Level structure of ^{10}C

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The level structure of ^{10}C

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The level structure of ^{10}C below 7 MeV is discussed. We suggest the spin-parity assignments for the unbound states in ^{10}C. The assignments are based on the observed widths, Coulomb displacement energies, the reported decay modes, the potential model prediction and the shell model calculations.

I. INTRODUCTION

Currently a lot of attention is attracted to the excited 0^+ state (7.65 MeV) in ^{12}C and possible band based on this level [1, 2]. There is evidence that this state (Hoyle state) is a dilute α-cluster state [3]. In this context, the first 0^+ excited state in ^{10}Be is also of interest. This state cannot be well described in shell model [4] or *ab initio* calculations [5]. On the other hand, predictions of antisymmetrized molecular dynamics plus Hartree-Fock model [6], molecular orbit model [7] and microscopic four cluster model [8] indicate that the second 0^+ (6.18 MeV) state in ^{10}Be corresponds to the spatially extended structure with large separation between the two α-particles. The fact that ^{10}Be is not a self-conjugate nucleus, like ^{12}C, provides for new possibilities to obtain information on α clustering. W. von Oertzen attracted attention to the effects of extra "valence" nucleons in ^{10}Be on the cluster structure of the states [9]. Cluster nature of some of the states in ^{10}Be was also considered recently in [10]. Due to the lower binding energy of nucleons in non-self-conjugate nuclei it is possible to observe nucleon decay of the α-cluster states and to obtain direct information on the relation between the cluster and single particle structure. One more new possibility related mainly with the developments in the studies of unstable nuclei is the analysis of the isotopic shift of the cluster levels in mirror nuclei [11]. To explore this possibility for the A=10 (T=1) nuclei one should know the ^{10}C level scheme. Unfortunately, the information on the ^{10}C level scheme is limited. This is due to difficulties of reaching ^{10}C and the fact that excitation energies of several levels above the first excited 2^+ state are nearly degenerate (Fig. 1). Two recent analyses of ^{10}C spectra [12, 13] focused on the constraints for the excitation energy of the second 0^+ level. Both works consider d^2/s^2 shell model structure of this level as was defined in [12], but in [13] the Coulomb displacement energies were calculated using R-matrix approach. The excitation energy of the second 0^+ level was obtained at 5.2 MeV in both works, excluding by this the 0^+ assignment for the 4.2 MeV resonance claimed in [14]. Later it was recognized that the 4.2 MeV state was an experimental artifact [15]. Recently a detailed study of ^{10}C [16] was made using resonance decay spectroscopy. The authors [16] observed levels below 7 MeV excitation energy in ^{10}C. They presented new data on the widths and decay modes of the resonances and gave some constraints on their spins. In this paper, we are attempting spin-parity assignments for the excited levels in ^{10}C below 7 MeV using the data [16, 17] and new results for the T=1 α-cluster states in ^{10}B [18]. Isotope invariance allows to relate the data available for ^{10}Be and ^{10}B with the ^{10}C spectrum. We apply the Coulomb displacement energies and most importantly, widths and decay modes of the resonances to suggest the spin-parity assignments.

II. POTENTIAL MODEL

The conventional Woods-Saxon potential was used to evaluate the Coulomb displacement energies that are sensitive to the orbital angular momentum, binding energy and structure of the states and also the single particle widths of the states. The depth of the well was adjusted for each state to fit the binding energy in ^{10}Be. Radius of $R=1.25\times\sqrt[3]{9}$ fm and diffuseness of a=0.65 fm was used for the central part of the potential. For the spin-orbit potential we use V_{so}=6.4 MeV, $R_{so}=1.3\times\sqrt[3]{9}$ and a_{so}=0.64 fm. The Coulomb potential was that of the homogeneously charged sphere of $R_c=1.17\times\sqrt[3]{9}$. The parameters of the single particle potential were initially taken from [19], and then modified slightly to fit the g.s. binding energies for mirror pairs ^{9}Be-^{9}B, and ^{13}C-^{13}N. The 2s1/2 and 1d5/2 single particle states were also included into the fit for the ^{13}C-^{13}N pair. We obtained agreement for the states included in the fit to within 100 keV. The Coulomb shifts of α cluster states (analogs of 0^+_2 and 2^+_3 in the ^{10}Be spectrum) were calculated using the Woods-Saxon potential from [18] with V=-119 MeV, radius and charge radius of 2.58 fm and 2.27 fm respectively and diffuseness of a=0.677 fm. Such potential generates the correct binding energy for the 0^+_2 level in ^{10}Be and the correct excitation energy of 2^+_3 and also produces deeply bound "forbidden" states to account for Pauli principle. Note that the results of potential model calculations are not very sensitive to the specific choice of potential parameters as long as the excitation energies of the states are reproduced. In what follows we assume that all states below 7.5 MeV excitation are known in ^{10}Be.

