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Phys. Rev. C **86**, 034902 — Published 4 September 2012

DOI: [10.1103/PhysRevC.86.034902](https://doi.org/10.1103/PhysRevC.86.034902)

Flow Fluctuations from Early-Time Correlations in Nuclear Collisions

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We propose that flow fluctuations have the same origin as transverse momentum fluctuations. The common source of these fluctuations is the spatially inhomogeneous initial state that drives hydrodynamic flow. Longitudinal correlations from an early Glasma stage followed by hydrodynamic flow quantitatively account for many features of multiplicity and p_t fluctuation data. We develop a framework for studying flow and its fluctuations in this picture. We then compute elliptic and triangular flow fluctuations, and study their connections to the ridge.

PACS numbers: 25.75.Gz, 25.75.Ld, 12.38.Mh, 25.75.-q

I. INTRODUCTION

Fluctuations in the early stages of nuclear collisions contribute to the anisotropic flow measured in RHIC and LHC experiments, particularly the odd harmonics. In Ref. [1] we find that this early-time variation can also account for the multiplicity and transverse momentum fluctuations measured in the same experiments. Nevertheless, flow and fluctuation observables reveal different aspects of the initial state. Harmonic flow is generated largely by the global spatial anisotropy of the initial state. In contrast, fluctuation observables probe local spatial correlations that are largely independent of the overall geometry [1].

In this paper we argue that the *fluctuations* of the harmonic flow coefficients are driven by the very same local correlations that give rise to p_t and multiplicity fluctuations. Moreover, if early time correlations are the only source of these fluctuations, then the two measurements are related with no free parameters.

Experimenters define flow fluctuations by studying the difference of the flow coefficients $v_n\{2\}$ and $v_n\{4\}$ measured using two and four particle correlations, respectively [2–5]. Jets, resonance decays, and HBT effects also contribute to that difference. Measurements using particles separated in rapidity by more than 1–2 units eliminate these short range effects. Significantly, a difference between $v_n\{2\}$ and $v_n\{4\}$ remains [6], suggesting that flow fluctuations are a long range phenomenon. Causality dictates that such long range correlations originate in the early stages of the collision [7, 8].

To compute flow fluctuations we build on an approach started in Refs. [8, 9] in which long range correlations result from the fragmentation of Glasma flux tubes. In this formulation local spatial correlations emerge from fluctuations in the number and distribution of flux tubes. These spatial correlations are then modified by transverse expansion, giving rise to azimuthal correlations. A similar physical picture motivates studies using a wide range of different techniques [10–17].

This paper is organized as follows. In Sec.II, we begin with a general description of multi-particle correla-

tions and flow coefficients. We relate the intrinsic correlations in the multi-particle system to flow and its fluctuations. To begin, we write expressions for $v_n\{2\}$ and $v_n\{4\}$ in terms of correlation functions, adapting the tools invented in Refs. [18, 19] to our framework. Further results are in the appendix. We next define flow fluctuations in terms of $v_n\{2\}$ and $v_n\{4\}$, and turn to discuss the physical interpretation of this definition.

We discuss the common influence of local correlations on flow and p_t fluctuations in Sec.III. This relationship is the heart of our work. The fluctuations of these quantities are both consequences the spatially inhomogeneous collision environment modified by hydrodynamic flow. The general arguments in Secs. II and III set the stage for the more phenomenological analysis that follows.

In Sec.IV we describe initial state fluctuations in a CGC-Glasma picture. The number and distribution of Glasma flux tubes relative to the geometrical shape of the system ultimately determine the energy, projectile-mass, and centrality dependence of flow fluctuations. Next, we explain how collective flow and the correlation to the reaction plane modify local correlations.

We calculate the contribution of long range correlations to the flow coefficients and their fluctuations in Sec.IV. Our results are in good agreement with the latest LHC and RHIC data. In Sec.V, we argue that these same flow fluctuations are also responsible for the ridge. The same factors that influence flow fluctuations also determine the multiplicity and momentum fluctuations calculated in [1]. We emphasize that no new parameters are introduced here, so that results can be compared directly with [1].

In Sec.VI we discuss the extent to which the flow coefficients determine the angular distribution of the ridge. We also discuss the possible factorization of the Fourier coefficients of the pair distribution into products of flow coefficients [20–23]. We show that flow fluctuations can violate factorization at low p_t , depending on how the flow coefficients are defined. We then speculate that factorization holds for momenta $> 1 - 2$ GeV regardless of the source of anisotropy.

Lastly, we summarize and discuss the broader implications of our results in Sec.VII.

II. FLOW AND ITS FLUCTUATIONS

Nuclear collisions at non-zero impact parameter b produce anisotropic flow [24, 25]. This anisotropy derives from the change in the shape of the collision volume with respect to the reaction plane, i.e., the plane spanned by \mathbf{b} and the beam direction. If the reaction plane is known, this anisotropy is characterized by the moments

$$\langle v_n \rangle = \langle \cos n(\phi - \psi_{RP}) \rangle, \quad (1)$$

where ϕ is the azimuthal angle, ψ_{RP} is the angle of the reaction plane, and the brackets denote an average over particles and events. While the reaction plane cannot be observed directly, its influence can be deduced from multi-particle correlation measurements. Many strategies have been employed for measuring these flow coefficients, and they all have strengths and weaknesses.

Experimenters often deduce the flow from the two-particle cumulant

$$v_n\{2\}^2 = \langle \cos n(\phi_1 - \phi_2) \rangle, \quad (2)$$

exploiting the reaction plane information implicit in the relative distribution of particle pairs. Specifically, they measure the relative azimuthal angle for each particle pair in each event, and then average over events to obtain

$$v_n\{2\}^2 = \frac{\langle \sum_{i \neq j} \cos n(\phi_i - \phi_j) \rangle}{\langle N(N-1) \rangle} \quad (3)$$

[26]. The cumulant method was developed in Ref. [18, 19] and has seen extensive use [24, 25].

In this section we apply the framework of [18, 19] to the computation of event-wise flow fluctuations. To facilitate work in the later sections we cast this framework in terms of our treatment of correlations in [1]. We start by writing (3) as

$$v_n\{2\}^2 = \int \frac{\rho_2(\mathbf{p}_1, \mathbf{p}_2)}{\langle N(N-1) \rangle} \cos n(\phi_1 - \phi_2) d\mathbf{p}_1 d\mathbf{p}_2, \quad (4)$$

where the integrals are over the momenta of particles \mathbf{p}_i for $i = 1, 2$ and the distribution of particle pairs is

$$\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \frac{dN}{d\mathbf{p}_1 d\mathbf{p}_2}. \quad (5)$$

In the absence of correlations, $\rho_2(\mathbf{p}_1, \mathbf{p}_2) \rightarrow \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)$, where the single particle distribution is $\rho_1(\mathbf{p}) = dN/d\mathbf{p}$. We stress that the densities ρ_1 and ρ_2 are event-averaged quantities that respect the reaction plane. The factorization of ρ_2 then allows for factorization of (4) such that $v_n\{2\}^2 \rightarrow \langle v_n \rangle^2$.

