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A Regge Model for Nucleon-Nucleon Spin Dependent Amplitudes

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Abstract

There are currently no models readily available that provide nucleon-nucleon spin dependent scattering amplitudes at high energies ($s \geq 6 \text{ GeV}^2$). This work aims to provide a model for calculating these high energy scattering amplitudes. The foundation of the model is Regge theory since it allows for a relativistic description and full spin dependence. We present our parameterization of the amplitudes, and show comparisons of our solutions to the data set we have collected. Overall the model works as intended, and provides an adequate description of the scattering amplitudes.

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I. INTRODUCTION

We present a model for calculating elastic, spin-dependent scattering amplitudes for the nucleon-nucleon system. While much work has been applied to this topic over the years there is no analysis available for both proton-proton and proton-neutron in the mid to high energy range, Mandelstam $s > 6 \text{ GeV}^2$. Thus far the most complete, highest energy, and readily available work is the SAID [1, 2] analysis which provides the proton-neutron amplitudes to $s \approx 6 \text{ GeV}^2$ and the proton-proton amplitudes up to $s \approx 9.8 \text{ GeV}^2$. Our goal is to calculate the amplitudes at higher energies. In order to accurately describe the nucleon-nucleon system at these energies we require a fully relativistic, spin dependent model. Furthermore, due to the scarcity of data, particularly in the proton-neutron case, we require a model which will provide some confidence in extrapolating the results to higher energies.

Our primary motivation in building this model is to utilize the amplitudes to describe the final state interactions of deuteron electrodisintegration. It has been shown that a complete description of the final state interactions is necessary in order to accurately describe this process [3–5]. Currently the final state interactions are given by the SAID amplitudes, however, the kinematics at Jefferson lab, where experiments have been performed, allow for final state nucleons with energies greater than can be described by SAID.

Our approach is to parameterize the helicity amplitudes in terms of Regge poles or exchanges [6–10]. Regge theory has had great phenomenological success over the years. The theory is fully relativistic and allows for a complete spin-dependent description. Regge theory is also ideal for us to use since it readily scales to higher energies. Furthermore, Regge theory has been utilized in the past to model proton-neutron scattering at mid-range energies with good results [11]. Overall Regge theory provides us a systematic method of parameterizing the scattering amplitudes, while meeting all the criteria of the model.

The fundamental idea of Regge theory is to study the analytic behavior of the amplitudes, when one allows the angular momentum J to be continuous and complex. While the analysis is rigorous[6] for non-relativistic scattering, the relativistic case is based on a series of assumptions. However, it is in the relativistic case, by exploiting crossing symmetry, that Regge theory is useful as a parameterization method. Performing the Regge analysis in the t -channel center of momentum frame, the large s , small t approximation to the amplitudes in the s -channel center of momentum frame is obtained [7–9].

While Regge theory excels at high energies, at lower energies it becomes more difficult to implement, as more and more Regge exchanges can contribute. Because of this feature, however, it naturally lends itself as a method for extrapolating to higher energies, since as one increases in energy the low energy poles are suppressed. In our approach we are able to fit to the low energy nucleon-nucleon data and extrapolate our results to higher energy regions where data is unavailable.

Section II discusses our method of parameterizing the helicity amplitudes in terms of Regge poles. Then in Section III we present comparisons between the data and our results for both a polarized and unpolarized fit solution. We conclude in Section IV with a summary and outlook.

II. THEORETICAL FRAMEWORK

All observables in the nucleon-nucleon system can be described by five independent helicity amplitudes [12]. These amplitudes are given as,

$$\begin{aligned}
a &= \phi_1 = \langle ++ | T | ++ \rangle \\
b &= \phi_5 = \langle ++ | T | +- \rangle \\
c &= \phi_3 = \langle +- | T | +- \rangle \\
d &= \phi_2 = \langle ++ | T | -- \rangle \\
e &= \phi_4 = \langle +- | T | -+ \rangle.
\end{aligned} \tag{1}$$

In order to calculate the amplitudes we look to Regge theory to provide us with a parameterization method. Applying Regge theory to the nucleon-nucleon system presents some challenges, primarily due to the inclusion of spin. The Regge analysis should be performed in the crossed (t) channel, and the result crossed back to the direct (s) channel. Because of the many helicity configurations the crossing relations are complicated. Also, Regge exchanges have definite quantum numbers, such as parity P , G -parity G , and isospin I , which must be taken into account, and because of the symmetries of the nucleon-nucleon system, any non-strange mesonic Regge exchange can contribute. Finally, since nucleons are fermions we need to properly take into account Fermi statistics, shown in Fig. 1.

Fortunately we can either avoid, or at least simplify, these complications by relating

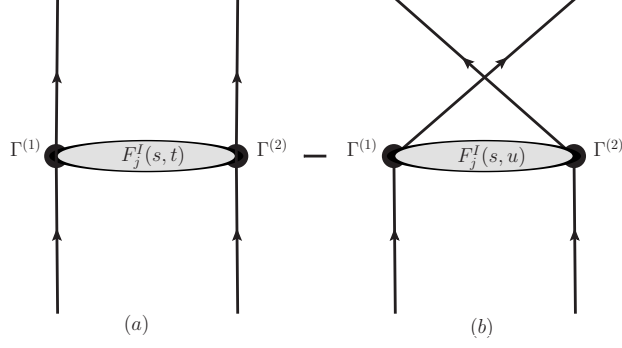


FIG. 1. Pictorial representation of the helicity amplitudes in terms of the Fermi invariants. Γ represents the various gamma matrices which contribute to this process.

the Regge exchanges to the Fermi invariants [13, 14],

$$\begin{aligned} \hat{T} = & F_S^I(s, t) 1^{(1)} \cdot 1^{(2)} - F_P^I(s, t) (i\gamma_5)^{(1)} \cdot (i\gamma_5)^{(2)} \\ & + F_V^I(s, t) \gamma^{\mu(1)} \gamma_\mu^{(2)} + F_A^I(s, t) (\gamma_5 \gamma^\mu)^{(1)} (\gamma_5 \gamma_\mu)^{(2)} \\ & + F_T^I(s, t) \sigma^{\mu\nu(1)} \sigma_{\mu\nu}^{(2)} \end{aligned} \quad (2)$$

where s and t are the Mandelstam variables, I is an isospin label, 1 and 2 correspond to the vertices shown in Fig. 1. This immediately benefits us since we get all spin dependence “out in the open”, and we can immediately do the Dirac algebra to get the s-channel helicity amplitudes,

$$T_i^{pp \rightarrow pp} = \sum_j \{ C_{ij}^t [F_j^0(s, t) + F_j^1(s, t)] - C_{ij}^u [F_j^0(s, u) + F_j^1(s, u)] \} \quad (3)$$

$$T_i^{pn \rightarrow pn} = \sum_j \{ C_{ij}^t [F_j^0(s, t) - F_j^1(s, t)] - 2C_{ij}^u F_j^1(s, u) \}, \quad (4)$$

where i corresponds to the helicity configurations $(++; ++)$, $(++; +-)$, $(+-; +-)$, $(++; --)$, $(+-; -+)$, and j to the different types of Fermi invariants S, V, T, P, A . The matrices C_{ij}^t and C_{ij}^u , containing all the spin dependence, are obtained from performing the Dirac algebra, and are given in the appendix. For convenience Mandelstam u is used in the terms corresponding to the interchange of the final state particles necessary to account for Fermi statistics, Fig. 1(b).

Our goal, instead of Reggeizing the helicity amplitudes directly, is to instead parameterize the Fermi invariants in terms of Regge exchanges. This is extremely beneficial since the crossing relations become trivial as the Fermi invariants are invariant, and since we have

taken care of spin explicitly, we will see that the Regge analysis will reduce to the spinless case.

A. Relation Between Fermi Invariants and Regge Exchanges

Regge exchanges are found in the crossed channel and have definite quantum numbers P , G , and I . I is easily factored out and is taken care of in (3) and (4), so we simply need to retain the label here. Also, we need only concern ourselves with exchanges related to Fig. 1(a), since Reggeization of Fig. 1(b) can easily be obtained by interchanging $t \leftrightarrow u$ in our final result. In order to find the Regge contributions to the nucleon-nucleon ($NN \rightarrow NN$) system, we need to analyze the t -channel, nucleon-anti-nucleon ($N\bar{N} \rightarrow N\bar{N}$) amplitudes of definite P and G .

We will focus only on the initial state since there is a simple relation between the initial and final states. Our basic state that we will work with is,

$$(\psi_{in})_{\alpha\beta} = \bar{v}_{\alpha}(-\vec{p}_t, \lambda_2) u_{\beta}(\vec{p}_t, \lambda_1) |I, M_I\rangle, \quad (5)$$

where p_t is the t -channel center of mass momentum, λ_1 and λ_2 are the helicities of particles 1, and 2, I is the total isospin of the state, M_I is the 3rd component of isospin, and we have labeled the Dirac indices explicitly with α and β . The goal is to symmetrize this state in terms of parity and G -parity. We begin with parity. In Dirac space the parity operator is γ_0 . Parity acting on this state yields,

$$\hat{P}(\psi_{in})_{\alpha\beta} = -\bar{v}_{\alpha}(\vec{p}_t, -\lambda_2) u_{\beta}(-\vec{p}_t, -\lambda_1) |I, M_I\rangle \quad (6)$$

We can construct a vertex with definite parity then as,

$$(\psi_{in}^P)_{\alpha\beta} \Gamma_{\alpha\beta}^{(1)} = \frac{1}{\sqrt{2}} [\bar{v}(-\vec{p}_t, \lambda_2) \Gamma^{(1)} u(\vec{p}_t, \lambda_1) - P \bar{v}(\vec{p}_t, -\lambda_2) \Gamma^{(1)} u_{\beta}(-\vec{p}_t, -\lambda_1)] |I, M_I\rangle \quad (7)$$

Defining $\gamma_0 \Gamma \gamma_0 = P_{\Gamma} \Gamma$, where P_{Γ} is ± 1 , we can simplify (7) to,

$$(\psi_{in}^P)_{\alpha\beta} \Gamma_{\alpha\beta}^{(1)} = \frac{1}{\sqrt{2}} (1 + P_{\Gamma} P) \bar{v}(-\vec{p}_t, \lambda_2) \Gamma^{(1)} u(\vec{p}_t, \lambda_1) |I, M_I\rangle \quad (8)$$

