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Nuclear electric dipole moment of three-body system

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Abstract

Background The existence of Electric Dipole Moment (EDM) of stable nuclei would be a direct evidence of time reversal invariance violation (TRIV). Therefore, it's measurement could be considered as a complementary to the search for neutron and atomic EDMs.

Purpose To clarify theoretical issues related to calculations of EDMs in many body systems we calculated the EDMs of the simplest nuclei.

Method For calculations of three-nucleon systems EDMs we used TRIV potentials based on the meson exchange theory, as well as the ones derived by using effective field theories (EFT) with and without explicit pions. Nuclear wave functions were obtained by solving Faddeev equations in configuration space for the complete Hamiltonians comprising both TRIV and realistic strong interactions.

Results The expressions for EDMs of ^{3}He and ^{3}H are given in terms of meson exchange couplings and low energy constants of EFT potentials.

Conclusions The obtained results are compared with the previous calculations of ^{3}He EDM and with time reversal invariance violating effects in neutron-deuteron scattering. The model dependence on strong interactions is discussed.

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I. INTRODUCTION

The electric dipole moment (EDM) of particles is a very important parameter in searching for Time Reversal Invariance Violation (TRIV) and for the possible manifestation of new physics. The discovery of non-zero value of the EDM would be a clear evidence of TRIV [1], therefore, it has been a subject for intense experimental and theoretical investigations for more than 50 years. The search for TRIV also has fundamental importance for the explanation of the baryon asymmetry [2] of the Universe which requires a source of CP-violation [3] beyond that entering the Cabibbo-Kobayashi- Maskawa matrix of the Standard Model. Any observation of EDM in the near future will be a direct indication of new physics beyond the Standard Model. However, theoretical estimates for the values of particle EDMs are extremely small which results in many difficulties in experimental search for neutron and electron EDMs. Therefore, it is desirable to consider more complex systems, where EDMs or another TRIV parameters could be enhanced. This also would provide assurance that there would be enough observations to avoid a possible "accidental" cancelation of T-violating effects due to unknown structural factors related to strong interactions. The study of the EDM is particularly important for the simplest few-nucleon systems, the result of which may lead to a better understanding of TRIV effects in heavier nuclei. Moreover, few-nucleon systems meet requirements for a number of proposals to measure EDMs of light nuclei in storage rings [4–7].

In this paper, we calculate nuclear EDMs of ^{3}He and ^{3}H using TRIV potential in meson exchange model, as well as the potentials derived from EFT with and without explicit pions. Weak TRIV potentials are used in conjunction with realistic strong interaction Hamiltonians. Several realistic nucleon-nucleon potentials have been tested to represent the strong interaction: Argonne v18 potential (AV18) [8], "Inside Non-local Outside Yukawa tail"(INOY) potential [9], Reid soft-core (Reid) and Nijmegen potentials (NijmII) [10]. Also, we have performed calculations with the AV18 potential in conjunction with the Urbana IX three-nucleon force (AV18+UIX) potential [11]. Three-nucleon wave functions have been obtained by solving Faddeev equations in the configuration space for the complete Hamiltonians, comprising both TRIV and strong interactions.

II. TIME REVERSAL VIOLATING POTENTIALS

The most general form of time reversal violating and parity violating part of nucleon-nucleon Hamiltonian in the first order of relative nucleon momentum can be written as the sum of momentum independent and momentum dependent parts, $H^{\not{T}\not{P}} = H^{\not{T}\not{P}}_{stat} + H^{\not{T}\not{P}}_{non-static}$ [12],

$$\begin{aligned} H^{\not{T}\not{P}}_{stat} &= g_1(r)\boldsymbol{\sigma}_- \cdot \hat{r} + g_2(r)\tau_1 \cdot \tau_2 \boldsymbol{\sigma}_- \cdot \hat{r} + g_3(r) T^z_{12} \boldsymbol{\sigma}_- \cdot \hat{r} \\ &\quad + g_4(r)\tau_+ \boldsymbol{\sigma}_- \cdot \hat{r} + g_5(r)\tau_- \boldsymbol{\sigma}_+ \cdot \hat{r} \end{aligned} \tag{1}$$

$$\begin{aligned} H^{\not{T}\not{P}}_{non-static} &= \left(g_6(r) + g_7(r)\tau_1 \cdot \tau_2 + g_8(r)T^z_{12} + g_9(r)\tau_+\right) \boldsymbol{\sigma}_\times \cdot \frac{\bar{\boldsymbol{p}}}{m_N} \\ &\quad + \left(g_{10}(r) + g_{11}(r)\tau_1 \cdot \tau_2 + g_{12}(r)T^z_{12} + g_{13}(r)\tau_+\right) \\ &\quad \times \left(\hat{r} \cdot \boldsymbol{\sigma}_\times \hat{r} \cdot \frac{\bar{\boldsymbol{p}}}{m_N} - \frac{1}{3}\boldsymbol{\sigma}_\times \cdot \frac{\bar{\boldsymbol{p}}}{m_N}\right) \\ &\quad + g_{14}(r)\tau_- \left(\hat{r} \cdot \boldsymbol{\sigma}_1 \hat{r} \cdot (\boldsymbol{\sigma}_2 \times \frac{\bar{\boldsymbol{p}}}{m_N}) + \hat{r} \cdot \boldsymbol{\sigma}_2 \hat{r} \cdot (\boldsymbol{\sigma}_1 \times \frac{\bar{\boldsymbol{p}}}{m_N})\right) \\ &\quad + g_{15}(r)(\tau_1 \times \tau_2)^z \boldsymbol{\sigma}_+ \cdot \frac{\bar{\boldsymbol{p}}}{m_N} \\ &\quad + g_{16}(r)(\tau_1 \times \tau_2)^z \left(\hat{r} \cdot \boldsymbol{\sigma}_+ \hat{r} \cdot \frac{\bar{\boldsymbol{p}}}{m_N} - \frac{1}{3}\boldsymbol{\sigma}_+ \cdot \frac{\bar{\boldsymbol{p}}}{m_N}\right), \end{aligned} \tag{2}$$

where the exact form of $g_i(r)$ depends on the details of the particular theory and $\boldsymbol{\sigma}_\pm = \boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2$, $\boldsymbol{\sigma}_\times = \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$ and $\tau_\pm = \tau_1 \pm \tau_2$. Because of the additional factor, the non-static potential contributions are suppressed by a factor $\frac{\bar{\boldsymbol{p}}}{m_N}$, therefore, we consider here only static TRIV interactions which could be obtained within three different approaches: in a meson exchange model, EFT with and without explicit pions. [1].