Including two-particle correlations, the pair distribution does not factorize. To identify the contributions of the mean anisotropic flow (1) and genuine two-particle correlations to $v_n\{2\}$, we follow [18] and write (5) in terms of a cumulant expansion,

$$\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2) + r(\mathbf{p}_1, \mathbf{p}_2), \quad (6)$$

where r is the two-particle correlation function. We then use (1), (4), and (6) to write

$$v_n\{2\}^2 = \langle v_n \rangle^2 + 2\sigma_n^2. \quad (7)$$

This result is standard, and the factor of two in σ is conventional [27]. The contribution to (7) from two-body correlations is

$$\sigma_n^2 = \int d\mathbf{p}_1 d\mathbf{p}_2 \frac{r(\mathbf{p}_1, \mathbf{p}_2)}{2\langle N(N-1) \rangle} \cos n\Delta\phi, \quad (8)$$

where $\Delta\phi = \phi_1 - \phi_2$ is the relative azimuthal angle of the correlated particles.

Strictly speaking, this quantity measures the event-wise fluctuations of anisotropic flow due to geometry and dynamics together with non-flow correlations from resonance decays, the HBT effect, and jets. We will not address non-flow effects here. Observe that all such correlation effects are of order $1/N$, where N is the number of particles in the system. We focus on contributions to fluctuations at leading order in N . Accordingly, we omit a factor $\langle N \rangle^2 / \langle N(N-1) \rangle \approx 1$ that multiplies $\langle v_n \rangle^2$ term in (7); see appendix A for exact formulae.

To reduce the effect of fluctuations on the flow signal, experimenters also measure the four particle cumulant $v_n\{4\}$ defined by the relation

$$v_n\{4\}^4 = 2v_n\{2\}^4 - \langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle \quad (9)$$

[18, 19]. The four-angle term depends of the four particle correlation function $\rho_4(1, 2, 3, 4) \equiv \rho_4(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)$. In systems with large numbers of particles, multi-particle correlations are dominated by two-particle correlations. We therefore write

$$\begin{aligned} \rho_4(1, 2, 3, 4) &= \rho_1(1)\rho_1(2)\rho_1(3)\rho_1(4) \\ &\quad + \rho_1(1)\rho_1(2)r(3, 4) + \dots \\ &\quad + r(1, 2)r(3, 4) + \dots \end{aligned} \quad (10)$$

where $\rho_1(1) \equiv \rho_1(\mathbf{p}_1)$, etc., and the ellipses represents the distinct permutations of the momenta. Computing $\langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle$, one finds that the second and third lines in (10) contribute respectively terms of order $\langle v_n \rangle^2 \sigma_n^2$ and σ_n^4 that precisely cancel the σ_n contributions from $v_n\{2\}^4$. This cancellation is by design – it dictates the form of (9) proposed in Ref. [18, 19]. One finds

$$v_n\{4\} \approx \langle v_n \rangle, \quad (11)$$

plus corrections that are in practice very small. The fact that $v_n\{4\}$ receives no contribution from σ_n is usually thought of as a consequence of the Bessel-Gaussian approximation [27], but we see that it is true whenever two-body correlations are dominant. However, note that definitions of $\langle v_n \rangle$ other than (1) would yield different results.

As an aside, we briefly discuss the corrections to (11). Strictly speaking, calculation of (9) using (10) yields

$$v_n\{4\}^2 \approx \langle v_n \rangle^2 - 2\sigma_n^2, \quad (12)$$

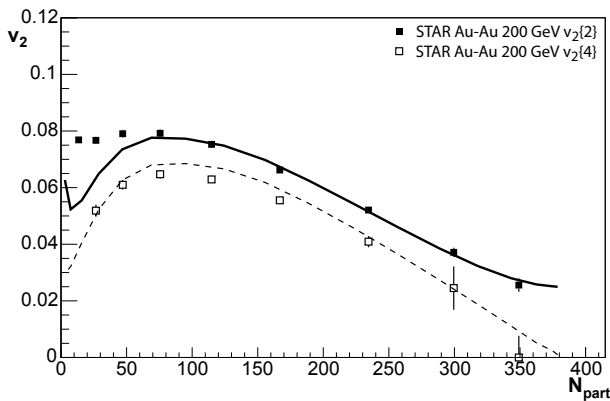


FIG. 1. Measured $v_2\{2\}$ and $v_2\{4\}$ from STAR [2] compared to calculations. Computed $v_2\{2\}$ (solid) uses (7) and (8), while $v_2\{4\}$ (dashed) uses (1) and (11). The difference between the calculated curves is due to flow fluctuations.

where

$$\Sigma_n^2 = \int d\mathbf{p}_1 d\mathbf{p}_2 \frac{r(\mathbf{p}_1, \mathbf{p}_2)}{\langle N(N-1) \rangle} \cos 2n(\Phi - \psi_{RP}) \quad (13)$$

for $\Phi = (\phi_1 + \phi_2)/2$ the average angle of the correlated pairs. Further corrections of higher order in $1/N$ are discussed in appendix A. Both Σ_n and σ_n are of the same order in $1/N$, but one expects Σ_n to be smaller than σ_n because it effectively is a higher harmonic, of order $2n$ rather than n . Indeed, we find the contribution of Σ_n to $v_n\{4\}$ smaller than 1.2% for $n = 2$ for peripheral collisions in our model. Note that $\Sigma_n = 0$ in central collisions. We comment that Σ_n represents fluctuations of harmonics of Φ , which are important in studying the Chiral Magnetic Effect [28].

Our aim is to understand the contribution of two-particle correlations to flow fluctuation measurements. If the only correlations are due to the reaction plane then $v_n\{2\} = v_n\{4\}$. This requires that no other sources of correlation exist, i.e., $r(\mathbf{p}_1, \mathbf{p}_2) = 0$, allowing $\rho_2(\mathbf{p}_1, \mathbf{p}_2)$ to factorize. However, experiments measure differences between $v_n\{2\}$ and $v_n\{4\}$ for several harmonic orders, n [2–5]. For example, Fig.1 shows $v_2\{2\}$ and $v_2\{4\}$ as measured by the STAR experiment in 200 GeV Au+Au collisions [2]. One can quantify this difference in terms of σ_n by substituting the square of (11) into (7), to obtain

$$\sigma_n^2 = \frac{v_n\{2\}^2 - v_n\{4\}^2}{2}. \quad (14)$$

Measurements with rapidity separations should suppress contributions to σ_n from jets, resonance decays, and other short range non-flow correlations, but do not explain the difference between $v_n\{2\}$ and $v_n\{4\}$, suggesting that $\langle v_n^2 \rangle$ -factorization is not a signature of flow or flow fluctuations.

