted by the state's Boltzmann factor. When $M_Q \gg T$, $\Lambda_{\rm QCD}$, the states containing heavy quarks are weighted by factors of roughly $\exp(-2M_Q/T)$ and are suppressed. Therefore, in this limit, the dominant con-

tribution to $G(\tau)$ comes from the usual vacuum expectation value in imaginary time made periodic with period β.

Because of the mass of the heavy quark, we ignore spin and consider the Green functions at finite temperature for a scalar field. The correlation function $G(\mathbf{x}, \mathbf{x}^{'}, \tau) = \langle \phi(\mathbf{x}, \tau)\phi(\mathbf{x}, 0)\rangle_{\beta}$, for a free non-relativistic field, satisfies the differential equation

$$\left(\hat{H}(\mathbf{x}) + \frac{\partial}{\partial \tau}\right) G(\mathbf{x}, \mathbf{x}^{'}, \tau) = \delta^3(\mathbf{x} - \mathbf{x}^{'})\delta(\tau), \tag{18}$$

subject to the periodicity condition $G(x, x^{'}, \tau+\beta) = G(x, x^{'}, \tau)$. The solution is analogous to the method of images: the proper periodicity is obtained by summing over zero-temperature correlation functions [25]:

$$\langle \phi(\mathbf{x}, \tau)\phi(\mathbf{x}, 0)\rangle_{\beta} = \sum_{n=-\infty}^{\infty} \langle \phi(\mathbf{x}, \tau + n\beta)\phi(\mathbf{x}, 0)\rangle_{T=0}\,. \tag{19}$$

The rest of the argument proceeds as above for the reduced density matrix. We now focus on the Green functions in imaginary time in this sum, which will be determined numerically with path-integral Monte Carlo. We consider the same system of one heavy particle interacting with a light degree of freedom:

$$S = \int_0^{\tau} d\tau^{'} \left[\frac{1}{2}M\dot{x}^2 + V(x) + \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m\omega^2 r^2 + Cxr\right]. \tag{20}$$

The propagator can be expressed as a path integral:

$$\langle x_f, r_f, \tau | x_i, r_i, 0\rangle = \int \mathcal{D}x\mathcal{D}r\ \exp(-S), \tag{21}$$

where the paths have the endpoints $x(0) = x_i$, $x(\tau) = x_f$, $r(0) = r_i$, and $r(\tau) = r_f$. The subscript $i(f)$ is again used to denote the initial (final) position. Note the difference between the reduced density matrix where the trace over the single variable $r_i = r_f$ is taken and the above propagator where r_i and r_f are independently integrated. As before the path integral over the light degree of freedom can be performed analytically

$$\begin{aligned}
\langle x_f, r_f, \tau | x_i, r_i, 0\rangle \;=\; & \int \mathcal{D}x\ \exp\left(-\int_0^{\tau} d\tau^{'}\left[\frac{1}{2}M\dot{x}^2 + V(x) + \frac{1}{2}Cx(\tau^{'})A[x,\tau^{'}]\right] + \frac{1}{2}m\dot{A}[x,\tau]A[x,\tau]\right) \\
& \times \sqrt{\frac{m\omega}{2\pi\sinh(\omega\tau)}}\exp(-m\dot{A}[x,\tau]r_f) \\
& \exp\left(-\frac{m\omega}{2\sinh(\omega\tau)}([r_i^2 + (r_f - A[x,\tau])^2]\cosh(\omega\tau) - 2r_i(r_f - A[x,\tau]))\right),
\end{aligned} \tag{22}$$

where $A[x,\tau]$ was given in Eq. 3. We now want to integrate over all values of r_i and r_f, leading to a *reduced* imaginary-time Green function for the heavy particle. The integrals are Gaussian and can be performed analytically:

$$\langle x_f,\tau|x_i,0\rangle_{\rm red} = \int_{x(0)=x_i}^{x(\beta)=x_f} \mathcal{D}x \exp\left(-\int_0^\tau d\tau'\left[\frac{1}{2}M\dot{x}^2+V(x)\right]\right.$$
$$\left.+\sum_k \frac{C_k^2}{2m\omega_k\sinh(\omega_k\tau)}\int_0^\tau d\tau'\int_0^{\tau'} ds\; x(\tau')x(s)\cosh\left[\omega_k(\tau-\tau')\right]\cosh(\omega_k s)\right). \tag{23}$$

The above expression is completely analogous to the reduced density matrix in Eq. 8 except with the difference in traces over the bath.