The TRIV meson exchange potential in general involves exchanges of pions ($J^P = 0^-$, $m_\pi = 138$ MeV), η-mesons($J^P = 0^-$, $m_\eta = 550$ MeV), and ρ- and ω-mesons ($J^P = 1^-$, $m_{\rho,\omega} = 770, 780$ MeV). To derive this potential, one can use strong $\mathcal{L}^{st}$ and TRIV $\mathcal{L}_{\not{T}\not{P}}$ Lagrangians, which can be written as [13, 14]

$$\begin{aligned} \mathcal{L}^{st} &= g_\pi \bar{N} i\gamma_5 \tau^a \pi^a N + g_\eta \bar{N} i\gamma_5 \eta N \\ &\quad - g_\rho \bar{N}\left(\gamma^\mu - i\frac{\chi_V}{2m_N}\sigma^{\mu\nu}q_\nu\right)\tau^a \rho^a_\mu N \\ &\quad - g_\omega \bar{N}\left(\gamma^\mu - i\frac{\chi_S}{2m_N}\sigma^{\mu\nu}q_\nu\right)\omega_\mu N, \end{aligned} \tag{3}$$

[1] We used term 'pionful' EFT for the EFT with explicit pion degrees of freedom in previous papers.

$$
\begin{aligned}
\mathcal{L}_{\not T \not P} \;=\;& \bar{N}[\bar{g}_\pi^{(0)}\tau^a\pi^a + \bar{g}_\pi^{(1)}\pi^0 + \bar{g}_\pi^{(2)}(3\tau^z\pi^0 - \tau^a\pi^a)]N \\
&+\bar{N}[\bar{g}_\eta^{(0)}\eta + \bar{g}_\eta^{(1)}\tau^z\eta]N \\
&+\bar{N}\frac{1}{2m_N}[\bar{g}_\rho^{(0)}\tau^a\rho_\mu^a + \bar{g}_\rho^{(1)}\rho_\mu^0 + \bar{g}^{(2)}(3\tau^z\rho_\mu^0 - \tau^a\rho_\mu^a)]\sigma^{\mu\nu}q_\nu\gamma_5 N \\
&+\bar{N}\frac{1}{2m_N}[\bar{g}_\omega^{(0)}\omega_\mu + \bar{g}_\omega^{(1)}\tau^z\omega_\mu]\sigma^{\mu\nu}q_\nu\gamma_5 N, \qquad (4)
\end{aligned}
$$

where $q_\nu = p_\nu - p'_\nu$, χ_V and χ_S are iso-vector and scalar magnetic moments of a nucleon ($\chi_V = 3.70$ and $\chi_S = -0.12$), and $\bar{g}_\alpha^{(i)}$ are TRIV meson-nucleon coupling constants.

Then, a TRIV potential obtained from these Lagrangians can be written as

$$
\begin{aligned}
V_{\not T \not P} \;=\;& \left[-\frac{\bar{g}_\eta^{(0)}g_\eta}{2m_N}\frac{m_\eta^2}{4\pi}Y_1(x_\eta) + \frac{\bar{g}_\omega^{(0)}g_\omega}{2m_N}\frac{m_\omega^2}{4\pi}Y_1(x_\omega)\right]\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+\left[-\frac{\bar{g}_\pi^{(0)}g_\pi}{2m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi) + \frac{\bar{g}_\rho^{(0)}g_\rho}{2m_N}\frac{m_\rho^2}{4\pi}Y_1(x_\rho)\right]\tau_1\cdot\tau_2\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+\left[-\frac{\bar{g}_\pi^{(2)}g_\pi}{2m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi) + \frac{\bar{g}_\rho^{(2)}g_\rho}{2m_N}\frac{m_\rho^2}{4\pi}Y_1(x_\rho)\right]T_{12}^z\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+\left[-\frac{\bar{g}_\pi^{(1)}g_\pi}{4m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi) + \frac{\bar{g}_\eta^{(1)}g_\eta}{4m_N}\frac{m_\eta^2}{4\pi}Y_1(x_\eta) + \frac{\bar{g}_\rho^{(1)}g_\rho}{4m_N}\frac{m_\rho^2}{4\pi}Y_1(x_\rho) + \frac{\bar{g}_\omega^{(1)}g_\omega}{4m_N}\frac{m_\omega^2}{4\pi}Y_1(x_\omega)\right]\tau_+\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+\left[-\frac{\bar{g}_\pi^{(1)}g_\pi}{4m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi) - \frac{\bar{g}_\eta^{(1)}g_\eta}{4m_N}\frac{m_\eta^2}{4\pi}Y_1(x_\eta) - \frac{\bar{g}_\rho^{(1)}g_\rho}{4m_N}\frac{m_\rho^2}{4\pi}Y_1(x_\rho) + \frac{\bar{g}_\omega^{(1)}g_\omega}{4m_N}\frac{m_\omega^2}{4\pi}Y_1(x_\omega)\right]\tau_-\boldsymbol{\sigma}_+\cdot\hat{r}, \\
&\qquad (5)
\end{aligned}
$$

where $T_{12}^z = 3\tau_1^z\tau_2^z - \tau_1\cdot\tau_2$, $Y_1(x) = (1+\frac{1}{x})\frac{e^{-x}}{x} = -\frac{d}{dx}Y_0(x)$, $Y_0(x) = \frac{e^{-x}}{x}$,$x_a = m_a r$.

