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The recent high-resolution measurement of the electric dipole ($E1$) polarizability $\alpha_{\rm D}$ in ^{208}Pb [Phys. Rev. Lett. **107**, 062502 (2011)] provides a unique constraint on the neutron-skin thickness of this nucleus. The neutron-skin thickness $r_{\rm skin}$ of ^{208}Pb is a quantity of critical importance for our understanding of a variety of nuclear and astrophysical phenomena. To assess the model dependence of the correlation between $\alpha_{\rm D}$ and $r_{\rm skin}$, we carry out systematic calculations for ^{208}Pb, ^{132}Sn, and ^{48}Ca based on the nuclear density functional theory (DFT) using both non-relativistic and relativistic energy density functionals (EDFs). Our analysis indicates that whereas individual models exhibit a linear dependence between $\alpha_{\rm D}$ and $r_{\rm skin}$, this correlation is not universal when one combines predictions from a host of different models. By averaging over these model predictions, we provide estimates with associated systematic errors for $r_{\rm skin}$ and $\alpha_{\rm D}$ for the nuclei under consideration. We conclude that precise measurements of $r_{\rm skin}$ in both ^{48}Ca and ^{208}Pb—combined with the recent measurement of $\alpha_{\rm D}$—should significantly constrain the isovector sector of the nuclear energy density functional.

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The Lead Radius Experiment (PREX) [1, 2] at the Jefferson Laboratory has recently determined the neutron root-mean-square (rms) radius r_n of ^{208}Pb [3]. Parity-violating electron scattering, a powerful technique used by the PREX collaboration, is particularly sensitive to the neutron distribution because the neutral weak-vector boson couples preferentially to the neutrons in the target [4]; the coupling to the proton is suppressed by the weak mixing angle. In spite of the many challenges that it faced, this purely electroweak measurement may be interpreted with as much confidence as conventional electromagnetic scattering experiments that have been used for decades to accurately map the electric charge distribution of the nucleus.

A quantity that is related to the neutron radius is the neutron-skin thickness $r_{\rm skin}{=}r_n{-}r_p$, namely, the difference between the rms neutron and proton radii. The importance of the neutron skin lies in its strong sensitivity to the poorly known isovector density $\rho_1{=}\rho_n{-}\rho_p$. Given that $r_{\rm skin}$ is a strong indicator of isovector properties, the determination of r_n of a heavy nucleus is a problem of fundamental importance with far-reaching implications in areas as diverse as nuclear structure [5–8], atomic parity violation [9], and neutron-star structure [10, 11]. By measuring the neutron form factor of ^{208}Pb at a moderate momentum transfer of $q{\approx}0.475\,{\rm fm}^{-1}$, and through an extrapolation to low-momentum transfers [6, 12], PREX was able to determine the following values for the neutron radius and neutron-skin thickness: $r_n{=}5.78^{+0.16}_{-0.18}\,{\rm fm}$ and $r_{\rm skin}{=}0.33^{+0.16}_{-0.18}\,{\rm fm}$ [3].

Prompted by the implications of a measurement of r_n, interest in the use hadronic probes to map the neutron distribution has been revived. Of particular relevance are experiments that employ antiprotonic atoms [13–15] and the elastic scattering of protons [16, 17]. Recent analyses from such experiments have determined the neutron-skin thickness of ^{208}Pb to be: $r_{\rm skin}{=}0.16\pm(0.02)_{\rm stat}\pm(0.04)_{\rm syst}\,{\rm fm}$ [14] and $r_{\rm skin}{=}0.211^{+0.054}_{-0.063}\,{\rm fm}$ [17]. Unfortunately, extraction of r_n from measurements based on hadronic probes is still a subject of significant model dependence and large theoretical uncertainties [18, 19]. Moreover, elastic proton scattering is highly insensitive to the isovector density as medium-energy protons probe preferentially the isoscalar density [20]. So while hadronic probes will continue to play a critical role in our understanding of novel nuclear properties, the complementary approach based on electroweak probes provides a clean and largely model-independent alternative.