Using the same density of states as given in Eq. 9, the integral over the density of states can be performed in the above reduced Green function. After taking the $\Omega\to\infty$ limit the result is

$$G_{\rm red}(x_f,x_i,\tau,\beta) = \sum_{n=-\infty}^{\infty}\langle x_f,|\tau+n\beta||x_i,0\rangle_{\rm red}$$
$$= \sum_{n=-\infty}^{\infty}\int_{x(0)=x_i}^{x(|\tau+n\beta|)=x_f}\mathcal{D}x\;\exp\left(-\int_0^{|\tau+n\beta|}d\tau'\left[\frac{1}{2}M\dot{x}(\tau')^2+V_R(x(\tau'))\right.\right.$$
$$\left.\left.-\frac{\eta}{2\pi}\int_0^{\tau'}ds\;\dot{x}(\tau')\dot{x}(s)\log\left[\frac{\sin(\frac{\pi}{2}\frac{\tau'-s}{|\tau+n\beta|})}{\sin(\frac{\pi}{2}\frac{\tau'+s}{|\tau+n\beta|})}\right]\right]\right). \tag{24}$$

In the case of two mutually interacting particles, the above path integrals can be re-expressed in terms of relative, $\mathbf{x}$, and absolute, $\mathbf{X}$, coordinates as

$$\langle \mathbf{X}_f,\mathbf{x}_f;\tau|\mathbf{X}_i,\mathbf{x}_i;0\rangle = \int\mathcal{D}\mathbf{X}\;\exp\left\{-\int_0^\tau d\tau' M\dot{\mathbf{X}}^2-\frac{2\eta}{\pi}\int_0^\tau d\tau'\int_0^{\tau'}ds\;\dot{\mathbf{X}}(\tau')\dot{\mathbf{X}}(s)\log\left(\frac{\sin\left(\frac{\tau'+s}{\tau}\right)}{\sin\left(\frac{\tau'-s}{\tau}\right)}\right)\right\}$$
$$\times\int\mathcal{D}\mathbf{x}\;\exp\left\{-\int_0^\tau d\tau'\left[\frac{1}{4}M\dot{\mathbf{x}}^2+V(\mathbf{x})\right]\right.$$
$$\left.-\frac{\eta}{2\pi}\int_0^\tau d\tau'\int_0^{\tau'}ds\;\dot{\mathbf{x}}(\tau')\dot{\mathbf{x}}(s)\log\left(\frac{\sin\left(\frac{\tau'+s}{\tau}\right)}{\sin\left(\frac{\tau'-s}{\tau}\right)}\right)\right\}, \tag{25}$$

where similar to before, $\mathbf{X}(0)=\mathbf{X}_i$, $\mathbf{X}(\tau)=\mathbf{X}_f$, $\mathbf{x}(0)=\mathbf{x}_i$, and $\mathbf{x}(\tau)=\mathbf{x}_f$. We focus now on the path integral for the relative coordinate, and for simplicity of presentation, we only determine and show the first term in this sum; examining only the first term in the sum also makes deconvolution easy.