An alternative measure of flow fluctuations can be constructed using event-by-event harmonics [27, 29, 30]. If each event produces a set of harmonics $\{\hat{v}_n\}$, then one

can compute the variance $\hat{\sigma}_n^2 = \langle \hat{v}_n^2 \rangle - \langle \hat{v}_n \rangle^2$ for an ensemble of such events. It is then reasonable to ask how our σ_n compares to this variance. Taking the mean square of \hat{v}_n to be $\langle \hat{v}_n^2 \rangle = \langle (\sum_i \cos n(\phi_i - \Psi_{RP}))^2 \rangle / \langle N \rangle^2$, we find

$$\hat{\sigma}_n^2 = \hat{\sigma}_{\text{stat}}^2 + \sigma_n^2 + \Sigma_n^2/2, \quad (15)$$

where σ_n and Σ_n are given by (8) and (13), and we keep only leading order in N . We define the statistical variance

$$\hat{\sigma}_{\text{stat}}^2 = \frac{1 + \langle v_{2n} \rangle}{2\langle N \rangle}. \quad (16)$$

These fluctuations arise because a finite number of particles are sampled in each event. In contrast, the dynamical fluctuations σ_n and Σ_n are caused by the correlations between particles [31]. To an excellent approximation the v_{2n} contribution to (16) and the Σ_n^2 contribution to (15) can be neglected.

We see that the event-wise variance of \hat{v}_n is very different from the quantity (14) that characterizes the difference between $v_n\{2\}$ and $v_n\{4\}$. The statistical contribution $\hat{\sigma}_{\text{stat}}$ can be comparable in magnitude to σ_n . We point out that the quantity $\hat{\sigma}_n$ may be measured directly from event-by-event harmonic coefficients.

It is instructive to compare our results to the model of Ref. [27, 30] in which flow fluctuations are exclusively due to the event-wise variation of the global geometry. In the purely geometric interpretation, one defines an eccentricity for each event $\hat{\epsilon}_n$. If one assumes that the relation between $\hat{\epsilon}_n$ and the resulting anisotropy of the fluid flow \hat{v}_n is approximately deterministic, then fluctuations of the ratio $\hat{v}_n/\hat{\epsilon}_n$ are negligible. The geometric contribution to the variance is then $\hat{\sigma}_n^2/\langle v_n \rangle^2 = (\langle \hat{\epsilon}_n^2 \rangle - \langle \hat{\epsilon}_n \rangle^2)/\langle \hat{\epsilon}_n \rangle^2$. We mention that Glauber model estimates of the relevant eccentricities are consistent with data for $n = 2$, where the correlation of \hat{v}_n with an event-wise eccentricity is highly plausible and well established [32]. However, the applicability of such an approach for $n \neq 2$ is less clear [33, 34].

We comment that the purely geometrical description of flow fluctuations in Ref. [30] requires $\langle \hat{v}_n \rangle \gg \hat{\sigma}_n$ in addition to $\hat{v}_n \propto \hat{\epsilon}_n$. Neither our general discussion here nor our specific calculations in Sec. IV require these assumptions. In Sec. IV, we include geometric fluctuations in our estimates of flow fluctuations through the distribution of flux tube sources ρ_{FT} . Correlations are then modified by the local transverse expansion of the system.

III. LOCAL CORRELATIONS

To understand the source of flow fluctuations, we recall some lessons from general fluctuation studies [31, 35–37]. Such studies typically focus on the variation of bulk observables such as multiplicity or transverse momentum within an ensemble of collisions. We draw most heavily from the study of p_t fluctuations in Ref. [1], to which the present work is essentially a sequel.

We measure p_t fluctuations using the covariance $\langle \delta p_{t1} \delta p_{t2} \rangle$, where $\delta p_{ti} = p_{ti} - \langle p_t \rangle$ is the deviation of each p_t from the average. Pairs in which both particles have higher than average momentum add to $\langle \delta p_{t1} \delta p_{t2} \rangle$. Lower-than-average pairs also add to the covariance, while high/low pairs subtract from it. In global equilibrium $\langle \delta p_{t1} \delta p_{t2} \rangle \equiv 0$. The presence of hot spots makes $\langle \delta p_{t1} \delta p_{t2} \rangle > 0$ (as would cold spots) [38]. Motion of the sources further enhances this quantity [10]. It follows that both jets and flow add to the p_t covariance.

Moving hot spots in the fluctuating system are sources of local correlations. We distinguish such correlations from those due to the global event-wise variation of the shape and size of the collision volume. We write the p_t covariance as

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \int d\mathbf{p}_1 d\mathbf{p}_2 \frac{r(\mathbf{p}_1, \mathbf{p}_2)}{\langle N(N-1) \rangle} \delta p_{t1} \delta p_{t2}. \quad (17)$$

In [1] we demonstrate that the p_t covariance is independent of the global spatial anisotropy, because (17) is integrated over azimuthal angle. It is also independent of size fluctuations by construction [39].

To see that flow fluctuations are also primarily driven by local correlations, observe that (17) has the same form as (8) with $\delta p_{t1} \delta p_{t2}$ replaced by $\cos n \Delta \phi$. As the p_t covariance probes correlations in momentum, so σ_n measures correlations of the angular separation $\Delta \phi$ of pairs.

This analogy makes physical sense because of the common influence of hot spots on $\Delta \phi$ and p_t . Flow pushes particles from a source into a particular opening angle, depending on the source position and the local fluid velocity. The same flow velocity boosts the p_t of these particles. Therefore multiple random hot spots give rise to both flow and p_t fluctuations. Based on this analogy, we expect σ_n in central collisions to be non-zero for all orders n , while $\langle v_n \rangle$ defined by (1) vanishes. Furthermore, it is not likely that σ_n is as strongly influenced by the global geometry as the mean $\langle v_n \rangle$.

In the next sections we discuss flow fluctuations due to early time local spatial correlations contributing to $r(\mathbf{p}_1, \mathbf{p}_2)$. We employ a Glasma-flux tube model in which particles are initially correlated at the point of production and modified by later stage transverse expansion. These flux tubes produce our ‘hot spots’. Transverse expansion is modeled with a blast wave scenario which inherits anisotropy from the average eccentricity $\langle \varepsilon_{RP} \rangle$.

