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Phys. Rev. C **85**, 034907 — Published 26 March 2012

DOI: [10.1103/PhysRevC.85.034907](https://doi.org/10.1103/PhysRevC.85.034907)

KNO scaling of fluctuations in pp and pA , and eccentricities in heavy-ion collisions

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Multiplicity fluctuations at midrapidity in pp collisions at high energies are described by a negative binomial distribution and exhibit approximate Koba-Nielsen-Olesen (KNO) scaling. We find that these KNO fluctuations are important also for reproducing the multiplicity distribution in $d + Au$ collisions observed at RHIC, adding to the Glauber fluctuations of the number of binary collisions or participants. We predict that the multiplicity distribution in $p + Pb$ collisions at the LHC also deviates little from the KNO scaling function. Finally, we analyze various moments of the eccentricity of the collision zone in $A + A$ collisions at RHIC and LHC and find that particle production fluctuations increase fluctuation dominated moments such as the triangularity ϵ_3 substantially.

Charged particle multiplicity distributions in the central region of inelastic (non-single diffractive) $\bar{p} + p$ collisions at high energies were shown by the UA1 and UA5 collaborations to follow a negative binomial distribution (NBD) [1, 2] exhibiting approximate “KNO scaling” [3] over at least a limited range of multiplicities excluding the tails. Within the framework of high-energy QCD they may be thought to arise from fluctuations of the density of large- x valence charges [4] and of stochastic emissions in the rapidity evolution ladders [5, 6] leading from the rapidity of the sources to the central region.

Collisions of hadrons or heavy ions at high energies release a large number of gluons from their wave functions. In fact, the wave function of a hadron boosted to (nearly) the light cone is so densely packed with gluons that they may “overlap”, leading to non-linear interactions [7]. Therefore, at high energies the colliding hadrons can be treated as a high occupancy gluon field. This dense system is nowadays referred to as Color Glass Condensate (CGC) [8].

Here, we use the “ k_{\perp} -factorization” approach [7] to compute particle production in high-energy collisions:

$$\left\langle \frac{dN^{A+B \rightarrow g}}{dy d^2\mathbf{r}_{\perp}} \right\rangle = K \frac{N_c}{N_c^2 - 1} \int \frac{d^2\mathbf{p}_{\perp}}{p_{\perp}^2} \int^{p_{\perp}} d^2\mathbf{k}_{\perp} \alpha_s(Q) \Phi\left(\frac{|\mathbf{p}_{\perp} + \mathbf{k}_{\perp}|}{2}, x_1\right) \Phi\left(\frac{|\mathbf{p}_{\perp} - \mathbf{k}_{\perp}|}{2}, x_2\right), \quad (1)$$

where $N_c = 3$ is the number of colors in QCD and $K \simeq 1.5 - 2$ is a multiplicative factor to account for corrections to this LO formula. Further, we assume that the hadron multiplicity is proportional to the multiplicity of gluons. These factors may depend somewhat on the initial condition for small- x evolution of $\Phi(k_{\perp})$ but were found to be approximately independent of collision energy or centrality (for heavy-ion collisions) [9]. Also, in eq. (1) $x_{1,2} = (p_{\perp}/\sqrt{s}) \exp \pm y$ and the strong coupling is evaluated at the scale $Q = \max(|\mathbf{p}_{\perp} + \mathbf{k}_{\perp}|, |\mathbf{p}_{\perp} - \mathbf{k}_{\perp}|)/2$.

We require the evolution of the so-called unintegrated gluon distribution $\Phi(k_{\perp}, x)$ (per unit transverse area) with the light-cone momentum fraction x , starting from an initial condition at $x_0 \simeq 10^{-2}$. This is obtained by solving the non-linear Balitsky-Kovchegov (BK) equation [11] with the running-coupling kernel according to Balitsky’s prescription [12]. Specifically, we use the unintegrated gluon distribution “set MV” from ref. [9].

For the case of heavy-ion projectiles and/or targets, we allow for fluctuations of the locations of the sources (i.e., of the valence charges at x_0) for the small- x fields in the transverse plane before the collision [9, 10]. This leads to fluctuations of the “geometry” of the collision zone from configuration to configuration, and to fluctuations of the number of participants N_{part} and the number of collisions N_{coll} which are determined within the well-known Glauber approach. Note that eq. (1) refers to a *single* such configuration. We computed these “geometry” fluctuations assuming that the *hard valence charges* are smeared over a finite and energy independent area $\sigma_0 \sim \sigma_{NN}(200 \text{ GeV}) = 4.2 \text{ fm}^2$. This reduces higher-order eccentricities as compared to point-like nucleons [13] which are used in some Monte-Carlo Glauber simulations. Our numerical simulations do not account for correlations (in the transverse plane) among the valence charges which could further suppress geometry fluctuations [14].

The unintegrated gluon densities $\Phi(k_{\perp}, x)$ from eq. (1) have already been averaged over the local fluctuations of the valence charges *in color space*, and over the evolution ladders. It is in this sense that we interpret eq. (1) as a mean (local) multiplicity. In each cell $\Delta^2\mathbf{r}_{\perp}$ of the transverse plane the actual multiplicity is a NBD random variable,

$$P(n) dn = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}} dn. \quad (2)$$

Here, $\bar{n} \equiv \langle dN/d\eta d^2\mathbf{r}_{\perp} \rangle \Delta^2\mathbf{r}_{\perp} \Delta\eta$ is the mean multiplicity from eq. (1) in a given cell and k is the fluctuation parameter: smaller k correspond to larger fluctuations about the mean and KNO scaling is obtained when $k \ll \bar{n}$

(see below). We finally average over geometric configurations of sources in the \mathbf{r}_\perp plane described above, and over the impact parameter of the collision.

