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# Role of the two- and three-body hyperon-nucleon interaction in $\Lambda$ -hypernuclei

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We discuss the effect of the hyperon-nucleon-nucleon interaction on the binding energy of a  $\Lambda$  hyperon in closed shell hypernuclei from  $A=5$  to 91. The  $\Lambda$  binding energy has been calculated using the Auxiliary Field Diffusion Monte Carlo method that we have used, for the first time, to study light and heavy hypernuclei within the same model. Our results show that including a three-body component in the hyperon-nucleon interaction leads to a saturation of the  $\Lambda$  binding energy remarkably close to experimental data. In contrast, the two-body force alone gives a very unphysical binding energy.

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The onset of strange baryons ( $\Sigma^-$  and  $\Lambda$ ) in neutron matter at densities of order  $(2-3)\rho_0$ , where  $\rho_0 = 0.16 \text{ fm}^{-3}$ , has been questioned for a long time. Recent theoretical calculations based on the Brueckner-Hartree-Fock theory suggest that any process generating new on-shell degrees of freedom in high density Fermionic matter leads to a substantial softening of its equation of state (EoS) (see for example [1, 2] and references therein). When occurring in the inner core of a neutron star, such a mechanism would reduce the value of its predicted maximum mass and of its radius. Until a few years ago, astrophysical observations of neutron stars were concentrated in a relatively narrow region in the neighborhood of the Chandrasekhar limit ( $M \simeq 1.41 M_\odot$ ). Most of the realistic EoS based on the hypothesis of matter made of nucleons only, while compatible with these observations, predict a maximum mass typically larger than  $2 M_\odot$ . This result can be considered very robust. As an example, recent Quantum Monte Carlo (QMC) calculations of the equation of state of Pure Neutron Matter (PNM), Symmetric Nuclear Matter (SNM) and baryonic matter at  $\beta$ - and  $\mu$ -equilibrium using realistic Density Dependent Potentials (DDP) [3], essentially confirm the behavior predicted by Akmal, Pandharipande and Ravenhall with a full AV18+three-body interaction [4]. With such nuclear Hamiltonian the predicted EoS supports a maximum neutron star mass larger than  $1.97 M_\odot$  recently observed [5].

In so far as the appearance of strange baryons is concerned, the situation is more controversial. Some authors (see e.g. [6, 7]), suggest that the appearance of hyperons in the EoS does not lead to very strong effects. Other recent papers, like Refs. [1, 2, 8–10], show a more substantial influence, but with contradictory outcomes in terms of the predicted maximum mass of neutron stars not compatible with the observations [11]. Therefore, the situation is far from being completely settled.

A combination of reasons leads to the uncertainty in the analysis of the influence of strangeness degrees of freedom in the EoS. First of all, the interaction between nucleons and hyperons is still far from being known with sufficient accuracy. The prospective measurements of

properties of light hypernuclei should improve the quality of the available data, making possible a realistic phenomenological analysis. Second, the theoretical tools employed are all affected by uncontrollable intrinsic approximations as soon as one tries to push the study beyond few body systems. As a consequence, so far it is not clear how well the model hyperon-nucleon (YN) potentials work in the limit of medium mass hypernuclei, and, as a consequence, in the extrapolation to homogeneous matter. However, in the last few years important advances have been made both on the experimental and on the theoretical side. Several experiments aim to measure the binding energy of different  $\Lambda$ -hypernuclei [12–14]. On the theoretical side, the development of Quantum Monte Carlo methods opened the way to study consistently nuclear systems from few-nucleons to infinite matter [15–17] within the same scheme/model.

In this paper we discuss the use the Auxiliary Field Diffusion Monte Carlo (AFDMC), to study a non-relativistic Hamiltonian based on a phenomenological  $\Lambda N$  interaction in order to show how the inclusion of explicit  $\Lambda N N$  terms provides the necessary repulsion to realistically describe the separation energy of a  $\Lambda$  hyperon in hypernuclei of intermediate masses. This point makes very clear the fact that the lack of an accurate Hamiltonian might be responsible for the unrealistic predictions of the EoS that would tend to rule out the appearance of strange baryons in high density matter.