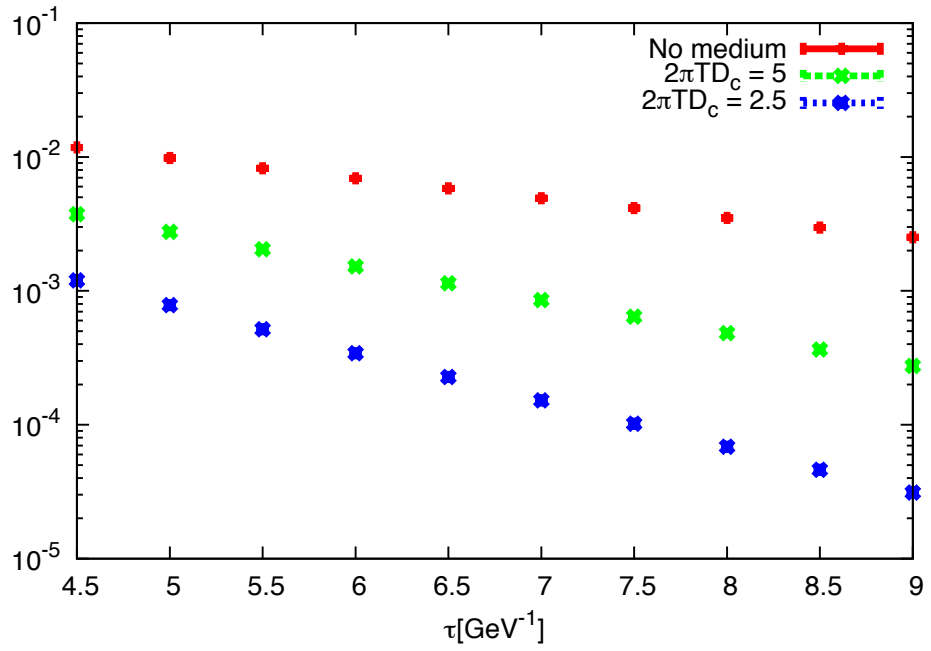

FIG. 1: (Color online) $G_0(\tau)$ for different diffusion coefficients.

At this point, the potential energy and drag coefficient for heavy quark bound states must be matched with their calculations from QCD. In the infinite mass limit, the trace over the light degrees of freedom has been computed on the lattice as the expectation value of two Polyakov loops [24]. The dissipative effects on this propagator have been studied, as we noted previously, by gauge-gravity duality. Here, we match the results of this trace, yielding a potential term and a heavy quark drag coefficient, onto terms in the path integral.

The function

$$G_0(\tau) = \langle \mathbf{x}_f = \mathbf{0}; \tau | \mathbf{x}_i = \mathbf{0}; 0 \rangle \tag{26}$$

can be calculated with a path integral Monte Carlo calculation, using the potential and diffusion coefficient that fits best (or is fitted) from phenomenology of heavy quarks, or from any other considerations. We use the Cornell potential for the interaction between the heavy quarks, regularized at $\mathbf{x} = \mathbf{0}$ with a harmonic potential, and set η for the cases of $2\pi TD = \infty$, 5, and 2.5. The results of such a calculation are shown in Figure 1.

Dissipative effects have a significant effect on this Green function, which is given by the Laplace transform of the spectral function for charmonium:

$$G_0(\tau) = \int d\omega \exp(-\omega\tau)\rho(\omega). \tag{27}$$

Although deconvolution of this Green function is difficult, the effect of dissipation can be read off: the peak corresponding to a bound state or resonance has been shifted up in energy, and its integral has decreased (indicating a weaker response of the medium to the charmonium creation operator).

IV. CONCLUSIONS

We have taken the approach of Caldeira and Leggett and determined expressions for quantities calculable in imaginary-time. Path-integral Monte Carlo techniques were developed and applied to the imaginary-time Green function. Interactions with the QCD medium above deconfinement clearly affect Euclidean heavy-heavy correlators, and any determination of these Green functions at finite-temperature must deal with the likely large heavy quark drag coefficient. These numerical methods are currently the only techniques available that can deal with large drag coefficients while describing quarkonium quantum-mechanically.

The binding energies and widths of quarkonia in these systems will finally be obtained when the results in Fig. 1 are deconvolved into spectral functions. $G(\tau)$ is the Laplace transform of the spectral function; deconvolution is non-trivial and often is done with the maximum entropy method: a procedure that combines data-fitting with information theory for probability distributions. Forthcoming work will show these results.

Let us argue one last time for treating quarkonium above deconfinement as an open quantum system: typically in particle physics, the rate for the scattering of an N-particle state into another is determined from the square of the matrix element whose indices are the initial and final states, where these states are in the momentum basis. This makes perfect sense in high-energy experiments, where the incoming and outgoing states are basically momentum eigenstates and the matrix element can be expanded in terms of a small coupling. Such an approach, however, seems entirely inappropriate for quarkonium formed in a heavy-ion collision, whose constituent quarks are localized in position relative to the surrounding medium and are strongly coupled both to the medium and each other. Non-perturbative techniques and experiment both suggest that quarkonium rapidly thermalizes and interacts strongly with the medium. The most reasonable approach for explaining the observables is to describe quarkonium with a reduced density matrix, whose evolution is determined by a potential *and* a drag coefficient which are both treated non-perturbatively.

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