IV. FLOW FLUCTUATIONS FROM GLASMA

Flow transforms early-time spatial correlations into observable momentum correlations. In this section we calculate the contribution of such correlations to flow fluctuations using the model developed in [1]. For the sake of brevity, we refer to [1] for detailed derivations, and only motivate those features needed to explain our assumptions and results.

We describe early time correlations using the correlation function $c(\mathbf{x}_1, \mathbf{x}_2)$, which is the difference between the density of pairs and the product of the single particle densities. This function describes local pair correlations in configuration space as well as global correlations due to fluctuating event eccentricities, shapes, and orientations (with respect to the reaction plane). Local correlations arise because the position and number of the colliding nucleons fluctuates in individual events. In general, these fluctuations result in high-density ‘hot spots’ distributed throughout the collision volume. It is more likely to find pairs of particles near hot spots.

Key to our analysis is the correlation strength \mathcal{R} , which we define using the integral of the correlation function over the collision volume. In Ref.[1] we construct a correlation function $c(\mathbf{x}_1, \mathbf{x}_2)$ that describes a specific early-time scenario based on Glasma theory. The correlation strength satisfies the scaling relation

$$\mathcal{R} dN/dy = \kappa \alpha_s^{-1} (Q_s^2), \quad (18)$$

which relates the product of the correlation strength \mathcal{R} times the rapidity density to the saturation scale Q_s . This relation is a rigorous consequence of Glasma theory [7]; our value of κ is consistent with ridge measurements [8, 9] and first-principles calculations [40]. The saturation scale depends on many collision variables including the density of participant nucleons and the collision energy [41, 42]. Any experimental indication of this scaling would support the underlying theory.

To relate the early-time spatial distribution to final-state momentum correlations, we write the two-particle correlation function $r(\mathbf{p}_1, \mathbf{p}_2)$ in (6) as the convolution

$$r(\mathbf{p}_1, \mathbf{p}_2) = \int c(\mathbf{x}_1, \mathbf{x}_2) f(\mathbf{x}_1, \mathbf{p}_1) f(\mathbf{x}_2, \mathbf{p}_2) d\Gamma_1 d\Gamma_2, \quad (19)$$

where $c(\mathbf{x}_1, \mathbf{x}_2)$ is the spatial correlation function, $f(\mathbf{x}_i, \mathbf{p}_i)$ is the local equilibrium Boltzmann distribution, and $d\Gamma_i$ are elements of the Cooper-Frye freeze out surface. We then describe the average anisotropic expansion of the system with a blast wave model [8, 9, 43–45]. In this formulation, particles passing through the freeze out surface have a velocity that is assumed to have a Hubble-like correlation with the position at which they were produced [1].

To compute flow fluctuations using this correlation function, we combine (8) and (19). We then use (7) and (11) to calculate $v_2\{2\}$ and $v_2\{4\}$, where the flow coefficient relative to the reaction plane is $\langle v_n \rangle = \int \rho_1(\mathbf{p}) \cos n(\phi - \Psi_{RP}) d\mathbf{p}$. Figure 1 compares $v_2\{2\}$ and $v_2\{4\}$ to STAR data from Au-Au at 200 GeV [2]. The $v_2\{4\}$ data is essentially an input to our model, as we use it to determine the mean eccentricity ε [1]. On the other hand, the fluctuation contribution to $v_2\{2\}$ is computed from the model and is in that sense a ‘prediction’. It is significant that this contribution satisfies $\sigma_n^2 \propto \mathcal{R}$; this is also true for the long range contribution to other fluctuation quantities [1].

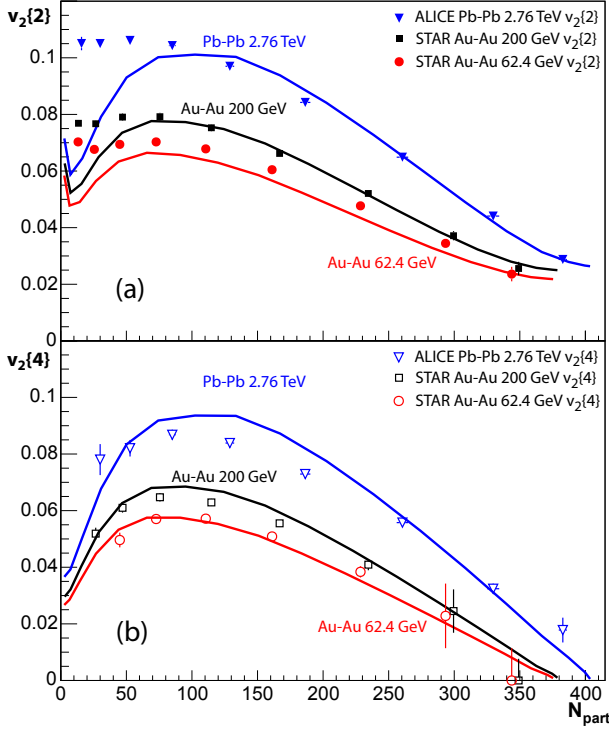


FIG. 2. (Color online) (a) Measured $v_2\{2\}$ from STAR [2] and ALICE [4, 5] compared to calculations using (7) and (8). (b) Same for $v_2\{4\}$ computed from (1) and (11)

The influence of event shapes and eccentricities enters (19) in two ways. First, the correlation function $c(\mathbf{x}_1, \mathbf{x}_2)$ is proportional to the probability distribution of flux-tube sources ρ_{FT} [1]. This distribution implicitly accounts for event-wise variation of the volume and eccentricity. Second, the azimuthal distribution of particles from each source is modified by flow. A pair emerging from a source that experiences a greater push will have a narrower opening angle. The magnitude of the push follows ρ_{FT} as the velocity depends on initial position.

The long range behavior introduced by Glasma correlations provides key centrality and collision energy dependence on the saturation scale Q_s^2 through \mathcal{R} in (18). The blast wave parameters provide significant centrality dependence, but their change with collision energy is minimal, as discussed in [1].