After the pioneristic work reported in Ref. [18], several models have been proposed to describe the hyperon-nucleon (YN) interaction. A number of potentials in the Nijmegen soft-core form have been developed in the past (like NSC89 and NSC97x). A recent review of these interactions, together with HF calculations have been published by Dapo et al. [19]. These potentials are accurate in describing the available scattering two body data, and have been used in the BHF studies previously quoted. Starting in the 1980s, a class of Argonne-like interactions have been developed by Bodmer, Usmani and Carlson on the grounds of Quantum Monte Carlo calculations. A possible form of a three-body YNN interaction was also proposed in the same context [20–24]. More recently

Polinder et al. [25] proposed a potential based on a chiral perturbation theory expansion. As an alternative a cluster model to study light hypernuclei has been recently proposed by Hiyama and collaborators (see for example Refs. [26, 27]). Interesting results on  $\Lambda$ -hypernuclei have also been obtained within a  $\Lambda$ -nucleus potential model, in which the need of a functional with a more than linear density dependence was shown, suggesting the importance of a many-body interaction [28]. Finally, other methods based on mean-field techniques has been used to study medium and heavy mass hypernuclei [29, 30].

An important point that needs to be made is that  $\Lambda N$  and  $\Lambda NN$  interactions are both at the two-pion exchange order. Another important difference with respect the nucleonic case is that the mass of the intermediate excited state  $\Sigma$  compared to the  $\Lambda$  is much smaller than in the pure nucleonic case, where the difference between the nucleon and the  $\Delta$  is much larger.  $\Lambda N$  and  $\Lambda NN$  interactions should therefore be considered necessary in any consistent theoretical calculation. In 2002 Nogga et al. [31] performed Fadeev-Yakubowsky calculations of the  $0^+$  and the  $1^+$  state of  ${}^4_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{He}$  in order to study charge symmetry breaking effects. In both cases they predict a  $\Lambda$ -separation energy that is too small, and claim that an attractive  $\Lambda NN$  interaction is necessary.

We have revisited the problem from a slightly different starting point. We employed a potential in coordinates space, including an explicit repulsion between baryons, with  $NN$ ,  $\Lambda N$  and  $\Lambda NN$  components. Keeping the parameters of the  $\Lambda N$  interaction at the values determined by Usmani et al., we computed the ground state energy of a set of hypernuclei, and calculate for each the quantity  $B_{\Lambda}$ , i.e. the separation energy of the  $\Lambda$  hyperon, by means of the AFDMC method, using a realistic nucleon-nucleon interaction. We select one of the possible set of parameters of the  $\Lambda NN$  interaction suggested in Bodmer et al., and then by Usmani and collaborators, that reasonably reproduces experimental data on a set of light hypernuclei.

Within this model, nuclei and hypernuclei are described as non-relativistic particles interacting via two- and three-body forces:

$$H_{nuc} = \sum_{i=1}^{A-1} \frac{p_i^2}{2m_N} + \sum_{i<j}^{A-1} v_{ij}, \quad (1)$$

$$H_{hyp} = H_{nuc} + \frac{p_{\Lambda}^2}{2m_{\Lambda}} + \sum_{i=1}^{A-1} v_{\Lambda i} + \sum_{i<j}^{A-1} v_{\Lambda ij}. \quad (2)$$

Here  $A$  refers to the total number of baryons, nucleons plus the  $\Lambda$  particle. In order to test the effect of using different nuclear Hamiltonians on the  $\Lambda$ -separation energy, and to test the compatibility of the  $NN$  interaction with the  $\Lambda N$  and  $\Lambda NN$  forces, we use three different two-nucleon potentials  $v_{ij}$ : the Argonne AV4' and AV6' [32], that are simplified versions of the Argonne AV18 [33] and the Minnesota potential from Ref. [34].