We now move on to G -parity. G -parity is defined as, $\hat{G} = \hat{C} e^{i\pi \hat{I}_2}$, where \hat{I}_2 is the y rotation matrix in isospin space and \hat{C} is the charge conjugation operator, given in Dirac space as,

TABLE I. Symmetries of γ matrices.

| Γ | I | γ^5 | γ^0 | $\vec{\gamma}$ | $\gamma^0\gamma^5$ | $\vec{\gamma}\gamma^5$ | $i\vec{\alpha}$ | $\vec{\Sigma}$ |
|----------------------|-----|------------|------------|----------------|--------------------|------------------------|-----------------|----------------|
| P_Γ | + | - | + | - | - | + | - | + |
| \mathcal{C}_Γ | + | - | - | - | + | + | - | - |

$\hat{\mathcal{C}} = \mathcal{C}\gamma_0 K$, where K is the complex conjugation operator, and

$$\mathcal{C} = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (9)$$

acting on a two particle state, (5), yields,

$$\hat{G}(\psi_{in})_{\alpha\beta} = \bar{u}_\alpha(-\vec{p}_t, \lambda_2) v_\beta(\vec{p}_t, \lambda_1) (-1)^I \eta_{\lambda_2} \eta_{\lambda_1} \eta_C |I, M_I\rangle \quad (10)$$

where $\eta_{\pm\lambda} = (-1)^{1/2-\lambda}$, and η_C is an arbitrary phase which is convenient to define as $\eta_C = (-1)^I \eta_{\lambda_2} \eta_{\lambda_1}$. We can then construct a vertex of definite parity and G -parity as,

$$(\psi_{in}^{PG})_{\alpha\beta} \Gamma_{\alpha\beta}^{(1)} = \frac{1}{2} (1 + P_\Gamma P) [\bar{v}(-\vec{p}_t, \lambda_2) \Gamma^{(1)} u(\vec{p}_t, \lambda_1) + G \bar{u}(-\vec{p}_t, \lambda_2) \Gamma^{(1)} v(\vec{p}_t, \lambda_1)] |I, M_I\rangle \quad (11)$$

Defining $C_\Gamma \Gamma = \mathcal{C}\gamma_0 K \Gamma \mathcal{C}\gamma_0 K$, where C_Γ is ± 1 , this simplifies to,

$$(\psi_{in}^{PG})_{\alpha\beta} \Gamma_{\alpha\beta}^{(1)} = \frac{1}{2} (1 + P_\Gamma P) (1 + \eta_{\lambda_1} \eta_{\lambda_2} \mathcal{C}_\Gamma G) \bar{v}(-\vec{p}_t, \lambda_2) \Gamma^{(1)} u(\vec{p}_t, \lambda_1) |I, M_I\rangle. \quad (12)$$

P_Γ and \mathcal{C}_Γ for the available couplings are given in Table I. Note that we use the decomposition $\sigma^{\mu\nu(1)} \sigma_{\mu\nu}^{(2)} = -2\vec{\alpha}^{(1)} \cdot \vec{\alpha}^{(2)} + 2\vec{\Sigma}^{(1)} \cdot \vec{\Sigma}^{(2)}$.

The result, (12) can also be used to calculate outgoing states of definite parity and G -parity by utilizing the relations,

$$(\bar{u}(\vec{p}, \lambda) \Gamma v(-\vec{p}, \lambda_2))^* = v^\dagger(-\vec{p}, \lambda_2) \Gamma^\dagger \bar{u}^\dagger(\vec{p}, \lambda) = \bar{v}(-\vec{p}, \lambda_2) \Gamma u(\vec{p}, \lambda). \quad (13)$$

With these results we can construct the symmetrized amplitudes,

$$\tilde{T}_{\lambda'_1, \lambda'_2; \lambda_1, \lambda_2}^{PGI} = S_{\lambda_i}^{PG} F_S^I(s, t) + P_{\lambda_i}^{PG} F_P^I(s, t) + V_{\lambda_i}^{PG} F_V^I(s, t) + A_{\lambda_i}^{PG} F_A^I(s, t) + T_{\lambda_i}^{PG} F_T^I(s, t), \quad (14)$$

where λ_i represents $\lambda_1, \lambda_2, \lambda'_1, \lambda'_2$, which are the helicities of the initial and final particles, and $S_{\lambda_i}^{PG}, P_{\lambda_i}^{PG}, V_{\lambda_i}^{PG}, A_{\lambda_i}^{PG}, T_{\lambda_i}^{PG}$ are obtained from the dirac algebra, and are dependent upon specific values of P, G , and helicity configurations.

Now, in order to Reggeize, we set up a partial wave expansion, of definite parity and G -parity, in the t -channel, center of momentum frame,

$$\tilde{T}_{\lambda'_1, \lambda'_2; \lambda_1, \lambda_2}^{PGI} = \sum_J (2J+1) [f_{\lambda'_1, \lambda'_2; \lambda_1, \lambda_2}^{GIJ}(E_t) - P(-1)^{J+\lambda'_1-\lambda_1} f_{\lambda'_1, \lambda'_2; -\lambda_1, -\lambda_2}^{GIJ}(E_t)] d_{\lambda_1-\lambda_2, \lambda'_1-\lambda'_2}^J(\theta_t), \quad (15)$$

where E_t and θ_t are the t -channel center of momentum energy and scattering angle, and the $f_{\lambda'_1, \lambda'_2; \lambda_1, \lambda_2}^{GIJ}(E_t)$ correspond to partial wave coefficients from expanding on to the Wigner-d functions.

We now equate (14) and (15) and select specific P , G and helicity values to isolate and solve for each of the Fermi invariants in terms of partial waves. Essentially, there are many redundant equations since the Fermi invariants do not depend on spin, giving us the freedom to only choose helicity combinations of $(++; ++)$ and $(++; --)$, where this notation refers to $(\lambda_1, \lambda_2; \lambda'_1, \lambda'_2)$. Once this is done all the Wigner-d functions reduce to Legendre polynomials, and the Reggeization procedure reduces to the spinless case. Each Fermi invariant trivially crosses back to the s -channel, (3) and (4), and we do not have to deal with any of the complications that are associated with a typical Regge analysis of particles with spin. From here we can follow the typical methods to Reggeize spinless amplitudes[7–9], the result is,

$$\begin{pmatrix} F_S^I(s, t) \\ F_V^I(s, t) \\ F_T^I(s, t) \\ F_P^I(s, t) \\ F_A^I(s, t) \end{pmatrix} = \begin{pmatrix} \frac{m^2}{2(t-4m^2)} & 0 & 0 & 0 & 0 \\ 0 & \frac{t-4m^2}{8(2s+t-4m^2)} & \frac{t}{8(2s+t-4m^2)} & 0 & 0 \\ 0 & 0 & -\frac{m^2}{4(2s+t-4m^2)} & 0 & 0 \\ 0 & 0 & 0 & \frac{-m^2}{2t} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} \end{pmatrix} \begin{pmatrix} R_{+1}^{I++}(s, t) \\ R_{-2}^{I--}(s, t) \\ R_{-3}^{I+-}(s, t) \\ R_{+4}^{I--}(s, t) \\ R_{+5}^{I-+}(s, t) \end{pmatrix} \quad (16)$$

where $m = .93895$ (GeV) is the nucleon mass, and the right-most vector is defined by a sum of Regge exchanges,

$$R_{\pm j}^{IPG}(s, t) = \zeta(s, t) \sum_k \xi_{k\pm}(t) \beta_k^{IPG}(t) \left(-1 + \frac{2s}{4m^2 - t} \right)^{\alpha_k(t)}, \quad (17)$$

where $\beta(t)$ and $\alpha(t)$ correspond to the residue and the trajectory of the Regge pole and are discussed in the following section, $\zeta(s, t)$ is a cutoff factor also discussed in the following section, j is simply the position of the Regge exchange in the vector, and

$$\xi_{\pm}(t) = \begin{cases} e^{-i(\pi\alpha(t)/2+\delta)} & + \\ -ie^{-i(\pi\alpha(t)/2+\delta)} & - \end{cases} \quad (18)$$

Here we have also introduced an additional phase for each Regge exchange, δ , which accounts for the fact that we have absorbed all extra t dependence into the residue, including the approximation we utilized for $\xi_{\pm}(t)$. Ultimately it provides an extra degree of freedom which is convenient when fitting certain Reggeons. Also note that while Reggeons with $PG = --$, enter into two different positions in (16), the residues of any contributing poles in these positions are not necessarily the same.

Similar constructions, which rely on expanding a set of invariant functions onto Legendre polynomials have been used in the past[15, 16]. While our approach utilizes different techniques, the concept is the same, that is the spin dependence can be dealt with explicitly, thereby simplifying the partial wave expansion, and making the Reggeization process much more transparent.

In order to take into account the u -channel exchanges of Fig. 1(b), we can simply substitute $t \rightarrow u$ in (16). We also utilize an additional factor of $\frac{t}{4m^2}$ for type 4 exchanges, guaranteeing that amplitude $d = 0$ at $t = 0$, which is required by conservation of angular momentum. In addition, we multiply type 5 exchanges with a factor of $\frac{4m^2}{s}$, which we assume we can factor from $F_A(s, t)$. This is necessary in order to cancel with an additional factor of s in the matrix C_{ij}^t and prevents amplitude e from blowing up at large s . This seems to be a general problem with expressing the amplitudes in terms of the Fermi invariants at large s , and $F_A(s, t)$ should either always be redefined or parameterized to explicitly cancel this factor of s in order to avoid this problem. Now that we have the Fermi invariants parametrized in terms of Regge exchanges we can plug this result into (3) and (4) for a Regge approximation of the s -channel helicity amplitudes.

B. Residue and Trajectory

We utilize linear Regge trajectories, $\alpha(t) = \alpha_0 + \alpha_1 t$. These are obtained from the meson masses available from the Particle Data Group[17]. In addition to the mesonic trajectories, we also utilize “effective” trajectories, which we discuss in more detail in the following section.

We use three different parametrizations for the residues,

$$\begin{aligned}\beta_I(t) &= \beta_0 e^{\beta_1 t} \\ \beta_{II}(t) &= (1 - e^{\gamma t}) \beta_0 e^{\beta_1 t} \\ \beta_{III}(t) &= \frac{t}{4m^2} \beta_0 e^{\beta_1 t}\end{aligned}\tag{19}$$

where β_0 , β_1 and γ are fit parameters. We utilize the different types of residues for different Regge exchanges, as well as different fit solutions, based on trial and error.