Comparing eq. (1) with this potential, one can see that $g_i(r)$ functions in a meson exchange model can be identified as

$$
\begin{aligned}
g_1^{ME}(r) \;=\;& \bar{g}_\eta^{(0)}g_\eta\widetilde{Y}_1(m_\eta,r) - \bar{g}_\omega^{(0)}g_\omega\widetilde{Y}_1(m_\omega,r) \\
g_2^{ME}(r) \;=\;& \bar{g}_\pi^{(0)}g_\pi\widetilde{Y}_1(m_\pi,r) - \bar{g}_\rho^{(0)}g_\rho\widetilde{Y}_1(m_\rho,r) \\
g_3^{ME}(r) \;=\;& \bar{g}_\pi^{(2)}g_\pi\widetilde{Y}_1(m_\pi,r) - \bar{g}_\rho^{(2)}g_\rho\widetilde{Y}_1(m_\rho,r) \\
g_4^{ME}(r) \;=\;& \frac{1}{2}\bar{g}_\pi^{(1)}g_\pi\widetilde{Y}_1(m_\pi,r) - \frac{1}{2}\bar{g}_\eta^{(1)}g_\eta\widetilde{Y}_1(m_\eta,r) - \frac{1}{2}\bar{g}_\rho^{(1)}g_\rho\widetilde{Y}_1(m_\rho,r) - \frac{1}{2}\bar{g}_\omega^{(1)}g_\omega\widetilde{Y}_1(m_\omega,r) \\
g_5^{ME}(r) \;=\;& \frac{1}{2}\bar{g}_\pi^{(1)}g_\pi\widetilde{Y}_1(m_\pi,r) + \frac{1}{2}\bar{g}_\eta^{(1)}g_\eta\widetilde{Y}_1(m_\eta,r) + \frac{1}{2}\bar{g}_\rho^{(1)}g_\rho\widetilde{Y}_1(m_\rho,r) - \frac{1}{2}\bar{g}_\omega^{(1)}g_\omega\widetilde{Y}_1(m_\omega,r), \qquad (6)
\end{aligned}
$$

where $\widetilde{Y}_i(\Lambda,r) \equiv -\frac{1}{2m_N}\frac{\Lambda^2}{4\pi}Y_i(\Lambda r)$.

The TRIV potentials in pionless EFT (without explicit pion contributions) contain only point-like nucleon-nucleon interactions which are proportional to the local delta functions.

Thus, one can write the corresponding $g_i(r)$ functions as

$$g^{\not\pi}_{i=1..5}(r) = \frac{c^{\not\pi}_{i=1..5}}{2m_N}\frac{d}{dr}\delta^{(3)}(r) \to \frac{c^{\not\pi}_{i=1..5}\mu^2}{2m_N}\left(-\frac{\mu^2}{4\pi}Y_1(\mu r)\right) = c^{\not\pi}_{i=1..5}\mu^2\widetilde{Y}_1(\mu, r), \qquad (7)$$

where low energy constants (LECs) $c^{\not\pi}_i$ of pionless EFT have the dimension of $[fm^2]$. Here, for numerical calculations we approximate the singular delta functions by the Yukawa type functions as $\delta^{(3)}(r) \simeq \frac{\mu^3}{4\pi}Y_0(\mu r)$, as it was done in [14], with a natural scale of the parameter $\mu \simeq m_\pi$. We use this general form of a leading order of TVPV potential with an assumption of naturalness of low energy constants (LECs) in the Lagrangian. However, LECs can be suppressed for some models of TRIV, which can lead to the importance of higher order contributions to the potential (see, for example [15]).

In the EFT with explicit pions, the long range terms of the potential are due to the one pion exchange whereas the short range terms are similar to the ones obtained within the pionless EFT. Then, by ignoring the contribution of the two pion exchange at the middle range scale, as well as other higher order corrections, one can write $g_i(r)$ functions for the EFT with explicit pion as

$$\begin{aligned}
g^\pi_1(r) &= c^\pi_1\mu^2\widetilde{Y}_1(\mu, r) \\
g^\pi_2(r) &= c^\pi_2\mu^2\widetilde{Y}_1(\mu, r) + \bar{g}^{(0)}_\pi g_\pi \widetilde{Y}_1(m_\pi, r) \\
g^\pi_3(r) &= c^\pi_3\mu^2\widetilde{Y}_1(\mu, r) + \bar{g}^{(2)}_\pi g_\pi \widetilde{Y}_1(m_\pi, r) \\
g^\pi_4(r) &= c^\pi_4\mu^2\widetilde{Y}_1(\mu, r) + \frac{1}{2}\bar{g}^{(1)}_\pi g_\pi \widetilde{Y}_1(m_\pi, r) \\
g^\pi_5(r) &= c^\pi_5\mu^2\widetilde{Y}_1(\mu, r) + \frac{1}{2}\bar{g}^{(1)}_\pi g_\pi \widetilde{Y}_1(m_\pi, r).
\end{aligned} \qquad (8)$$

For this potential, the cutoff scale μ is larger than pion mass, because pion is an explicit degree of freedom of the theory.[2] The following identities might be useful in order to compare this potential with the one used in reference [16]:

$$\frac{g_\pi}{2m_N} \leftrightarrow \frac{g_A}{F_\pi} \text{ in [16]}, \quad \bar{g}^{(0,1)}_\pi \leftrightarrow \left(-\frac{\bar{g}_{0,1}}{F_\pi}\right) \text{ in [16]}, \quad \frac{c^\pi_{1,2}}{2m_N} \leftrightarrow \frac{1}{2}\bar{C}_{1,2} \text{ in [16]}. \qquad (9)$$

[2] These expressions do not include the cutoff for the pion exchange terms. The introduction of this cutoff with the corresponding Fourier transformation will modify the Yukawa functions. This will modify the contributions from the short distance terms, as well as the form of the scalar functions in contact terms. However, these corrections are of a higher order. It should be mentioned that the values of the LECs and their scaling behavior, as a function of a cutoff parameter $c^\pi_i(\mu)$, differ from the corresponding behavior of $c^{\not\pi}_i(\mu)$ terms obtained in pionless EFT.

