

CHCRUS

This is the accepted manuscript made available via CHORUS. The article has been published as:

Resonant α scattering of 6 He: Limits of clustering in 10 Be

D. Suzuki, A. Shore, W. Mittig, J. J. Kolata, D. Bazin, M. Ford, T. Ahn, F. D. Becchetti, S. Beceiro Novo, D. Ben Ali, B. Bucher, J. Browne, X. Fang, M. Febbraro, A. Fritsch, E. Galyaev, A. M. Howard, N. Keeley, W. G. Lynch, M. Ojaruega, A. L. Roberts, and X. D. Tang Phys. Rev. C 87, 054301 — Published 2 May 2013 DOI: 10.1103/PhysRevC.87.054301

1	Resonant α scattering of ⁶ He: limits of clustering in ¹⁰ Be
2	D. Suzuki, ^{1,2,*} A. Shore, ^{1,3} W. Mittig, ^{1,3} J.J. Kolata, ⁴ D. Bazin, ¹ M. Ford, ^{1,†}
3	T. Ahn, ¹ F.D. Becchetti, ⁵ D. Ben Ali, ⁶ B. Bucher, ⁴ J. Browne, ^{4,1,3}
4	X. Fang, ⁴ M. Febbraro, ⁵ A. Fritsch, ^{1,3} E. Galyaev, ¹ A.M. Howard, ⁴
5	N. Keeley, ⁷ W.G. Lynch, ^{1,3} M. Ojaruega, ⁵ A.L. Roberts, ⁴ and X.D. Tang ⁴
6	¹ National Superconducting Cyclotron Laboratory,
7	Michigan State University, East Lansing, Michigan 48824-1321, USA
8	² Institut de Physique Nucléaire, IN2P3-CNRS,
9	Université Paris-Sud, F-91406 Orsay, France
10	³ Department of Physics and Astronomy, Michigan State University,
11	East Lansing, Michigan 48824-1321, USA
12	⁴ Department of Physics, University of Notre Dame,
13	Notre Dame, Indiana 46556-5670, USA
14	⁵ Department of Physics, University of Michigan,
15	Ann Arbor, Michigan 48109-1040, USA
16	⁶ Département de Physique, Université Paris-Sud, F-91403 Orsay, France
17	⁷ National Centre for Nuclear Research,
18	ul. Andrzeja Sołtana 7, 05-400 Otwock, Poland
19	(Dated: April 7, 2013)

Abstract

The structure of ¹⁰Be was studied via resonant α -particle scattering of a neutron-rich ⁶He beam. 21 A time projection chamber, PAT-TPC, was operated in an active-target mode to provide a gaseous 22 ⁴He target and trace the beam and reaction products traversing its active tracking volume. This 23 significantly lowered the detection threshold of reaction products at low energies. Elastic scattering, 24 inelastic scattering to the ⁶He 2⁺ state, and the ⁶He(α , 2n)⁸Be reaction were measured below an 25 energy of 6 MeV in the center-of-mass frame. Continuous spectra of excitation functions and 26 angular distributions were obtained from unambiguously-identified recoiling α particles for the 27 elastic and inelastic channels. While a resonance of the 4^+ state at 10.15 MeV in ¹⁰Be previously 28 reported was confirmed, no other resonances were identified in the elastic channel over the measured 29 energy region. The results are in line with antisymmetric molecular dynamics calculations that 30 predict the limits of α clustering in high-spin states due to a spin-orbit force. 31

32 PACS numbers: 24.30.Gd, 25.55.Ci, 27.20.+n, 29.40.Cs

^{*} e-mail: suzuki@ipno.in2p3.fr

[†] Present address: Air Force Institute of Technology (AFIT), WPAFB, Ohio 45433, USA

33 I. INTRODUCTION

Clustering of α particles is a unique aspect of nuclear correlations. It is known that α 34 clustering often occurs in light nuclei along the N = Z line as already suggested since 1930s 35 to explain their level schemes [1-3]. Archetypal examples are the ground state of ⁸Be and 36 the second 0⁺ state of ¹²C (referred to as "Hoyle" state) [4, 5], manifesting well-developed 2α 37 and 3α clusters, respectively, and likewise further heavier systems such as 4α states in ¹⁶O [6] 38 or 6α states in ²⁴Mg [7]. These cluster states, which exclusively consist of α particles, have 39 been providing a unique playground to discuss bosonic condensations [6, 8, 9] or geometries 40 of multiple quantum objects [3, 7] in femtometer-scale systems. 41

The structure of the nucleus, a quantum many-body system, can drastically change with 42 addition or removal of nucleons. How, if at all, do these nucleons affect α clustering in nuclei? 43 The structure of ¹¹B, the nucleus with one proton removed from ¹²C, was recently studied 44 via the (d, d') reaction [10]. The large monopole strength extracted for the $3/2^{-}$ state at 45 8.56 MeV indicates a well-developed $2\alpha + t$ cluster structure, suggesting that the clustering 46 nature of the Hoyle state in ¹²C persists in the presence of a proton hole. Theoretical studies 47 on neutron-rich beryllium isotopes that trace the cluster evolution away from ⁸Be also predict 48 the persistence of 2α clusters in the neutron-excess systems ¹⁰Be [11–24] and ¹²Be [25, 26]. 49 It is further suggested that the unique correlation of neutrons and 2α clusters diversifies the 50 evolution of structure. The formation of predicted structures, such as neutron-molecular 51 orbitals [15, 16, 25, 26] or a di-neutron pair around 2α cores [24], plays an important role 52 to stabilize 2α cores [16, 25, 26], break the axial symmetry [24], or possibly quench the shell 53 gap at the magic number 8 [25–28]. However, there are limited experimental data to support 54 such interplay between α clusters and valence neutrons. 55

In the present paper, we report on the excitation properties of ¹⁰Be via resonant α 56 scattering of ⁶He. The neutron-rich ¹⁰Be nucleus is a simple system consisting of ⁸Be plus 57 2n. Nevertheless, ¹⁰Be involves a complex level scheme that features three rotational bands 58 built on the 0^+ ground state, the 1^- state at 5.96 MeV, and the second 0^+ state at 6.18 MeV. 59 Theoretically, the origin of these bands is explained in terms of molecular orbitals of valence 60 neutrons [15, 16]. In this picture, these neutrons are delocalized over the 2α cores and 61 occupy different orbital levels, thus different intrinsic states. There has been a number of 62 studies conducted in search of experimental signatures of the predicted intrinsic structures. 63

Particularly high-spin members of the $0^+_{g.s.}$ and 0^+_2 bands have been searched using different 64 reaction probes. The 4^+ member of the 0^+_2 band was first speculated for the state at 10.2 MeV 65 found in a study of the $^{7}\text{Li} + ^{7}\text{Li}$ reaction [29]. While this state was later confirmed at 66 10.15(2) MeV using the ⁷Li + 6,7 Li reactions, a spin-parity of 3⁻ was assigned from the 67 angular correlation of α and ⁶He particles following the α decay [30]. Another angular 68 correlation study using the ${}^{6}\text{He} + {}^{6,7}\text{Li}$ reactions, however, gave a conflicting assignment 69 of 4^+ , indicating that the method depends on the model assumed in reaction analyzes [31]. 70 A spin-parity of 4^+ was assigned without such assumptions in a recent measurement of α 71 scattering from ⁶He, in which the 10.15-MeV state in ¹⁰Be was resonantly populated and 72 the angular distribution was characteristic of the angular momentum 4 [32]. The associated 73 large width for α emission was interpreted as a strong indication of the predicted molecular 74 structure. The 4^+ member of the $0^+_{g.s.}$ band is considered the 4^+ state at 11.76 MeV [33]. The 75 α clustering of the ground state band is naïvely assumed from its level scheme being nearly 76 equal to ⁸Be, having the 2^+ state at 3.37 MeV (⁸Be 2_1^+ 3.03 MeV) and the 4^+ state at 11.76 77 MeV (⁸Be 4_1^+ 11.4 MeV) [33]. This assumption has been supported by theoretical studies 78 that describe well the level scheme within the molecular orbital picture [15, 16, 18, 19, 21, 79 22, 24]. The possibility of a shell-model-like structure was, however, recently discussed in a 80 study of 10 Be via inelastic scattering with 12 C [34], where the excitation to the 4⁺ member 81 of the $0^+_{g.s.}$ band was not observed. This suggests there is a delicate balance between the 82 persistence and the dissociation of α clusters in ¹⁰Be, and merits further elucidation. 83

In this study, we measured scattering of ⁶He on α particles at low energies to resonantly 84 populate states in ¹⁰Be. The resonance strength, which is related to the decay width for 85 α emission, is an important indicator of the degree of α clusterization [32, 35]. Elastic α 86 scattering of ⁶He was measured in a few previous studies [32, 36–38]. The measurement by 87 Ter-Akorpian *et al.* was performed at a center-of-mass energy $(E_{c.m.})$ of 60.3 MeV [36], while 88 those of Raabe et al. were at 11.6 and 15.9 MeV [37, 38]. At these higher energies, the 89 data are well reproduced by direct reaction analyses based on the coupled-discretized con-90 tinuum channels formalism [39] and the coupled-reaction channel formalism [40]. Freer et al. 91 recently measured scattering at low energies below 5 MeV, and identified a resonance orig-92 inating from the 4⁺ state at 10.15 MeV [32]. Their measurement was, however, performed 93 at only three center-of-mass energies, namely $E_{\rm c.m.} = 2.44$, 3.00 and 4.44 MeV. Therefore, 94 there was no continuous excitation spectrum, which hampers the ability to thoroughly scan 95

resonances over a given excitation energy region. In addition, the reconstruction of the reac-96 tion kinematics had an ambiguity because recoiling α particles and scattered ⁶He particles 97 were not differentiated, while the obtained angular distribution was correctly analyzed using 98 simulations that took this ambiguity into account. In the present study, we measured contin-99 uous excitation functions over a wide energy range of $E_{\rm c.m.} = 2-6$ MeV and reconstructed 100 angular distributions from unambiguously-identified recoiling α particles. This allowed us to 101 survey resonances over the energy domain where the 4^+ members of the ground-state band 102 and the second 0^+ band are located, as well as the region where some other resonances have 103 been predicted [14, 23]. 104