Equation (17) differs from the usual expression in that we have kept the full expression for $\cos(\theta_t)$. Generally the Regge limit assumes that $\cos(\theta_t) \gg 1$ which implies that $s \gg 4m^2 - t$. In extrapolating from the region where the SAID partial wave analysis has been performed to higher s , we are violating this condition in two respects. First, data where s is of the same order of magnitude as $4m^2$ are included. Second, in the same region there are significant data for $4m^2 - s < t < 0$. So at backward angles t is of the same order of magnitude as s . For this reason we keep the exact expression for $\cos(\theta_t)$.

We have chosen to fit the low s data using the same form as the Regge parameterization, but in doing so it is necessary to introduce additional “effective” trajectories that are not derived from the meson spectrum.

A practical problem associated with fitting these forms at low s is that the u channel contributions necessarily overlap those from the t channel. Fitting to data near $\theta = 0^\circ$, where $t = 0$ and $u = 4m^2 - s$, and near $\theta = 180^\circ$, where $u = 0$ and $t = 4m^2 - s$, can be affected substantially by the tail of the crossed channel. This can cause the fitting procedure to become very sensitive, if not unstable. As a result we have found it useful to introduce a cutoff factor,

$$\zeta(s, t) = \left(1 - e^{20\left(\frac{t}{4m^2 - s} - 1\right)}\right),\tag{20}$$

to decouple the t and u channel contributions at the endpoints in order to simplify the fitting procedure. This has no effect at large s where the two channels have no significant overlap,

but is extremely useful for smaller values of s .

C. Electromagnetic Effects

In order to properly describe the proton-proton interaction we need to account for electromagnetic effects. We use the full proton vertex (E1), with a one photon exchange. The one photon exchange amplitudes are given in the appendix (E4). In order to account for higher order effects we utilize a helicity-dependent constant and phase. Since the electromagnetic contribution is dominated by “no flip” and “single flip” contributions we redefine the one photon exchange amplitudes as follows,

$$a'_{EM}(s, t) = \beta_a e^{i\delta_a} a_{EM}(s, t) \quad (21)$$

$$b'_{EM}(s, t) = \beta_b e^{i\delta_b} b_{EM}(s, t) \quad (22)$$

$$c'_{EM}(s, t) = \beta_c e^{i\delta_c} c_{EM}(s, t), \quad (23)$$

where β_a , β_b , β_c , δ_a , δ_b , and δ_c are fit to available polarization and differential cross section data. Utilizing this approach allows us to keep the electromagnetic effects under control, and smoothly fit between the Coulomb and hadronic regions.

III. RESULTS

We present here two solutions to our fits, an unpolarized version as well as a polarized version. While one of our primary goals is to include all spin dependence, we also foresee applications for which an unpolarized solution will prove useful.

In comparison with the SAID analysis, there is overlap between the two models at lower energies, and we expect the SAID solution to be more precise, i.e. lower χ^2 . The emphasis on our fit is the ability to extrapolate to higher energies. Using a Regge model over the entire angular region may give less precise results than the SAID parameterization, but it does allow this extrapolation. A further test of our results will be when we implement them into the electrodisintegration of the deuteron process, as it will allow us to see how much the results vary in comparison with the use of the SAID amplitudes through the kinematic region of overlap.

The data set was assembled from the SAID analysis [2], the Durham database [18], the

Cudell dataset [19], and the Particle Data Group [17]. We also reference here the original papers [20–186]. The dataset that we have collected can currently be obtained by contacting the authors. Where both the statistical and systematic errors are available, we use the larger of the two in fitting the model to the data.

All observables were fit simultaneously. The χ^2 values are given in Table II, and the parameter values are given in Tables III and IV for the polarized and unpolarized solutions respectively. In order to avoid the largest data set dominating the fit, we implemented weights which were varied in order to keep all observables on the same footing. This was especially useful since the proton-neutron data is so limited in comparison to the proton-proton data. Because our data set includes differential cross section data from many sources, there is a potential problem with normalization. In order to correct for this we fit to the shape of the differential cross section data, and allow the overall magnitude of the data to float by plus or minus 15%.

The trajectories labeled X_i are effective trajectories that are introduced to obtain a fit to the data. Trajectories X_1 and X_2 are introduced to reproduce the diffractive structure in the proton-proton differential cross sections at large values of s . Several of the remaining effective trajectories have relatively large negative values of α_0 . These are required to provide more rapid variation of observables at small values of s . The need for this is most obvious for the total cross sections, which reach a peak and then decrease in value with decreasing s . Although less obvious, these are also important for other observables at low s .

The parameters of the fit are generally quite tightly constrained for all trajectories. However, both β_1 and α_1 contribute to the t dependence of the amplitudes; β_1 through the exponential factor, and α_1 by changing the exponent of $(-1 + \frac{2s}{4m^2 - t})$. If β_1 is large, it controls the t dependence and α_1 is not tightly constrained. Examination of the uncorrelated errors indicates that, with the exception of the situation mentioned above, errors in the fitting parameters are less than one percent. A more sophisticated error analysis is planned as a future work.

The total cross sections for both proton-proton and proton-neutron are presented in Fig. 2. As these are calculated at $t = 0$, the Regge approximation works extremely well.

In order to constrain the model at large s , we also fit to high energy proton-proton data. In Fig. 3 we show differential cross sections through both the Coulomb and dip regions, as well as the polarization parameter. These results illustrate the ability of the Regge model

TABLE II. χ^2 values for both the unpolarized and polarized solutions. The unpolarized solution is based on 6111 data points and has 131 parameters. The polarized solution is based on 13869 data points and has 136 parameters.

| Observable | N | Unpolarized | | Polarized | |
|-------------------------------------|-----------|-------------|------------|-----------|------------|
| | | χ^2 | χ^2/N | χ^2 | χ^2/N |
| σ | pp 181 | 151.3 | 0.8 | 160.4 | 0.9 |
| | pn 69 | 9.3 | 0.1 | 11.1 | 0.2 |
| $\frac{d\sigma}{dt}(s > 20(GeV^2))$ | pp 1635 | 2872.5 | 1.8 | 2853.5 | 1.7 |
| $\frac{d\sigma}{dt}$ | pp 3481 | 6513.5 | 1.9 | 8353.3 | 2.4 |
| | pn 745 | 1338.4 | 1.8 | 1963.4 | 2.6 |
| $P(A_N)$ | pp 3410 | | | 8411.2 | 2.5 |
| | pn 508 | | | 1600.2 | 3.1 |
| A_{YY} | pp 1587 | | | 7371.3 | 4.6 |
| | pn 117 | | | 306.0 | 2.6 |
| A_{ZX} | pp 568 | | | 3159.2 | 5.6 |
| | pn 81 | | | 96.6 | 1.2 |
| A_{ZZ} | pp 608 | | | 3505.9 | 5.8 |
| | pn 89 | | | 229.8 | 2.6 |
| A_{XX} | pp 276 | | | 2616.7 | 9.5 |
| D | pp 188 | | | 919.5 | 4.9 |
| | pn 37 | | | 111.3 | 3.0 |
| D_T | pp 281 | | | 1885.5 | 6.7 |
| | pn 8 | | | 3.1 | 0.4 |
| total | | 10885.1 | 1.8 | 43558.0 | 3.1 |

to scale to higher energies.

Low energy differential cross sections for proton-proton and proton-neutron are shown in Figs. 4 and 5. The model works very well, especially considering that we describe the data over the entire angular region, and for relatively low s , well outside of where one would typically expect the Regge approximation to be valid. We have also included the unpolarized

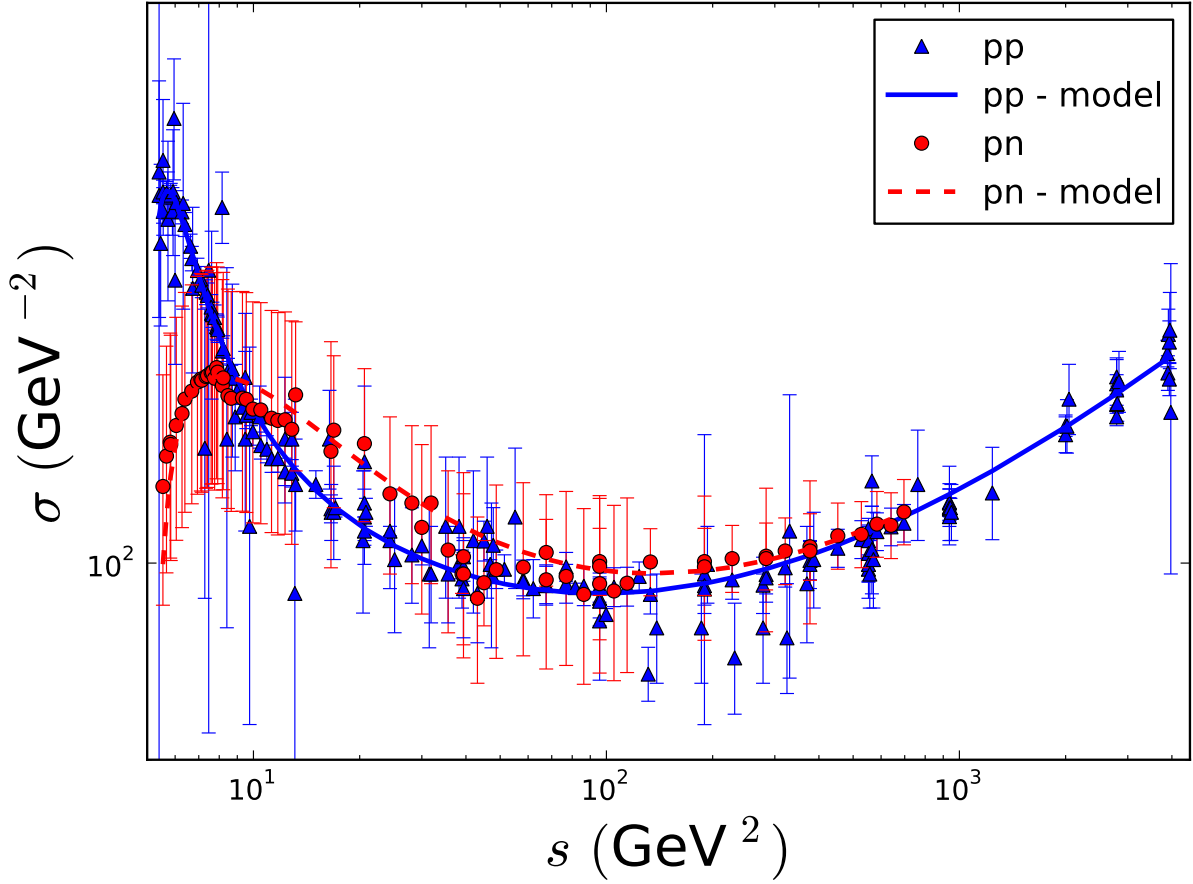


FIG. 2. (Color online) Total cross sections for proton-proton and proton-neutron as a function of Mandelstam s .

results in these graphs. While the unpolarized results yield a very low χ^2 , there is limited data to constrain the fit in the pn case.