However, the parameters $c^{\pi}_{3,4,5}$ and $\bar{g}^{(2)}_{\pi}$ were not included at the leading order potential in [16] because they are suppressed for the model of CP-violation considered in [16].

It is important that all these three potentials which come from different approaches have exactly the same operator structure. Thus, the only difference between them is related to the difference in corresponding scalar functions which, in turn, differ only by the values of the characteristic masses: m_π, m_η, m_ρ, and m_ω. Therefore, to unify the notation, it is convenient to define the new constants C^a_n such that

$$V_{\not{T}\not{P}} \equiv \sum_n \left(\sum_a C^a_n \widetilde{Y}_1(\mu_a, r) \right) \mathcal{O}_n \tag{10}$$

where the expressions for constants C^a_n and for operators $\mathcal{O}_n$ are provided in eqs. (6), (7), and (8).

Since the non-static TRIV potential terms, with $g_{n>5}$, do not appear either in a meson exchange model or in the lowest order EFTs, they can be considered as a higher order correction to the lowest order EFT or be related to heavy meson or multi-meson contributions in the meson exchange model.

III. EDM OPERATORS

The value of nuclear EDM is defined as

$$d = \langle JJ|\hat{D}|JJ\rangle = \sqrt{\frac{J}{(2J+1)(J+1)}}\langle J||\hat{D}||J\rangle, \tag{11}$$

where $|JJ\rangle$ is a nuclear wave function with a total spin and its projection equal to J. The EDM operator $\hat{D}$ contains direct contributions from the intrinsic nucleon EDMs (current operators)

$$\hat{D}^{nucleon}_{\not{T}\not{P}} = \sum_i \frac{1}{2}[(d_p + d_n) + (d_p - d_n)\tau^z_i]\boldsymbol{\sigma}_i \tag{12}$$

and contributions from the nuclear EDM polarization operator

$$\hat{D}^{pol}_{\not{T}\not{P}} = \sum_i Q_i \boldsymbol{r}_i, \tag{13}$$

which describe polarization of the nuclei due to TRIV potentials. Here, d_n and d_p are neutron and proton EDMs, and Q_i and r_i are charge and position of i-th nucleon. We neglect 2-body meson exchange currents since their contributions to deuteron EDM are small [14].

Therefore, the value of nuclear EDM can be expressed as

$$d = \frac{1}{\sqrt{6}} \left[\langle\Psi||\hat{D}^{nucleon}_{\not{T}\not{P}}||\Psi\rangle + \langle\Psi_{\not{T}\not{P}}||\hat{D}^{pol}_{TP}||\Psi\rangle + \langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle \right], \tag{14}$$

where $|\Psi\rangle$ and $|\Psi_{\not{T}\not{P}}\rangle$ represent the time reversal invariant and TRIV parts of the nuclear wave function.

In order to calculate polarization contributions to the nuclear EDM, we solve Faddeev equations in a configuration space [17] by including TRIV potentials. We consider neutrons and protons as isospin-degenerate states of the same particle nucleon whose mass is fixed to $\hbar^2/m = 41.471$ MeV·fm. By using the isospin formalism, the three Faddeev equations become formally identical, which for pairwise interactions reads

$$(E - H_0 - V_{ij})\,\psi_k = V_{ij}(\psi_i + \psi_j), \tag{15}$$

where (ijk) are particle indexes, H_0 is the kinetic energy operator, V_{ij} is a two body force between particles i, and j, and $\psi_k = \psi_{ij,k}$ is the so-called Faddeev component. In the last equation, the potential formally contains both a strong interaction (TRI conserving) part (V^{TC}_{ij}) and a TRIV (parity violating) part ($V^{\not{T}\not{P}}$), i.e.: $V_{ij} = V^{TC}_{ij} + V^{\not{T}\not{P}}_{ij}$. Due to the presence of TRIV potential, the system's wave function does not have a definite parity and contains both positive and negative parity components. As a consequence, the Faddeev components of the total wave function can be split into the sum of positive- and negative-parity parts:

$$\psi_k = \psi^+_k + \psi^-_k. \tag{16}$$

Three-nucleon bound state wave function has a strongly predominant positive-parity component. The TRIV interaction is weak ($V^{\not{T}\not{P}}_{ij} << V^{TC}_{ij}$). Then, by neglecting second-order terms in TRIV potential, one obtains a system of two differential equations:

$$\left(E - H_0 - V^{TC}_{ij}\right)\psi^+_k \;=\; V^{TC}_{ij}(\psi^+_i + \psi^+_j), \tag{17}$$

$$\left(E - H_0 - V^{TC}_{ij}\right)\psi^-_k \;=\; V^{TC}_{ij}(\psi^-_i + \psi^-_j) + V^{\not{T}\not{P}}_{ij}(\psi^+_i + \psi^+_j + \psi^+_k) \tag{18}$$

One can see that the first equation (17) defines only the positive-parity part of the wave function. This equation contains only a strong nuclear potential and corresponds to the standard three-nucleon problem: a bound state of helium or triton. The solution of the second differential equation (18), which contains an inhomogeneous term $V^{\not{T}\not{P}}_{ij}(\psi^+_i + \psi^+_j + \psi^+_k)$, gives us the negative-parity components of the wave functions.