III. ANALYSIS

The analysis starts with fitting the well depth of the potential to reproduce the binding energy of the states in ^{10}Be. Then the excitation energy for the correspond-

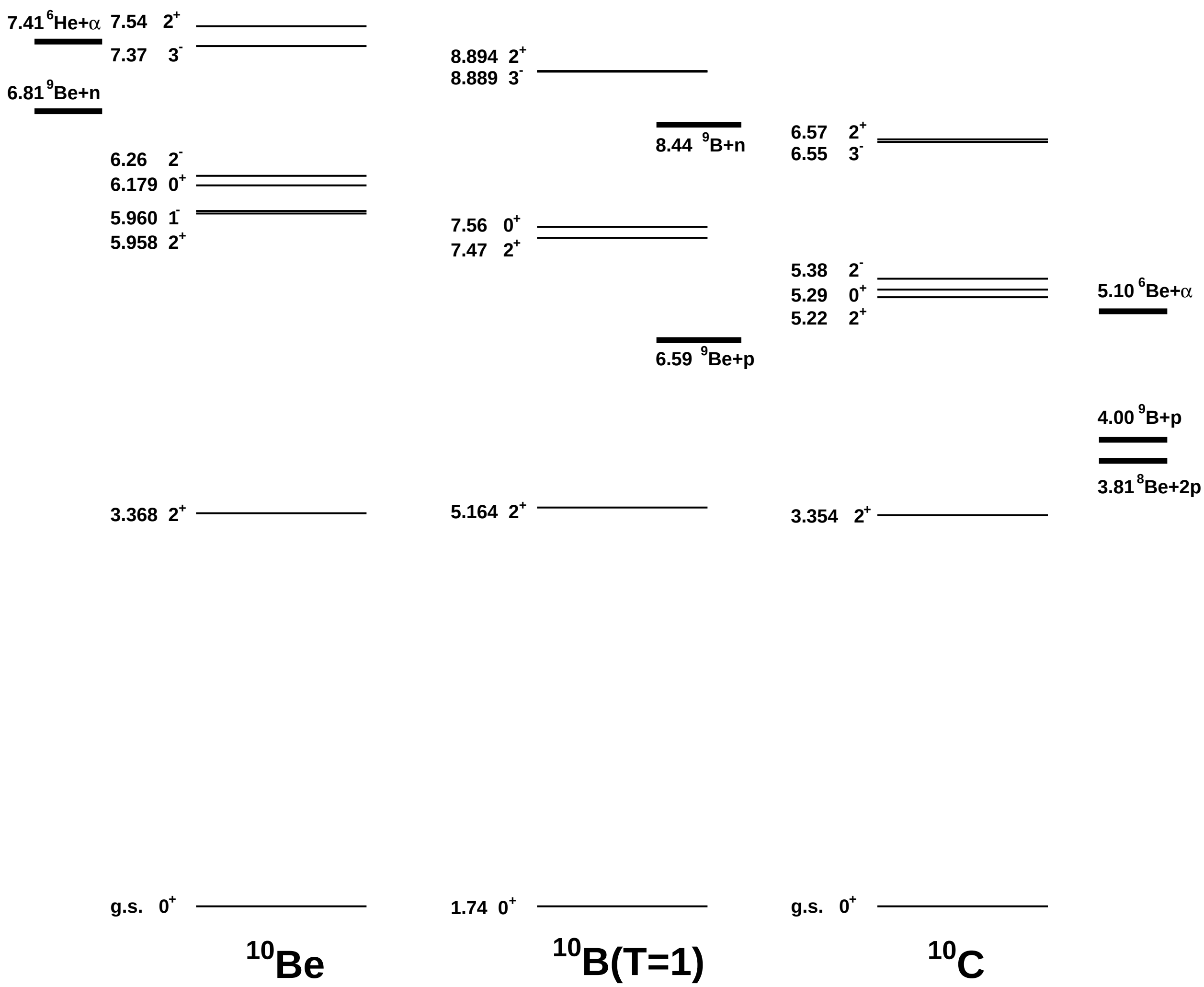

FIG. 1. Levels diagram for the A=10 T=1 isobaric multiplet.