In Fig.2 we show comparisons of our calculated $v_2\{2\}$ and $v_2\{4\}$ to measured data [2, 4, 5] and find the agreement concerning change in collision energy is quite good. Similarly in Fig.3 we calculate $v_4\{2\}$ and $v_4\{4\}$, represented as the solid and dashed lines, respectively. Comparisons to $v_n\{4\}$ in both figures show how the eccentricity ε affects the results. The effect of eccentricity is to reduce the even harmonics as the collision area becomes circular and $\varepsilon \rightarrow 0$. The non-zero $v_n\{2\}$ in central collisions are due to fluctuations (14), which in our case are due to Glasma correlations (19). The effect of the change

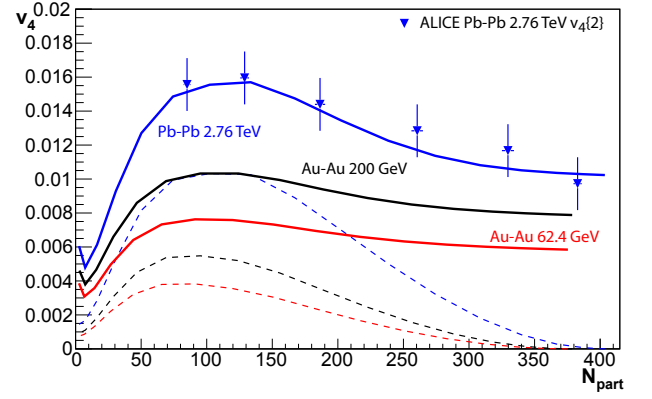


FIG. 3. (Color online) Same calculation as in Fig.2 but for the fourth order flow coefficients $v_4\{2\}$ (solid) and $v_4\{4\}$ (dashed). Data is from the ALICE collaboration [4].

in blast wave parameters with collision energy is apparent from the separation of the solid lines in the bottom panel of Fig.2 and the dashed lines in Fig. 3. The importance of the Glasma contribution is similarly evident by the change in separation between $v_n\{4\}$ and $v_n\{2\}$ with collision energy. The increase in growth of flow fluctuations (8) is coupled to the growth in the saturation scale.

As a caveat we emphasize our neglect of the non-flow contribution to flow fluctuations. Given that omission, it is perhaps surprising that our results agree with data as well as they do. Related concerns were discussed in Ref. [1], where we treated different observables. Experiments can reduce non-flow by measuring $v_n\{2\}$ and $v_n\{4\}$ with rapidity separations and comparing them to harmonic extractions from ridge measurements; see Secs. V and VI. However, further experimental and theoretical work is needed to pin down the non-flow contribution in the broad context we consider here and in [1].

STAR has also measured v_2 fluctuations in the form of

$$\frac{\sigma_{v_n}}{\langle v_n \rangle} = \sqrt{\frac{v_n\{2\}^2 - v_n\{4\}^2}{v_n\{2\}^2 + v_n\{4\}^2}} \quad (20)$$

resembling the so-called “coefficient of variation” defined as the standard deviation divided by the mean. Care should be taken here since the definition of σ_n^2 , Eq.(14), is not strictly the variance. In Fig.4 we compare our calculation of (20) to measurement. Calculations at RHIC energies seem to agree reasonably well and we include the calculation for Pb+Pb 2.76 GeV as a prediction.

Observe that local correlations that contribute to σ_n lead to fluctuations of all harmonic orders in n including v_3 . As a result, Glasma correlations generate a measurable $v_3\{2\}$ even in the absence of $v_3\{4\}$ or any triangular flow. In Fig.5 we show v_3 fluctuations from Glasma. We further emphasize that the energy dependence in Fig.5 is in good accord with data, supporting the Glasma scaling with Q_s^2 . The shape with centrality reflects contributions not only from Glasma, but also our parameterizations of

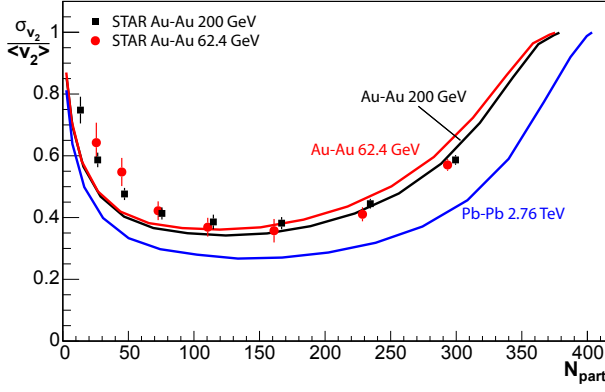


FIG. 4. (Color online) Flow fluctuations in terms of the coefficient of variation for elliptic flow (20) compared to STAR data [2].

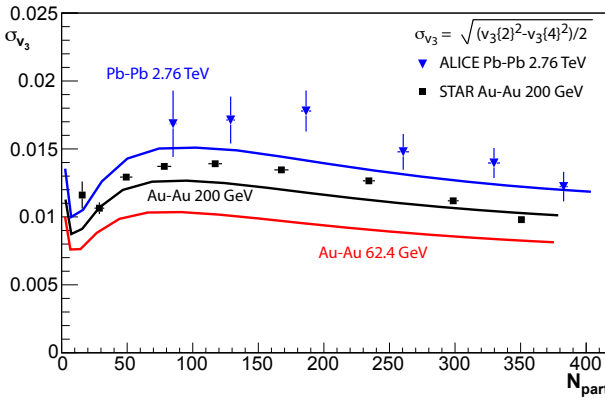


FIG. 5. (Color online) Triangular flow fluctuations, σ_{v_3} , calculated from (8) and compared to data computed from $v_3\{2\}$ and $v_3\{4\}$ measurements by the STAR and ALICE collaborations [3, 4].

the average reaction plane eccentricity ε and our choice the flux tube distribution ρ_{FT} .

To use (8) and (14) to calculate $v_3\{2\}$, we must come to grips with the fact that our blast wave parametrization assumes $v_3\{4\} = 0$. While this seems to be the case for the STAR measurements, ALICE has measured a non-zero $v_3\{4\}$ for Pb+Pb collisions at 2.76 TeV. To correct for this possible discrepancy, we can provide an ad-hoc parameterization of $v_3\{4\}$ and use (8) and (14) to calculate $v_3\{2\}$. Agreement shown in Fig.6 is reasonable. As a preferable alternative, we compare our calculated σ_n to fluctuations extracted from ALICE and STAR measurements in Fig.5.

V. THE RIDGE

Early analyses of the soft or untriggered ridge, including our own, focus on the idea that the ridge is composed of correlations in excess of momentum conserva-

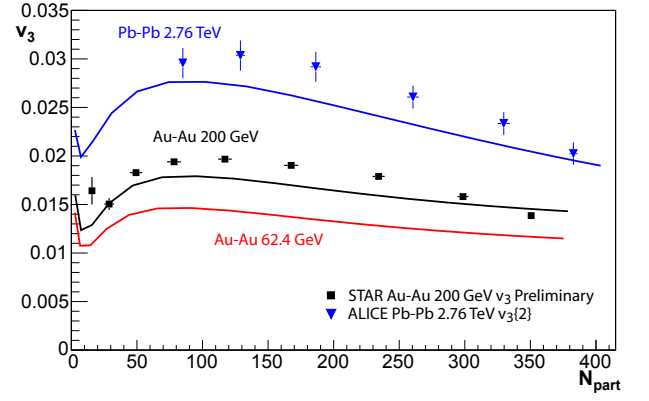


FIG. 6. (Color online) The triangular flow coefficient, $v_3\{2\}$ compared to STAR and ALICE data [3, 4].

tion and elliptic flow. The novel observation of Alver and Roland that correlations resulting from geometrical fluctuations result in odd flow harmonics such as triangular flow, v_3 , suggests that v_3 , together with flow harmonics of all higher orders, explains this excess [46]. In this paper we have studied how angular correlations tied to common production points influence two-particle correlation measurements of flow harmonics $v_n\{2\}$ through flow fluctuations σ_n^2 . Most notably, as discussed in Sec.IV, we find significant contributions to $v_3\{2\}$ even in the absence of triangular flow. In fact, the same correlations contributing to flow fluctuations were initially proposed as explanations of the ridge [7–12, 14–17, 47, 48], although their role in reproducing the full $\Delta\phi$ azimuthal structure was not understood.