An experimental challenge is the detection of low-energy helium particles from scattering, 105 which quickly lose their energy in the target medium. We used a time projection chamber 106 (TPC) in the "active-target" mode to address this issue, where the gas of the TPC serves 107 simultaneously as the α -particle target as well as the tracking medium of reaction products. 108 This allows measurements of the energy and scattering angle of particles which stop inside 109 the target. This setup is capable of measuring not only elastic scattering, but also inelastic 110 scattering and neutron-emission channels such as ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$, in which negative reaction 111 Q values further lower the energy of the reaction products. This new method also enables 112 the determination of reaction energies from the direct measurement of reaction positions. In 113 the thick target method [41], the established and widely-used method to measure continuous 114 excitation functions in inverse kinematics, the reaction energy is indirectly obtained from 115 the energy of recoiling particles assuming a given kinematical scenario. This usually requires 116 several different settings for beam energies to avoid mixing elastic and inelastic scattering. In 117 our method, the unambiguous identification of the reaction energy allows us to differentiate 118 reaction channels, thus enabling us to cover a wide energy range in a single measurement. 119

120 II. EXPERIMENT

The experiment was performed at the *TwinSol* radioactive nuclear beam facility [42] at the University of Notre Dame. Scattering of ⁶He on α particles was measured using the Prototype Active-Target Time-Projection Chamber (PAT-TPC) [43]. A secondary ⁶He beam with an energy of 15 MeV was produced using the ⁷Li(d, ³He)⁶He reaction. A stable ⁷Li beam was accelerated to 29.2 MeV by the FN Tandem accelerator and impinged on a

gaseous deuterium target. The pressure of the deuterium gas was 1200 mm-Hg on average. 126 The gas cell had windows consisting of two 4.2-mg/cm² Havar foils 2.5 cm apart along 127 the beam axis. The effective path length in the cell was longer by a few millimeters due to 128 bowing of the foils under pressure. Outgoing reaction products were collected and focused by 129 a pair of superconducting solenoidal magnets of the TwinSol device [42]. A 6-mg/cm² CH₂ 130 foil was placed at the cross-over point between the two magnets, which significantly slowed 131 down higher Z contaminants, namely Li, from the primary beam, and greatly improved the 132 purity of the ⁶He beam. During the beam tuning, a silicon $E-\Delta E$ telescope was used in 133 front of the PAT-TPC as a beam monitor. The beam was predominantly composed of ⁶He 134 and the main contaminant was ⁴He with an energy of about 22 MeV. The telescope was 135 then removed from the beam line once the beam tuning was completed. 136



137

FIG. 1. (Color online) Schematic drawing of the experimental setup of the PAT-TPC. The inset
is a magnified view of the segmented anode pad plane of the Micromegas detector near the beam
axis.

The PAT-TPC was installed 1.5 m downstream of the end of *TwinSol*. A schematic 141 drawing of the setup is shown in Fig. 1. The detector encompasses a cylindrical field cage of 142 50 cm in length and 28 cm in diameter, which is centered on the beam axis. The cage was 143 filled with a $He:CO_2$ 90:10 mixture gas at atmospheric pressure, which serves simultaneously 144 as the tracking medium as well as the reaction target containing ⁴He. To limit impurities, 145 the gas was kept continuously flowing at a rate of $4.5 \text{ cm}^3/\text{s}$, which replaces the whole volume 146 of gas every 2 hours. The ⁶He particles entering through an entrance window travel down 147 along the symmetry axis of the field cage and induce nuclear reactions with the gas nuclei. 148 Both the beam particles and outgoing charged particles from the reactions ionize gas atoms 149 while traveling across the gas volume. Their paths are reconstructed from the ionization 150 electrons, which are transported downstream by an electric field parallel to the beam axis, 151 and multiplied by the gas electron amplifier, Micromegas [44]. The latter was fabricated by 152 the SEDI/IRFU, CEA-Saclay. An electric field of 0.8 kV/cm was generated by a negative 153 potential of -40 kVDC applied to a cathode plate at the upstream end of the cage. The 154 initial potential was stepped down by a series of equipotential rings toward the Micromegas. 155 The resulting electron drift velocity was measured to be about 2.4 cm/ μ s [43]. The field 156 cage is gas tight and is surrounded by nitrogen gas at atmospheric pressure, which provides 157 a high dielectric strength to mitigate the risk of discharge to the chamber walls at ground 158 potential. 159

The secondary beam was focused on the entrance window of the PAT-TPC. The spot 160 size of the 6 He beam at the window was estimated to be 25 mm in diameter from solenoid 161 optics calculations, while the entrance aperture is circular with a diameter of about 8 mm. 162 The beam was thus collimated by the window, which changed the intensity and purity of 163 ⁶He in the TPC. These quantities were estimated using signals from the micromesh of the 164 Micromegas, of which a detailed description is given later. The ⁶He particles are decelerated 165 in the gas and stop at the end of the field cage about 50 cm downstream of the beam 166 window, fully depositing 15 MeV of energy. In contrast, ⁴He has higher energy and punches 167 through the TPC while losing only 6 MeV in the detector volume. This results in a pulse-168 height difference in the micromesh signals, thus allowing identification and counting of the 169 respective nuclei. The average intensity of the collimated ⁶He beam was 2×10^3 counts per 170 second with a purity of 90% at a primary ⁷Li(3⁺) beam intensity of 0.5 electric μ A. 171

¹⁷² The Micromegas consists of an anode readout plane and a micromesh stretched over it.

Electron avalanches occur in the narrow amplification gap of 128 μ m defined by the anode 173 and the micromesh. The micromesh was biased at -320 V to create a high field gradient 174 over the gap, while the anode plane was grounded. The gas gain was measured to be about 175 75 as also reported elsewhere [43]. The anode plane of the Micromegas has a circular-shaped 176 active area of 250 mm in diameter. It is segmented into multiple pads to read out and locate 177 the avalanche electrons. Specifically, the anode pads consist of a 5-mm-diameter central 178 pad and 2-mm-pitch coaxial strips, each spaced by a gap of 0.25 mm (inset of Fig. 1). The 179 coaxial strips are divided into quadrants and the five innermost strips into octants. The 180 beam particles travel approximately perpendicular to the anode plane. The central pad and 181 its neighboring coaxial strips record the energy deposit profile as a function of time, and 182 the position along the beam axis (z) is deduced from the drift time of ionization electrons. 183 The particles emitted from a reaction travel away from the center to the perimeter. Each 184 particle traverses a series of coaxial strips in a given quadrant. This set of strips thus gives 185 the energy deposit profile as a function of the radius r of the strip as well as z, providing 186 the polar angle and the range of the reaction products. Examples of energy deposit profiles 187 are found in our previous report [43]. 188

A total of 253 signals from the anode plane were read out by a combined preamplifier/shaper/wave-189 digitizer system developed for the neutrino-flux monitor of the T2K experiment [45, 46], 190 which is referred to as T2K electronics hereafter. The T2K electronics were set to record 191 the waveform of input signals at 12.5 MHz over the full time range of 40 μ s. A time bin 192 width thus corresponds to 80 ns, or a spatial size of 2 mm along the beam axis given the 193 electron drift velocity of 2.4 cm/ μ s. To generate external triggers for the T2K electronics. 194 signals from the micromesh as well as from some coaxial anode strips were routed to an 195 auxiliary circuit consisting of NIM-standard modules. Two sets of data were taken with 196 different triggers. The first trigger was optimized for the elastic and inelastic scattering 197 of ⁶He on α particles. The primary signature of these reactions is the observation of two 198 helium isotopes, ⁴He and/or ⁶He, involved in the final state. Because of the low atomic 199 number (Z = 2), both particles have a long range and traverse many strips away from the 200 center. The trigger was thus designed to record events which fire specific off-center strips in 201 different quadrants. The four coaxial strips, each belonging to different quadrants, located 202 at a radius of 18 mm were used as trigger sources. The data acquisition was triggered on 203 tracks that extend 18 mm or more in radius in two or more quadrants. For monitoring 204

purposes, the trigger from one quadrant, or multiplicity equal to one, was also added after 205 downscaling by a factor of 32. The second trigger was used to record $^{6}\text{He} + \alpha$ reactions 206 involving beryllium in the final state. These reactions produce a large energy deposit in a 207 short distance due to the high Z of Be and its low recoil velocity. The time structure of 208 the micromesh signal was used to generate a trigger when the signal had (a) a higher Bragg 209 peak and (b), later times with respect to the Bragg peak of the fully stopped ⁶He beam. 210 The late times imply that the observed Bragg peak is located closer to the beam entrance, 211 making it a good indicator of the occurrence of a reaction during the deceleration of the 212 beam. The average rate of Trigger 1 was 15 Hz, while that of Trigger 2 was 50 Hz. The 213 live-time ratio of the data acquisition was 70% and 40%, respectively, for these two triggers. 214

215 III. ANALYSIS

A. Elastic and inelastic scattering

In the following section, we describe the analysis procedure of the elastic and inelastic scattering data taken with Trigger 1 (optimized for these processes). Part of the analysis procedure, particularly the tracking of the ⁶He beam and reaction products (⁶He and/or ⁴He), is the same as reported in Ref. [43]. Energy deposition was calculated using the SRIM code [47].

