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# $0_{2}^{+}$band in ${ }^{102} \mathrm{Ru}$ and the evolution of nuclear deformation in Ru isotopes. 

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#### Abstract

Two measurements of $\gamma$ rays from the slow-neutron capture reaction on a ${ }^{101} \mathrm{Ru}$ target were performed at the PF1B cold-neutron facility and the GAMS5 DuMond spectrometer of ILL Grenoble, to study in detail excitations in the transitional nucleus ${ }^{102} \mathrm{Ru}$. A band structure on top of the $0_{2}^{+}$level has been identified and its nature discussed. Mixing between the $0_{1}^{+}$and $0_{2}^{+}$levels in Ru isotopes with $50<\mathrm{N}<66$ neutrons and its role in the development of deformation in the ground state of the Ru nuclei have been studied.


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The emergence of collective effects in complex systems is of general interest in physics, in particular when observed in systems governed by laws of quantum mechanics, like atomic nuclei. In certain regions of the nuclear chart rapid changes of nuclear deformation are observed when the number of nucleons changes just by a few percent. Such pronounced effects offer a deeper insight into the appearance of the nuclear deformation.

The mechanism of the sudden onset of deformation at $\mathrm{N} \approx 60$ neutrons in mass $\mathrm{A} \sim 100$ nuclei has been studied in numerous works. It has been first proposed $[1,2]$ that the effect is due to the proton-neutron interaction between the spin-orbit partner (SOP) orbitals, $\nu g_{7 / 2}$ and $\pi g_{9 / 2}$, causing a gradual increase of the deformation of the, initially spherical, ground state when neutrons are added to the $\nu g_{7 / 2}$ orbital. Although likely, this process alone could not reproduce the scale and the rate of the observed deformation change. Therefore, it has been proposed that the sudden deformation change is due to a lowering of a deformed structure based on the third $0^{+}$level, which becomes the deformed ground-state in nuclei with $\mathrm{N} \geq 60[3,4]$. However, such deformed structures on top of the $0_{3}^{+}$level were not confirmed [5]. Instead, weakly deformed bands based on the second $0^{+}$ level have been reported in Sr and Zr isotopes, with the deformation growing with the neutron number [6]. Further deformation increase, at $\mathrm{N} \geq 60$, has been attributed to the population of the deformation-driving $\nu h_{11 / 2}$ intruder [7-9]. It has been proposed that this process is reinforced by vacating, at the same time, the sphericaldriving $\nu 9 / 2[404]$ extruder orbital [10, 11]. The subshell closures at $\mathrm{Z}=40$ and $\mathrm{N}=56$ most likely enhance the sudden nature of this deformation change [12].

It is interesting to ask about the nature of the weakly deformed bands at $\mathrm{N}=58$ on top of the $0_{2}^{+}$level, which are the "seeds" for the deformed bands at $\mathrm{N}>58$. The evolution of such bands should be seen more clearly away
from the $\mathrm{Z}=40$ closure, which preserves the non-collective structures up to $N=56$. Furthermore, it is expected that at $\mathrm{Z} \geq 42$ the $\nu 9 / 2[404]$ extruder departs from the Fermi level and does not contribute to the deformation change. Therefore, Ru isotopes, are good candidates to study the evolution of a collective structure based on the $0_{2}^{+}$level.

With excitations suggesting both, spherical and deformed configurations, ${ }^{102} \mathrm{Ru}$ marks the point on the $\mathrm{N}=58$ and $\mathrm{Z}=44$ lines, which separates spherical from deformed nuclei [13]. A good knowledge of the structure of this transitional nucleus is essential for understanding the evolution of deformation in the $\mathrm{A} \sim 100$ region. However, the nature of the crucial cascade on top of the $0_{2}^{+}$ level in ${ }^{102} \mathrm{Ru}$ is still not clear [13]. In this work we report on experimental studies of non-yrast excitations in ${ }^{102} \mathrm{Ru}$, performed in order to determine the properties of this cascade as well as its role in the development of nuclear deformation.