Flow and its fluctuations both contribute to the two-particle correlation landscape. The pair distribution is

$$\rho_2(\Delta\phi) = \rho_{\text{ref}} \left(1 + 2 \sum_{n=1}^{\infty} \langle v_n \rangle^2 \cos n\Delta\phi \right) + r(\Delta\phi), \quad (21)$$

where ρ_{ref} is the experimental mixed-event background. Observe that the Fourier coefficients of (21) reproduce (7). We take $\rho_{\text{ref}} = \frac{1}{2\pi} \int \overline{\rho_1} \int \overline{\rho_1}$ where the overbar indicates an event plane average, to mimic the experimental mixed event technique for constructing ρ_{ref} . The quantity $\Delta\rho(\Delta\phi) = \rho_2(\Delta\phi) - \rho_{\text{ref}}$ characterizes the angular correlations. STAR measures the ratio

$$\frac{\Delta\rho(\Delta\phi)}{\sqrt{\rho_{\text{ref}}}} = \frac{1}{2\pi} \frac{dN}{d\eta} \left(2 \sum_{n=1}^{\infty} \langle v_n \rangle^2 \cos n\Delta\phi + \frac{r(\Delta\phi)}{\rho_{\text{ref}}} \right), \quad (22)$$

where the $\langle v_n \rangle$ terms follow from (1).

STAR performs a multi-parameter fit to this distribution to separate the ridge peak from the elliptic flow contributions (along with momentum conservation and HBT contributions, which we omit for clarity). They report a flow-subtracted ridge amplitude $(\Delta\rho(\Delta\phi, \Delta\eta)/\sqrt{\rho_{\text{ref}}})|_{FS}$ on the near side, centered at $\Delta\phi = \Delta\eta = 0$ [49–51]. We use (18) and (19) to calculate the flow-subtracted ridge

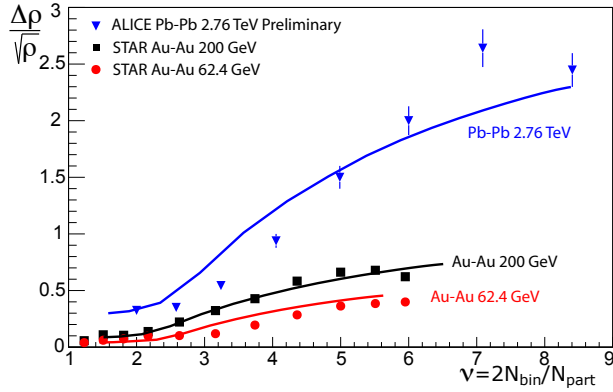


FIG. 7. (Color online) Near side ridge amplitude calculation from Glasma source correlations. Experimental data is from (STAR) [49–51] and (ALICE) [52]

amplitude

$$\left. \frac{\Delta\rho}{\sqrt{\rho_{\text{ref}}}} \right|_{FS} = \mathcal{R} \frac{dN}{dy} F(\Delta\phi), \quad (23)$$

where $F(\Delta\phi) \propto r(\Delta\phi)$ is the angular correlation function normalized so that $\int F(\Delta\phi) d\Delta\phi = 1$. The energy and centrality independent scale constant κ in (18) is fixed by Au-Au 200 GeV data as in [8, 9].

The blast wave parameters have little energy dependence and the Glasma factor (18) allows for strong agreement with the 62 GeV data without adjustment of κ . Additionally, the ALICE collaboration has measured (23) in Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV [52], providing a further test of the CGC-Glasma energy dependence. We confront the ALICE data in the same way, without adjusting κ , however the ALICE [52] measurement procedure differs slightly than that from STAR [51]. This difference in fitting procedures is mostly insignificant, especially in central collisions, but not necessarily so in peripheral collisions to which we normalize our calculation (at 200 GeV). To adjust for this possible effect, we multiply (23) by an additional scale factor of ~ 1.5 to return agreement in peripheral collisions; see note [53]. In Fig. 7 we compare calculated ridge amplitudes from (23) with measured data for 200 and 62.4 GeV Au+Au collisions from STAR as well as 2.76 TeV Pb+Pb collisions from ALICE. In this paper we only compare to measurements from which elliptic flow has been subtracted, because we find that fluctuations dominate v_3 at STAR.

The characterization of the ridge in terms of Fourier coefficients can prove a valuable tool for analysis of such discrepancies as well as relate the magnitude its effect with respect to other phenomena. Viewing (14) in terms of (8) and (23), one can write

$$2 \frac{dN}{dy} \sigma_n^2 \approx \int \frac{\Delta\rho(\Delta\phi)}{\sqrt{\rho_{\text{ref}}}} \cos(n\Delta\phi) d\Delta\phi. \quad (24)$$

A similar equation was pointed out by Sorensen et. al.

[54, 55], and discussed in terms of event plane eccentricity fluctuations.

We stress, that (24) is a general result that is model independent. Direct measurement of this quantity can isolate the effects of flow fluctuations on the ridge by comparing the results to independent flow fluctuation measurements. Furthermore, flow fluctuation measurements can be an effective way to quantify the ridge. Deviations from (24) at large $\Delta\eta$. can indicate the degree at which other phenomena contribute to long range correlation measurements. At smaller $\Delta\eta$ shorter range phenomenon such as jets or diffusion also come into play. Combined flow fluctuation and ridge studies analyzed with increasing lower p_t limits as done in [56] can reveal the emergence of jets and their influence. For example, one could test contributions to $v_n\{2\}$ from correlations induced by jet quenching as suggested in [9, 57].