Another observable that is a strong indicator of isovector properties is the electric dipole polarizability $\alpha_{\rm D}$ related to the response of the nucleus to an externally applied electric field. For stable medium-to-heavy nuclei with a moderate neutron excess, the dipole response is largely concentrated in the giant dipole resonance (GDR) of width 2-4 MeV that exhausts almost 100% of the energy-weighted sum rule [21]. For this isovector mode of excitation – perceived as an oscillation of neutrons against protons – the symmetry energy $a_{\rm sym}$ acts as the restoring force. Models with a soft symmetry energy, namely, those that change slowly with density, predict larger values for $a_{\rm sym}$ at the lower densities of relevance to the excitation of this mode [22, 23]. In this context, the inverse energy-weighted $E1$ sum rule m_{-1} – a quantity directly proportional to $\alpha_{\rm D}$ – is of particular interest as it is highly sensitive to the density dependence of the symmetry energy. This sensitivity suggests the existence of a correlation: the larger $r_{\rm skin}$, the larger $\alpha_{\rm D}$. Indeed, the approximate proportionality of these two quantities is expected based on both macroscopic arguments [24, 25] and microscopic calculations [8, 26]. The recently completed high-resolution $(\vec{p},\vec{p}')$ measurement at RCNP of the distribution of $E1$ strength in ^{208}Pb over a wide range of excitation energy [27] has, therefore, created considerable excitement. Of particular relevance to our work is the precise value of the measured electric dipole polarizability of ^{208}Pb: $\alpha_{\rm D}$=(20.1±0.6) fm^3.

The purpose of this work is fourfold. First, we examine the robustness of the correlation between the dipole polarizability and the neutron-skin thickness of ^{208}Pb. Second, in order to provide a meaningful estimate of $r_{\rm skin}$ from $\alpha_{\rm D}$, we compute the associated systematic error. Third, we predict $\alpha_{\rm D}$ in ^{48}Ca and ^{132}Sn with quantified uncertainties. Finally, we assess the importance of the follow-up PREX measurement of $r_{\rm skin}$ in ^{48}Ca.

Generally, to assess a linear correlation between two observables A and B within *one given model*, one resorts to a least-squares covariance analysis, with the correlation coefficient

$$C_{AB} = \frac{|\overline{\Delta A\,\Delta B}|}{\sqrt{\overline{\Delta A^2}\ \overline{\Delta B^2}}}, \qquad (1)$$

providing the proper statistical measure [28]. In Eq. (1) the overline means an average over the statistical sample. A value of $|C_{AB}| = 1$ means that the two observables are fully correlated whereas $C_{AB} = 0$ implies that they are totally uncorrelated. Recently, the statistical measure C_{AB} was used to study correlations between various nuclear observables [8] in the context of the Skyrme SV-min model [29]. In particular, it was concluded that good isovector indicators that strongly correlate with the neutron radius of ^{208}Pb are its electric dipole polarizability as well as neutron skins and radii of neutron-rich nuclei [8]. Indeed, by relying on the strong correlation between $\alpha_{\rm D}$ and $r_{\rm skin}$ (C_{AB}=0.98) predicted by such DFT calculations, Tamii *et al.* deduced a value of $0.156^{+0.025}_{-0.021}$ fm for the neutron-skin thickness of ^{208}Pb.

However, the correlation coefficient C_{AB} cannot assess systematic errors that reflect constraints and limitations of a given model [8]. Such systematic uncertainties can only emerge by comparing different models (or sufficiently flexible variants of a model) and this is precisely what has been done in this Letter. To assess the linear dependence between two observables A and B for a sample of *several models*, the correlation coefficient $C_{AB}^{\rm models}$ is now obtained by averaging over the predictions of those models. Although the correlation coefficient $C_{AB}^{\rm models}$ determined in such a way may not have a clear statistical interpretation, it is nevertheless an excellent indicator of linear dependence.