With spin $\mathrm{I}^{\pi}=5 / 2^{+}$of the ground state in ${ }^{101} \mathrm{Ru}$, the decay of the neutron-capture level in ${ }^{102} \mathrm{Ru}$ at 9219.7 keV has sufficient spin and energy to populate levels with spins from $\mathrm{I}=0$ to $\mathrm{I}=6$. This allows a search for bands in ${ }^{102} \mathrm{Ru}$. In this work excited levels in ${ }^{102} \mathrm{Ru}$ were populated in the ${ }^{101} \mathrm{Ru}(\mathrm{n}, \gamma){ }^{102} \mathrm{Ru}$ reaction at the PF1B coldneutron facility of the Institut Laue-Langevin. A neutron beam, collimated to about 1 cm in diameter, irradiated a 4 mg target of ${ }^{101} \mathrm{Ru}$, isotopically enriched to $99 \%$. The $\gamma$ radiation from the reaction has been measured using 8 Ge detectors of about $60 \%$ relative efficiency each, placed in a plane perpendicular to the neutron-beam direction. The angle between two neighboring detectors was 45 degree, which has enabled angular correlation measurement at three angles of $90^{\circ}, 135^{\circ}$ and $180^{\circ}$. Details of the experimental setup are given in Ref. [14].

About $2 \times 10^{9}$ triggerless events, consisting of a $\gamma$ energy and the time of its registration (by a 40 MHz clock), have been collected. Out of this data we sorted $4 \times$


FIG. 1: Partial level scheme of ${ }^{102} \mathrm{Ru}$ as observed in the present work. New levels are marked with an asterisk. More accurate level and transition energies are given in Table I.
$10^{8}$ double- $\gamma$ and $5 \times 10^{6}$ triple- $\gamma$ coincidence events, applying a 600 ns time window to define a coincidence event. Various two- and three-dimensional histograms were sorted and used to build the level scheme of ${ }^{102} \mathrm{Ru}$.

We have identified and placed in the decay scheme of ${ }^{102} \mathrm{Ru}$ over $10^{3} \gamma$ transitions. A full report on these results will be given elsewhere [15] while in this paper we present the data relevant to the discussion of the cascade on top of the $0_{2}^{+}$level. A partial level scheme of ${ }^{102} \mathrm{Ru}$ as observed in the present work is shown in Fig. 1 and the properties of the $\gamma$ lines are collected in Tables I and II.

The energies of levels and transitions in Table I agree with the literature values [16] and are, in most cases, more accurate. Two examples illustrate the high quality of the angular correlations shown in Table II, (i) the measured $A_{2}$ and $A_{4}$ values for the very anisotropic, 468.6-$475.1-\mathrm{keV}$ cascade agree well with the theoretical values of 0.357 and 1.143 , respectively and (b) the $\delta$ values of $+0.25(3)$ and $-5.7(3)$ reported previously [16] for the $1105.3-$ and $1046.5-\mathrm{keV}$ transitions, respectively, agree with our values, both, in amplitude and sign.

Decays of the $1580.5-\mathrm{keV}$ level to the 1103.1 - and $1106.4-\mathrm{keV}$ levels, not observed till now, are expected in the vibrational limit [13]. In a fit to the singles spectrum we have found a new line at $477.32(9) \mathrm{keV}$. Its energy agrees with the (1580.48-1103.09)- keV difference between the corresponding level energies (Table I). Because of the particular location of the 477.3 decay branch, double- $\gamma$ coincidences involving this line are obscured by the strong $475.1-\mathrm{keV}$ line. More helpful are triple- $\gamma$ co-

TABLE I: Energies and intensities $\gamma$ transitions in ${ }^{102} \mathrm{Ru}$, as measured in the present work in the ${ }^{101} \mathrm{Ru}+n_{t h}$ reaction, using the Ge array ${ }^{a}$. Uncertainties of $\gamma$ intensity include a $3 \%$ systematic error due to the efficiency calibration.