The total kinetic energy (TKE) and the emission angle (θ_{lab}) of reaction products were obtained by analyzing the set of information on charge Q_i , radial position r_i and signal timing t_i from the coaxial strips, denoted by i, which the particle tracks traveled over. First, the angle θ_{lab} , which is defined with respect to the beam axis, or the axis of time projection (Fig. 1), was determined from the slope of the t vs. r plot. A linear fitting function was adopted and defined as:

$$t = \frac{r}{v_{\rm drift} \tan \theta_{\rm lab}} + t_1 \tag{1}$$

with v_{drift} being the electron drift velocity and t_1 the time at r = 0. Once θ_{lab} was determined from the fit, the Q vs. r plot, which represents the energy deposition profile in the radial direction, was compared to calculated curves at a given TKE using the θ_{lab} previously obtained. The optimal TKE was determined to minimize χ^2 with respect to the experimental data. Some of the particles escaped from the active volume of the PAT-TPC, particularly

the ones with large or small scattering angles where the energies are higher. To obtain 233 the acceptance over a wide angular range, we also analyzed the events involving particle 234 escapes. While the range cannot be determined for these particles due to the missing position 235 information of the Bragg peak, the amplitude and slope of the energy deposit profile towards 236 the Bragg peak are still sensitive to the TKE, which allows TKE determination by the same 237 χ^2 minimization procedure. The quality of the TKE determination of escaping particles 238 will be discussed later when the reconstruction of the excitation energy spectrum of ⁶He 239 is presented. When a given particle stops inside the active volume, ⁶He and α particles 240 can be differentiated from the amplitude of the tail of Bragg peak as demonstrated in our 241 previous report [43]. In the present analysis, however, we differentiated ⁶He and ⁴He from 242 the reaction kinematics as detailed later. 243

Reconstruction of reaction kinematics by the missing mass method requires knowing 244 the energy of the beam particles. In the present measurement, the energy continuously 245 decreases as the beam particle travels along the beam axis. To determine the reaction 246 energy, the energy deposition before reaching the reaction position (z_{reac}) needs to be taken 247 into account. In this analysis, z_{reac} was determined from the recorded drift times of ionization 248 electrons released at the positions of the reaction vertex and the beam entrance. The drift 249 time corresponding to the reaction position was given by that of the reaction vertex t_1 250 previously deduced in the analysis of the trajectory of the reaction products. The drift time 251 of electrons released at the entrance (t_0) was obtained by analyzing the waveform of anode 252 signals near the central region, which represents the energy deposit profile of beam particles 253 along the beam axis. The anode signals of the central pad and its neighboring coaxial strips 254 were summed when the beam charge spreads over multiple pads. The difference between 255 the two times $t_0 - t_1$ was then translated into z_{reac} using the electron drift velocity. The 256 corresponding energy loss was calculated and subtracted from the initial beam energy to 257 define the energy at the moment of the reaction (E_{reac}) . 258

The ⁶He ions and the contaminant ⁴He ions in the secondary beam were differentiated from the energy deposit per unit length (dE/dz) averaged over the track from t_0 to t_1 . The value of dE/dz provides a good measure of isotope separation as the ⁶He particles have lower energies (3.8 MeV/*u* or less) compared to the ⁴He contaminants (4.4–5.6 MeV/*u*). Figure 2(a) shows the scatter plot between dE/dz and z_{reac} , where two loci corresponding to ⁶He and ⁴He are clearly separated. The separation of the loci, which becomes smaller at

shorter travel lengths, is uncertain below $z_{\text{reac}} = 50 \text{ mm}$, where the finite value of the shaping 265 time (0.5 μ s) smears the difference in dE/dz. In this region, the properties of the reaction 266 kinematics were further used to eliminate elastic scattering events of ⁴He on α particles. A 267 gate was set on the sum of the angles $\theta_{lab}^{(i)}$ of the two reaction products labeled i = 1 and 2. 268 Figure 2(b) shows the $\theta_{\text{lab}}^{(1)} + \theta_{\text{lab}}^{(2)} vs. dE/dz$ plot for the region of $z_{\text{reac}} < 50$ mm. A cluster 269 is seen at 90 degrees. This corresponds to ⁴He beam scattered by the ⁴He in the gas. The 270 opening angle of two identical particles from elastic scattering always equals 90 degrees in 271 the laboratory frame regardless of the center-of-mass scattering angle. The gate displayed 272 in the figure was adopted to exclude these ⁴He-beam events. 273



274

FIG. 2. (Color online) Identification of the secondary beam particles. (a) Scatter plot between the energy loss per unit length of beam path dE/dz and the reaction position z_{reac} . (b) Scatter plot between dE/dz and the opening angle of reaction products $\theta_{\text{lab}}^{(1)} + \theta_{\text{lab}}^{(2)}$ for the events at $z_{\text{reac}} < 50$ mm. The gate to select ⁶He scattering is indicated.

The excitation energy of ⁶He (E_x) and the center-of-mass scattering angle ($\theta_{c.m.}$) were



FIG. 3. (Color online) TKE/ $u vs. \theta_{lab}$ plot for ⁶He + α scattering at 15 MeV. Elastic scattering and inelastic scattering to the first 2⁺ state of ⁶He are denoted by the solid and dashed lines, respectively. Calculated curves are shown both for the recoiling α (thick blue) and scattered ⁶He (thin magenta) particles.

obtained by the missing-mass method using E_{reac} as well as TKE and θ_{lab} of the recoiling 280 α particles. An inherent issue in deducing excitation-energy spectra from ⁶He + α -particle 281 scattering is that another α particle is produced in the final state when inelastically scattered 282 ⁶He decays via 2n emission. All excited states of ⁶He are unbound above the 2n separation 283 energy at 0.972(1) MeV [48]. It is therefore necessary to correctly select the α particle 284 recoiling from the target. The information on kinematical properties of the reaction was 285 used to eliminate the ambiguity in identifying the recoiling particle. Figure 3 shows the 286 calculated plots of TKE per nucleon (TKE/u) vs. θ_{lab} for elastic and inelastic scattering 287 to the 2^+ state at a beam energy of 15 MeV. It is found that TKE/u, or the velocity of 288 ⁶He, is always smaller than that of the recoiling α particle at a given laboratory angle 289 except for the very forward region below 10 degrees in the center-of-mass frame, which is 290 outside the present detector acceptance. Therefore, the range of the α particle following 291 2n emission decay tends to be shorter than that of the recoiling α particle. According to 292 these characteristics, we adopted the following procedure. First, two excitation energies 293 $(E_{\mathbf{x}}^{(i)})$ were obtained individually from two reaction products by assuming the particle i as 294

²⁹⁵ the recoiling α particle. The set of $E_{\rm x}^{(i)}$ were then compared. If the ⁶He particle, or the α ²⁹⁶ particle after 2*n* emission decay, is identified as the recoiling α particle, its shorter range ²⁹⁷ results in an underestimate of TKE and thus gives higher excitation energies than the true ²⁹⁸ value. Accordingly, we adopted the particle with a smaller $E_{\rm x}$ as the recoiling α particle.



299

FIG. 4. (Color online) Scatter plot between $E_{\rm x}^{(1)}$ and $E_{\rm x}^{(2)}$. The events to the left of the solid lines were excluded from the analysis. The contents of the region inside the dashed lines are weighted by a factor of 3 for presentation purposes.

Figure 4 shows the scatter plot between $E_{\rm x}^{(1)}$ and $E_{\rm x}^{(2)}$, where the indices 1 and 2 were 303 randomly assigned. Loci corresponding to the ground state and the first 2^+ state at 1.79 MeV 304 are visible. It is evident in the locus of elastic scattering that the smaller $E_{\rm x}^{(i)}$ value of a 305 given pair is mostly found near 0 MeV and well separated from the larger $E_{\rm x}^{(i)}$ which is 306 wrongly estimated from the scattered ⁶He. The result is similar for the 2^+ state, of which 307 the locus appears above 1.9 MeV only. However, the separation in the region close to the 308 $E_{\rm x}^{(1)} = E_{\rm x}^{(2)}$ line is not as clear as that of the ground state because of the velocity shift due to 309 particle emission and the contribution of breakup events. Therefore, above $E_x = 1$ MeV, the 310 events with $E_{\rm x}^{(1)}$ and $E_{\rm x}^{(2)} < 2.5$ MeV were eliminated from the analysis to ensure a sufficient 31 difference in energy for properly selecting α particles. 312

The resulting excitation-energy spectrum of ⁶He is shown in Fig. 5(a). Two peaks visible near 0 and 2 MeV correspond to the 0^+ ground state and the first 2^+ state at



FIG. 5. (Color online) (a) Excitation-energy spectrum of ⁶He (unshaded histogram). The best-fit result (thin red line) is shown together with the contribution of the background from the ⁶He breakup reaction (thick green line). The spectrum in the shaded histogram was made by selecting backward scattering ($\theta_{c.m.} > 90^{\circ}$) with particle escape from the TPC. (b) Scatter plot of E_x and E_x^{θ} . E_x^{θ} is the excitation energy of ⁶He obtained from the angles of the two reaction products.

1.797(25) MeV [49]. A fit to the spectrum gives excitation energies of -0.01(1) and 315 1.90(1) MeV, respectively, where the quoted errors are statistical. The FWHM resolu-316 tion is about 0.85 MeV. The fitting function consisted of a Gaussian function for the ground 317 state, a Voigt function [50] for the unbound 2^+ state, and background from the breakup 318 reaction of ⁶He involving the final state of an α particle and 2 neutrons. The Voigt function 319 is a convolution of a Breit-Wigner function and a Gaussian, which allows the natural width 320 of an unbound state and the resolution of the detection system to be taken into account. 321 The width of the 2^+ state was set to the adopted value, 0.113(20) MeV [49]. The background 322 shape was simulated by the Monte-Carlo method, where the α particle and neutrons in the 323

final state were generated in a uniform phase space. The procedure of identification based on the comparison between a pair of $E_{\rm x}^{(i)}$ was taken into account. The best-fit curve shown in the figure well reproduces the experimental data. The background of the breakup reaction denoted by the thick line accounts for the tail towards higher energies.