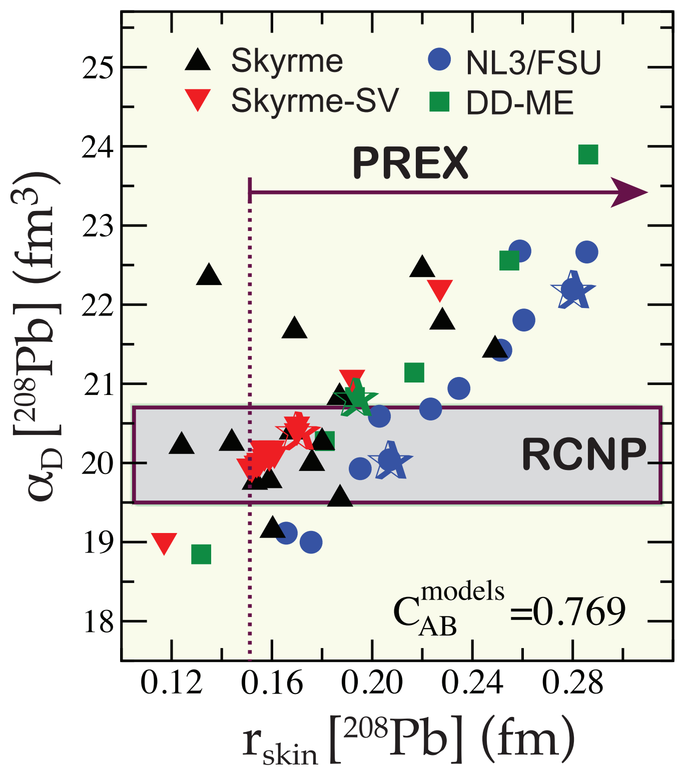

FIG. 1: (Color online) Predictions from 48 nuclear EDFs discussed in the text for the electric dipole polarizability and neutron-skin thickness of ^{208}Pb. Constrains on the neutron-skin thickness from PREX [3] and on the dipole polarizability from RCNP [27] have been incorporated into the plot.

To this end, we have computed the distribution of $E1$ strength using both relativistic and non-relativistic DFT approaches with different EDFs. In all cases, these self-consistent models have been calibrated to selected global properties of finite nuclei and some parameters of nuclear matter. Once calibrated, these models are used without any further adjustment to compute the $E1$ strength $R_{\rm E1}$ using a consistent random-phase approximation. The electric dipole polarizability is then obtained from the inverse energy-weighted sum [8, 26, 30]:

$$\alpha_{\rm D} = \frac{8\pi}{9}e^2 \int_0^\infty \omega^{-1} R_{\rm E1}(\omega)\, d\omega\ . \qquad (2)$$

The relation between $\alpha_{\rm D}$ and $r_{\rm skin}$ for ^{208}Pb is displayed in Fig. 1 using the predictions from the 48 EDFs chosen in this work. In particular, the up-triangles mark predictions from a broad choice of Skyrme EDFs that have been widely used in the literature: SGII, SIII, SkI3, SkI4, SkM*, SkO, SkP, SkX, SLy4, SLy6, (see Refs. [31, 32] for the original references), Sk255 [33], BSk17 [34], LNS [35], and UNEDF0 and UNEDF1 [36]. In addition, we consider a collection of relativistic and Skyrme EDFs that have been systematically varied around an optimal model without a significant deterioration in the quality of the fit. (This is particularly true for the case of the isovector interaction which at present remains poorly constrained.) Those results are marked in Fig. 1 as NL3/FSU [26, 37] (circles), DD-ME [38] (squares), and Skyrme-SV [29] (down-triangles). Note that the "stars" in the figure are meant to represent the predictions from the optimal models within the chain of systematic variations of the symmetry energy. At first glance a clear (positive) correlation between the dipole polarizability and the neutron skin is discerned.

Yet on closer examination, one observes a significant scatter in the results, especially for the standard Skyrme models. In particular, by including the predictions from all the 48 EDFs considered here, the correlation $C_{AB}^{\rm models}$=0.77 is obtained. However, as seen in Table I, within each set of the systematically varied models an almost perfect correlation is found. Note that by imposing the recent experimental constraints on $r_{\rm skin}$ and $\alpha_{\rm D}$, several of the models – especially those with either a very soft or very stiff symmetry energy – may already be ruled out. Thus, if we average our theoretical results over the set of 25 EDFs ("Set-25") whose predictions fall within the RCNP value of $\alpha_{\rm D}$, we obtain $r_{\rm skin}$=(0.168±0.022) fm, a value that is fairly close to the one obtained in Ref. [27]. It is to be noted that 23 of those 25 EDFs are consistent with the PREX constraint of $r_{\rm skin}$ greater than 0.15 fm. However, the average theoretical value is significantly below the current PREX mean of 0.33 fm [3]. If confirmed by the anticipated higher-precision (1%) PREX run, this large difference could either indicate the need for significant revisions of current nuclear structure models or of the models employed by PREX to deduce $r_{\rm skin}$ from the neutron form factor, or both. Provided that the new PREX and theoretical average values of $r_{\rm skin}$ are closer, in order to discriminate between theoretical models of Fig. 1 and further constraint theory, an accuracy of at least 0.03 fm on the experimental value of the neutron radius will be required. Based on the central PREX value of r_n=5.78 fm [3], this translates to a 0.5% measurement.