| $\begin{gathered} \mathrm{E}_{\gamma} \\ (\mathrm{keV}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{I}_{\gamma} \\ \text { (rel.) } \\ \hline \end{gathered}$ | $\begin{aligned} & \hline \mathrm{E}_{e x c}^{2} \\ & (\mathrm{keV}) \\ & \hline \end{aligned}$ | $\begin{gathered} \hline \mathrm{E}_{\gamma} \\ (\mathrm{keV}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{I}_{\gamma} \\ \text { (rel.) } \end{gathered}$ | $\begin{gathered} \mathrm{E}_{\text {exc }}^{i} \\ (\mathrm{keV}) \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 277.1(1) | 0.07(1) | 1798.68(2) | 697.41(2) | 1.7(1) | 2219.03(4) |
| 415.36(5) | 0.71(7) | 1521.62(3) | 711.4(1) | 0.04(1) | 2584.90 |
| 418.60(3) | 3.8(2) | 1521.62(3) | 766.84(3) | 1.6(1) | 1873.21(3) |
| 420.6(2) | 0.03(1) | 2219.03(4) | 786.27(4) | 0.42(5) | 2584.90(5) |
| 468.58(1) | 4.8(2) | 943.68(2) | 815.4(2) | 0.02(1) | 3034.4(3) |
| 475.10(1) | 100(3) | 475.10(1) | 927.05(8) | 0.11(1) | 2800.3(1) |
| 477.41(6) ${ }^{\text {a }}$ | 1.3(4) | 1580.48(4) | 1046.48(2) | 10.4(5) | 1521.62(3) |
| 601.30(5) | 0.27(2) | 2181.74(4) | 1075.34(3) | 1.9(1) | 2181.74(4) |
| 627.99(1) | 16.9(6) | 1103.09(2) | 1078.51(9) | 0.08(1) | 2181.74(4) |
| 631.27(1) | 28.9(7) | 1106.37(2) | 1103.09(2) | 10.0(5) | 1103 |
| 636.84(1) | 3.3(1) | 1580.48(4) | 1105.31(3) | 5.6(4) | 1580.48(4) |
| 660.15(9) | 0.15(2) | 2181.74(4) | 1112.74(4) | 0.63(9) | 2219.03(4) |
| 692.29(3) | 2.4(2) | 1798.68(2) | 1323.60(3) | 1.1(21) | 1798.68(2) |
| 695.60(2) | 3.6(2) | 1798.68(3) | 1580.57 | 0.9(1) | 1580.48 |

${ }^{a}$ energy taken from the GAMS5 measurement

TABLE II: Angular correlation coefficients for $\gamma-\gamma$ cascades in ${ }^{102} \mathrm{Ru}$ populated in the ${ }^{101} \mathrm{Ru}+n_{t h}$ reaction in this work.

| $\mathrm{E}_{\gamma 1}-\mathrm{E}_{\gamma 2}$ <br> cascade | $A_{2} / A_{0}$ <br> exp. | $A_{4} / A_{0}$ <br> exp. | $\delta^{e x p}\left(\mathrm{E}_{\gamma 1}\right)$ |
| :---: | :---: | :---: | :---: |
| $468.6-475.1$ | $0.364(13)$ | $1.156(26)$ |  |
| $628.0-475.1$ | $-0.074(11)$ | $0.337(21)$ | $-287(-\infty,+234)$ |
| $631.3-475.1$ | $0.104(8)$ | $0.009(15)$ |  |
| $1105.3-475.1$ | $0.117(9)$ | $0.014(19)$ | $0.17(2)$ |
| $1046.5-475.1$ | $-0.301(6)$ | $-0.087(11)$ | $-7.8(6)$ |
| $636.8-468.6$ | $0.001(9)$ | $0.018(18)$ |  |
| $601.3-638.8$ | $0.05(11)$ | $0.12(21)$ |  |
| $601.3-1105.3$ | $0.27(8)$ | $-0.16(18)$ | $0.6(-0.4,+0.6)$ |
| $1075.3-631.3$ | $0.177(18)$ | $0.022(36)$ | $0.06(6)$ or $-1.02(12)$ |