In the present analysis, TKE for an input to the missing-mass method was deduced 328 from the slope of the Bragg curve when recoiling α particles escape from the active region 329 and hence the Bragg peak is unavailable. Without the precise position information of the 330 Bragg peak, the analysis might result in inaccurate excitation energies. The gated spectrum 331 for one-particle escape events is shown in the shaded spectrum of Fig. 5(a). The spectrum 332 clearly shows a two peak structure, confirming good reconstruction of the reaction kinematics. 333 To further confirm the results, the adopted excitation energies are compared to excitation 334 energies (E_x^{θ}) that were obtained from the correlation of laboratory angles, $\theta_{lab}^{(1)}$ and $\theta_{lab}^{(2)}$ 335 between the two reaction products. This method does not require the knowledge of the 336 TKE of reaction products. Thus we can correctly extract excitation energies from the 337 escape events as long as the scattered ⁶He particle is stable against particle emission and 338 keeps its initial angle unchanged after scattering. The scatter plot of $E_{\mathbf{x}}^{\theta}$ against $E_{\mathbf{x}}$ is shown 339 in Fig. 5(b). A gate was set to select backward scattering angles of $\theta_{\rm c.m.} > 90$ degrees, where 340 most of the escaping particles are recoiling α nuclei. The presence of a cluster centered at 341 $E_{\rm x}^{\theta} = E_{\rm x} = 0$ MeV indicates that most of the events involving the ground state have $E_{\rm x}$ 342 well correlated with $E_{\rm x}^{\theta}$. The ratio of the number of events in the cluster is nearly 90% with 343 respect to that of the peak in the shaded spectrum in Fig. 5(a), limiting the systematic 344 uncertainty in yields to 10%. The locus for the 2^+ state widely spreads along the vertical 345 axis of $E_{\mathbf{x}}^{\theta}$ because the angles of scattered particles shift after the 2n emission. 346

347 B. Two-neutron emission channel

To investigate branching to the two-neutron emission ${}^{6}\text{He}(\alpha, 2n)^{8}\text{Be}$ channel, the trigger was set for reactions having a peak in the energy loss profile which was taller than the Bragg peak of the ${}^{6}\text{He}$ beam particles (Trigger 2). In the data analysis a peak was observed for Bragg amplitudes that corresponded closely to 4.5 MeV mg⁻¹ cm², the value expected for two highly-correlated α particles, such as originating from the decay of ${}^{8}\text{Be}$. Since this amplitude is lower than 5.9 MeV mg⁻¹ cm² for ${}^{9}\text{Be}$ ions with a higher Z recoiling from



FIG. 6. (Color online) Total charge collected Q_{total} as a function of the location of the reaction vertex z_{reac} . (a) Experimental data. The elastic events are clearly distinguishable near $Q_{\text{total}} = 15$ MeV. Events above the locus of elastic scattering indicate pile up of beam particles. Only events inside the contour line were used in calculating the excitation function. (b) Simulated results for the decay into the 0⁺ ground state and (c) the 2⁺ state at 3.03 MeV of ⁸Be.

the ${}^{6}\text{He}(\alpha, n){}^{9}\text{Be}$ reaction, we concluded the main decay channel that contributed was ${}^{10}\text{Be} \rightarrow 2n + {}^{8}\text{Be}$, with ${}^{8}\text{Be}$ unbound to α decay by 92 keV. Beam contaminants were eliminated offline by comparing the particle's energy loss profile near the beginning of its track to the experimentally-measured energy-loss profile for ${}^{6}\text{He}$. The location of the reaction vertex z_{reac} was determined from the time where the enhancement of the energy deposits starts due to the creation of reaction products. Since the beam particles and the charged

reaction products were stopped in the detector, we could use the total charge collected in the 360 detector (Q_{total}) as the energy sum signal, which was obtained by summing all anode signals. 361 Q_{total} is plotted as a function of the location of the reaction vertex z_{reac} in Fig. 6(a). The total 362 charge is converted to energy by normalizing the experimental data to the allowed energies 363 from a kinematical simulation of the sequential decay of ${}^{10}\text{Be} \rightarrow 2n + {}^{8}\text{Be} \rightarrow 2n + 2\alpha$ 364 described later. The events shown are selected by the Bragg amplitude for two- α correlated 365 events. The events in the $z_{\text{react}} < 50 \text{ mm}$ region were rejected because it was difficult to 366 distinguish ⁸Be events from beam contaminants, particularly ⁷Li, when the incident energy 367 loss curve was relatively short. As can be seen in Fig. 6(a), there is an accumulation of 368 events near 15 MeV for the charge collected for full beam energy deposition corresponding 369 to elastic scattering. Below this line is a broad range of energies deposited in the detector. 370 These events are attributed to the ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$ reaction since the neutrons were not 371 detected in our experiment and the energy taken away by them will be missing in the total 372 energy signal. We simulated events corresponding to the decay of ¹⁰Be into ⁸Be with the 373 unobserved two neutrons, for which we assumed isotropic emission in the center-of-mass 374 system and phase-space sharing of the available energy between the two neutrons. It is also 375 assumed that the decay of ${}^{10}\text{Be} \rightarrow 2n + {}^{8}\text{Be} \rightarrow 2n + 2\alpha$ is sequential. The decay into the 376 2^+ state at 3.03 MeV of ⁸Be, which is energetically allowed at higher energies, is simulated 377 as well. The results for the ground state and the 2^+ state are shown in Fig. 6(b) and (c), 378 respectively. The kinematical simulation for the ground state is seen to agree well with the 379 data, confirming the interpretation of the events. The simulation also served as a guide 380 for defining the contour line seen in Fig 6(a). Only the events inside the contour line were 381 subsequently used to calculate the excitation function. 382

383 IV. RESULTS

³⁸⁴ A. Elastic and inelastic scattering

To obtain excitation functions and angular distributions, cross sections $(d\sigma/d\Omega)$ were deduced for a given bin of the reaction position z_{reac}^i and the scattering angle $\theta_{\text{c.m.}}^j$ specified by the indices (i, j), respectively. Yields $(Y^{(i, j)})$ of the ground state and the 2⁺ state were obtained by fitting excitation energy spectra using the same function as adopted for the fit to the angle-integrated spectrum of Fig. 5(a). $Y^{(i, j)}$ was then translated into $d\sigma/d\Omega^{(i, j)}$ following the expression:

$$d\sigma/d\Omega^{(i, j)} = Y^{(i, j)} / \left(\epsilon_{eff}^{(i, j)} \cdot \epsilon_{\text{DAQ}} \cdot N_{^{6}\text{He}} \cdot N_{\text{He}}^{(i)} \cdot \Delta\Omega^{(j)} \right),$$
(2)

where the indices i and j denote that the associated parameter depends on z_{reac} and $\theta_{\text{c.m.}}$, 391 respectively. The detection efficiency $\epsilon_{eff}^{(i, j)}$ was estimated by simulations, where the geome-392 tries of the detector, the condition of the trigger, and the energy losses of beam and reaction 393 products were taken into account. The live-time ratio of the data acquisition ϵ_{DAQ} was 70% 394 on average. The integrated count of 6 He beam particles N_{6}_{He} was estimated using the counts 395 of micromesh signals measured by a scaler. This was about 2×10^8 after taking into account 396 the beam purity of 90%. The number of He atoms in the gaseous target $N_{\text{He}}^{(i)}$ was calculated 397 from the density of the He:CO₂ 90:10 gas at atmospheric pressure (0.33 mg/cm^3) and the 398 bin size of z_{reac} . The solid angle $\Delta \Omega^{(j)}$ was calculated from the bin size of $\theta_{\text{c.m.}}$. 399

The resulting excitation functions for elastic scattering are shown in Fig. 7. The beam 400 energy in the laboratory frame, E_{reac} , was converted into the center-of-mass energy by 401 $E_{\text{c.m.}} = 0.4 \times E_{\text{reac.}}$ Given the separation threshold of an α particle at 7.42 MeV [48], 402 $E_{\rm c.m.}$ is related to the excitation energy in ¹⁰Be by $E_{\rm x} = E_{\rm c.m.} + 7.42$ MeV. The systematic 403 error of $E_{\rm c.m.}$ is estimated to be ± 0.1 MeV, which arises from the deviation of $E_{\rm reac}$ with 404 respect to the energy sum of the ⁶He and α particles from elastic scattering. A strong en-405 hancement of cross sections is observed around 2.7 MeV in the angle-integrated spectrum of 406 $\theta_{\rm c.m.} = 65^{\circ} - 135^{\circ}$ in Fig. 7(a). A peak consistently exists around this energy in the different 407 angular regions shown in Figs. 7(b)-(h), indicating the presence of a resonance. A fit with a 408 Voigt function [50] and a linear background gives a resonance energy of $E_{\rm c.m.} = 2.57(15)$ MeV, 409 where the quoted uncertainty is the sum of the statistical and systematic errors. This is 410 consistent with the resonance of the state at $E_x = 10.15(2)$ MeV, or $E_{c.m.} = 2.73(2)$ MeV, 411 observed in a previous measurement of elastic scattering [32]. No peak is visible in the 412 excitation functions except for this strong resonance. The absence of sizable resonances 413 indicates that no other states have a large decay width for α emission within the energy 414 window of the present study $(E_{\text{c.m.}} = 2 - 6 \text{ MeV}).$ 415

The excitation functions for inelastic scattering are shown in Fig. 7. A sizable decay width to the 2⁺ state of ⁶He was inferred for the 2.73-MeV resonance in the analysis of the previous study [32]. However, this could not be measured in the present study due to



FIG. 7. (Color online) Excitation functions of ${}^{6}\text{He} + \alpha$ scattering: (a) angle-integrated spectrum over $\theta_{\text{c.m.}} = 65^{\circ} - 135^{\circ}$, and (b-h) spectra for 10° angular bins. The data for elastic scattering are shown by the filled circles, while those for inelastic scattering to the 2⁺ state are shown by the open circles. The inelastic scattering data are scaled by a factor of 1.5. The dashed and dotted lines denote $E_{\text{c.m.}} = 2.7$ and 4.4 MeV, respectively.