Using either lighter nuclei measured at larger momentum transfers or nuclei with a larger neutron excess will increase the parity-violating asymmetry. Therefore, it is pertinent to ask whether parity-violating experiments in other nuclei may be warranted [39]. To this end, we have computed data-to-data relations between the neutron-skin thickness of ^{208}Pb and the neutron-skin thickness of two doubly magic neutron-rich nuclei: stable ^{48}Ca and unstable ^{132}Sn. While parity-violating experiments on radioactive nuclei are unlikely to happen in the foreseeable future, such experiments on stable targets may serve to calibrate experiments with hadronic probes that could eventually be used to extract neutron radii of short-lived systems such as ^{132}Sn.

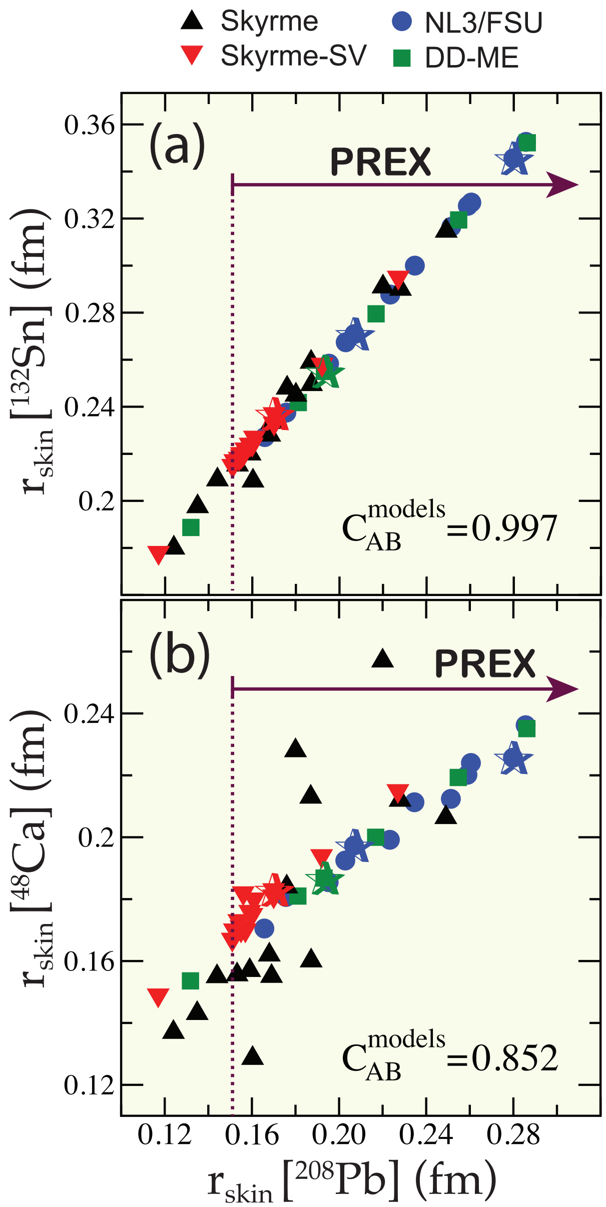

FIG. 2: (Color online) Predictions from the 48 nuclear EDFs used in the text for the neutron-skin thickness of ^{208}Pb and ^{132}Sn (a) and ^{48}Ca (b). Constrains on the neutron-skin thickness from PREX [3] have been incorporated into the plot.

Figure 2a displays model predictions for the neutron-skin thickness of ^{132}Sn as a function of the corresponding $r_{\rm skin}$ in ^{208}Pb. The displayed correlation is both strong and fairly model independent. Indeed, $C_{AB}^{\rm models}$=0.997 for the set of 48 EDFs used in this work, and it is even closer to unity for the systematically varied forces listed in Ta-

	$\alpha_D[^{208}\text{Pb}]$			$r_{\text{skin}}[^{132}\text{Sn}]$			$r_{\text{skin}}[^{48}\text{Ca}]$		
Model	C_{AB}^{model}	Slope(fm^2)	Intercept(fm^3)	C_{AB}^{model}	Slope	Intercept(fm)	C_{AB}^{model}	Slope	Intercept(fm)
Skyrme	0.996	29.08	15.53	0.999	1.06	0.06	0.977	0.60	0.08
DD-ME	0.994	31.99	14.52	1.000	1.06	0.05	1.000	0.53	0.08
NL3/FSU	0.994	29.89	13.97	1.000	1.04	0.05	0.987	0.50	0.09