Single polarization or analyzing power are presented for proton-proton, Fig. 6, and for proton-neutron Fig. 7. Again the model describes the data well, although more proton-neutron data would be useful to constrain the model further.

Finally we present the double-polarization observables, proton-proton in Fig. 8 and proton-neutron in Fig. 9. These were fit with minimal priority, due to the lack of data. For each of these observables we roughly describe the data, and in certain cases the model works very well.

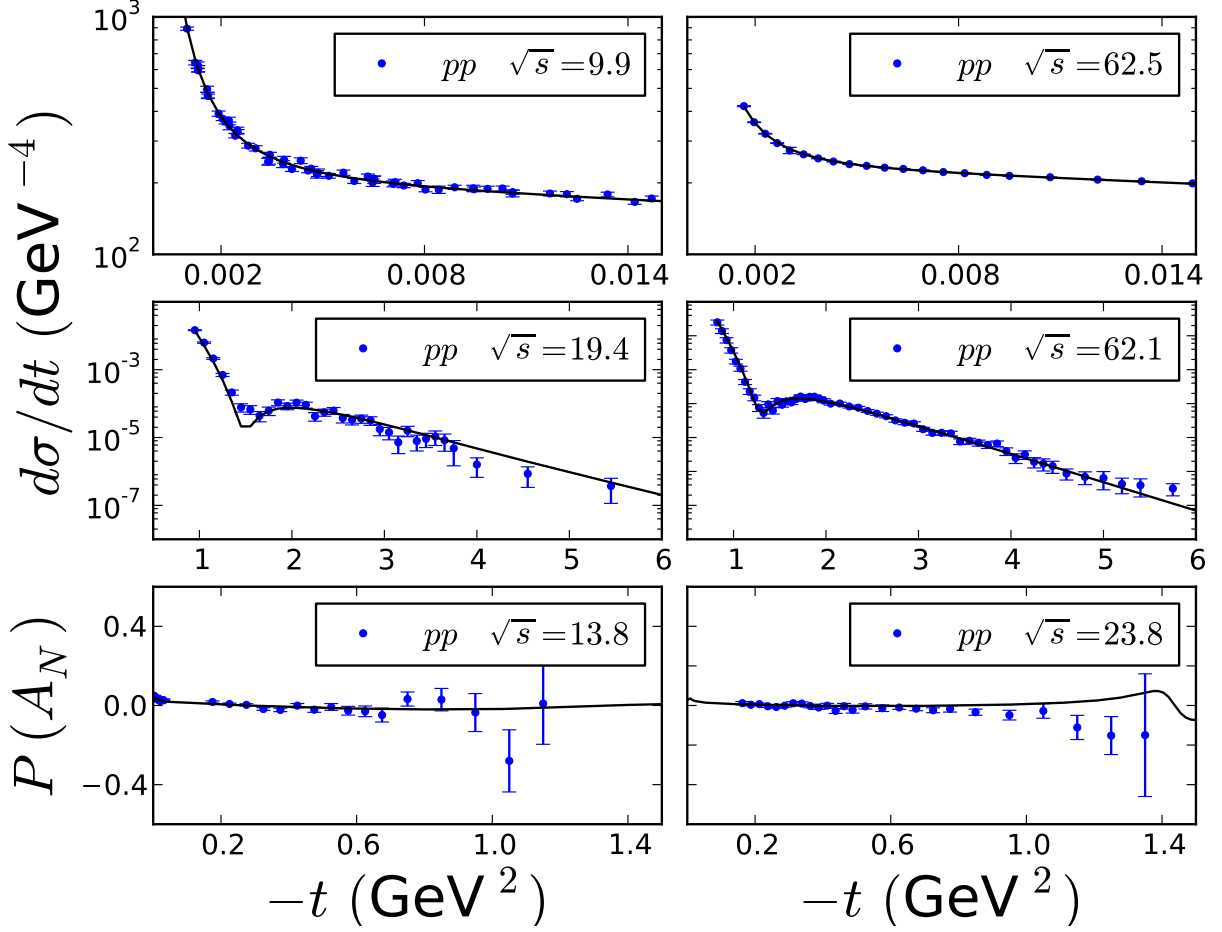


FIG. 3. (Color online) High energy results for proton-proton differential cross sections in both the Coulomb and dip region, as well as high energy polarization.

IV. SUMMARY AND OUTLOOK

We have parameterized the nucleon-nucleon scattering amplitudes in terms of Regge exchanges. Relating to the Fermi invariants allows us to calculate all spin dependence directly, while ensuring that a Regge exchange with definite quantum numbers contributes appropriately to the amplitude. Because of the ability of Regge theory to scale, we are able to extrapolate our results to $s \approx 20 \text{ GeV}^2$ with reasonable confidence.

Our next step is to utilize these amplitudes to describe the final state interactions of the $D(e,e'p)n$ process. We also plan to provide these amplitudes, as well as the data set we have collected to the nuclear physics community. Currently the amplitudes and data set can be

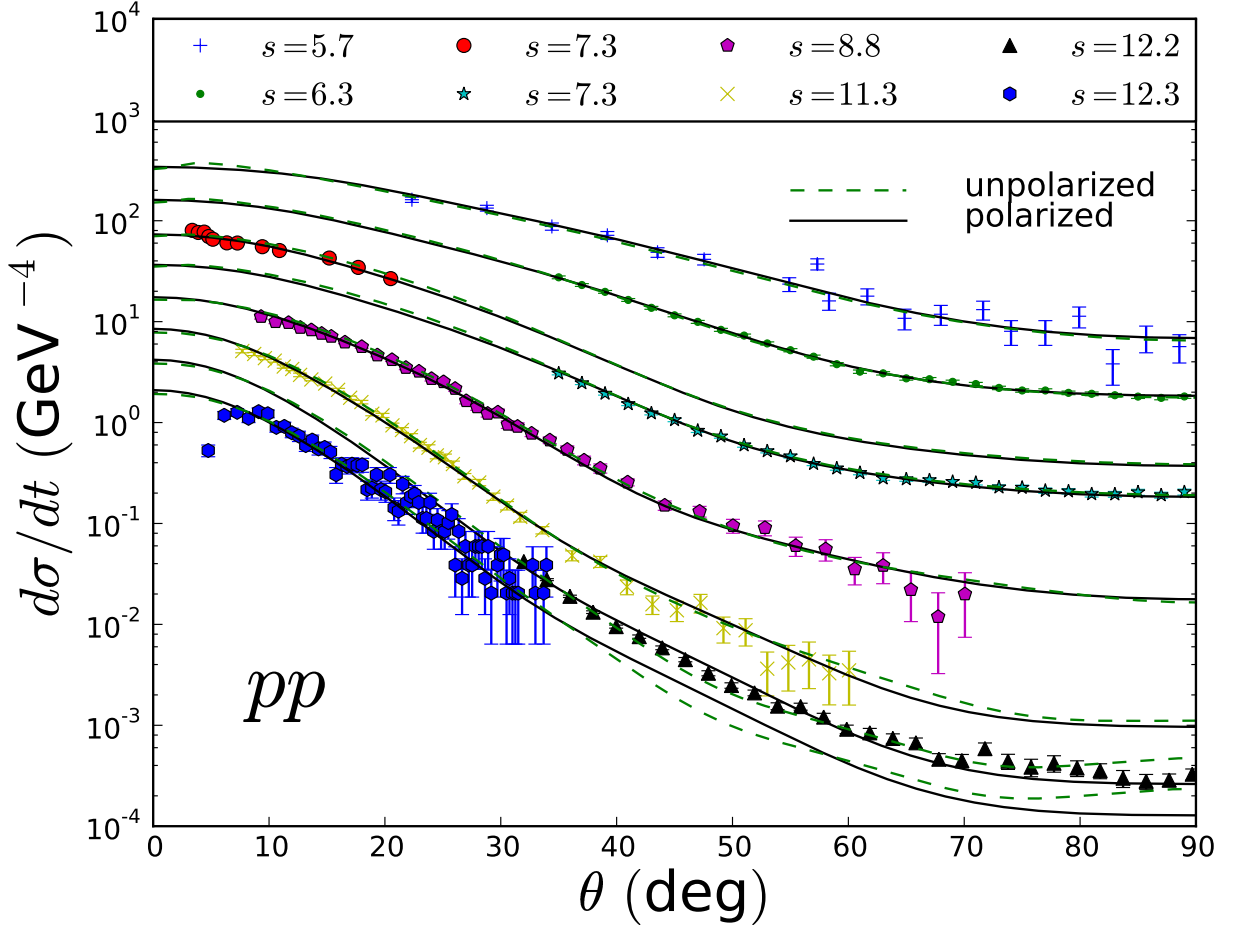


FIG. 4. (Color online) Differential cross section for proton-proton as a function of center of mass angle θ . Each data set is offset by a factor of two.

obtained by contacting the authors.