ing isobaric analog state in ^{10}B is calculated. For this, we use the average between the energy relative to the threshold for ^{10}B decay into n+^{9}B (the same as relative to the ^{10}Be decay into ^{9}Be+n) and the energy relative to the threshold for ^{10}B decay into ^{9}Be+p (calculated with the potential found for ^{10}Be and changing the neutron for the proton). The single particle nucleon (or α for the 0_2^+ and 2_3^+ levels) widths Γ_{sp} are determined from the potential model. The width of the resonance is defined using behavior of the wave function in the interior as in [24], for narrow resonances ($\Gamma < 300$ keV) this definition is identical to the energy interval between which phase shift changes from 45° to 135°. The known widths of the T=1 resonances in ^{10}B (mainly ^{9}Be+p) [20] are proportional to the Spectroscopic Factors. We used the expression $C^2S=\Gamma_{exp}/\ \Gamma_{sp}$, where C is an isospin Clebsch-Gordan coefficient ($C^2=1/2$ for the ^{9}Be+p decay of ^{10}B). The ratios of the experimental proton widths (when known) of the resonances to the calculated single particle widths were considered as the Spectroscopic Factors (^{9}Be(g.s)+n for ^{10}Be or ^{9}B(g.s)+p for ^{10}C). Reduced widths for proton and neutron decay should be the same for the T=1 states in ^{10}B if isospin is conserved. When the known partial proton widths in ^{10}B were used to obtain the SF, the corresponding $\Gamma_{exp}/\ \Gamma_{sp}$ ratio is multiplied by a factor of 2 to get a SF in ^{10}Be and ^{10}C. These values were considered as experimental values and summarized in the seventh column of Table I. SF from the ^{9}Be(d,p) reactions [22, 23] are given in the fourth column of Table I. We then calculated theoretical SFs in the framework of the Shell Model using code CoSMo [25] (column 3 of Table I). The psd valence space with WBP interaction [4] was used. 0-2 $\hbar\omega$ excitations were considered for the positive parity states and 1-3 $\hbar\omega$ for the negative parity state. As it is clear from Table I, generally there is reasonable agreement between the experimental

TABLE I. T=1 Excited states above 5 MeV in A=10 nuclei.

J^π	ℓ_N	S_{th} [a]	$C^2S_{exp}(^{10}Be)$ [b]	$\Gamma_p(^{10}B)$ (keV)	$\Gamma_{sp}(^{10}B)$ (keV)	$2\times\frac{\Gamma_p}{\Gamma_{sp}}$ [c]	^{10}Be E_{exp} (MeV)	^{10}B(T=1) E_{exp} (E_{calc}) (MeV)	^{10}C E_{calc} (MeV)	^{10}C Γ_{calc} [g] (keV)	^{10}C E_{exp} (MeV)	^{10}C Γ_{exp} [16] (keV)	^{10}C Γ_{exp} [20] (keV)
2_2^+	1	0.73	0.54	65(10)	200	0.65(10)	5.958	7.47 (7.50)	5.16	210(30)/170	5.22	294(16)	225(45)
1_1^-	0	0.40	-	100(10)	1240	0.16(2) [d]	5.960	7.43 (7.49)	5.1 [e]	>180			
	2	0.15	-	-	-	-							
0_2^+	1	0.07	-	2.65(18)	250	0.021(2)	6.179	7.56 (7.58)	5.39	10 [h]	5.287	106(11)	
2_1^-	0	0.11	0.132	210(60)	2700	0.15(4)	6.263	7.74 (7.79)	5.4 [e]	370(100)	5.38[21]		300(60)
	2	0.53	0.065	-	-	-							
3_1^-	2	0.57	0.53	75(10) [f]	350	0.43(6)	7.371	8.89 (8.93)	6.70	140(20)/175	6.553	214(31)	
2_3^+	1	0.03	0.007	7(2) [f]	2200	0.006(2)	7.542	8.89 (8.87)	6.70	90 [h]	6.568	172(31)	190(35)