incidences. The presence of the $477.3-\mathrm{keV}$ decay is supported by a spectrum doubly-gated on the 475.1- and $628.0-\mathrm{keV}$ lines, in which there is a doublet around 476 keV with two components fitted at at $475.0(2) \mathrm{keV}$ and $478.1(3) \mathrm{keV}$. However, the intensity of the $475.0(2)-\mathrm{keV}$ component is more than twice that of the $478.1(3)-\mathrm{keV}$ component, while they should be equal. This may be due to chance self-coincidences of the very strong $475.1-\mathrm{keV}$ line or due to another line in this multiplet.

To clear these doubts we have measured a singles spectrum from the neutron capture on ${ }^{101} \mathrm{Ru}$, using the high resolution Bragg spectrometer GAMS5 of ILL, equipped with a curved crystal in DuMond diffraction geometry [17, 18]. Figure 2 shows a fragment of a spectrum around the $475-\mathrm{keV}$ multiplet. In the inset one sees a line at $477.41(6) \mathrm{keV}$. Its intensity is $0.013(4)$ as compared to the $475.095-\mathrm{keV}$ reference line [16]. The decay to the $1106.4-\mathrm{keV}$ level, expected at 474.11 keV is not observed.


FIG. 2: Singles- $\gamma$ spectrum from GAMS5, measured in the third diffraction order. In the picture one channel corresponds to 100 eV . The energy scale has been adjusted to reproduce the energy of the $475.095(1)-\mathrm{keV}$ reference line [16], with the accuracy of 0.003 keV .

The error-bar for the intensity of the $477.41-\mathrm{keV}$ line may serve as the estimate of the upper intensity limit of the unobserved 474.11-keV line.

Our coincidence data indicate the existence of a new, $2181.7-\mathrm{keV}$ level. Its decay branches limit the possible spin for this level to $2^{+}, 3^{+}, 3^{-}$and $4^{+}$. The $3^{-}$and $3^{+}$options are eliminated by angular correlations. The $4^{+}$solution is consistent with angular correlations (all fits with $\chi^{2} \leq 1.0$ ). Spin $2^{+}$is less likely due to $\chi^{2}=5.4$ for the $1075.3-631.3-\mathrm{keV}$ cascade and the absence of any decay to the ground and the $475.1-\mathrm{keV}$ states. Therefore, we propose that the $2181.7-\mathrm{keV}$ level has spin $4^{+}$and is the next excitation in the band on top of the $943.7-\mathrm{keV}$ level. The $2800.2-\mathrm{keV}$ level, which decays only to the $6^{+}$ level at 1873.2 keV , is a candidate for the $6^{+}$member of the band on top of the $943.7-\mathrm{keV}$ level. The nonobservation of a decay to the $2181.7-\mathrm{keV}$ level may be due to the detection limit.

In Ref. [13] it was suggested, that the decay properties of the $1580.5-\mathrm{keV}$ level are what one expects in a rotor. However the present data are in favor of a vibrational picture. The selection rules for a vibrator predict $B(E 2)$ branching ratios of 1.4:0.57:1.03 for the decay of the $2_{3}^{+}$ level to the $0_{2}^{+}, 2_{2}^{+}$and $4_{1}^{+}$levels, respectively [19]. With the upper limit of $\gamma$ intensity for the $474.1-\mathrm{keV}$ decay to the $1106.4-\mathrm{keV}$ level the upper limit for the $\mathrm{B}(\mathrm{E} 2)$ rate for this pure E 2 branch is $11.5 \mathrm{~W} . \mathrm{u}$. This value is similar to what is observed in ${ }^{100} \mathrm{Ru}$ [20], a nucleus which is closer to the vibrational limit. A similar analysis for the $477.4-\mathrm{keV}$ is more difficult because we do not know the mixing ratio, $\delta$, for this $\mathrm{M} 1+\mathrm{E} 2$ transition. We have calculated the $\delta$ value using the formalism of the generalized Bohr Hamiltonian [21, 22] (the full account of these calculations will be published elsewhere [15]). The resulting $\mathrm{B}(\mathrm{E} 2)$ value for the $477.4-\mathrm{keV}$ decay branch is 6.8 W.u., to be compared with the expectation of 8.8 W.u. for the vibrator. This result is again close to the vibrational limit.