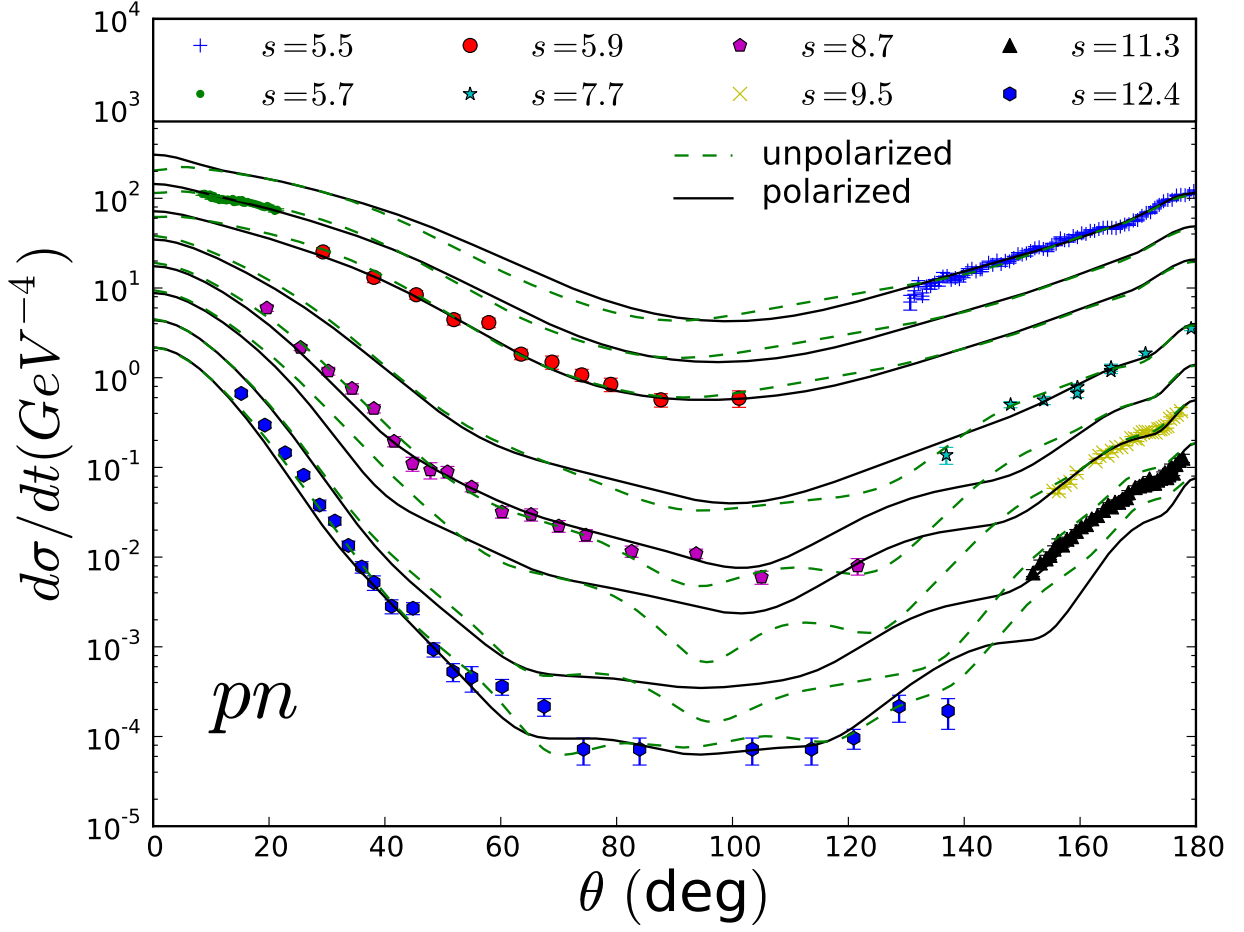


FIG. 5. (Color online) Differential cross section for proton-neutron as a function of center of mass angle θ . Each data set is offset by a factor of two.

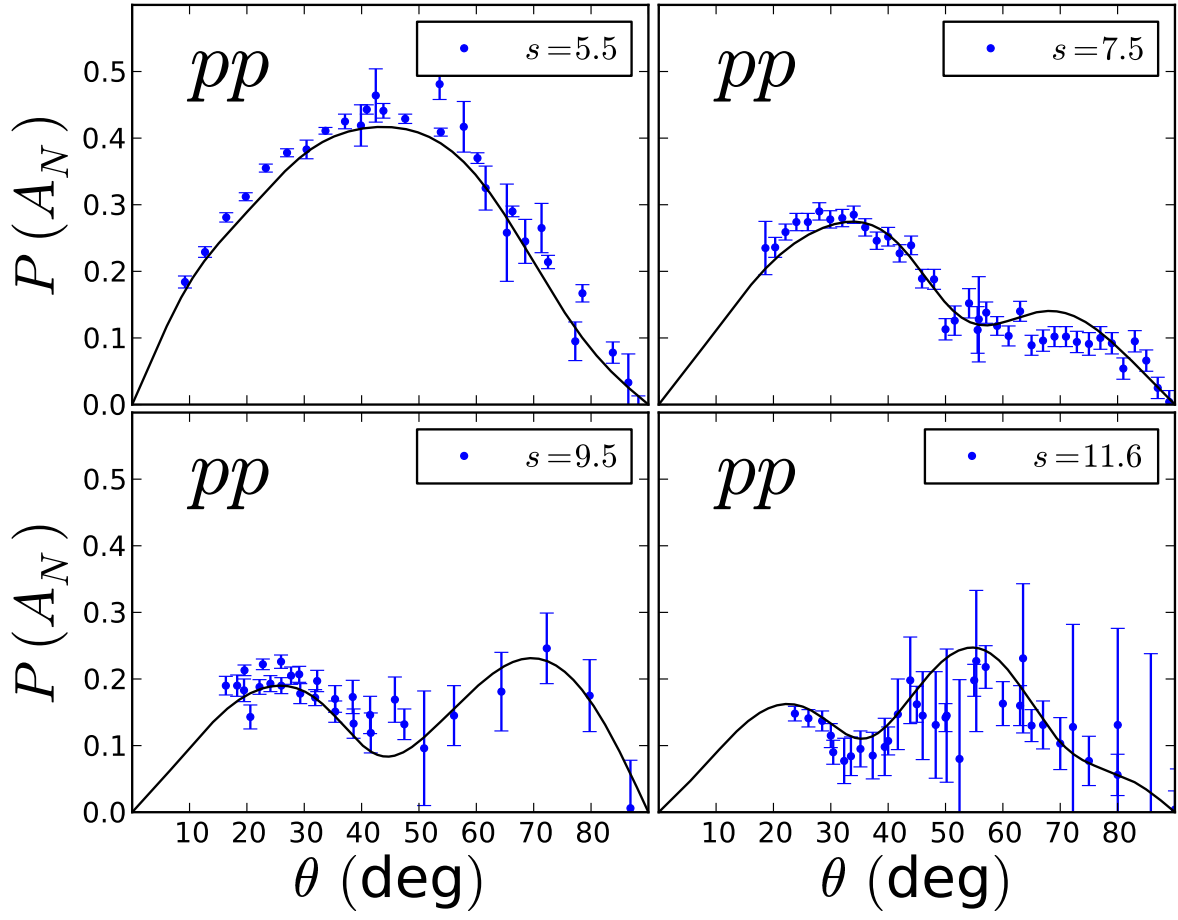


FIG. 6. (Color online) Polarization for proton-proton as a function of center of mass angle θ .

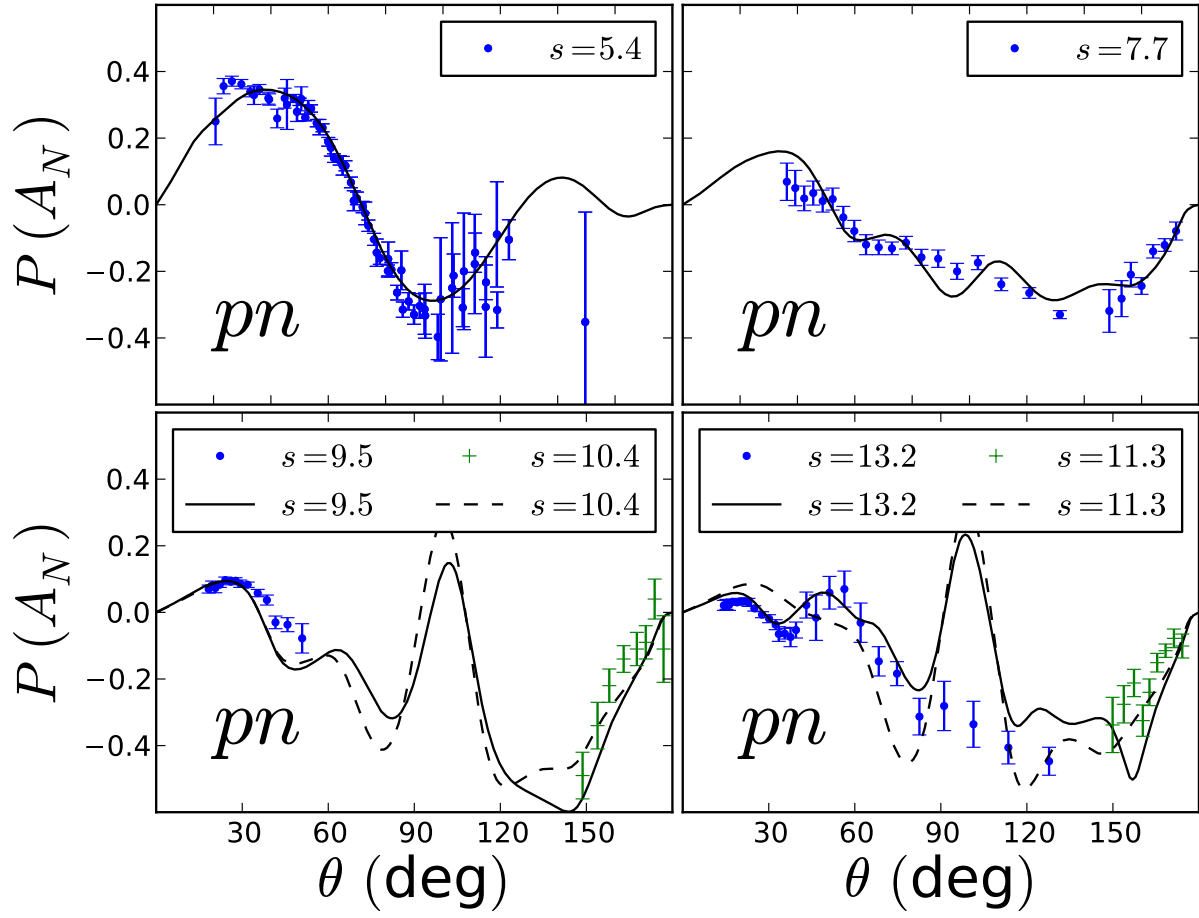


FIG. 7. (Color online) Polarization for proton-neutron as a function of center of mass angle θ .