VI. FACTORIZATION

We now ask whether the azimuthal dependence of the pair distribution is entirely determined by the flow coefficients. This is somewhat of a circular question, because flow coefficients are themselves derived from correlation analyses. A meaningful answer depends on how the flow coefficients are defined. Experimentalists expand the transverse-momentum-dependent pair distribution as

$$\rho_2 \propto 1 + 2 \sum_n V_{n\Delta}(p_{t1}, p_{t2}) \cos(n\Delta\phi). \quad (25)$$

The term “factorization” is often applied when one can express the Fourier coefficients $V_{n\Delta}(p_{t1}, p_{t2})$ as a product of flow coefficients $v_n(p_{ti})$. Authors often cite factorization as a signature of collective flow [20–22, 58]; see also [23] for a theoretical perspective. Problems stem from the fact that $V_{n\Delta} \equiv v_n\{2\}^2$ as an exact consequence of the definition (4). In other words, $V_{n\Delta}$ *always* factorizes as $v_n\{2\} \times v_n\{2\}$, regardless of the source of anisotropy.

To define factorization in a useful way, one must compare $V_{n\Delta}$ to the coefficients $\langle v_n(p_{ti}) \rangle$ defined by the reaction plane as in (1). Proxies like $v_n\{4\}$ can also be used. To see why this is true, we define r_n to be the Fourier coefficient of the two particle correlation function $r(\Delta\phi)$ in (6). We then compare (25) to (21) to find

$$V_{n\Delta}(p_{t1}, p_{t2}) = \langle v_n(p_{t1}) \rangle \langle v_n(p_{t2}) \rangle + \frac{r_n(p_{t1}, p_{t2})}{\rho_{\text{ref}}(p_{t1}, p_{t2})}, \quad (26)$$

where, for simplicity, we keep only the leading order in the number of particles in the relevant p_t ranges. The first term in (26) measures the effect of global geometric anisotropy, while the second term is due to local fluctuations. Integrating over momenta and using (7), we indeed find

$$V_{n\Delta} = \langle v_n \rangle^2 + 2\sigma_n^2 \equiv v_n\{2\}^2, \quad (27)$$

as noted above. However, (26) shows that the Fourier coefficients of the correlation function $r_n(p_{t1}, p_{t2})$ break factorization in terms of $\langle v_n(p_{t2}) \rangle$.

We see that hydrodynamic flow does not generally imply factorization, as follows from (19) and (26). Specifically, (24) shows that flow fluctuations violate factorization. In addition, jets, HBT, and resonances contribute to r_n along with the long range correlations we have considered here.

Flow coefficients at the LHC are found to factorize for pairs of high p_t particles, but not when high and low p_t particles are mixed [58]. This may seem puzzling if you view factorization as a hydrodynamic signature. To understand this result, observe that the contribution r_n/ρ_{ref} in (26) can be large compared to anisotropy only when a substantial fraction of pairs in the selected momentum range are correlated. RHIC energy measurements show that the overall magnitude of the correlation function drops precipitously relative to mixed-event pairs when a lower p_t cutoff is increased above ~ 1 GeV [56]. Our calculations in [9], which include both flow and jet-medium interactions, agree with that trend; see, e.g., fig. 9 of [9]. We therefore do not expect significant violations of factorization from the components r_n due to long range correlations from flow or jets above $p_{ti} > 1 - 2$ GeV. An analogous trend at higher beam energy may explain the LHC results.

VII. DISCUSSION

In this paper we study the connection between flow fluctuations and initial state correlations. Flow fluctuations are defined by (14) using two and four-particle correlation measurements of the harmonic flow coefficients. Section II provides a general discussion of the calculation of flow observables from multiparticle momentum space distributions. Following the cumulant expansion method introduced in [18, 19], multiparticle distributions can be expanded into factorized and correlated parts, e.g. (5) and (10). We derive explicit expressions relating the correlated parts to the fluctuation observables σ_n and Σ_n . Such genuine correlations were neglected in Refs. [18, 19], which were primarily aimed at isolating non-flow effects from flow measurements.

We are the first to calculate the contribution of genuine correlations to flow fluctuations. Treating such fluctuations on the same footing as p_t and multiplicity fluctuations, we obtain eqs. (8) and (13) for σ_n and Σ_n . In view of the approximation (11), we cast (7) to superficially resemble the Bessel-Gaussian approximation from Voloshin et al. [27]. Nevertheless, we stress that our local picture of the origin of fluctuations is more general from the global geometrical view in [27]. The relation between these works is discussed in Sec. II in the paragraphs surrounding eqs. (15) and (16).

We study the contribution of local long range correlations to σ_n based on parton production at common

transverse positions. Section IV presents calculations of σ_n in our Glasma flux tube model. We mention that our model incorporates much of the same physics as used by event-by-event hydro in [17, 59–63]. Those authors obtain the initial state in individual events by sampling a probability distribution analogous to the ρ_{FT} described in [1]. In all cases, Cooper Fry freeze out is used. We use a blast wave constrained by data to approximate the velocity distribution and the freeze-out surface. The key differences are that 1) our correlated regions are point-like in the transverse plane, while theirs may be larger and 2) they evolve their initial distributions using deterministic hydrodynamics in each event. We expect that they will eventually achieve a better description of the v_n at each energy. Our complementary aim is to study the big picture by exploring the energy range with a variety of variables.

The Glasma formulation provides key collision system, energy, and centrality dependences resulting in reasonable agreement with experimental measurements of $v_n\{2\}$ from 62.4 GeV Au+Au to 2.76 TeV Pb+Pb collisions for $n=2, 3$, and 4. The presence of σ_n provides two key results: non-zero values of even harmonics in central (circular) collisions, and the existence of $v_3\{2\}$ without global triangular flow.

In Sec.V we turn to discuss two particle correlations and, in particular, the ridge. We see that flow fluctuations and the ridge are facets of the same phenomenon, related by a Fourier transform (24). The computed peak ridge amplitude in Fig. 7 is in reasonable agreement with calculations from 62.4 GeV Au+Au to 2.76 TeV Pb+Pb, consistent with our flow results. Further investigation is needed to understand the full correlation landscape. In particular, we did not discuss the $n = 1$ harmonic, in which momentum conservation plays an important role. Characterization of the ridge in terms of flow fluctuations (i.e., harmonics) combined with earlier fit procedures as in Ref. [51] will surely prove a useful tool.

We discuss the question of whether the Fourier coefficients of the correlation function factorize into a product of flow coefficients in Sec.VI. In general, irreducible two-particle correlations from any mechanism violate this factorization [20–22]. In particular, the long range correlations that we compute violate factorization at low p_t , as would jets, resonance decays, and HBT effects. We expect any such correlations to become negligible for pairs above $p_t \sim 1 - 2$ GeV, due to the rapid decrease of the number of correlated pairs relative to the mixed-event background that is observed experimentally [56].