To learn more about the vibrational cascade on top of


FIG. 3: Systematics of $0_{2}^{+}$bands in Ru isotopes. The $2_{1}^{+}$and $4_{1}^{+}$levels are also shown to assist further discussion (see the text). Lines are drawn to guide the eye. The data are taken from Refs. [16, 20, 23-28].
the $843.7-\mathrm{keV}, 0_{2}^{+}$level we have drawn at the systematics of $0_{2}^{+}$bands in Ru isotopes, shown in Fig. 3. In ${ }^{108} \mathrm{Ru}$, at the neutron number $\mathrm{N}=64$, the cascade on top of the $0_{2}^{+}$level has level spacing characteristic of a rotational band. This changes gradually with the decreasing neutron number towards vibrational spacings in ${ }^{102} \mathrm{Ru}$. The vibrational character and a low deformation of the $0_{2}^{+}$band in ${ }^{102} \mathrm{Ru}$ may be a surprise. With 8 valence neutrons one would expect high collectivity in this state. Below we present a simple picture, which may explain the low collectivity in this cascade and the role of the $0_{2}^{+}$configuration in the rapid deformation change in this region.

In spherical nuclei one observes a collective excitation, a phonon state with spin and parity $2^{+}$, corresponding to a quadrupole vibration of the nuclear surface. At twice its energy, the $0_{2}^{+}, 2_{2}^{+}$and $4_{1}^{+}$triplet of collective twophonon states is formed. The $4_{1}^{+}$and $2_{2}^{+}$level of the triplet are progenitors of the $4^{+}$and $2^{+}$excitations of the ground-state and $\gamma$-cascade, respectively (see Fig. 1). The role of the $0_{2}^{+}$level is less clear. One expects that due to its two-phonon character this level should be more collective than the ground state.

Figure 4 shows positions of the $0_{1}^{+}$and $0_{2}^{+}$levels in Ru isotopes (filled circles) relative to the $2_{1}^{+}$phonon excitation. In the neutron range from $\mathrm{N}=50$ to $\mathrm{N}=58$ the $0_{1}^{+}$ and $0_{2}^{+}$levels display a characteristic, two-level mixing pattern as a function of neutron number. The symmetry seen in the figure supports the two-phonon contribution to the $0_{2}^{+}$level, because in vibrational nuclei this level is expected at twice the excitation energy of the $2_{1}^{+}$phonon excitation. More interesting is the fact that the two levels interact strongly in a limited range, $\mathrm{N}=55$ to 59 .

We have performed a two-level mixing calculation (see e.g. Ref. [19]) for the data points in Fig. 4, under simple assumptions that the excitation energy of the, unperturbed $0^{+}$level, $0_{1 u}^{+}$(or $0_{2 u}^{+}$), raises (or falls) linearly with the increasing number of valence neutrons, $n=\mathrm{N}-50$, as $\mathrm{E}\left(0_{1 u}^{+}\right)=\Delta \mathrm{E}(-1+n / \Delta \mathrm{N})\left(\right.$ or $\left.\mathrm{E}\left(0_{2 u}^{+}\right)=\Delta \mathrm{E}(1-n / \Delta \mathrm{N})\right)$, as shown by dotted lines in Fig. $4(\Delta \mathrm{~N}$ is the distance