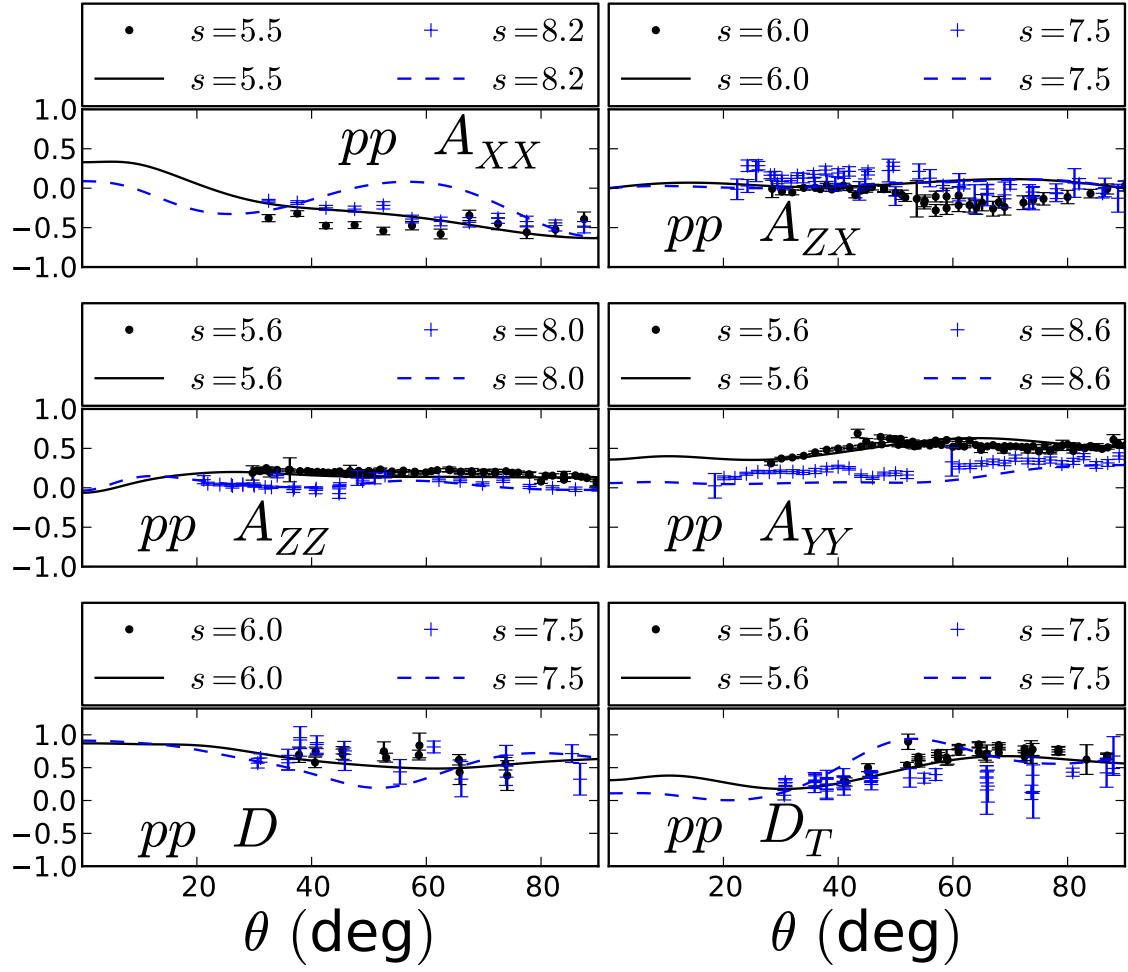


FIG. 8. (Color online) Double polarization observables for proton-proton as a function of center of mass angle θ .

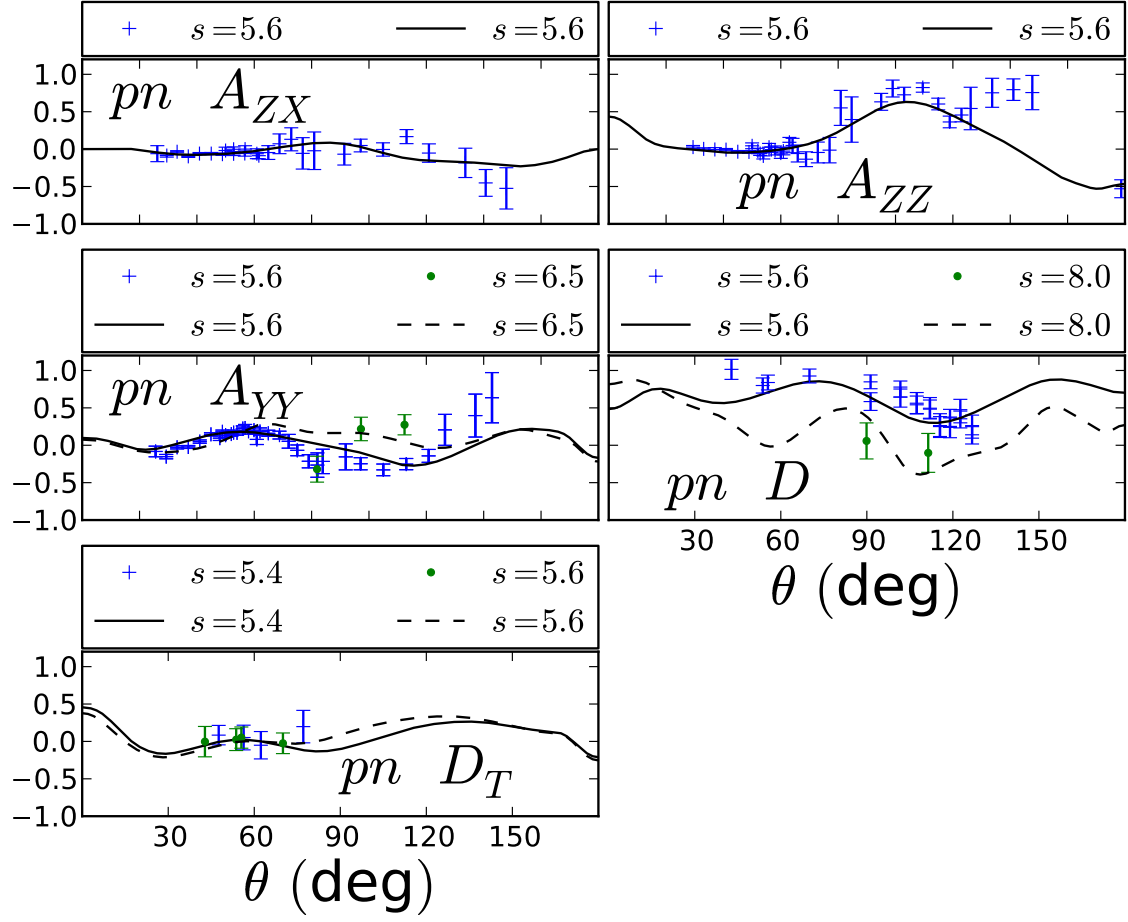


FIG. 9. (Color online) Double polarization observables for proton-neutron as a function of center of mass angle θ .

Appendix A: Parameters

Parameter values for polarized and unpolarized results are given in Table III, and Table IV respectively.

Appendix B: Amplitudes and Observables

All observables can be written in terms of the five independent helicity amplitudes [12] given in (1). We present here the observables relevant to this paper,

$$\sigma = \frac{-2m^2}{\sqrt{s(s-4m^2)}} \Im[a+c]_{t=0} \quad (\text{B1})$$

$$\frac{d\sigma}{dt} = \frac{m^4}{2\pi s(s-4m^2)} (|a|^2 + 4|b|^2 + |c|^2 + |d|^2 + |e|^2) \quad (\text{B2})$$

$$\tilde{\sigma} = \frac{1}{2} (|a|^2 + 4|b|^2 + |c|^2 + |d|^2 + |e|^2) \quad (\text{B3})$$

$$\frac{d\sigma}{dt} = \frac{m^4}{\pi s(s-4m^2)} \tilde{\sigma} \quad (\text{B4})$$

$$\tilde{\sigma}P = \tilde{\sigma}A_N = -\Im[b^*(a+c+d-e)] \quad (\text{B5})$$

$$\tilde{\sigma}A_{XX} = \Re(a^*d + c^*e) \quad (\text{B6})$$

$$\tilde{\sigma}A_{ZX} = -\Re[b^*(a+d-c+e)] \quad (\text{B7})$$

$$\tilde{\sigma}A_{ZZ} = -\frac{1}{2} (|a|^2 + |d|^2 - |c|^2 - |e|^2) \quad (\text{B8})$$

$$\tilde{\sigma}A_{YY} = \Re(a^*d - c^*e) + 2|b|^2 \quad (\text{B9})$$

$$\tilde{\sigma}D = \Re(a^*c - d^*e) + 2|b|^2 \quad (\text{B10})$$

$$\tilde{\sigma}D_T = \Re(a^*e - d^*c) + 2|b|^2 \quad (\text{B11})$$

Appendix C: Helicity Spinors

In the center of momentum frame the helicity spinors are,

$$u(\pm\mathbf{p}, \lambda) = N \begin{pmatrix} 1 \\ 2\lambda\tilde{p} \end{pmatrix} \chi_{\pm\lambda}(\hat{\mathbf{p}}), \quad (\text{C1})$$

$$v(\pm\mathbf{p}, \lambda) = N \begin{pmatrix} -2\lambda\tilde{p} \\ 1 \end{pmatrix} \chi_{\mp\lambda}(\hat{\mathbf{p}}), \quad (\text{C2})$$

TABLE III. Parameter values for polarized solution. The fit parameters are indicated in bold.