To solve these equations numerically, we use our standard procedure described in detail in [18]. Using a set of Jacobi coordinates, defined by $\boldsymbol{x}_k = (\boldsymbol{r}_j - \boldsymbol{r}_i)$ and $\boldsymbol{y}_k = \frac{2}{\sqrt{3}}(\boldsymbol{r}_k - \frac{\boldsymbol{r}_i+\boldsymbol{r}_j}{2})$, we expand each Faddeev component of the wave function in bipolar harmonic basis:

$$\psi_k^{\pm} = \sum_{\alpha} \frac{F_{\alpha}^{\pm}(x_k, y_k)}{x_k y_k} \left| \left(l_x \left(s_i s_j \right)_{s_x} \right)_{j_x} \left(l_y s_k \right)_{j_y} \right\rangle_{JM} \otimes \left| \left(t_i t_j \right)_{t_x} t_k \right\rangle_{TT_z}, \tag{19}$$

where index α represents all allowed combinations of the quantum numbers presented in the brackets, l_x and l_y are the partial angular momenta associated with respective Jacobi coordinates, and s_i and t_i are spins and isospins of the individual particles. Functions $F_\alpha(x_k, y_k)$ are called partial Faddeev amplitudes. In the expansion (19), we consider both possible total isospin channels $T = 1/2$ and $T = 3/2$, regardless of the fact that positive-parity components ψ_k^+ have predominant contribution of $T = 1/2$ state.

Equations (17) and (18) must be supplemented the appropriate boundary conditions for Faddeev partial amplitudes $F_\alpha^{\pm}$: partial Faddeev amplitudes are regular at the origin

$$F_\alpha^{\pm}(0, y_k) = F_\alpha^{\pm}(x_k, 0) = 0, \tag{20}$$

and the system's wave function vanishes exponentially as either x_k or y_k becomes large. This condition is imposed by setting Faddeev amplitudes to vanish at the borders (x_{max}, y_{max}) of a chosen grid, i.e.:

$$F_\alpha^{\pm}(x_k, y_{max}) = 0, \quad F_\alpha^{\pm}(x_{max}, y_k) = 0. \tag{21}$$

This formalism can be easily generalized to accommodate three-nucleon forces, as is described in paper [19].

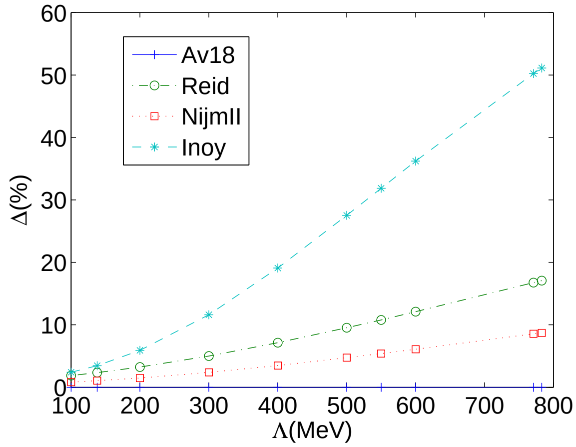
A. Deuteron EDM

First, we analyze the contribution of the intrinsic nucleon EDMs to the deuteron EDM. Then, eq.(12) becomes

$$\hat{D}_{T\!\!\!/P\!\!\!/}^{nucleon} \to (d_p + d_n)\frac{\boldsymbol{\sigma}_{(+)}}{2} + (d_p - d_n)\frac{\tau_{(-)}^z \boldsymbol{\sigma}_{(-)} + \tau_{(+)}^z \boldsymbol{\sigma}_{(+)}}{4}.$$

Since the total isospin of deuteron is zero, only the iso-scalar part of the EDM operator can contribute to the nuclear EDM. Therefore, it's value $d_d^{nucleon} = d_p + d_n$ does not depend on nuclear wave functions and, consequently, on the choice of strong interaction potentials. The

FIG. 1: (Color Online) The relative deviations of the d_d^{pol} value from the one obtained for AV18 potential, $\Delta \equiv \frac{d^{pol}-d^{pol}(AV18)}{d^{pol}(AV18)} \times 100$. Results are presented as a function of the cutoff parameter.



situation is different for three-body systems, ^{3}H or ^{3}He nuclei, where the EDM contributions from intrinsic nucleon EDMs become wave function dependent.

For the choice of the AV18 strong potential the contribution of EDM polarization operator to the deuteron EDM gives

$$d_d^{(pol)} = 18.95 \times 10^{-2}\bar{g}_\pi^1 + 3.52 \times 10^{-3}\bar{g}_\eta^1 + 17.13 \times 10^{-4}\bar{g}_\rho^1 - 49.09 \times 10^{-4}\bar{g}_\omega^1, \qquad (22)$$

which is in excellent agreement with the result of [14]

$$d_d^{(pol)} = 18.69 \times 10^{-2}\bar{g}_\pi^1 + 3.56 \times 10^{-3}\bar{g}_\eta^1 + 17.19 \times 10^{-4}\bar{g}_\rho^1 - 49.17 \times 10^{-4}\bar{g}_\omega^1. \qquad (23)$$

It should be noted that only operator 5 contributes to the $d_d^{(pol)}$ polarization part of deuteron EDM.

To show strong interaction model dependence of EDM due to the polarization operator, we plot a relative deviation of EDMs from the value calculated with the AV18 potential $\Delta \equiv \frac{d^{pol}-d^{pol}(AV18)}{d^{pol}(AV18)} \times 100$ (see Fig.1). One can see that this deviation is very small at the pion mass scale, which is consistent with the observed small model dependence between local potentials and non-local separable potentials [20]. However, when mass scale increases, the parameter Δ becomes larger. The strongest deviation is observed for the INOY potential, which is non-local and has the softest core and tensor strength compare to the other potentials.

B. ^{3}He and ^{3}H EDMs

The the contribution of the intrinsic nucleon EDM momenta to the nuclear EDM for $A = 3$ nuclei are summarized in Table I. Unlike for the case of deuteron, the EDM contributions from intrinsic nucleon EDMs become wave function dependent for ^{3}H or ^{3}He nuclei. One can see that nucleonic contributions to the nuclear EDMs are in rather good agreement for all strong potentials with local interactions: the AV18, the Reid93 and the Nijm II. Nevertheless, these values differ for the models that include non-locality or for the ones where a three-nucleon force is added (see Table I where EDM calculations from the references [16, 21] are also listed).