Isospin conservation implies that a  $\Lambda$  hyperon can ex-

change a pion only with a  $\Lambda\pi\Sigma$  vertex. This fact leads to the consequence that one pion exchange (OPE) processes are not allowed. The lowest order  $\Lambda N$  coupling must therefore involve the exchange of two pions, with the formation of a virtual  $\Sigma$  hyperon, as illustrated in Fig. 1(a). One meson exchange processes can only occur through the exchange of a  $K$  or  $K^*$ . This process has the effect of exchanging the strangeness between the two baryons, as shown in Fig. 1(b). The  $\Lambda N$  interaction can therefore be modeled with a central term, which includes the  $\Lambda N$  exchange operator  $\varepsilon(\hat{P}_x - 1)$ , plus a spin-dependent contribution:

$$v_{\Lambda i} = v_0(r_{\Lambda i})(1 - \varepsilon + \varepsilon\hat{P}_x) + \frac{1}{4}v_{\sigma}T_{\pi}^2(m_{\pi}r_{\Lambda i})\boldsymbol{\sigma}_{\Lambda}\cdot\boldsymbol{\sigma}_i, \quad (3)$$

where  $\hat{P}_x$ ,  $v_0$  and  $T_{\pi}^2$  are defined in Ref. [24] and references therein, and  $\boldsymbol{\sigma}_{\Lambda}$  and  $\boldsymbol{\sigma}_i$  are Pauli matrices acting on the  $\Lambda$  and nucleons. Both the spin-dependent and the central terms contain the usual tensor operator  $T_{\pi}$  acting twice. All the pion exchange interaction is therefore active at intermediate range. The short range contributions are as usual included by means of a phenomenological central repulsive factor, included in  $v_0(r)$ . For more details see for example Ref. [24].

The remaining diagrams in Fig. 1, are two-nucleon one-hyperon interactions, which are at the same TPE order, and should therefore be included together with the two-body part in order to have a consistent description. The three-body potential  $v_{\Lambda ij}$  can be conveniently decomposed in a contribution that we label as  $v_{\Lambda ij}^{2\pi} = v_{\Lambda ij}^P + v_{\Lambda ij}^S$ , and that corresponds to the p-wave and s-wave two-pion exchange diagrams (respectively 1(c) and 1(d) in Fig. 1), and a dispersive term that includes short range contributions, labeled as  $v_{\Lambda ij}^D$ . They can be expressed as:

$$\begin{aligned} v_{\Lambda ij}^D &= W^D T_{\pi}^2(m_{\pi}r_{\Lambda i}) T_{\pi}^2(m_{\pi}r_{\Lambda j}) \left[ 1 + \frac{1}{6}\boldsymbol{\sigma}_{\Lambda}\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \right], \\ v_{\Lambda ij}^P &= -\left(\frac{C^P}{6}\right)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\{X_{i\Lambda}, X_{\Lambda j}\}, \\ v_{\Lambda ij}^S &= C^S Z(m_{\pi}r_{\Lambda i}) Z(m_{\pi}r_{\Lambda j})(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{i\Lambda}\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{j\Lambda})\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j. \end{aligned} \quad (4)$$

The definition of the functions  $X_{i\Lambda}$  and  $Z(x)$  as well as the range of parameters for the three-body force can be found in [24] and references therein.

The ground-state energy of the many-body nuclear and hypernuclear Hamiltonians, is computed by means of the AFDMC method. The algorithm was originally introduced by Schmidt and Fantoni [35] in order to deal in an efficient way with spin-dependent Hamiltonians. A trial wave function  $\Psi_T$  is propagated in imaginary-time  $\tau$  by sampling configurations of the system in coordinate-spin-isospin space. Expectation values are computed averaging over the sampled configurations. In the  $\tau \rightarrow \infty$  limit, the evolved state approaches the ground-state of  $H$  and thus the ground state properties of the system can be obtained.