FIG. 8. (Color online) Angular distributions of elastic ⁶He + α scattering. The results of CRC calculations (solid lines) are compared to the data. The CRC results at the higher energy bins are also shown for reference with the dashed lines. The blue shaded areas denote the systematic errors from the ambiguities in the beam angle. The inset (h) shows the data at $E_{\rm c.m.} = 2.7$ (full circles) and 3.3 MeV (open circles) on a linear scale. The angles where the Legendre polynomials $P_L(\cos \theta_{\rm c.m.})$ for L = 4 and 6 become zero are denoted by the solid and dashed lines, respectively.



FIG. 9. (Color online) Angular distributions of inelastic ⁶He + α scattering to the ⁶He 2⁺ state. The results of CRC calculations (solid lines) are compared with the data. The blue shaded areas denote the systematic errors from the ambiguities of the beam angle.

the lack of acceptance at this energy. While there are no resonances as strong as the one 419 observed in the elastic channel, a small peak is visible at 4.4 MeV in the inelastic channel 420 consistently over the angular region of $\theta_{c.m.} = 65^{\circ} - 105^{\circ}$, which might be due to a resonance. 421 The differential cross sections were deduced for eight different incident energies to study 422 angular distributions. The resulting data for elastic scattering are displayed in Fig. 8. The 423 vertical error bars are only statistical, and the horizontal bars denote the size of the angular 424 bins. The leading contributions of the systematic error are the uncertainty of yields for 425 the escape events (10%) and the detection efficiency. The latter mostly comes from the 426 uncertainties of the beam angle, which could be inclined by about 1 degree with respect to 427 the central axis. The blue shaded areas in Figs. 8 denote the variation of cross sections when 428

the incident beam angle varies by 1 degree. As seen in the figures, a series of differential 429 cross sections for elastic scattering show a gradual and continuous change in their angular 430 distributions as a function of $E_{\text{c.m.}}$ until the cross sections are enhanced at $E_{\text{c.m.}} = 2.7$ MeV. 431 In the inset of Fig. 8(h), the on-resonance data at $E_{\rm c.m.} = 2.7$ MeV (filled circles) are 432 compared to the off-resonance data at $E_{\rm c.m.} = 3.3$ MeV (open circles) shown on a linear scale. 433 At $E_{\rm c.m.} = 2.7$ MeV, the cross section clearly peaks at 90° and dips at 60° and 110°. In 434 contrast, the data at $E_{\rm c.m.} = 3.3$ MeV lack such a steep rise and drop, indicating that the peak 435 and dip structures seen in the 2.7-MeV data are due to the resonance. Angular distributions 436 of resonance scattering primarily follow the square of the Legendre polynomial $[P_L(\cos\theta_{c.m.})^2]$ 437 corresponding to the angular momentum (L) involved. The diffractive pattern observed is 438 therefore a useful measure to identify L. The presence of a peak at 90° rules out the odd 439 angular momenta, of which the corresponding Legendre polynomials always become zero at 440 this angle. The dips at 70° and 110° more agree with the polynomial of L = 4 than that of 441 L = 2 having zeros at 55° and 125°, or L = 6 at 76° and 104°. The dip angles of L = 4, 6442 are shown for reference by the solid and dashed lines, respectively, in the inset of Fig. 8(h). 443 The diffractive pattern of the experimental data is seen closer to that of L = 4 and clearly 444 deviated from that of L = 6 despite the larger angular bins and the lower statistics compared 445 to a previous measurement [32]. We therefore assign L = 4 for the resonance observed. Since 446 the initial and final states involve spinless particles only, the spin-parity of the corresponding 447 resonant state in ¹⁰Be is assigned as 4⁺. The present results thus confirm the resonance of 448 the 4⁺ state at $E_{\rm x} = 10.15$ MeV in ¹⁰Be observed in a previous measurement of elastic 449 $^{6}\text{He} + \alpha$ scattering [32], and rule out the spin-parity of 3⁻ claimed in an angular correlation 450 study using the ⁷Li + ^{6,7}Li reactions [30]. The angle-integrated cross section ($\sigma_{4\pi}$) can be 451 translated into the partial α decay width (Γ_{α}) via the relation [52]: 452

$$\sigma_{4\pi} = (2L+1)\frac{\pi}{k^2} \frac{\Gamma_{\alpha}^2}{(E_{\text{c.m.}}^0 - E_{\text{c.m.}})^2 + (\Gamma/2)^2},\tag{3}$$

with k being the wavenumber, $E_{c.m.}^0$ the resonance energy, and Γ the total decay width. The total width of 296(15) keV measured in a previous study [30] was adopted for Γ . $\sigma_{4\pi}$ was estimated from the cross section of the resonance in the angular range $\theta_{c.m.} = 65^{\circ} - 135^{\circ}$ in Fig. 7(a) given the $P_4(\cos \theta_{c.m.})^2$ dependence of the angular distribution. The resulting ratio of Γ_{α}/Γ is 0.49(5), which corresponds to $\Gamma_{\alpha} = 145(15)$ keV. This Γ_{α}/Γ ratio agrees with 0.45(3) from the previous study [32].

$E_{\rm c.m.}$ (MeV)	$N_{ m R}$	$W(r) ({ m MeV})$	r_W (fm)	a_W (fm)	$\mathrm{SA}_{\mathrm{g.s.}}$
5.82	1.0	60.0	1.26	0.27	0.80
5.45	1.0	60.0	1.27	0.25	0.80
5.06	1.0	60.0	1.27	0.25	0.80
4.64	1.0	60.0	1.27	0.25	0.80
4.21	1.0	60.0	1.27	0.25	0.80
3.76	1.0	60.0	1.28	0.25	0.80
3.26	1.0	65.0	1.28	0.32	0.80
2.69	1.0	65.0	1.38	0.20	0.80

TABLE I. Best-fit values of the optical model potential parameters and the $\langle {}^{6}\text{He}|^{4}\text{He} + 2n \rangle$ overlap spectroscopic factor. See text for details.

To assess the contribution of direct reactions, coupled reaction channels (CRC) calculations were performed using the code FRESCO [53]. These calculations are similar to those of Khoa and von Oertzen [40], although some simplifications were made in light of their results. We adopted the same basic model as Ref. [40] for the optical potentials, i.e. a double-folded real potential and a Woods-Saxon imaginary potential, giving a potential of the form:

$$U(r) = N_{\rm R} V_{\rm DF}(r) + i W(r) + V_{\rm C}(r),$$
(4)

where $V_{\rm C}(r)$ is the usual Coulomb potential with radius $R_{\rm C} = 1.25(A_1^{1/3} + A_2^{1/3})$ in fm, 464 $V_{\rm DF}(r)$ is the double-folded real potential with normalization parameter $N_{\rm R}$ and W(r) is the 465 imaginary potential of depth W in MeV, radius $R_{\rm W} = r_W (A_1^{1/3} + A_2^{1/3})$ in fm and diffuseness 466 a_W in fm. The parameters N_R , W(r), r_W and a_W were adjusted to obtain the best agreement 467 with the data at each energy. The double-folding potentials were calculated with the code 468 DFPOT [54] using the M3Y effective interaction [55]. The ⁴He nuclear-matter density was 469 derived from the three-parameter Fermi distribution charge density of Ref. [56] by unfolding 470 the proton charge distribution as described in Ref. [57], assuming that $\rho_n = (N/Z)\rho_p$, and 471 the ⁶He nuclear-matter density was the FC6 model density of Ref. [58]. Inelastic excitation 472 of the 1.8-MeV 2^+ state of ⁶He was included in a similar way as detailed in Ref. [40]. The 473 real part of the inelastic scattering form factor was calculated using the double-folding 474 model, the proton and neutron parts of the transition density being calculated using the 475

Tassie Model [59], and the proton and neutron deformation lengths taken from Ref. [40]. 476 The imaginary part of the inelastic form factor was obtained by deforming the Woods-477 Saxon imaginary potential using the isoscalar deformation length of Ref. [40]. Following the 478 conclusions of Ref. [40], we adopted the simplifying assumption of a pure $(1p_{3/2})^2$, S-wave 479 (J = L = 0) configuration for the two neutrons in the ⁶He ground state, where J is the 480 internal angular momentum of the 2n cluster and L its orbital angular momentum with 481 respect to the ⁴He core. The $\langle {}^{6}\text{He} | {}^{4}\text{He} + 2n \rangle$ overlaps were calculated using the Bayman-482 Kallio method [60] and the spectroscopic amplitude was adjusted to give the best description 483 of the data. Also following the conclusions of Ref. [40], the two-step sequential (⁶He, ⁵He; ⁵He, 484 ⁴He) and indirect (⁶He, ⁶He_{2⁺}; ⁶He_{2⁺}, ⁴He) transfer paths were omitted as having negligible 485 influence at the incident energies concerned. Test calculations for the $E_{\rm c.m.} = 5.82$ MeV data 486 supported this conclusion. Values of $N_{\rm R}$, W(r), r_W , a_W and the spectroscopic amplitude for 487 the $\langle {}^{6}\text{He} | {}^{4}\text{He} + 2n \rangle$ overlap SA_{g.s.} are given in Table I. The best-fit value of SA_{g.s.} is somewhat 488 smaller than the expected value of about 1.25 quoted in Khoa and von Oertzen [40], but is 489 close to 0.85, the smallest empirical value they obtained. 490