One can see that strong potential with weaker tensor forces result in nuclear EDM values which are closer to the vector sum of individual nucleon EDMs (i.e. the neutron EDM for ^{3}He case, and the proton EDM for ^{3}H)[3]. Let us mention that our results for this single-nucleonic operator are in excellent agreement with those from references [16, 21].

TABLE I: The nucleon electric dipole moment contributions to nuclear EDMs calculated for different strong interaction potentials.

model	^{3}He	^{3}H
AV18	$-0.0468d_p + 0.877d_n$	$0.877d_p - 0.0480d_n$
Reid93	$-0.0465d_p + 0.878d_n$	$0.879d_p - 0.0475d_n$
NijmII	$-0.0458d_p + 0.880d_n$	$0.880d_p - 0.0468d_n$
AV18UIX	$-0.0542d_p + 0.868d_n$	$0.868d_p - 0.0552d_n$
INOY	$-0.0229d_p + 0.927d_n$	$0.928d_p - 0.0236d_n$
CD-BONN[16, 21]	$-0.0370d_p + 0.897d_n$	-
AV18[16, 21]	$-0.0470d_p + 0.877d_n$	-
EFT NN[16, 21]	$-0.0310d_p + 0.905d_n$	-
EFT NN+NNN[16, 21]	$-0.0350d_p + 0.901d_n$	-

[3] To illustrate the intensity of tensor forces for these potentials we can consider the deuteron D-state probability. The local interaction models give 5.64 % and to 5.76 % for probability for for the NijmII and the Av18 potential, respectively. This value drops to 4.85 % for the CD-BONN potential and is only 3.6 % for the INOY.

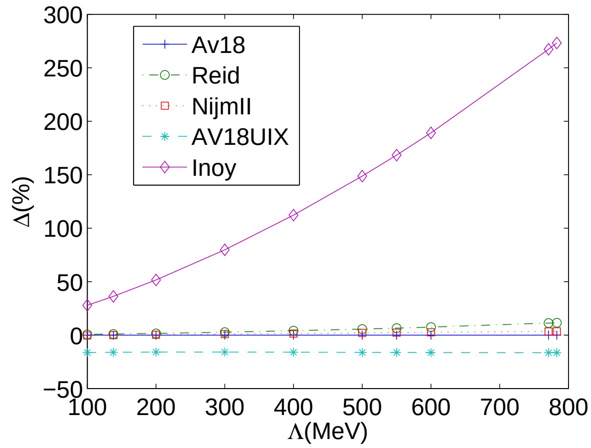
In Table II, we summarize the calculation of EDM from polarization operator, $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$, using potentials $V_{\not{T}\not{P}} = \widetilde{Y}_1(\Lambda, r)\mathcal{O}_n$. The values of the matrix elements for each TRIV operator from Eq.(5) are obtained for a different choice of the strong interaction and calculated for various values of the parameter Λ, corresponding to the masses of π, η, ρ and ω mesons. Therefore, using eqs. (6), (7), and (8), this table can be applied for the analyze of TRIV potentials in the meson exchange model as well as the TRIV potentials in pionless EFT or EFT with explicit pion.

TABLE II: Contribution of the different TRIV operators in Eq.(5) to the expectation value of $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$. Calculations has been performed for several different strong potentials and for ^{3}He (^{3}H) nucleus; values are given in 10^{-3} e-fm units.

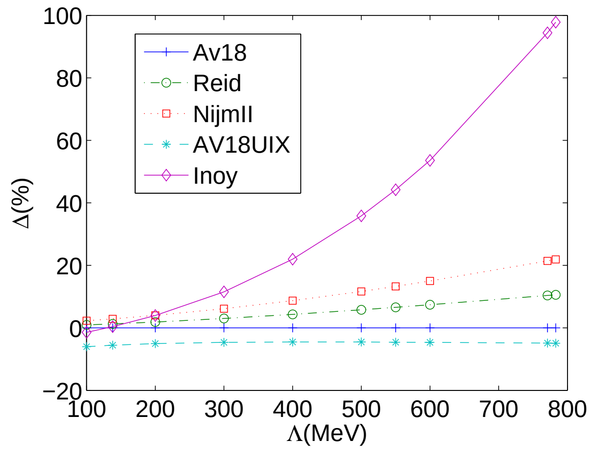
operator	Λ	AV18	Reid93	NijmII	AV18UIX	INOY
1	m_π	−5.32(5.28)	−5.37(5.33)	−5.31(5.28)	−4.46(4.42)	−7.24(7.23)
	m_η	−0.571(0.572)	−0.608(0.609)	−0.584(0.585)	−0.478(0.477)	−1.53(1.54)
	m_ρ	−0.233(0.234)	−0.26(0.261)	−0.241(0.242)	−0.195(0.195)	−0.857(0.862)
	m_ω	−0.223(0.224)	−0.249(0.25)	−0.231(0.232)	−0.187(0.186)	−0.833(0.838)
2	m_π	5.9(−5.89)	6.08(−6.07)	6.12(−6.11)	5.5(−5.48)	10.3(−10.2)
	m_η	0.673(−0.681)	0.803(−0.81)	0.771(−0.777)	0.629(−0.635)	2.72(−2.73)
	m_ρ	0.292(−0.296)	0.387(−0.391)	0.351(−0.354)	0.27(−0.273)	1.6(−1.6)
	m_ω	0.281(−0.284)	0.374(−0.378)	0.337(−0.341)	0.259(−0.262)	1.56(−1.56)
3	m_π	6.76(−7.02)	6.78(−7.01)	6.76(−6.98)	6.66(−6.89)	7.46(−7.72)
	m_η	0.775(−0.814)	0.773(−0.804)	0.762(−0.794)	0.784(−0.819)	1.25(−1.31)
	m_ρ	0.304(−0.32)	0.3(−0.312)	0.295(−0.307)	0.308(−0.322)	0.645(−0.674)
	m_ω	0.29(−0.305)	0.285(−0.297)	0.281(−0.293)	0.294(−0.307)	0.625(−0.653)
4	m_π	2.17(2.42)	2.2(2.41)	2.25(2.46)	2.81(3.03)	2.27(2.48)
	m_η	0.286(0.319)	0.291(0.317)	0.296(0.322)	0.372(0.403)	0.397(0.436)
	m_ρ	0.112(0.125)	0.114(0.125)	0.116(0.127)	0.146(0.159)	0.202(0.223)
	m_ω	0.107(0.12)	0.109(0.119)	0.111(0.121)	0.139(0.152)	0.196(0.216)
5	m_π	19.4(19.6)	19.6(19.8)	20(20.2)	18.3(18.5)	19.5(19.6)
	m_η	2.43(2.47)	2.59(2.63)	2.75(2.8)	2.32(2.35)	3.5(3.56)
	m_ρ	0.985(1.01)	1.09(1.11)	1.2(1.22)	0.937(0.953)	1.92(1.95)
	m_ω	0.942(0.961)	1.04(1.06)	1.15(1.17)	0.896(0.911)	1.86(1.9)