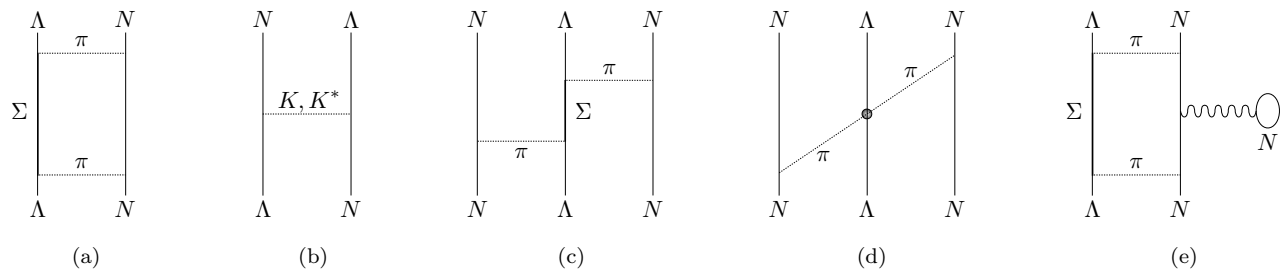


Figure 1. Meson exchange processes between nucleons and hyperons. 1(a) and 1(b) represent the  $\Lambda N$  channels. 1(c), 1(d) and 1(e) are the three-body ANN channels included in the potential by Usmani et al. [22, 24].

For a system with  $A$  nucleons, the quadratic operatorial structure  $\mathcal{O}_n^2$  of the nuclear Hamiltonians leads to a number of spin-isospin states in the propagated wave function which grows exponentially with  $A$ . This number quickly becomes intractable as  $A$  gets large. Standard Green's Function Monte Carlo (GFMC) calculations are in fact limited to up to 12 nucleons [36] or 16 neutrons [17]. By applying the Hubbard-Stratonovich transformation the computational cost of the calculation becomes proportional to  $A^3$  and systems with a larger number of particles can be studied [16]. The AFDMC algorithm can be applied to nuclear systems interacting via the Argonne V6-type potentials, for which the two-body force can be separated into a spin-independent and a spin-dependent part. The latter can be written as a sum of real matrices which contain proper combinations of the components of V6. By means of the diagonalization of such matrices it is possible to write the imaginary time propagator in the Hubbard-Stratonovich form (see Refs. [16, 37, 38] for a detailed discussion). However a realistic three-body force cannot be included in the propagator.

A straightforward variant of AFDMC can be applied to  $\Lambda$ -hypernuclear systems, including the two-body (3) and three-body (4) hyperon-nucleon interactions. It is indeed possible to recast the  $\Lambda N$  and ANN interactions so that they contain at most two-body operators. These terms can directly be included in the AFDMC propagator. The rest of the algorithm closely follows the nucleon-only version [16].

We assume that the wave function of a single  $\Lambda$ -hypernucleus is a nuclear Slater determinant (the same of Ref. [38]), multiplied by a single particle wave function for the  $\Lambda$  hyperon. As nucleon single particle states we use the radial solutions of the Hartree Fock problem with the Skyrme force and we consider a  $1s_{1/2}$  single particle state for the  $\Lambda$  particle. With the wave function defined we consider nucleons and the hyperon as distinct particles. In this way, we do not include the  $\Lambda N$  exchange term of the  $\Lambda N$  potential directly in the AFDMC propagator, because it mixes hyperon and nucleon states. A perturbative treatment of this factor is however possible.

A direct comparison of energy calculations with ex-

perimental results is given for the  $\Lambda$ -separation energy, defined as:

$$B_\Lambda = B_{nuc} - B_{hyp}, \quad (5)$$

where  $B_{nuc}$  and  $B_{hyp}$  are respectively the total binding energies of a nucleus with  $A$  nucleons and the corresponding hypernucleus with  $A$  nucleons plus one  $\Lambda$ . The most significant outcome of the calculation is the fact that the inclusion of the three-body ANN interaction qualitatively changes the saturation properties of the  $\Lambda$ -separation energy. However, this result might depend on the particular choice of the NN interaction used to describe both the nucleus and the hypernucleus. In particular, one might expect a strong influence from the different nucleon density generated by disparate models. In order to discuss this possible dependence, we have performed calculations with different NN interactions having very different saturation properties. The nuclear Hamiltonians considered here are semi-realistic, and can be easily implemented within the AFDMC scheme. We should point out that in neither case did we use a three nucleon interaction.