The calculated differential cross sections of elastic scattering are compared to the exper-491 imental data in Fig. 8. To display the variation of the angular distribution as a function of 492 energy, the CRC calculations for the previous energy bins are shown together by the dashed 493 lines. Except for the last energy bin at 2.7 MeV, the calculated cross sections reproduce the 494 data. The calculated cross sections vary smoothly as a function of $E_{\text{c.m.}}$, while the reaction 495 parameters are nearly constant. At 2.7 MeV, the cross sections are suddenly enhanced and 496 the radius and diffuseness parameters of the imaginary potential drastically change. This 497 result indicates that elastic scattering predominantly proceeds via direct reactions outside 498 of the region near $E_{\rm c.m.} = 2.7$ MeV. It supports the conjecture made from the excitation 499 functions that there is no sizable resonance in the region from 9.4 to 13.4 MeV in $E_{\rm x}$ of ¹⁰Be 500 except for the 4^+ state at 10.15 MeV. 501

The resonance of another 4⁺ state at 11.76 MeV, therefore, was not identified at the corresponding energy of $E_{\rm c.m.} = 4.4$ MeV. The upper limit of the partial decay width was estimated by searching the limit where the resonance can be identified as a peak over the contribution of non-resonant scattering in the spectrum. The resulting value is $\Gamma_{\alpha}/\Gamma < 0.15$, or $\Gamma_{\alpha} < 20$ keV for the known Γ value of 121(10) keV [51].

⁵⁰⁷ The differential cross sections of inelastic ⁶He + α scattering to the ⁶He 2⁺ state are

shown in Fig. 9 together with the CRC calculations denoted by the solid lines. The CRC 508 calculations for the magnitude of the cross sections are in good agreement with the experi-509 mental data, suggesting the validity of the framework of reaction analysis and the adopted 510 optical-model potential parameters. The diffractive pattern is slightly out of phase with 511 respect to the CRC predictions in the energy bin of $E_{\rm c.m.} = 4.2$ MeV, where a resonance is 512 inferred from the excitation function spectra. The experimental angular distribution peaks 513 at 90° and symmetrically decreases towards 60° and 120° . This is characteristic of an angular 514 momentum of L = 2. 515

516 B. Two-neutron emission channel

The angle-integrated excitation function was deduced for the ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$ channel. 517 The reaction vertex was converted to reaction energy E_{reac} after correcting for the energy 518 loss using a program based on the SRIM code [47]. The cross section of the reaction as 519 a function of center-of-mass energy $E_{\rm c.m.}$ is shown in Fig. 10. Note that above 3 MeV, 520 the spectrum includes also the contribution from the ${}^{8}\text{Be} 2^{+}$ state. The contribution of 521 the ground state, however, more likely dominates as the z_{reac} - Q_{total} data (Fig. 6(a)) agree 522 well with the simulation for the ground state (Fig. 6(b)). These two components would be 523 differentiated by analyzing the correlations between two α particles that were not measured 524 in this study. The vertical error bars are statistical only, while the horizontal error bars 525 denote the sum of statistical and systematic errors. The latter comes from the ambiguity in 526 locating the reaction vertex which is estimated to be ± 10 mm. The cross section increases 527 from the reaction threshold of the ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$ reaction at 1.06 MeV toward higher energies. 528 It is clear that no significant resonance strength exists in the energy region covered in 529 the present study. There is a minor peak structure at 2.5 MeV. It is difficult to judge 530 whether this is due to a resonance or to a statistical fluctuation, particularly without the 531 corresponding information from an angular distribution. In the former case, it would be 532 possible that it originates from the 10.15-MeV 4^+ state with nearly the same resonance 533 energy. A fit with a Voigt function [50] was made to estimate the possible partial width 534 $\Gamma_{^{8}\text{Be}}$. Three different backgrounds, namely linear, quadratic, and exponential functions, were 535 tested. The resonance energy was set to the result from the elastic channel (2.56 MeV) and 536 varied within the error (0.15 MeV), while the experimental resolution was fixed to 0.25 MeV 537

rms, which arises from the uncertainty in reaction energy (0.1 MeV) and that in vertex determination (0.2 MeV). The resulting $\Gamma_{^{8}\text{Be}}/\Gamma$ value is 0.09(5) and this gives an upper limit of $\Gamma_{^{8}\text{Be}}/\Gamma \sim 0.15$ for this possible decay branch.



541

FIG. 10. Angle-integrated excitation function for the ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$ channel.

542 V. DISCUSSION

The present study identified a 4⁺ state with a large α decay width $\Gamma_{\alpha}/\Gamma = 0.49(5)$ at 543 9.98(15) MeV in ¹⁰Be. The observed state most likely corresponds to the known 4⁺ level 544 at 10.15(2) MeV [31, 32] given the observed excitation energy and spin-parity. In previous 545 studies [24, 32, 33], this state is considered the 4^+ member of a rotational band built on the 546 second 0^+ state at 6.1793(7) MeV [51]. The excitation energies of ¹⁰Be states are plotted 547 against J(J+1) in Fig. 11. The linear extrapolation from the 0^+_2 state and the 2^+ state 548 at 9.560(20) MeV [51] indeed nicely agrees with the 10.15-MeV state in energy. The large 549 moment of inertia from the narrow level spacing of the band members is well explained by 550 the σ -type molecular orbital structure from both cluster-model calculations [21, 22, 25] and 551 microscopic calculations based on the antisymmetric molecular dynamics (AMD) method 552 [15, 24, 26]. In this picture, the valence neutrons are delocalized over the two clusterized α 553 cores and the extension along the α cores' axis gives strong deformative characteristics to 554 ¹⁰Be. The large decay width for α emission indicates a high degree of clusterization in this 555



FIG. 11. (Color online) Plot of $E_x vs. J(J+1)$ for ¹⁰Be. The band members of the ground and the second 0⁺ states are shown by the circles and squares, respectively. The linear extrapolation using the 0⁺ and 2⁺ states is shown for each band. The horizontal lines at J = 4 denote predicted level energies of the 4⁺ member of the ground state band from the β - γ constraint AMD method [24] (solid line), the variational AMD method [15] (dashed line), the four-body cluster model [21] (dotted line), the molecular orbital model [16] (dot-dashed line), the semi-microscopic algebraic cluster model [18], (double-dot-dashed line), and the multi-cluster generator coordinate method [19], (triple-dot-dashed line). The data of Refs.[16, 21] were obtained from the calculated values with respect to the threshold energy of $2\alpha + 2n$ at 8.386 MeV. The shaded area denotes the energy domain covered by the present study.

⁵⁵⁶ 4⁺ state and supports this type of cluster structure. An α spectroscopic factor of 3.1(2) is ⁵⁵⁷ estimated in a recent analysis of the measured partial width [61]. This value is as large ⁵⁵⁸ as the spectroscopic factors of about 1.5 for the ground-state band members of ⁸Be with ⁵⁵⁹ well-developed two α clusters [61, 62].

In addition to the 0_2^+ state, theoretical studies [15, 16, 22, 24] predict a π -type cluster 560 structure for the 0^+ ground state, in which valence neutrons are extending perpendicular 561 to the axis of the two α cores. Given the 2⁺ state at 3.37 MeV, the 4⁺ state of the $0^+_{\sigma,s}$ 562 band is anticipated at around 11 MeV as seen in the linear extrapolation shown in Fig. 11. 563 In previous studies [24, 33], the 4⁺ state at 11.76(2) MeV is considered the most likely 564 candidate for the 4^+ member of the $0^+_{g.s.}$ band because of its excitation energy and spin-parity. 565 In the present study, however, there was no resonance observed around $E_x = 11.8$ MeV 566 $(E_{\rm c.m.} = 4.4 \text{ MeV})$. This is in stark contrast with the significant resonance strength of the 567 4^+ state of the 0^+_2 band at 10.2 MeV. The α decay width of the 11.8-MeV state is estimated 568 less than 20 keV and is much smaller than $\Gamma_{\alpha} = 145(15)$ keV deduced for the 10.2-MeV 569 state. Such a difference is unexpected as both 4^+ states belong to the rotational bands of 570 the clusterized 0^+ states. Nearly the same spectroscopic amplitudes of ${}^{6}\text{He} + \alpha$ are predicted 571 for these 4⁺ states in the microscopic $2\alpha + 2n$ four-cluster-model [21]. The present result 572 does not agree with this prediction. The small spectroscopic amplitude of the 4⁺ member is 573 also unlike the ground state 0^+ band of ⁸Be, despite what appears to be a similar moment 574 of inertia. The α spectroscopic factors are predicted to be equally large in all 0^+ , 2^+ , and 575 4^+ states in ⁸Be [62], which is supported by the folding potential model that well describes 576 the level energies and widths of these states [63]. 577