Similar to the intrinsic nucleon EDM contributions, presented in table I, nuclear EDMs due to the polarization operator also are rather insensitive to the choice of the local strong interaction potential. The addition of the three-nucleon force affects the results by 10-20%, whereas the presence of the strong non-locality in two-nucleon interaction (as in the INOY model) has a large impact. This fact is also confirmed by the cutoff dependence behavior of the relative deviations of the $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$ matrix elements for operators 1

FIG. 2: (Color Online) The relative deviations of the $d^{pol}_{^3\mathrm{He}}$ value from the one obtained for AV18 potential, $\Delta \equiv \frac{d^{pol} - d^{pol}(AV18)}{d^{pol}(AV18)} \times 100$. Results are presented for the operators 1(upper) and 5(lower) and as a function of the cutoff parameter.



(a)Δ for operator 1



(b)Δ for operator 5

and 5 calculated for different strong interaction potentials in relation to the matrix elements calculated with the AV18 potential (see Fig. (2)), where the first plot corresponds to the operator 1 and second one to the operator 5. The general behavior of the deviation Δ as a function of a mass scale parameter Λ is similar to the deuteron EDM results. As increasing the parameter Λ, the result become more sensitive to the short range details of a potential model. The largest deviation for the INOY potential was also observed for matrix elements calculated for parity violating radiative neutron capture [22]. However, unlike to the case of parity violating radiative capture, the EDM matrix elements do not show significant dependence at short distances for local strong potentials. One can see that INOY provides systematically the largest values of the matrix elements, while the AV18UIX potential gives the smallest ones. This must be related to the short-range differences of these potentials discussed in the relation of Table I, in particular, to the differences in tensor force strength and locality.

TABLE III: Contributions to $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$ for ^{3}He(^{3}H) EDMs from different terms of meson exchange TRIV potential in 10^{-3} e-fm units. We use the following values for strong couplings constants: $g_\pi = 13.07$, $g_\eta = 2.24$, $g_\rho = 2.75$, $g_\omega = 8.25$. A similar table can be inferred for the case of EFT with and without explicit pion, Table II.

Couplings	AV18	Reid93	NijmII	AV18UIX	INOY	AV18[21]
$\bar{g}^0_\pi$	77.2(-76.9)	79.5(-79.3)	80.0(-79.8)	71.9(-71.6)	134(-134)	157
$\bar{g}^1_\pi$	141(144)	143(145)	145(148)	138(141)	142(145)	288
$\bar{g}^2_\pi$	88.3(-91.8)	88.7(-91.6)	88.3(-91.3)	87.1(-90.1)	98.5(-102)	444
$\bar{g}^0_\rho$	-0.803(0.814)	-1.06(1.08)	-0.964(0.974)	-0.742(0.751)	-4.40(4.41)	-1.65
$\bar{g}^1_\rho$	1.20(1.21)	1.34(1.35)	1.49(1.50)	1.09(1.09)	2.36(2.37)	2.48
$\bar{g}^2_\rho$	-0.836(0.879)	-0.824(0.858)	-0.811(0.845)	-0.846(0.885)	-1.77(1.85)	4.13
$\bar{g}^0_\omega$	1.84(-1.85)	2.05(-2.06)	1.91(-1.91)	1.54(-1.54)	6.88(-6.91)	4.13
$\bar{g}^1_\omega$	-4.33(-4.46)	-4.74(-4.86)	-5.19(-5.32)	-4.27(-4.38)	-8.49(-8.71)	-9.08
$\bar{g}^0_\eta$	-1.28(1.28)	-1.36(1.36)	-1.31(1.31)	-1.07(1.07)	-3.43(3.45)	-
$\bar{g}^1_\eta$	2.40(2.41)	2.57(2.59)	2.75(2.77)	2.18(2.18)	3.48(3.50)	-

In terms of meson-nucleon couplings in meson exchange TRIV potential from Eq.(5), the nuclear EDM from polarization, $d^{(pol)}$, are shown in Table III. The last column in Table III shows the results for ^{3}He EDM obtained in reference [21]. Comparing results of [21] with our calculations for AV18 potential, one can see that there is a systematic discrepancy for all the values of the matrix elements. For these calculations, we have used the same strong and TRIV potential as in [21]. It, therefore, points at the possible systematic error in one of the calculations. We use solutions of the Faddeev equations, while the calculations of the wave functions in [21] have been done in a no-core shell model framework using perturbative expansion for the negative parity states. Based on the presented results, it is impossible to figure out if the discrepancy is the result of a numerical error in one of the algorithms or if there is an intrinsic limitation of the perturbative expansion used in reference [21] with no-core shell model approach[4]. On the other hand, note that our calculations for the deuteron (two-body system) using the same formalism and employing AV18 strong interaction gives excellent agreement with the result of reference [14].