| β_0 | β_1 | δ | γ | α_0 | α_1 | Isospin | Parity | G-Parity | Type | Residue | Name |
|--|---|---|-------------------------|---|---|---------|--------|----------|------|---------|------------|
| -2.3014×10^2 | 3.0982×10^0 | 0.0000×10^0 | | 1.0800×10^0 | 2.5000×10^{-1} | 0 | + | + | 1 | I | IP |
| 3.3606×10^1 | 2.5208×10^0 | -1.3505×10^0 | 3.0000×10^0 | 1.2915×10^0 | 3.0031×10^{-1} | 0 | + | + | 1 | II | X_1 |
| -1.4315×10^0 | 4.2364×10^{-1} | -3.2163×10^0 | 3.3330×10^{-1} | 1.2228×10^0 | 7.6208×10^{-2} | 0 | + | + | 1 | II | X_2 |
| -4.2550×10^2 | 8.7558×10^0 | -2.8650×10^{-1} | | 6.7000×10^{-1} | 8.2000×10^{-1} | 0 | + | + | 1 | I | f |
| 3.0579×10^3 | 3.5797×10^0 | -5.0940×10^0 | | -7.1114×10^{-1} | 1.1570×10^0 | 0 | + | + | 1 | I | X_3 |
| -5.1011×10^1 | 3.9362×10^{-1} | 0.0000×10^0 | | 4.3000×10^{-1} | 9.2000×10^{-1} | 0 | - | - | 2 | I | ω_a |
| -8.3319×10^2 | 6.0000×10^0 | -1.8189×10^0 | | 1.3000×10^{-1} | 8.3000×10^{-1} | 0 | - | - | 2 | I | ϕ_a |
| 3.3968×10^5 | 4.0113×10^1 | 2.0543×10^0 | | -8.3722×10^0 | 1.1658×10^{-3} | 0 | - | - | 2 | I | X_4 |
| 3.6954×10^2 | 1.0385×10^1 | -2.8048×10^0 | | -2.3000×10^{-1} | 8.6000×10^{-1} | 0 | + | + | 3 | I | h |
| -1.7985×10^2 | 1.4258×10^0 | 1.4192×10^0 | | 5.5908×10^{-1} | 8.2000×10^{-1} | 0 | + | - | 3 | I | X_5 |
| -3.2225×10^1 | 3.5689×10^{-2} | -4.1374×10^0 | | 4.3000×10^{-1} | 9.2000×10^{-1} | 0 | - | - | 4 | I | ω_b |
| -8.5937×10^3 | 6.0000×10^0 | 9.0817×10^{-1} | | 1.3000×10^{-1} | 8.3000×10^{-1} | 0 | - | - | 4 | I | ϕ_b |
| 1.8226×10^3 | 9.5443×10^{-1} | 3.2746×10^0 | | -6.5816×10^0 | 8.1649×10^{-4} | 0 | - | - | 4 | I | X_6 |
| -5.0967×10^2 | 1.6424×10^0 | -4.0676×10^{-1} | | -2.3000×10^{-1} | 8.6000×10^{-1} | 0 | - | + | 5 | I | η |
| 3.4250×10^2 | 1.2447×10^0 | -1.4211×10^0 | | 5.9469×10^{-3} | 2.4531×10^{-1} | 0 | - | + | 5 | I | X_7 |
| 7.7744×10^1 | 3.5462×10^1 | 2.5834×10^{-1} | | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | + | + | 1 | I | b |
| -5.9219×10^2 | 1.3809×10^0 | -3.0997×10^0 | | -5.5996×10^{-1} | 4.4269×10^{-1} | 1 | + | + | 1 | I | X_8 |
| 3.4534×10^2 | 1.2989×10^0 | -5.2535×10^0 | | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | - | - | 2 | I | π_a |

TABLE III. (Continued)

| β_0 | β_1 | δ | γ | α_0 | α_1 | Isospin | Parity | G-Parity | Type | Residue Name |
|-------------------------|-------------------------|--------------------------|--------------------------|-------------------------|------------|---------|--------|----------|-------|--------------|
| 9.1120×10^2 | 6.8368×10^{-1} | 1.1157×10^{-1} | -1.1372×10^1 | 1.7908×10^{-5} | 1 | - | - | 2 | I | X_9 |
| 6.8240×10^2 | 4.3461×10^{-1} | -5.3158×10^{-1} | 4.7000×10^{-1} | 8.9000×10^{-1} | 1 | + | - | 3 | I | a |
| 6.6961×10^2 | 5.4994×10^{-1} | 2.3584×10^0 | 4.5559×10^{-1} | 8.9000×10^{-1} | 1 | + | - | 3 | I | X_{10} |
| 2.9363×10^2 | 4.4182×10^{-1} | -2.3057×10^0 | 2.8195×10^{-1} | 8.9330×10^{-1} | 1 | + | - | 3 | I | X_{11} |
| -2.6347×10^1 | 1.7260×10^{-3} | 3.1945×10^0 | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | - | - | 4 | I | π_b |
| -1.5941×10^3 | 1.6051×10^0 | -3.8402×10^0 | -1.3492×10^0 | 1.5166×10^{-4} | 1 | - | - | 4 | I | X_{12} |
| 1.1653×10^2 | 6.0278×10^1 | -5.5234×10^0 | 7.0679×10^{-2} | 8.7412×10^{-2} | 1 | - | + | 5 | I | X_{13} |
| 1.3195×10^4 | 4.1072×10^0 | -1.5099×10^0 | 1.3000×10^{-1} | 8.3000×10^{-1} | 0 | - | - | 2 | III | X_{14} |
| -1.4708×10^3 | 1.3829×10^0 | 1.9703×10^0 | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | + | + | 1 | III | X_{15} |
| -2.2575×10^3 | 1.7071×10^0 | -2.5369×10^0 | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | - | - | 2 | III | X_{16} |
| 4.2280×10^3 | 1.0389×10^0 | -5.7058×10^{-1} | -1.8954×10^0 | 2.0775×10^{-1} | 0 | + | + | 1 | III | X_{17} |
| -6.3292×10^2 | 7.2339×10^{-1} | 5.0303×10^0 | -6.0089×10^0 | 4.8252×10^{-1} | 0 | - | - | 2 | III | X_{18} |
| -2.5071×10^4 | 1.9199×10^0 | 2.3593×10^{-1} | -1.2202×10^1 | 1.0805×10^{-5} | 1 | + | + | 1 | III | X_{19} |
| 9.6450×10^2 | 7.5662×10^{-1} | 1.1350×10^{-1} | -4.4005×10^0 | 4.4305×10^{-3} | 0 | + | - | 3 | III | X_{20} |
| 4.6080×10^1 | 1.5845×10^{-1} | 0.0000×10^0 | -2.1192×10^{-1} | 3.7327×10^{-1} | 0 | - | - | 4 | III | X_{21} |
| 1.3204×10^0 | | 3.0270×10^0 | | | | | | | | EM_a |
| 5.2979×10^{-1} | | 3.1599×10^0 | | | | | | | | EM_b |
| 4.6382×10^{-1} | | 3.7668×10^0 | | | | | | | | EM_c |

TABLE IV. Parameter values for unpolarized solution. The fit parameters are indicated in bold.

| β_0 | β_1 | δ | γ | α_0 | α_1 | Isospin | Parity | G-Parity | Type | Residue type | Name |
|--|---|---|-------------------------|---|---|---------|--------|----------|------|--------------|------------|
| -2.3014×10^2 | 3.0982×10^0 | 0.0000×10^0 | | 1.0800×10^0 | 2.5000×10^{-1} | 0 | + | + | 1 | <i>I</i> | <i>IP</i> |
| 3.3606×10^1 | 2.5208×10^0 | -1.3505×10^0 | 3.0000×10^0 | 1.2915×10^0 | 3.0031×10^{-1} | 0 | + | + | 1 | <i>II</i> | X_1 |
| -1.4315×10^0 | 4.2364×10^{-1} | -3.2163×10^0 | 3.3330×10^{-1} | 1.2228×10^0 | 7.6208×10^{-2} | 0 | + | + | 1 | <i>II</i> | X_2 |
| -4.2550×10^2 | 8.7558×10^0 | -2.8650×10^{-1} | | 6.7000×10^{-1} | 8.2000×10^{-1} | 0 | + | + | 1 | <i>I</i> | <i>f</i> |
| 2.6491×10^3 | 4.4051×10^0 | -3.4582×10^0 | | -1.7172×10^0 | 2.9128×10^{-5} | 0 | + | + | 1 | <i>I</i> | X_3 |
| -3.2822×10^1 | 1.4253×10^{-1} | -7.7596×10^{-2} | | 4.3000×10^{-1} | 9.2000×10^{-1} | 0 | - | - | 2 | <i>I</i> | ω_a |
| 2.1614×10^2 | 9.2400×10^2 | 1.0919×10^0 | | 1.3000×10^{-1} | 8.3000×10^{-1} | 0 | - | - | 2 | <i>I</i> | ϕ_a |
| 4.1651×10^4 | 3.7156×10^0 | 6.8897×10^{-1} | | -7.8837×10^0 | 1.9233×10^0 | 0 | - | - | 2 | <i>I</i> | X_4 |
| 2.0270×10^2 | 1.0385×10^1 | -4.2739×10^0 | | -2.3000×10^{-1} | 8.6000×10^{-1} | 0 | + | - | 3 | <i>I</i> | <i>h</i> |
| -1.2879×10^0 | 8.3758×10^{-6} | 8.9998×10^{-1} | | 1.3844×10^0 | 8.2000×10^{-1} | 0 | + | - | 3 | <i>I</i> | X_5 |
| 1.9353×10^1 | 3.5689×10^{-2} | -5.4629×10^0 | | 4.3000×10^{-1} | 9.2000×10^{-1} | 0 | - | - | 4 | <i>I</i> | ω_b |
| -1.7345×10^4 | 6.0000×10^0 | 1.2766×10^0 | | 1.3000×10^{-1} | 8.3000×10^{-1} | 0 | - | - | 4 | <i>I</i> | ϕ_b |
| 1.3702×10^3 | 5.8600×10^{-1} | 5.2754×10^0 | | -4.4008×10^0 | 6.6653×10^{-1} | 0 | - | - | 4 | <i>I</i> | X_6 |
| -2.9867×10^1 | 2.0935×10^{-3} | 3.0541×10^{-4} | | -2.3000×10^{-1} | 8.6000×10^{-1} | 0 | - | + | 5 | <i>I</i> | η |
| 7.7053×10^1 | 2.4688×10^{-5} | -1.1493×10^0 | | 5.4644×10^{-1} | 8.9621×10^{-1} | 0 | - | + | 5 | <i>I</i> | X_7 |
| 2.6850×10^2 | 3.5462×10^1 | 3.1197×10^{-1} | | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | + | + | 1 | <i>I</i> | <i>b</i> |
| -8.0481×10^2 | 1.5700×10^1 | -2.5542×10^0 | | -1.8343×10^0 | 4.9032×10^{-4} | 1 | + | + | 1 | <i>I</i> | X_8 |