[a] Theoretical spectroscopic factor S_{th} for N$\otimes 3/2^-_{g.s.}$ configuration. Square of isospin Clebsch-Gordan coefficient, which is unity for ^{10}Be and ^{10}C, and 1/2 for T=1 states in ^{10}B is omitted.
[b] SF from ^{9}Be(d,p) experiments [22, 23].
[c] This column gives the SF for the corresponding state determined from the ratio of the known proton partial width of the T=1 states in ^{10}B to a single particle width calculated with the potential model. The ratio is then multiplied by a factor of 2 to account for isospin Clebsch-Gordan coefficient.
[d] The 1^- at 7.43 MeV in ^{10}B has mixed isospin and experimental spectroscopic factor determined from the width of this state may be unreliable. See text for additional comments.
[e] Assuming experimental $\ell = 0$ SF and theoretical $\ell = 2$ SF, we estimate uncertainty of $\pm$200 keV for this value due to uncertainties in SF.
[f] Proton partial width in ^{10}B that was used to determine the SF is from [18].
[g] The widths for all states except for the cluster 0_2^+ and 2_3^+ were calculated as a product of Spectroscopic Factor given in column 7 and the single particle width for the p+^{9}B system. If SF from (d,p) reaction (column 4) is outside of the uncertainty given in column 7, then the second width that corresponds to the SF from (d,p) is also shown (after the slash).
[h] Width for this state was calculated as a sum of α single-particle width from α+^{6}Be potential model and the partial width for the proton decay to the ^{9}B(g.s.).

and calculated SFs. Two significant discrepancies should be pointed out, however. The experimental ℓ=0 SF for the 1_1^- state is appreciably smaller than the SM prediction. Another discrepancy is seen for the 2_1^- state. While agreement between the SM and the experimental value for the $\ell = 0$ SF is excellent, the $\ell = 2$ SF, determined from the (d,p) reaction [23], is much smaller than the SM prediction. The origin of these discrepancies is not clear. (Isospin mixing may be important in the case of the 1^- state.) We used the SM predictions to calculate the Coulomb displacement energies of these negative parity states in ^{10}C, but we realize that uncertainty of these calculations is much larger than for the other states. Fortunately, the width of the 2_1^- state in ^{10}C is determined by the $\ell = 0$ SF and is not affected by this discrepancy. As for the width of the 1^- state we present the lower limit. It is worthwhile to note remarkable stability of the differences in excitation energies for the cluster 0^+ and 2^+ levels calculated in ^{10}Be and ^{10}C. This equidistance is quite different from what should be expected for single particle nucleon resonances with $\ell = 0$ and $\ell = 2$. The well known Thomas-Ehrman effect [26, 27] shifts down the $\ell = 0$ unbound single particle levels in mirror proton rich nuclei. It is different for the α-particle resonances mainly due to larger reduced mass decreasing the role of the orbital momenta. This behavior can be considered as a specific characteristic of the cluster states.

Now we can compare the calculations with the experimental data for ^{10}C [16]. It is seen (Table I, column 11) that the α-cluster states may be relatively narrow resonances in ^{10}C. Thus, the 100 keV resonance at 5.29 MeV [16] can only be the 0_2^+ state, and the 170 keV resonance at 6.6 MeV is likely the 2_3^+ cluster state. The calculated widths of the cluster states, 0_2^+ and 2_3^+, are too small however, if only the cluster decay (with S_α=1) and the proton decay to the ground state in ^{9}B are taken into account. Several charged particle decays are energetically possible for the ^{10}C excited states, while the mirror decays are not possible in ^{10}Be. The account of the decays to the excited states in ^{9}B for all states (except for the 0_2^+ and 2_3^+) results in 10-15% increase of the widths shown in Table I. As for the cluster 0_2^+ and 2_3^+ states, the lowest 2p decay is a new and important channel. It is the only channel which can provide for the increase of the width of the 0_2^+ state. A simplified consideration of the 2p decay as a di-proton decay in the potential model shows that if the spectroscopic factor for this decay is about 0.15, then it provides for the 100 keV of the total width. In this case the 2p decay will be dominant in agreement with the experimental observation [16]. A similar consideration for the 2_3^+ cluster state would result in the increase of its width by $\sim$100 keV, also improving the agreement with the experimental data. Now our interpretation of the results [16] is the following.