IV. CONCLUSIONS

Using the data from Table III, one can obtain the polarization parts of ^{3}He and ^{3}H EDMs for the choice of AV18UIX strong potential. Then, the expressions for ^{3}He and ^{3}H EDMs can be written as

$$\begin{aligned} d_{^3\mathrm{He}} = & (-0.0542 d_p + 0.868 d_n) + 0.072[\bar{g}_\pi^{(0)} + 1.92\bar{g}_\pi^{(1)} + 1.21\bar{g}_\pi^{(2)} - 0.015\bar{g}_\eta^{(0)} + 0.03\bar{g}_\eta^{(1)} \\ & -0.010\bar{g}_\rho^{(0)} + 0.015\bar{g}_\rho^{(1)} - 0.012\bar{g}_\rho^{(2)} + 0.021\bar{g}_\omega^{(0)} - 0.06\bar{g}_\omega^{(1)}] e\cdot fm \end{aligned} \quad (24)$$

and

$$\begin{aligned} d_{^3\mathrm{H}} = & (0.868 d_p - 0.0552 d_n) - 0.072[\bar{g}_\pi^{(0)} - 1.97\bar{g}_\pi^{(1)} + 1.26\bar{g}_\pi^{(2)} - 0.015\bar{g}_\eta^{(0)} - 0.030\bar{g}_\eta^{(1)} \\ & -0.010\bar{g}_\rho^{(0)} - 0.015\bar{g}_\rho^{(1)} - 0.012\bar{g}_\rho^{(2)} + 0.022\bar{g}_\omega^{(0)} + 0.061\bar{g}_\omega^{(1)}] e\cdot fm. \end{aligned} \quad (25)$$

It should be noted that in general neutron and proton EDMs can not be related to the meson-nucleon TRIV constants from TRIV potential. However, it is convenient to present expressions for these EDMs, obtained in the chiral limit [23] with the assumption that nucleon EDM are resulted from TRIV potential

$$d_n = -d_p = \frac{e}{m_N}\frac{g_\pi(\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)})}{4\pi^2}\ln\frac{m_N}{m_\pi} \simeq 0.14(\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)}), \quad (26)$$

which could be used for some models of CP-violation.

Finally, one can compare the obtained expressions for nuclear EDMs with TRIV effects in neutron deuteron elastic scattering related to the $\boldsymbol{\sigma}_n \cdot (\boldsymbol{p} \times \boldsymbol{I})$ correlation, where $\boldsymbol{\sigma}_n$ is the neutron spin, $\boldsymbol{I}$ is the target spin, and $\boldsymbol{p}$ is the neutron momentum, which can be observed in the transmission of polarized neutrons through a target with polarized nuclei.

[4] Curiously enough, our results differ from the ones of reference [21] roughly by the factor 2 for all the isospin rank-0 and rank-1 TRIV potential terms, whereas isospin rank-2 operator results differ by factor 5.

This correlation leads to the difference [24] between the total neutron cross sections for $\boldsymbol{\sigma}_n$ parallel and anti-parallel to $\boldsymbol{p} \times \boldsymbol{I}$

$$\Delta\sigma_{\not{T}\not{P}} = \frac{4\pi}{p}\mathrm{Im}(f_+ - f_-), \tag{27}$$

and to neutron spin rotation angle [25] ϕ around the axis $\boldsymbol{p} \times \boldsymbol{I}$

$$\frac{d\phi_{\not{T}\not{P}}}{dz} = -\frac{2\pi N}{p}\mathrm{Re}(f_+ - f_-). \tag{28}$$

Here, $f_{+,-}$ are the zero-angle scattering amplitudes for neutrons polarized parallel and anti-parallel to the $\boldsymbol{p} \times \boldsymbol{I}$ axis, respectively, z is the target length, and N is the number of target nuclei per unit volume. Using results of [26], one can write

$$\begin{aligned} \frac{1}{N}\frac{d\phi^{\not{T}\not{P}}}{dz} = & \ (-65 \text{ rad} \cdot \ \text{fm}^2)[\bar{g}_\pi^{(0)} + 0.12\bar{g}_\pi^{(1)} + 0.0072\bar{g}_\eta^{(0)} + 0.0042\bar{g}_\eta^{(1)} \\ & -0.0084\bar{g}_\rho^{(0)} + 0.0044\bar{g}_\rho^{(1)} - 0.0099\bar{g}_\omega^{(0)} + 0.00064\bar{g}_\omega^{(1)}] \end{aligned} \tag{29}$$

and

$$\begin{aligned} P^{\not{T}\not{P}} = \frac{\Delta\sigma^{\not{T}\not{P}}}{2\sigma_{tot}} = & \ \frac{(-0.185 \text{ b})}{2\sigma_{tot}}[\bar{g}_\pi^{(0)} + 0.26\bar{g}_\pi^{(1)} - 0.0012\bar{g}_\eta^{(0)} + 0.0034\bar{g}_\eta^{(1)} \\ & -0.0071\bar{g}_\rho^{(0)} + 0.0035\bar{g}_\rho^{(1)} + 0.0019\bar{g}_\omega^{(0)} - 0.00063\bar{g}_\omega^{(1)}]. \end{aligned} \tag{30}$$

One can see that both nuclear EDMs and elastic scattering TRIV effects are mostly sensitive to TRIV pion coupling constants. However, while the EDM values are equally sensitive to all isospin parts of the pion coupling constant, the elastic scattering effects are mainly defined by the isoscalar interactions. This fact clearly demonstrates the complementarity of different TRIV effects in three-nucleon systems. Thus, the relative values of these TRIV parameters may vary for differen models of CP-violation and, therefore, the measurement of a number of TRIV observables can help to avoid a possible accidental cancelation of TRIV.

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