FIG. 4: Positions of $0_{1}^{+}$and $0_{2}^{+}$levels (full dots), relative to the $2_{1}^{+}$phonon excitation (open dots) in Ru isotopes. Solid lines represent a two-level mixing calculation. Symbols shown in the figure are explained in the text.
to the crossing point as shown in Fig. 4). The perturbed energies, calculated using $\Delta \mathrm{E}=1500 \mathrm{keV}$ (which is half of the separation energy at $n=0$ ), $\Delta \mathrm{N}=7$, and the interactions strength of $V=400 \mathrm{keV}$, are shown in Fig. 4 by solid lines. The calculation reproduces the experimental points surprisingly well, supporting the picture of the $0_{1}^{+}$ and $0_{2}^{+}$states crossing.

When crossing, the two levels exchange their properties [19]. One may expect that the ground state in ${ }^{102} \mathrm{Ru}$ will gain collectivity, while the collectivity in the cascade on top of the $0_{2}^{+}$will decrease. A schematic simulation shown in Fig. 5 illustrates this process. Taking the $\Delta \mathrm{E}$, $\Delta \mathrm{N}$ and $V$ parameters, as found above, we have simulated the exchange between the $0_{1}^{+}$and $0_{2}^{+}$levels of their certain property, $D$. We assumed that for the unperturbed $0_{2}^{+}$state, $0_{2 u}^{+}$, its $D$ value increases linearly with $n$ from 1.0 to 3.0 arbitrary units over the distance $2 \Delta \mathrm{~N}$, $D_{2 u}=1.0+n / \Delta \mathrm{N}$, while the $D$ value of the unperturbed $0_{1}^{+}$state, $0_{1 u}^{+}$, stays constant, $D_{1 u}=1.0$ (the $D$ parameter could be seen as a measure of a collectivity). The expectation value of $D$ in the $0_{1}^{+}$perturbed level is calculated as

$$
D_{1}=<0_{1}^{+}|D| 0_{1}^{+}>=\alpha D_{1 u}+\beta D_{2 u}
$$

where $\alpha$ and $\beta$ are the usual mixing amplitudes in the perturbed level, $\left|0_{1}^{+}>=\alpha\right| 0_{1 u}^{+}>+\beta \mid 0_{2 u}^{+}>$[19], while $D_{1 u}=<0_{1 u}^{+}|D| 0_{1 u}^{+}>, D_{2 u}=<0_{2 u}^{+}|D| 0_{2 u}^{+}>$and $<0_{1 u}^{+}|D| 0_{2 u}^{+}>=0$. The analogous formula for the $0_{2}^{+}$ level reads $D_{2}=-\beta D_{1 u}+\alpha D_{2 u}$.

The simulation shows that the $D_{1}$ value (collectivity of the ground state) increases rapidly over a small range of neutrons $54<\mathrm{N}<60$ where, effectively, the interaction takes place. This result, obtained with realistic $\Delta \mathrm{E}, \Delta \mathrm{N}$ and $V$ parameters, coincides with the 4-6 neutron range of the deformation change observed in Sr and Zr isotopes $[6,11]$. In contrast, the $D_{2}$ value falls and at $N=58$ has low $D$ value, coinciding with the low collectivity in the cascade on top of the $0_{2}^{+}$level in the heavier Ru isotopes.

An analogous crossing between $0_{1}^{+}$and $0_{2}^{+}$states has been reported in Gd nuclei [29] of the $\mathrm{A} \approx 150$ region, an-


FIG. 5: Exchange of properties between the crossing $0_{1}^{+}$and $0_{2}^{+}$levels, as simulated in two-level mixing calculation. See text for the explanation of $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ and further comments.