TABLE IV. (Continued)

| β_0 | β_1 | δ | γ | α_0 | α_1 | Isospin | Parity | G-Parity | Type | Residue type | Name |
|-------------------------|-------------------------|--------------------------|----------|--------------------------|-------------------------|---------|--------|----------|------|--------------|----------|
| 2.1070×10^1 | 9.8069×10^{-3} | -3.1377×10^0 | | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | — | — | 2 | I | π_a |
| 4.0621×10^1 | 5.9427×10^{-1} | -2.4486×10^{-1} | | -6.3368×10^{-1} | 2.9832×10^{-4} | 1 | — | — | 2 | I | X_9 |
| 8.3864×10^2 | 7.5961×10^{-1} | -3.0230×10^{-1} | | 4.7000×10^{-1} | 8.9000×10^{-1} | 1 | + | — | 3 | I | a |
| 5.9883×10^2 | 5.4362×10^{-1} | 2.7222×10^0 | | 5.0397×10^{-1} | 8.9000×10^{-1} | 1 | + | — | 3 | I | X_{10} |
| 4.1328×10^2 | 2.5177×10^0 | -2.8391×10^0 | | 1.1444×10^{-1} | 1.3527×10^{-5} | 1 | + | — | 3 | I | X_{11} |
| 2.2867×10^1 | 1.1023×10^{-6} | 4.6222×10^0 | | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | — | — | 4 | I | π_b |
| -4.3636×10^3 | 4.1122×10^0 | -3.3747×10^0 | | -6.0418×10^{-2} | 2.3409×10^{-5} | 1 | — | — | 4 | I | X_{12} |
| 8.0174×10^1 | 5.0000×10^0 | -1.1248×10^{-2} | | 4.7000×10^{-1} | 8.9000×10^{-1} | 1 | — | + | 5 | I | ρ |
| 1.2615×10^2 | 3.7157×10^1 | -2.3941×10^0 | | -5.3900×10^{-3} | 2.5158×10^{-1} | 1 | — | + | 5 | I | X_{13} |
| 1.1947×10^4 | 4.1937×10^0 | -1.0997×10^0 | | 1.3000×10^{-1} | 8.3000×10^{-1} | 0 | — | — | 2 | III | X_{14} |
| -1.5180×10^1 | 1.6027×10^{-3} | -1.6412×10^{-1} | | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | + | + | 1 | III | X_{15} |
| -1.9266×10^1 | 1.1984×10^{-2} | -1.2916×10^{-1} | | -4.0000×10^{-2} | 7.2000×10^{-1} | 1 | — | — | 2 | III | X_{16} |
| 3.1526×10^2 | 4.4662×10^{-1} | 4.9006×10^{-2} | | -1.5001×10^0 | 1.5091×10^{-3} | 0 | + | + | 1 | III | X_{17} |
| 1.3989×10^1 | 5.8787×10^{-2} | 7.9986×10^0 | | -6.1197×10^0 | 4.8648×10^{-1} | 0 | — | — | 2 | III | X_{18} |
| -2.5159×10^2 | 8.2800×10^{-2} | 3.6994×10^0 | | -8.7920×10^0 | 2.5026×10^{-4} | 1 | + | + | 1 | III | X_{19} |
| 1.3204×10^0 | | 3.0270×10^0 | | | | | | | | | EM_a |
| 5.2979×10^{-1} | | 3.1599×10^0 | | | | | | | | | EM_b |
| 4.6382×10^{-1} | | 3.7668×10^0 | | | | | | | | | EM_c |

TABLE V. Two component spinors of (C1)

| | $\chi_{\frac{1}{2}}(\hat{\mathbf{p}})$ | $\chi_{-\frac{1}{2}}(\hat{\mathbf{p}})$ |
|---------------|--|---|
| initial state | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ | $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ |
| final state | $\begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} \end{pmatrix}$ | $\begin{pmatrix} -\sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} \end{pmatrix}$ |

where $N = \sqrt{\frac{E+m}{2m}}$, $\tilde{p} = \frac{|\mathbf{p}|}{E+m}$, $\hat{\mathbf{p}}$ is a unit vector in the direction of \mathbf{p} , and $\chi_{\pm\lambda}(\hat{\mathbf{p}})$ are given in Table V.

Appendix D: Amplitudes to Fermi Invariants

The helicity dependent matrices which relate the Fermi invariants to the helicity amplitudes are,

$$C_{ij}^t = \begin{pmatrix} 1 + \frac{t}{s-4m^2} & -1 + \frac{s}{2m^2} + \frac{t}{s-4m^2} & -2 + \frac{2t}{s-4m^2} & 0 & -1 + \frac{s}{2m^2} - \frac{t}{s-4m^2} \\ C_{21}^t & C_{21}^t & 2C_{21}^t & 0 & -C_{21}^t \\ 1 + \frac{t}{s-4m^2} & C_{32}^t & 2 + \frac{2t}{s-4m^2} & 0 & -C_{32}^t \\ \frac{st}{4m^2(s-4m^2)} & \frac{t}{s-4m^2} & \frac{s-2m^2}{m^2} \left(2 + \frac{t}{s-4m^2}\right) & \frac{t}{4m^2} & -2 - \frac{t}{s-4m^2} \\ \frac{-st}{4m^2(s-4m^2)} & \frac{-t}{s-4m^2} & \frac{-2t}{s-4m^2} & \frac{t}{4m^2} & \frac{t}{s-4m^2} \end{pmatrix} \quad (\text{D1})$$

$$C_{21}^t = -\frac{\sqrt{s}}{4m} \sin(\theta) = -\frac{\sqrt{s}}{2m} \sqrt{\frac{-t}{s-4m^2}} + \frac{\sqrt{s}}{4m} \left(\frac{-t}{s-4m^2}\right)^{\frac{3}{2}} \quad (\text{D2})$$

$$C_{32}^t = \frac{1}{2m^2} (s-2m^2) \left(1 + \frac{t}{s-4m^2}\right) \quad (\text{D3})$$

$$C_{ij}^u = \begin{pmatrix} -1 - \frac{u}{s-4m^2} & 1 - \frac{s}{2m^2} - \frac{u}{s-4m^2} & 2 - \frac{2u}{s-4m^2} & 0 & 1 - \frac{s}{2m^2} + \frac{u}{s-4m^2} \\ -\sqrt{\frac{(4m^2-s-u)su}{4m^2(s-4m^2)^2}} & -\sqrt{\frac{(4m^2-s-u)su}{4m^2(s-4m^2)^2}} & -2\sqrt{\frac{(4m^2-s-u)su}{4m^2(s-4m^2)^2}} & 0 & \sqrt{\frac{(4m^2-s-u)su}{4m^2(s-4m^2)^2}} \\ \frac{-su}{4m^2(s-4m^2)} & \frac{-u}{s-4m^2} & \frac{-2u}{s-4m^2} & \frac{u}{4m^2} & \frac{u}{s-4m^2} \\ \frac{-su}{4m^2(s-4m^2)} & \frac{-u}{s-4m^2} & \frac{s-2m^2}{m^2} \left(-2 - \frac{u}{s-4m^2}\right) - \frac{u}{4m^2} & 2 + \frac{u}{s-4m^2} \\ 1 + \frac{u}{s-4m^2} & \frac{s-2m^2}{m^2} \left(-1 - \frac{u}{s-4m^2}\right) & 2 + \frac{2u}{s-4m^2} & 0 & \frac{s-2m^2}{m^2} \left(-1 - \frac{u}{s-4m^2}\right) \end{pmatrix} \quad (\text{D4})$$

Appendix E: Electromagnetic Effects

We utilize the full proton vertex given as,

$$\Gamma_{EM}^\mu = F_1(t)\gamma^\mu - \frac{F_2(t)}{2m}i\sigma^{\mu\nu}q_\nu, \quad (\text{E1})$$

$$F_1(t) = \frac{G_E(t) - G_M(t)t/4m^2}{1 - t/4m^2} \quad F_2(t) = \frac{G_M(t) - G_E(t)}{1 - t/4m^2} \quad (\text{E2})$$

$$G_E = G_M/2.79 = (1 - t/.71)^{-2}, \quad (\text{E3})$$

where q_ν is the four momentum of the photon.

The one photon-exchange contribution to the helicity amplitudes is then given as,

$$a_{EM}(s, t) = \frac{4\pi/137}{2t(4m^2 - s)(-4m^3 + mt)^2} \times \quad (E4)$$

$$(-8G_E G_M m^2 stu - G_M^2 t(32m^6 + s^2 t + 2m^2 t(s + t) - 8m^4(s + 2t))) \quad (E5)$$

$$- 8G_E^2 m^4(16m^4 + 2s^2 + 3st + t^2 - 4m^2(3s + 2t))) \quad (E6)$$

$$b_{EM}(s, t) = -\frac{4\pi/137}{2mt(-4m^2 + t)^2} \sqrt{\frac{stu}{(s - 4m^2)^2}} \times \quad (E7)$$

$$((s - u)(4m^2 G_E^2 + G_M^2 t) + 2G_E G_M(16m^4 - st - 4m^2(s + t))) \quad (E8)$$

$$c_{EM}(s, t) = -\frac{(4\pi/137)u}{2t(4m^2 - s)(-4m^3 + mt)^2} \times \quad (E9)$$

$$(8G_E^2 m^4(u - s) + 8G_E G_M m^2 st + G_M^2 t(-8m^4 + 2m^2 t - st)) \quad (E10)$$

$$d_{EM}(s, t) = \frac{4\pi/137}{(s - 4m^2)(t - 4m^2)^2} \times \quad (E11)$$

$$(4G_E G_M su + G_E^2 s(s - u) + G_M^2(16m^4 + 2s^2 + 3st + t^2 - 4m^2(3s + 2t))) \quad (E12)$$

$$e_{EM}(s, t) = -\frac{4\pi/137}{(s - 4m^2)(t - 4m^2)^2} \times \quad (E13)$$

$$(4G_E G_M su + G_E^2 s(s - u) + G_M^2(16m^4 + 2s^2 + 3st + t^2 - 4m^2(3s + 2t))). \quad (E14)$$

where $u = 4m^2 - s - t$.

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