A. States near 6.6 MeV in ^{10}C

The group at 6.6 MeV excitation energy in ^{10}C consists of two nearly degenerated levels: 3^- at 6.55 MeV and 2^+ at 6.57 MeV. (Here and below we are using experimental excitation energies given in column 12 of Table I which are known with precision of 50 keV [16].) The major mode of decay for the 2^+ level is into α+^{6}Be (this state was found to be extreme α-cluster state in [18]). The analog 3^- state in ^{10}B has substantial reduced α+^{6}Li(0^+;T=1) width as well (SF$_\alpha$=0.42 [18]). We expect much stronger population of the 3^- state than the 2^+ in the inelastic scattering [16], because of the collective enhancement of ΔL=3 transitions in light nuclei in this energy region [28]. Therefore, we suppose that the authors [16] observed the structure at 6.56 MeV which looked like a broad level decaying to α+^{6}Be with a width of <370 keV that is due to unresolved 3^-_1 and 2^+_3. However, the dominant mode of the 3^- level decay is to the ^{9}B+p. This decay is resulted in observation of the strong population of the level at 6.553 MeV with the width of 214(31) keV [16] (which is the real width of the 3^- level because the ^{9}B+p partial width of the 2^+ level is negligible). We estimated, that the admixture of the 2p+^{8}Be decay for the 2^+_3 level is $\sim$1/2 of the total width. This results in observation of the narrow structure of 172 keV (the width of the 2^+ level) in the 2p+^{8}Be channel, reported in [16]. The energy of the cluster 0^+_2 and 2^+_3 levels depends on the presence of the 2p+^{8}Be configuration. The admixture of this configuration at the level of 15% of the maximum di-proton width, which was needed to explain the widths of the 0^+_2 and 2^+_3 resonances, results in a decrease of $\sim$100 keV of the excitation energy of the 0^+_2 and 2^+_3, improving the agreement between the calculated and experimental Coulomb displacement energies. The maximum di-proton width was calculated using 2p+^{8}Be potential model. As for the 3^- state, the calculated excitation energy in ^{10}C should be corrected for the admixtures of the cluster and the collective configurations.

B. States near 5.3 MeV in ^{10}C

The narrowest resonance in the 5.2-5.3 MeV group is 0^+ at 5.29 MeV. The dominant decay mode for this state is 2p+^{8}Be because of small penetrability for the α+^{6}Be channel. (Similar decay is observed for the 2^+_3 level at 6.6 MeV). All other resonances close to 5.2 MeV should decay into the ^{9}B+p channel. The dominant population of this structure can be explained if 2^+_2 and 1^-_1 contribute to the peak at 5.2 MeV. While the measurements [16] presented a more detailed information on the ^{10}C states in question than the former experiments, there is an evident difference at 5.38 MeV where a peak with width of 300(60) keV was reported in ^{10}B(^{3}He,^{3}H) reaction [21]. The excitation energy and width of this peak are close to our calculations for the 2^- state. The states with abnormal parity can be populated in inelastic scattering experiment [16] only due to the second order effects. Therefore, we suppose that the 2^- state was not observed in [16].

IV. SUMMARY

We considered states in 5-7 MeV excitation energy region in ^{10}C and proposed spin-parity assignments for these states. In particular we showed that the states (0^+ and 2^+) with the cluster (α+^{6}Be) structure have the narrowest widths in this excitation region. We apply a rather common procedure of using shell model wave functions to calculate the Coulomb shifts and widths for the states with evident single particle spectroscopic factors. Similar procedure with cluster potentials accounting for the configurations forbidden by the Pauli principle was used to explore the isospin invariance for the cluster states. While different cluster potentials are conventional instruments to consider cluster states, it is difficult to find examples of the applications for the mirror nuclei. It is because the experimental data on cluster states in mirror nuclei are very rare. We noticed a remarkable (in comparison with the behavior of the nucleon single particle states) equidistance of 0^+ - 2^+ cluster states energies in mirror nuclei. Our test calculations showed that more complete data on the unknown members of cluster band (we expect 4^+ state at 10.1 MeV with a width of $\sim$600 keV) would provide for important information on the details of the cluster potential, first of all on the number of nodes of the cluster wave function. This number related with the details of the shell model structure appears to be related with the moment of inertia of the band.

The spin-parity assignments suggested here became possible due to recent experimental data containing information on the different decay modes of the states. As it is seen in Fig. 1, more decay channels are open for the states in the proton rich member of the T=1 multiplet, the exotic 2p decay being the lowest one. We have shown that the observation of this channel in [16] appeared to be very useful for the identification of the cluster levels. The 2p partial width is much larger than the single particle width for the cluster states. When we began this work we hoped that we would obtain an indication for the need to increase the Coulomb radius to an unusually large value for some (cluster) states. Indeed, the calculated excitation energies for the 0^+ and the 2^+ are higher than the experimental ones by $\sim$100 keV. 50% increase of the Coulomb radius would be needed to match the experimental data. However, the results of calculations depends on the proper accounting for the presence of 2p channel and also on the number of nodes of the cluster wave function. Based on our analysis we conclude that the partial widths for the 2p decay of the cluster 0^+ and 2^+ states are $\sim$100 keV. It is interesting to see if these can be reproduced by the microscopic many-body calculations. We presented evidence that novel measurements of the properties of the proton rich nuclei could be very useful. Even if the quantum characteristics determination is not directly possible in these experiments, the comprehensive analysis of the properties of the states in the isobaric multiplet can be reliable, and a test of the theoretical approaches can be more complete.

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