There are two possible scenarios to account for the hindered strength of the 4^+ member 578 of the $0^+_{g.s.}$ band. First is the possibility that the 4⁺ state at 11.8 MeV does not belong to the 579 $0^+_{\rm g.s.}$ band, and the real band member exists outside the energy window of the present study 580 $(E_{\rm c.m.} = 2-6 \text{ MeV or } E_{\rm x} = 9.4-13.4 \text{ MeV})$. This scenario implies an unusual level spacing 581 for the ground state band. On the contrary, regardless of the framework, most theoretical 582 studies [15, 16, 18, 19, 21, 24] predict the 4^+ state of the $0^+_{g.s.}$ band in the energy range 583 $E_{\rm x} = 10-13$ MeV (Fig. 11), the region anticipated from the proportionality to J(J+1). 584 The second scenario is that the 4^+ state does belong to the $0^+_{g.s.}$ band, but with a reduced 585 degree of clusterization that hinders the decay branch for α emission. The weakening of 586 clustering in the 4^+ state is pointed out by an early AMD study of ¹⁰Be [15]. In this 587 prediction, the 0^+ ground state of 10 Be is highly clusterized, as confirmed in another later 588 AMD study [64]. However, the component of two- α cores in the rotational band members 589 is gradually reduced as the total spin increases. The origin of the dissociation of α clusters 590 is attributed to the nuclear spin-orbit force. A stronger spin-orbit force yields a smaller 591

amplitude of two α cores in the 2⁺ and 4⁺ states of the 0⁺_{g.s.} band. This scenario naturally 592 explains why the four cluster model of $2\alpha + 2n$ overpredicts the spectroscopic amplitude of 593 ⁶He + α . In the molecular model, α clusters are assumed as the basis of wave functions, 594 and thus the model does not incorporate the effects of their dissociation. On the other hand, 595 the AMD method adopts Slater determinants where the spatial part of each single-particle 596 wave function is a Gaussian function, and does not require the assumption of inert cluster 597 cores [65]. This allows one to study the formation and dissociation of α clusters within one 598 framework. The absence of resonance strength of the 4⁺ state at 11.8 MeV agrees more with 599 the interpretation of AMD calculations, which predict the limits of α clustering in higher 600 spin members of the $0^+_{g.s.}$ band due to the spin-orbit force. 601

It is known that the spin-orbit force dissociates α clusters in high-spin states of ²⁰Ne [66– 602 68] or ⁴⁴Ti [67]. Such an effect is considered to be hindered in ⁸Be as it is a lighter nucleus 603 with a weaker spin-orbit force [67]. On the other hand, recent AMD studies on ${}^{12}C$ [64, 69]. 604 an isotone of ¹⁰Be, point out that the effect of the spin-orbit force is pronounced by the 605 sub-shell closure at N = 6 and this explains the small degree of clusterization for its ground 606 state. In ¹⁰Be, this effect might be more important for the $0^+_{g.s.}$ band, which primarily 607 consists of p-shell configurations [15], than for the intruder 0_2^+ band. The present result for 608 ¹⁰Be implies a delicate competition between the persistence of the clusters in ⁸Be and their 609 dissociation by the spin-orbit force as the number of neutrons increases toward the neutron 610 drip line. 611

Finally, we discuss the present results in comparison with theoretical calculations of res-612 onant α scattering of ⁶He in the framework of the four-body cluster model [14, 23]. The 613 study using the resonating-group method predicts drastic changes in the scattering phase 614 shift of L = 3 at $E_{\text{c.m.}} = 3.7$ MeV and L = 0 around 4.5 MeV for elastic scattering [14]. The 615 angular distribution at 3.7 MeV is predicted to show enhanced cross sections with an oscil-616 lation characteristic of L = 3. However, in the present measurement, sizable resonances are 617 clearly absent in the corresponding energy region in the elastic channel. Inelastic scattering 618 to the ${}^{6}\text{He} 2^{+}$ state was studied in a more recent work based on the generalized two-center 619 cluster approach, where the reaction of ${}^{6}\text{He} + \alpha$ and the structure of ${}^{10}\text{Be}$ are described in a 620 unified manner [23]. Excitation functions for L = 1, 2 were calculated and a few resonances 621 are proposed in the energy region of $E_{\rm c.m.} = 2-7$ MeV. The possible resonance of L = 2622 inferred at 4.4 MeV in the present study is in line with the L = 2 resonance predicted at 623

⁶²⁴ 4 MeV. It originates from a 0⁺ state in ¹⁰Be with a large amplitude of the α + ⁶He(2⁺) com-⁶²⁵ ponent, thus having a sizable strength in the inelastic channel. Further theoretical studies, ⁶²⁶ particularly on excitation functions for the elastic channel that are not presented in Ref. [23], ⁶²⁷ might be useful to interpret the inferred resonance.

628 VI. CONCLUSION

Scattering of neutron-rich ⁶He nuclei on α particles was studied at the TwinSol facil-629 ity [42] at the University of Notre Dame. A time projection chamber, PAT-TPC [43], using 630 $He:CO_2$ 90:10 gas at atmospheric pressure was operated in the active-target mode to pro-631 vide a gaseous ⁴He target and track the beam and reaction products traversing the target. 632 This unique capability significantly lowered the threshold for the energy of helium nuclei. 633 allowing a continuous measurement of the excitation functions over a wide energy range 634 with the unambiguous reconstruction of angular distributions, which are difficult to obtain 635 in conventional studies using non-active targets. 636

Excitation functions and angular distributions were thus measured for elastic scattering 637 and inelastic scattering to the ${}^{6}\text{He} 2^{+}$ state below a center-of-mass energy of 6 MeV. The 638 present system also allowed us to measure the excitation function of the ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$ 639 channel, which requires the detection of low-energy α particles following the decay of un-640 bound ⁸Be. The resulting excitation function of elastic scattering shows a resonance at 641 $E_{\rm c.m.} = 2.56(15)$ MeV, or an excitation energy $E_{\rm x} = 9.98(15)$ MeV in ¹⁰Be. A spin-parity 642 of 4^+ was assigned from the angular distribution. The partial α decay width was estimated 643 to be $\Gamma_{\alpha}/\Gamma = 0.49(5)$. These results obtained from the unambiguously-identified recoiling 644 α particles confirm the previous results obtained for the state at 10.15(2) MeV without 645 differentiating the ⁶He and α particles, but with considerably better statistics [32]. The 646 assignment of 4^+ supports the claim of Ref. [32] that the 10.15-MeV state corresponds to a 647 member of the 0^+_2 band. 648

The large partial width for α decay indicates a highly-developed α cluster structure in this 4⁺ state. It is consistent with most theoretical predictions proposing the σ -type molecular orbital structure for the 0^+_2 band. On the contrary, no resonance strength for another 4⁺ state was observed in this energy region, where the 4⁺ member of the ground state 0⁺ band is expected. Except for the region near $E_{\rm c.m.} = 2.6$ MeV, the angular distributions vary

smoothly as a function of energy, which is well reproduced by coupled-reaction channels 654 calculations. The hindered resonance strength is incompatible with theoretical calculations 655 done in the framework of the four-body cluster model, which predict a sizable spectroscopic 656 amplitude of ⁶He + α due to a π -type molecular orbital structure [21]. However, it is in 657 line with the results from an AMD study where the α clusters of the 0⁺ ground state are 658 gradually dissociated by the nuclear spin-obit force as the spin increases to its 2^+ and 4^+ 659 rotational band members [15]. The present results support the limits of clustering in ${}^{10}\text{Be}$ 660 due to the spin degree-of-freedom, and calls for more detailed spectroscopy of individual 661 cluster states in ¹⁰Be and related microscopic theoretical studies. 662

The excitation function of the inelastic channel shows a slight enhancement at $E_{\text{c.m.}} = 4.4 \text{ MeV}$ that may be due to a resonance. An angular momentum of L = 2 is suggested from the angular distribution. This resonance may be related to a 0⁺ state with a large ${}^{6}\text{He}(2^{+}) + \alpha$ component, which is predicted around 4 MeV [23].

The angle-integrated excitation function was obtained for the ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$ channel, 667 with no strong resonance observed in the region of $E_{\rm c.m.} = 1-5$ MeV. While the cross sec-668 tion is slightly enhanced around $E_{\text{c.m.}} = 2.5 \text{ MeV}$, it is uncertain whether this is a statistical 669 fluctuation or a possible resonance originating from the 4^+ state at 10.15 MeV. As mea-670 sured in this study, this 4⁺ state has a large partial decay width of $\Gamma_{\alpha}/\Gamma = 0.49(5)$ to the 671 ${}^{6}\text{He}(0^{+}) + \alpha$ final state, which is considered to be an indication of α clustering. The result of 672 the ${}^{6}\text{He}(\alpha, 2n){}^{8}\text{Be}$ channel, in contrast, gives an upper limit of $\Gamma_{{}^{8}\text{Be}}/\Gamma \sim 0.15$ for the decay 673 to ⁸Be, a nucleus with a pronounced 2α structure. It will be interesting to see if the partial 674 width to such a three body system (⁸Be + 2n) is sensitive to various α -cluster structures 675 predicted for ¹⁰Be. 676

677 VII. ACKNOWLEDGEMENT

We would like to thank J. Yurkon of the NSCL for his technical support on the PAT-TPC. We would also like to thank the staff of the Nuclear Science Laboratory (NSL), University of Notre Dame for assistance in the operation of the FN Tandem accelerator. We are also grateful to the following staff members of IRFU, CEA-Saclay: D. Calvet, F. Druillole and A. Shebli for supporting us in operating the T2K electronics, and S. Aune, M. Anfreville and R. Durand for the fabrication of the Micromegas. One of the authors (D.S.) would like to thank R. Raabe for fruitful discussions. The present work was partly supported by the
US National Science Foundation under Grants No. MRI09-23087 and No. PHY09-69456.