TABLE III: Truncated quadrupole shape invariants for $0_{1}^{+}$ and $\mathrm{O}_{2}^{+}$levels in Ru isotopes. The data are taken from Refs.[16, 20, 21, 27, 28]. See text for further explanation.

|  | ${ }^{98} \mathrm{Ru}$ | ${ }^{100} \mathrm{Ru}$ | ${ }^{102} \mathrm{Ru}$ | ${ }^{104} \mathrm{Ru}$ |
| :---: | :---: | :---: | :---: | :---: |
| $q_{2}\left(0_{1}^{+}\right)$ | $0.43(1) e^{2} b^{2}$ | $0.52(9) e^{2} b^{2}$ <br> $q_{2}\left(0_{2}^{+}\right)$ |  | $0.65(2)$$e^{2} b^{2}$ |
| $0.61(14) e^{2} b^{2}$ | $0.89(2){ }^{a} e^{2} b^{2}$ <br> $<0.98$$e^{2} b^{2}$ | $0.62(14) e^{2} b^{2}$ |  |  |

${ }^{a}$ without Ref. [21]; $q_{2}\left(0_{2}^{+}\right)=1.41(6) e^{2} b^{2}$ if Ref. [21] is used, only.
other place where a rapid change of nuclear deformation occurs. In Ref. [29] the crossing has been traced with the help of the quadrupole shape invariant, $q_{2}$, which measures quadrupole collectivity (or deformation, due to its proportionality to $<\beta_{2}>$ ) [30]. Interestingly, the behavior of the $q_{2}$ invariants for the $0_{1}^{+}$and $0_{2}^{+}$states in Gd nuclei is similar to the behavior of $D_{1}$ and $D_{2}$ values in Fig. 5. As shown in Fig. 1 of Ref. [29], the $q_{2}\left(0_{1}^{+}\right)$ invariant increases quickly over a range of four neutrons while the $q_{2}\left(0_{2}^{+}\right)$invariant, which first grows faster than $q_{2}\left(0_{1}^{+}\right)$, starts to drop just before the crossing point and after the crossing is smaller than $q_{2}\left(0_{1}^{+}\right)$.

The available $\mathrm{B}(\mathrm{E} 2)$ rates in Ru isotopes allow one to construct truncated $q_{2}\left(0_{1}^{+}\right)$and $q_{2}\left(0_{2}^{+}\right)$values, as described in Ref. [29]. We used the $\mathrm{B}(\mathrm{E} 2)$ rates between the $0_{1}^{+}, 0_{2}^{+}, 2_{1}^{+}, 2_{2}^{+}$and $2_{3}^{+}$levels in the ${ }^{98-104} \mathrm{Ru}$ isotopes $[16,20,21,27,28]$. The $q_{2}$ values obtained are shown in Table III. One observes that the $q_{2}\left(0_{1}^{+}\right)$invariant increases with the increasing neutron number. In contrast the $q_{2}\left(0_{2}^{+}\right)$does not increase, and in ${ }^{104} \mathrm{Ru}$ is significantly lower than $q_{2}\left(0_{1}^{+}\right)$. It is of high interest to improve the accuracy of the experimental $\mathrm{B}(\mathrm{E} 2)$ values in the Ru isotopes, especially those used to construct $q_{2}\left(0_{2}^{+}\right)$in ${ }^{100} \mathrm{Ru}$ and ${ }^{102} \mathrm{Ru}$.

It is worth noting that the truncated $q_{2}\left(0_{1}^{+}\right)$and $q_{2}\left(0_{2}^{+}\right)$ values in Table III are dominated by the $\mathrm{B}\left(\mathrm{E} 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)$ and the $\mathrm{B}\left(\mathrm{E} 2 ; 2_{3}^{+} \rightarrow 0_{2}^{+}\right)$rates, respectively. Therefore, they reflect the "in-band" $\mathrm{B}(\mathrm{E} 2)$ rates, and thus collectivity in the two cascades.