TABLE I: Least-square correlation coefficient, slope, and intercept between various observables and the neutron-skin thickness of ^{208}Pb for the systematically varied models: NL3/FSU, DD-ME, and Skyrme-SV. Slope and intercept are obtained by fitting a straight line through the data.

ble I. This suggests that new experimental information on r_{skin} in ^{132}Sn is not likely to provide additional constraints on the theoretical models used here, provided that an accurate measurement of the neutron-skin thickness of ^{208}Pb is available. Averaging our results, a theoretical estimate for r_{skin} in ^{132}Sn of (0.232 ± 0.022) fm is obtained with Set-25. In addition, we predict a value of $(10.081\pm0.150)\,\text{fm}^3$ for α_D.

The situation for the case of the neutron-skin thickness in ^{48}Ca shown in Fig. 2b is different. Whereas the correlation coefficient among the three systematically varied models remains close to unity (see Table I) there is a significant spread in the predictions of all 48 models that is driven primarily by the traditional Skyrme forces. This suggests that an accurate measurement of r_{skin} in ^{208}Pb is not sufficient to significantly constrain r_{skin} in ^{48}Ca. Conversely, by measuring the neutron-skin thickness of both ^{48}Ca and ^{208}Pb, and incorporating the recent measurement of α_D in ^{208}Pb, one should be able to significantly constrain the isovector sector of the nuclear EDF. The theoretical model-averaged estimate for r_{skin} in ^{48}Ca is (0.176 ± 0.018) fm for Set-25. Moreover, a prediction of $(2.306\pm0.089)\,\text{fm}^3$ for α_D in ^{48}Ca is obtained.

In summary, we have examined the correlation between the electric dipole polarizability and neutron-skin thickness of ^{208}Pb using a large ensemble of 48 reasonable nuclear energy density functionals. Physical arguments based on a macroscopic analysis suggest that these two isovector observables should be correlated, although this correlation may display some systematic model dependence. In fact, we have found that as accurately calibrated models are systematically varied around their optimal value, strong correlations between r_{skin} and α_D in ^{208}Pb do emerge. As these models are combined, however, the correlation weakens. To study the associated systematic errors, we have performed calculations of α_D and r_{skin} using the subset of models that are consistent with the experimental value of α_D in ^{208}Pb [27]. Using this subset we predict the following "model-averaged" values of r_{skin}: (0.168 ± 0.022) fm in ^{208}Pb, (0.232 ± 0.022) fm in ^{132}Sn, and (0.176 ± 0.018) fm in ^{48}Ca—as well as an electric dipole polarizability of: $(10.081\pm0.150)\,\text{fm}^3$ in ^{132}Sn and $(2.306\pm0.089)\,\text{fm}^3$ in ^{48}Ca. We note that these predictions are consistent with the experimental values determined from both antiprotonic atoms and proton elastic scattering for ^{132}Sn [13] and ^{208}Pb [13–15, 17]. Given these results, we conclude that the follow-up PREX measurements of r_{skin} in ^{208}Pb will be of great value in further constraining the poorly known isovector sector of the nuclear EDF. Moreover, the analysis carried out in this work has enabled us to identify additional critical observables that could help discriminate among theoretical models. Specifically, we endorse a measurement of the neutron radius in ^{48}Ca, as it provides information that is complimentary to the ^{208}Pb measurement. Finally, in the near future we aim to present a complementary study of r_{skin}, α_D, and the low-energy $E1$ strength by means of a detailed statistical covariance analysis within the realm of accurately calibrated models [8].