- ⁶⁸⁶ [1] J.A. Wheeler, Phys. Rev. **52**, 1083 (1937).
- ⁶⁸⁷ [2] D.M. Dennison, Phys. Rev. **96**, 378 (1954).
- ⁶⁸⁸ [3] H. Morinaga, Phys. Rev. **101**, 254 (1956).
- ⁶⁸⁹ [4] F. Hoyle, Astrophys. J. Suppl. Ser. 1, 121 (1954).
- ⁶⁹⁰ [5] C.W. Cook, W.A. Fowler, C.C. Lauritsen, and T. Lauritsen, Phys. Rev. **107**, 508 (1957).
- ⁶⁹¹ [6] T. Wakasa *et al.*, Phys. Lett. B **653**, 173 (2007).
- [7] A.H. Wuosmaa, R.R. Betts, B.B. Back, M.Freer, B.G. Glagola, Th. Happ, D.J. Henderson, P.
- ⁶⁹³ Wilt, and I.G. Bearden, Phys. Rev. Lett. **68**, 1295 (1992).
- ⁶⁹⁴ [8] A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. Lett. 87, 192501 (2001).
- [9] Y. Funaki, T. Yamada, H. Horiuchi, G. Röpke, P. Schuck, and A. Tohsaki, Phys. Rev. Lett.
 101, 082502 (2008).
- ⁶⁹⁷ [10] T. Kawabata *et al.*, Phys. Lett. B **646**, 6 (2007).
- ⁶⁹⁸ [11] M. Seya, M. Kohno, and S. Nagata, Prog. Thor. Phys. **65**, 204 (1981).
- ⁶⁹⁹ [12] Y. Kanada-En'yo, H. Horiuchi, and A. Ono, Phys. Rev. C **52**, 628 (1995).
- ⁷⁰⁰ [13] W. von Oertzen, Z. Phys. A **354**, 37 (1996).
- [14] K. Fujimura, D. Baye, P. Descouvemont, Y. Suzuki, and K. Varga, Phys. Rev. C 59, 817
 (1999).
- ⁷⁰³ [15] Y. Kanada-En'yo, H. Horiuchi, and A. Dote, Phys. Rev. C **60**, 064304 (1999).
- ⁷⁰⁴ [16] N. Itagaki and S. Okabe, Phys. Rev. C **61**, 044306 (2000).
- ⁷⁰⁵ [17] Y. Ogawa, K. Arai, Y. Suzuki, and K. Varga, Nucl. Phys. A 673, 122 (2000).
- ⁷⁰⁶ [18] L. Hernández de la Peña, P.O. Hess, G. Lévai, and A. Algora, J. Phys. G 27, 2019 (2001).
- ⁷⁰⁷ [19] P. Descouvemont, Nucl. Phys. A **699**, 463 (2002).
- ⁷⁰⁸ [20] N. Itagaki, S. Hirose, T. Otsuka, S. Okabe, and K. Ikeda, Phys. Rev. C 65, 044302 (2002).
- ⁷⁰⁹ [21] K. Arai, Phys. Rev. C **69**, 014309 (2004).
- ⁷¹⁰ [22] M. Ito, K. Kato, and K. Ikeda, Phys. Lett. B **588**, 43 (2004).
- ⁷¹¹ [23] M. Ito, Phys. Lett. B **636**, 293 (2006).
- ⁷¹² [24] T. Suhara and Y. Kanada-En'yo, Prog. Theo. Phys. **123**, 303 (2010).

- ⁷¹³ [25] N. Itagaki, S. Okabe, and K. Ikeda, Phys. Rev. C **62**, 034301 (2000).
- ⁷¹⁴ [26] Y. Kanada-En'yo and H. Horiuchi, Phys. Rev. C 68, 014319 (2003).
- ⁷¹⁵ [27] D. Suzuki *et al.*, Phys. Rev. Lett. **103**, 152503 (2009).
- ⁷¹⁶ [28] D. Suzuki, Eur. Phys. J A 48, 130 (2012).
- ⁷¹⁷ [29] N. Soić *el al.* Europhys. Lett. **34**, 7 (1996).
- [30] N. Curtis, D.D. Caussyn, N.R. Fletcher, F. Maréchal, N. Fay, and D. Robson, Phys. Rev. C
 64, 044604 (2001).
- ⁷²⁰ [31] M. Milin *et al.*, Nucl. Phys. A **753**, 263 (2005).
- ⁷²¹ [32] M. Freer *et al.*, Phys. Rev. Lett. **96**, 042501 (2006).
- ⁷²² [33] H.G. Bohlen, T. Dorsch, Tz. Kokalova, W. von Oertzen, Ch. Schulz, and C. Wheldon, Phys.
- ⁷²³ Rev. C **75**, 054604 (2007).
- ⁷²⁴ [34] S. Ahmed *et al.*, Phys. Rev. C **69**, 024303 (2004).
- ⁷²⁵ [35] H. Yamaguchi *et al.*, Phys. Rev. C **83**, 034306 (2011).
- ⁷²⁶ [36] G.M. Ter-Akopian *et al.*, Phys. Lett. B **426**, 251 (1998).
- ⁷²⁷ [37] R. Raabe *et al.*, Phys. Lett. B **458**, 1 (1999).
- ⁷²⁸ [38] R. Raabe *et al.*, Phys. Rev. C **67**, 044602 (2003).
- ⁷²⁹ [39] K. Rusek and K.W. Kemper, Phys. Rev. C **61**, 034608 (2000).
- ⁷³⁰ [40] D.T. Khoa and W. von Oertzen, Phys. Lett. B **595**, 193 (2004).
- [41] K.P. Artemov, M.S. Golovkov, V.Z. Goldberg, V.V. Pankratov, A.E. Pakhomov, I.N. Serikov,
 and V.A. Timofeev, Sov. J. Nucl. Phys. 55, 1460 (1992).
- ⁷³³ [42] F. Becchetti, M.Y. Lee, T.W. O'Donnell, D.A. Roberts, J.J. Kolata, L.O. Lamm, G. Rogachev,
- V. Guimarães, P.A. DeYoung, and S. Vincent, Nucl. Instr. and Meth. Phys. Res., Sect. A 505,
 377 (2003).
- ⁷³⁶ [43] D. Suzuki *et al.*, Nucl. Instr. and Meth. Phys. Res., Sect. A **691**, 39 (2012).
- [44] Y. Giomataris, Ph. Rebourgeard, J.P. Robert, and G. Charpak, Nucl. Instr. and Meth. Phys.
 Res., Sect. A 376, 29 (1996).
- [45] P. Baron, D. Calvet, E. Delagnes, X. de la Broise, A. Delbart, F. Druillole, E. Mazzucato,
 E. Monmarthe, F. Pierre, and M. Zito, IEEE Transactions on Nuclear Science NS-55, 1744
 (2008).
- [46] Y. Kudenko, Representing the T2K Collaboration, Nucl. Instr. and Meth. Phys. Res., Sect. A
 598, 289 (2009).

- [47] J.F. Ziegler, M.D. Ziegler, and J.P. Biersack., Nucl. Instr. and Meth. Phys. Res., Sect. B 268, 1818 (2010).
- ⁷⁴⁶ [48] G. Audi, A.H. Wapstra, C. Thibault, Nucl. Phys. A **729**, 337 (2003).
- 747 [49] D.R. Tilley, C.M. Cheves, J.L. Godwin, G.M. Hale, H.M. Hofmann, J.H. Kelley, C.G. Sheu,
- ⁷⁴⁸ and H.R. Weller, Nucl., Phys. A **708**, 3 (2002).
- ⁷⁴⁹ [50] W. Voigt, Münch. Ber., 603 (1912).
- ⁷⁵⁰ [51] D.R. Tilley, J.H. Kelley, J.L. Godwin, D.J. Milener, J.E. Purcell, C.G. Sheu, and H.R. Weller,
- ⁷⁵¹ Nucl., Phys. A **745**, 155 (2004).
- ⁷⁵² [52] A.M. Lane and R.G. Thomas, Phys. Mod. Phys. **30**, 257 (1958).
- ⁷⁵³ [53] I.J Thompson, Comput. Phys. Rep. 7, 167 (1988).
- ⁷⁵⁴ [54] J. Cook, Comput. Phys. Commun. **25**, 125 (1982).
- ⁷⁵⁵ [55] G. Bertsch, J. Borysowicz, H. McManus, and W.G. Love, Nucl. Phys. A 284, 399 (1977).
- ⁷⁵⁶ [56] J.S. McCarthy, I. Sick, and R.R. Whitney, Phys. Rev. C 15, 1396 (1977).
- ⁷⁵⁷ [57] G.R. Satchler and W.G. Love, Phys. Rep. 55, 183 (1979).
- ⁷⁵⁸ [58] J.S. Al-Khalili, J.A. Tostevin, and I.J. Thompson, Phys. Rev. C 54, 1843 (1996).
- ⁷⁵⁹ [59] L.J. Tassie, Australian J. Phys. 9, 407 (1956).
- ⁷⁶⁰ [60] B.F. Bayman and A. Kallio, Phys. Rev. **156**, 1121 (1967).
- ⁷⁶¹ [61] H.T. Fortune and R. Sherr, Phys. Rev. C 84, 024304 (2011).
- ⁷⁶² [62] D. Kurath, Phys. Rev. C 7, 1390 (1973).
- [63] P. Mohr, H. Abele, V. Kölle, G. Staudt, H. Oberhummer, and H. Krauss, Z. Phys. A 349,
 339 (1994).
- ⁷⁶⁵ [64] N. Itagaki, S. Aoyama, S. Okabe, and K. Ikeda, Phys. Rev. C 70, 054307 (2004).
- ⁷⁶⁶ [65] Y. Kanada-En'yo, Phys. Rev. Lett. **81**, 5291 (1998).
- ⁷⁶⁷ [66] T. Tomoda and A. Arima, Nucl. Phys. A **303**, 217 (1978).
- ⁷⁶⁸ [67] T. Yamada, Phys. Rev. C **42**, 1432 (1990).
- ⁷⁶⁹ [68] M. Kimura, Phys. Rev. C **69**, 044319 (2004).
- ⁷⁷⁰ [69] Y. Kanada-En'yo, Prog. Theo. Phys. **117**, 655 (2007).