In Tab. I we show the results of the AFDMC simulations for the  $\Lambda$ -separation energy in  ${}^5_\Lambda\text{He}$  and  ${}^{17}_\Lambda\text{O}$ . For each hypernucleus, the two columns correspond to calculations using the  $\Lambda N$  interaction only or both the  $\Lambda N + \text{ANN}$  force of Ref. [24] with different NN interactions. As it can be seen, for  ${}^5_\Lambda\text{He}$  the extrapolated values of  $B_\Lambda$  with the two-body  $\Lambda N$  interaction alone are about 10% off, and well outside statistical errors. In contrast the inclusion of the three-body ANN force gives a similar  $\Lambda$  binding energy independently to the choice of the NN force. On the grounds of this observation, we feel confident that the use of AV4', that makes AFDMC calculations less expensive and more stable, will in any case return realistic estimates of  $B_\Lambda$  for larger masses when including the ANN interaction. We have checked this assumption performing simulations in  ${}^{17}_\Lambda\text{O}$ , where the discrepancy between the  $\Lambda$ -separation energy computed using the different NN interactions and the full  $\Lambda N + \text{ANN}$  force is less than few per cent (last column of Tab. I). The various NN forces considered here are quite different. The AV6' includes a tensor force, while AV4' and Minnesota have a simpler structure. We compared the AV4' and

Minnesota, which have a similar operatorial structure but very different intermediate- and short-range correlations. The fact that the inclusion of the ANN force does not depend too much on the nuclear Hamiltonian is quite remarkable, because the different NN forces produce a quite different saturation point for the nuclear matter EoS, suggesting that our results are pretty robust. The discrepancies between our results and the experimental data are likely due to the ANN force that could be improved, while the term due to K-exchange not included in our calculation are expected to be small.

NN potential	${}^5_{\Lambda}\text{He}$		${}^{17}_{\Lambda}\text{O}$	
	$V_{\Lambda N}$	$V_{\Lambda N}+V_{\Lambda NN}$	$V_{\Lambda N}$	$V_{\Lambda N}+V_{\Lambda NN}$
Argonne V4'	7.1(1)	5.1(1)	43(1)	19(1)
Argonne V6'	6.3(1)	5.2(1)	34(1)	21(1)
Minnesota	7.4(1)	5.2(1)	50(1)	17(2)
exp	3.12(2)		13.0(4)	

Table I.  $\Lambda$ -separation energies (in MeV) for  ${}^5_{\Lambda}\text{He}$  and  ${}^{17}_{\Lambda}\text{O}$  obtained using different nucleon potentials (AV4', AV6', Minnesota) and different hyperon-nucleon interaction (two-body alone and two- plus three-body). In the last line the experimental  $B_{\Lambda}$  for  ${}^5_{\Lambda}\text{He}$  is from Ref. [39]. Since no experimental data for  ${}^{17}_{\Lambda}\text{O}$  exists, the reference separation energy is the semiempirical value reported in Ref. [22].

The results on the  $\Lambda$  separation energies are summarized in Fig. 2. We compare the prediction of the hyperon binding energy in the AV4'+ $\Lambda N$  and AV4'+ $\Lambda N$ +ANN models for a few closed shell hypernuclei with the experimental values observed in the same mass range. While the results for lighter hypernuclei might be inconclusive in terms of the physical consistency of the ANN contribution to the hyperon binding energy, the computations for  ${}^{41}_{\Lambda}\text{Ca}$  and  ${}^{91}_{\Lambda}\text{Zr}$  reveal a completely different picture. The saturation binding energy provided by the  $\Lambda N$  force alone is completely unrealistic, while the inclusion of the ANN force gives results that are qualitatively much closer to the experimental behavior. We should notice that the results might be further improved by a refitting of the terms in the ANN force. In particular, according to Ref. [22], in the present calculations the  $s$ -wave contribution is not present. Moreover, we are missing the explicit inclusion of the kaon exchange term. This contribution (see Eq. (3)) can be estimated at first order in perturbation theory by computing the expectation of the corresponding term. As an example, the values of the correction on the total energy we obtained for  $\varepsilon = 0.1$  [24] is  $-0.33(6)$  MeV in  ${}^5_{\Lambda}\text{He}$  and  $+0.2(4)$  MeV in  ${}^{17}_{\Lambda}\text{O}$ , the latter negligible compared to the corresponding binding energy.