In summary, we observe that early-time fluctuations have broad implications beyond the flow fluctuations and azimuthal correlations studied here. In Ref. [1] we studied the impact of these correlations on multiplicity and p_t fluctuations. We argued in Sec. III that p_t and flow fluctuations are intimately related because they are both driven by local hydrodynamic fluctuations. While the average flow coefficients are primarily determined by the global event shape, the fluctuations of these coefficients

have more in common with other fluctuation observables. Importantly, the overall magnitude of the contribution from long range correlations to all of these fluctuations – flow, multiplicity, and p_t – is set by a single scale factor \mathcal{R} . In Glasma theory, the dependence of \mathcal{R} on energy, centrality, and projectile mass are fixed its variation with the saturation scale Q_s . Results here and in Ref. [1] provide a survey of the ridge, and multiplicity, momentum and flow fluctuations that reveals a common energy and centrality dependence that we attribute to the production mechanism. Glasma calculations are consistent with this dependence.

This broad agreement is somewhat puzzling, because resonances, HBT, and jets must also contribute to two particle correlations. We refer to these effects as ‘short range’, since they only modify correlations for $\Delta\eta \sim 1-2$ units or less [1]. Short range correlations are responsible for the non-flow contribution to quantities like $v_n\{2\}$. These effects can be reduced by measuring flow, correlations, and fluctuations only for pairs with a large $\Delta\eta$ separation. This is already being done in flow measurements [58]. Measurements of all quantities over a range of energies with $\Delta\eta$ cuts can eventually allow us to refine our extraction of early-time behavior. However, it is imprudent to remove non-flow without understanding its cause, because the QCD phase transition also contributes to short range correlations. To gain further information on the sources of short-range correlations, observe that long range effects are charge independent. Net-charge correlations are therefore sensitive primarily to short range effects [64].

ACKNOWLEDGMENTS

We thank M. Bleicher, H. Caines, C. DeSilva, J. Jia, A. Majumder, P. Mota, C. Pruneau, P. Sorensen, R. Snellings, A. Timmins and S. Voloshin for discussions. S.G. thanks the Institute for Nuclear Theory in Seattle, Washington for hospitality during the completion of this work. S.G. thanks the Institute for Nuclear Theory in Seattle, Washington for hospitality during the completion of this work. This work was supported in part by the U.S. NSF grant PHY-0855369 (SG) and The Alliance Program of the Helmholtz Association (HA216/EMMI) (GM)

Appendix A: Four-Particle Flow Coefficients

In this appendix we calculate corrections to the four-particle flow coefficient measurement $v_n\{4\}$ from two-particle correlation sources. Trying to keep the notation as general as possible we follow [18] using the cumulant expansion method to write

$$\langle e^{in(\phi_1 - \phi_2)} \rangle = \frac{\int \rho_2(\mathbf{p}_1, \mathbf{p}_2) e^{in(\phi_1 - \phi_2)} d\mathbf{p}_1 d\mathbf{p}_2}{\int \rho_2(\mathbf{p}_1, \mathbf{p}_2) d\mathbf{p}_1 d\mathbf{p}_2} \quad (\text{A1})$$

$$= \frac{\langle N \rangle^2 \langle v_n \rangle^2}{\langle N(N-1) \rangle} + \frac{\int r(\mathbf{p}_1, \mathbf{p}_2) e^{in\Delta\phi} d\mathbf{p}_1 d\mathbf{p}_2}{\langle N(N-1) \rangle}, \quad (\text{A2})$$

where $v_n\{2\}^2 = \langle e^{in(\phi_1 - \phi_2)} \rangle$. Notice that since $\sin(n\Delta\phi)$ is odd, the second term in (A2) is equivalent to (8) when the pair distribution is symmetric about the normal to the reaction plane. The factor $\langle N \rangle^2 / \langle N(N-1) \rangle$ in the first term of (A2) contributes a correction of $1/(1+\mathcal{R}) \approx 1$ as long as the multiplicity, N , is large. For example, in central 200 GeV Au+Au collision we calculate $\mathcal{R} \approx 0.003$ [1].

Our goal is to calculate the four-particle flow coefficient

$$v_n\{4\}^4 = 2\langle e^{in(\phi_1 - \phi_2)} \rangle^2 - \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle, \quad (\text{A3})$$

where the final term is calculated analogously to (A1) but from the four-particle cumulant expansion (10). Keeping only contributions from two-particle correlations we have

$$\langle N(N-1)(N-2)(N-3) \rangle \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle = \quad (\text{A4a})$$

$$= \langle N \rangle^4 \langle v_n \rangle^4 \quad (\text{A4b})$$

$$+ \langle N \rangle^2 \langle v_n \rangle^2 \cdot 2Re \left\{ \int r(\mathbf{p}_1, \mathbf{p}_2) e^{i2n(\Phi - \psi_{RP})} d\mathbf{p}_1 d\mathbf{p}_2 \right\} \quad (\text{A4c})$$

$$+ 4\langle N \rangle^2 \langle v_n \rangle^2 \int r(\mathbf{p}_1, \mathbf{p}_2) e^{in\Delta\phi} d\mathbf{p}_1 d\mathbf{p}_2 \quad (\text{A4d})$$

$$+ \left| \int r(\mathbf{p}_1, \mathbf{p}_2) e^{i2n(\Phi - \psi_{RP})} d\mathbf{p}_1 d\mathbf{p}_2 \right|^2 \quad (\text{A4e})$$

$$+ 2 \left| \int r(\mathbf{p}_1, \mathbf{p}_2) e^{in\Delta\phi} d\mathbf{p}_1 d\mathbf{p}_2 \right|^2. \quad (\text{A4f})$$

where $\Delta\phi = \phi_1 - \phi_2$ and $\Phi = (\phi_1 + \phi_2)/2$ are the relative and average coordinates. To reduce the equations further it is necessary to make the approximation that $\langle N(N-1)(N-2)(N-3) \rangle \approx \langle N(N-1) \rangle^2$ then the terms (A4d) and (A4f) will cancel with corresponding terms emerging from the twice the square of (A2), leaving the corrections

$$v_n\{4\}^4 = \langle v_n \rangle^4 - \langle v_n \rangle^2 \cdot 2Re \{ \Sigma_n^2 \} - |\Sigma_n^2|^2, \quad (\text{A5})$$

where

$$\Sigma_n^2 = \frac{\int r(\mathbf{p}_1, \mathbf{p}_2) e^{i2n(\Phi - \psi_{RP})} d\mathbf{p}_1 d\mathbf{p}_2}{\langle N(N-1) \rangle} \quad (\text{A6})$$

which is equivalent to (13). Notice that these corrections are effectively of order $2n$ and depend on the reaction plane. As discussed in the text, the corrections in (A5) maximally modify (11) by 1.2% in our model.

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