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[1] C.J. Horowitz *et al.*, Phys. Rev. C **63**, 025501 (2001).
[2] R. Michaels *et al.*, Lead Radius Experiment PREX proposal 2005; http://hallaweb.jlab.org/parity/prex/.
[3] S. Abrahamyan *et al.*, Phys. Rev. Lett. **108**, 112502 (2012).
[4] T. Donnelly, J. Dubach, and I. Sick, Nucl. Phys. A **503**,589 (1989).
[5] B.A. Brown, Phys. Rev. Lett. **85**, 5296 (2000); S. Typel and B.A.Brown, Phys. Rev. C **64**, 027302 (2001).
[6] R.J. Furnstahl, Nucl. Phys. A **706**, 85 (2002).
[7] M. Centelles *et al.*, Phys. Rev. Lett. **102**, 122502 (2009); Phys. Rev. C **82**, 054314 (2010).
[8] P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C **81**, 051303(R) (2010).
[9] S.J. Pollock, E.N. Fortson, and L. Wilets, Phys. Rev. C **46**, 2587 (1992); T. Sil *et al.*, Phys. Rev. C **71**, 045502 (2005).
[10] C. Horowitz and J. Piekarewicz, Phys. Rev. Lett. **86**, 5647 (2001); Phys. Rev. C **66**, 055803 (2002).
[11] A.W. Steiner *et al.*, Phys. Rept. **411**, 325 (2005); B.-A. Li and A. W. Steiner, Phys. Lett. B **642**, 436 (2006); F.J. Fattoyev and J. Piekarewicz, Phys. Rev. C **82**, 025810 (2010).

[12] X. Roca-Maza *et al.*, Phys. Rev. Lett. **106**, 252501 (2011).
[13] A. Trzcińska *et al.*, Phys. Rev. Lett. **87**, 082501 (2001).
[14] B. Kłos, *et al.*, Phys. Rev. C **76**, 014311 (2007).
[15] B. A. Brown, *et al.*, Phys. Rev. C **76**, 034305 (2007).
[16] B.C. Clark, L.J. Kerr, and S. Hama, Phys. Rev. C **67**, 054605 (2003).
[17] J. Zenihiro *et al.*, Phys. Rev. C **82**, 044611 (2010).
[18] L. Ray and G. W. Hoffmann, Phys. Rev. C **31**, 538 (1985).
[19] L. Ray, G. W. Hoffmann, and W. R. Coker, Phys. Rept. **212**, 223 (1992).
[20] J. Piekarewicz and S.P. Weppner, Nucl. Phys. A **778**, 10 (2006).
[21] M.N. Harakeh and A. van der Woude, *Giant Resonances – Fundamental High-frequency Modes of Nuclear Excitation* (Clarendon, Oxford, 2001).
[22] P.-G. Reinhard, Nucl. Phys. A **649**, 305c (1999).
[23] L. Trippa, G. Colò, and E. Vigezzi, Phys. Rev. C **77**, 061304(R) (2008).
[24] E. Lipparini and S. Stringari, Phys. Lett. B **112**, 421 (1982); S. Stringari and E. Lipparini, Phys. Lett. B **117**, 141 (1982).
[25] W. Satuła, R. Wyss, and M. Rafalski, Phys. Rev. C **74**, 011301(R) (2006).
[26] J. Piekarewicz, Phys. Rev. C **83**, 034319 (2011).
[27] A. Tamii *et al.*, Phys. Rev. Lett. **107**, 062502 (2011).
[28] S. Brandt, *Statistical and computational methods in data analysis*, Third English Edition (Springer, New York 1997).
[29] P. Klüpfel, P.-G. Reinhard, T. Burvenich, and J.A. Maruhn, Phys. Rev. C **79**, 034310 (2009).
[30] E. Lipparini and S. Stringari, Phys. Rep. **175**, 103 (1989).
[31] M. Bender, P.-H. Heenen, and P.-G. Reinhard, Rev. Mod. Phys. **75**, 121 (2003).
[32] P.-G. Reinhard *et al.*, Phys. Rev. C **73**, 014309 (2006).
[33] B.K. Agrawal, S. Shlomo, and V. Kim Au, Phys. Rev. C **68**, 031304(R) (2003).
[34] S. Goriely, N. Chamel, and J. M. Pearson, Phys. Rev. Lett. **102**, 152503 (2009).
[35] L.G. Cao *et al.*, Phys. Rev. C **73**, 014313 (2006).
[36] M. Kortelainen *et al.* Phys. Rev. C **82**, 024313 (2010).
[37] B.K. Agrawal, Phys. Rev. C **81**, 034323 (2010).
[38] D. Vretenar, T. Nikšić, and P. Ring, Phys. Rev. C **68**, 024310 (2003); G.A. Lalazissis *et al.*, Phys. Rev. C **71**, 024312 (2005).
[39] S. Ban, C. Horowitz, and R. Michaels, J. Phys. G **39**, 015104 (2012).