For  ${}^{91}_{\Lambda}\text{Zr}$  we should also consider a charge symmetry breaking (CSB) potential. The latter can be easily included as a term of the form

$$v_{\Lambda i}^{\text{CSB}} = \tau_i^z C_0^{\text{CSB}} T_{\pi}^2 (m_{\pi} r_{\Lambda i}), \quad (6)$$

amounting to an isospin dependent correction to the cen-

tral potential. The inclusion of the CSB term using perturbation theory would be zero in isospin-symmetric hypernuclei. The value  $C_0^{\text{CSB}} = -0.050(5)$  MeV reported in literature [23], is fitted in order to reproduce the difference in  $\Lambda$ -separation energy of the  $A = 4$  mirror hypernuclei ( ${}^4_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{He}$ ). According to Eq. (6), the contribution of the charge symmetry breaking term depends on the difference between the number of neutrons and protons. For  $N > Z$  the CSB term is strictly positive. This implies a repulsive contribution per neutron excess that would further lower the  $B_{\Lambda}$  for  ${}^{91}_{\Lambda}\text{Zr}$ , where there are 10 more neutrons than protons, thereby reducing the discrepancy with the experimental result.

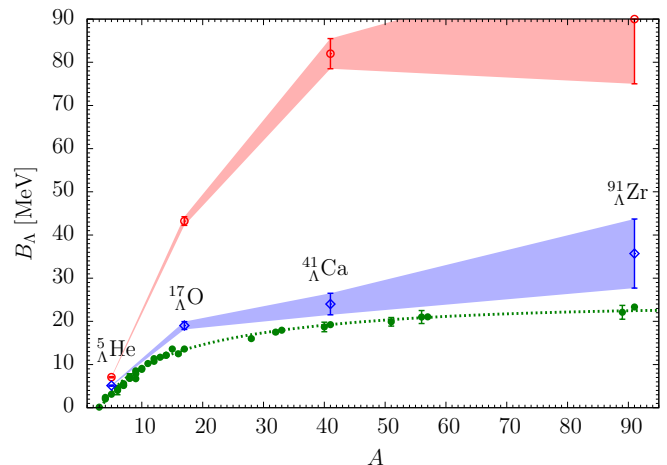


Figure 2.  $\Lambda$ -separation energy as a function of the baryon number  $A$ . Plain green dots [dashed curve] are the available  $B_{\Lambda}$  experimental values. Empty red dots [upper banded curve] refer to the AFDMC results for the nuclear AV4' potential plus the two-body  $\Lambda N$  interaction alone. Empty blue diamonds [lower banded curve] are the results with the inclusion of the three-body hyperon-nucleon force.

In this paper we have presented the first accurate calculation of the  $\Lambda$ -separation energy for closed-shell  $\Lambda$ -hypernuclei using the available microscopic interactions. Using the AFDMC method we were able to extend the calculation in the medium-heavy range of hypernuclei up to  $A=91$ , providing, for the first time, a consistent calculation of light and heavy hypernuclei. The main outcome of the study is that the inclusion of the three-body ANN interaction is fundamental in order to reproduce the saturation properties of the  $\Lambda$  binding energy in hypernuclei. The leading contribution to the three-body interaction is strictly repulsive in the range of hypernuclei studied. Within the model that we have studied, the inclusion of the  $\Lambda N$  force without a three-body force gives a very unphysical  $\Lambda$  binding energy.

We speculate that this would lead to a stiffer EOS for the  $\Lambda$ -neutron matter when the presented interaction is applied to the study of the homogeneous medium. This fact might eventually reconcile the onset of hyperons in the inner core of a neutron star with the observed masses

of order  $2 M_{\odot}$ . A study along this direction is in progress.

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