It is interesting to ask about the mechanism which cre-
ates deformation in the $0_{2}^{+}$bands in the $50<\mathrm{N}<60$ range. The proposed two-phonon excitation may be only one of the contributing effects. While the ground states at $\mathrm{N} \leq 56$ should be dominated by the $d_{5 / 2}$ pair of neutrons, at the excitation energy of the $0_{2}^{+}$level one expects the $\left(g_{7 / 2}\right)_{0^{+}}^{2}$ configuration, which may contribute to the $0_{2}^{+}$ level. With the $\pi g_{9 / 2}$ proton orbital well populated in the Ru isotopes, the SOP mechanism proposed in this region for the ground state [1, 2] may, actually, become the source of the collectivity in the $0_{2}^{+}$level. Such a possibility has been suggested for Sr isotopes [31] and recently supported by calculations of various $0^{+}$levels in ${ }^{96} \mathrm{Sr}$ and ${ }^{98} \mathrm{Zr}$ isotones of ${ }^{102} \mathrm{Ru}$ [32].

Finally, we note that above $\mathrm{N}=58$ the picture changes. As shown in Fig. 4, the $2_{1}^{+}$excitation energy (the difference between the $0_{1}^{+}$points and the $2_{1}^{+}$line) drops significantly. This suggests that another deformationgenerating mechanism starts around $\mathrm{N}=60$, which is most likely due to the population of low- $\Omega$, deformationdriving orbitals, originating from the $\nu h_{11 / 2}$ intruder [79]. It has been proposed in Ref. [13], that the rather low energy of the $2_{1}^{+}$level in ${ }^{102} \mathrm{Ru}$ and the $\mathrm{B}\left(\mathrm{E} 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)$ about $50 \%$ higher in ${ }^{102} \mathrm{Ru}$ than in lighter Ru isotopes, may indicate a rotational nature for the g.s. band in ${ }^{102} \mathrm{Ru}$. In fact, the $0_{1}^{+}$point for ${ }^{102} \mathrm{Ru}$ in Fig. 4 might be seen as the lowest- N family member of deformed Ru iso-
topes. However, its deformation is probably of a dynamic nature in contrast to the heavier Ru isotopes, which have static deformation. Therefore, it is more likely that ${ }^{102} \mathrm{Ru}$ is the highest-N, spherical Ru isotope. Figures 3 and 4 show that the $0_{2}^{+}$band in ${ }^{102} \mathrm{Ru}$ can be seen as a 'seed' for the $0_{2}^{+}$deformed bands at $\mathrm{N}>58$. It would be interesting to study the mechanism which generates the deformation in $0_{2}^{+}$bands at $\mathrm{N}>58$. One possibility is that the SOP mechanism is still at work at $\mathrm{N}>58$. Considering the fact that the energy separation between the $0_{1}^{+}$and $0_{2}^{+}$levels is not larger there than in ${ }^{102} \mathrm{Ru}$, it is also possible that the two levels still interact and the strong deformation generated in the $0_{1}^{+}$ground state by the $\nu h_{11 / 2}$ intruder population is passed to the $0_{2}^{+}$level.

At the end, let us remark that the role of the third $0^{+}$level, proposed earlier as a bandhead of a deformed structure, is not clear. In ${ }^{94} \mathrm{Ru},{ }^{100} \mathrm{Ru}$ and ${ }^{102} \mathrm{Ru}$ the $0_{3}^{+}$ level scatters at 750 keV above the $0_{2}^{+}$level. At $\mathrm{N}=60$, it drops to be found only 347 keV above the $0_{2}^{+}$in ${ }^{104} \mathrm{Ru}$ but at $\mathrm{N}=62$ it rises suddenly to 1643 keV above the $0_{2}^{+}$level. It is of interest to search for the unknown $0_{3}^{+}$ excitation energy in ${ }^{96} \mathrm{Ru},{ }^{98} \mathrm{Ru}$ and ${ }^{108} \mathrm{Ru}$, to see if any interaction pattern is emerging between the $0_{2}^{+}$and $0_{3}^{+}$ levels.

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