There have been numerous theoretical discussions of multiplicity fluctuations in high-energy collisions. Ref. [4], in particular, argued that NBD multiplicity fluctuations arise in a semi-classical calculation of gluon production from dense valence charge sources. They obtain that the fluctuation parameter k is proportional to the density per unit transverse area of valence charge squared, i.e. to the saturation momentum Q_s^2 at x_0 .

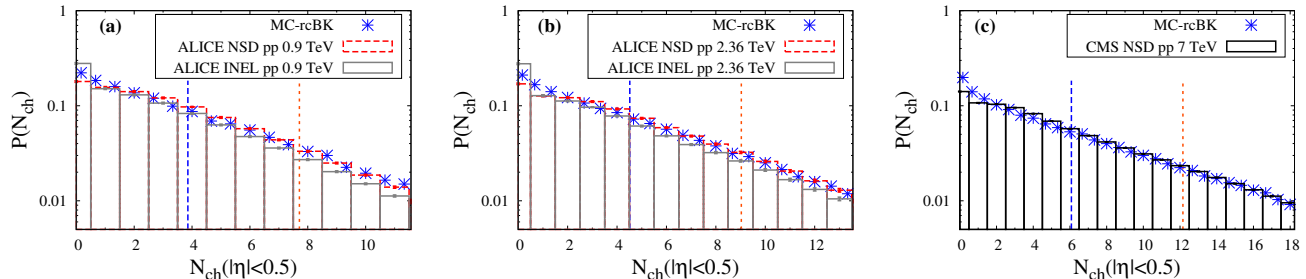


FIG. 1: (Color online) **Left**: multiplicity distribution of charged particles at $|\eta| < 0.5$ in pp collisions at $\sqrt{s} = 900$ GeV. Stars show the result of our calculation (see text) while solid and dashed histograms correspond to data taken by the ALICE collaboration with the “NSD” and “INEL” triggers, respectively [15]. The dashed vertical lines indicate the average and two times the average multiplicity, respectively. **Center**: Same at $\sqrt{s} = 2360$ GeV. **Right**: Same at $\sqrt{s} = 7000$ GeV compared to CMS NSD data [16].

We first analyze the multiplicity distributions in proton-proton collisions at LHC energies (fig. 1). We concentrate on the bulk of the distributions, $N_{\text{ch}} \lesssim 3\langle N_{\text{ch}} \rangle$ where $\langle N_{\text{ch}} \rangle$ denotes the average charged particle multiplicity at a given energy. Over this range the data can be described reasonably well by a NBD with constant

$$k_{pp} = \frac{1}{\pi} \Delta^2 \mathbf{r}_\perp \Delta \eta \Lambda_{\text{QCD}}^2. \quad (3)$$

Here $\Delta \eta = 1$ and $\Delta^2 \mathbf{r}_\perp$ is the area of a cell in transverse coordinate space over which we integrate eq. (1). Also, we choose $\Lambda_{\text{QCD}} = 0.24$ GeV. Numerically, $k/\bar{n} \simeq 0.16$ for $p + p$ collisions at 2.36 TeV. We have checked that a weak energy dependence of k is allowed as long as it does not change the distribution $P(N_{\text{ch}})$ appreciably over the range that we are interested in. The tails of $P(N_{\text{ch}})$ could be more sensitive to the detailed dependence of k on energy¹ but we do not explore the region $N_{\text{ch}} > 3\langle N_{\text{ch}} \rangle$ here; see, for example, ref. [17].

The most important consequence from (3) is that since $k = \text{const}$ and smaller than the average multiplicity \bar{n} , it follows that our multiplicity distributions satisfy Koba-Nielsen-Olesen (KNO) scaling [3]. That is, the probability distribution $P(N_{\text{ch}})$ is independent of energy if expressed in terms of $z \equiv N_{\text{ch}}/\langle N_{\text{ch}} \rangle$; for $\bar{n} \gg k$ and in the region $z > k/\bar{n}$ the NBD (2) can be written in the form of a Gamma distribution

$$\bar{n} P(n) dz \sim z^{k-1} e^{-kz} dz. \quad (4)$$

We show the KNO scaling function in pp collisions explicitly in fig. 3 below.

In fig. 2 we compare the calculated charged particle multiplicity distribution in $d + Au$ collisions at $\sqrt{s} = 200$ GeV to uncorrected data from STAR [18]. As described above, here we include also fluctuations of the number of participants N_{part} and of the number of binary collisions N_{coll} which arise for different configurations of nucleons in the target nucleus. Within our formalism, N_{coll} fluctuations alone are insufficient to reproduce the experimental multiplicity distribution. In this case we obtain a peak in $P(N_{\text{ch}})$ before the cutoff of the distribution which can be traced back to the fact that N_{ch} does not increase linearly with the density of sources when the latter is high. This “saturation” of particle production is also responsible for the higher elliptic eccentricity of the collision zone than obtained from simple linear estimates [19].

Additional intrinsic fluctuations with

$$k_{d+Au} = k_{pp} \cdot \min(T_A(\mathbf{r}_\perp), T_B(\mathbf{r}_\perp)) \sigma_0 \quad (5)$$

¹ The same applies to rapidity intervals bigger than $|\eta| < 0.5$; UA5 found that k then actually *decreases* with energy [2].

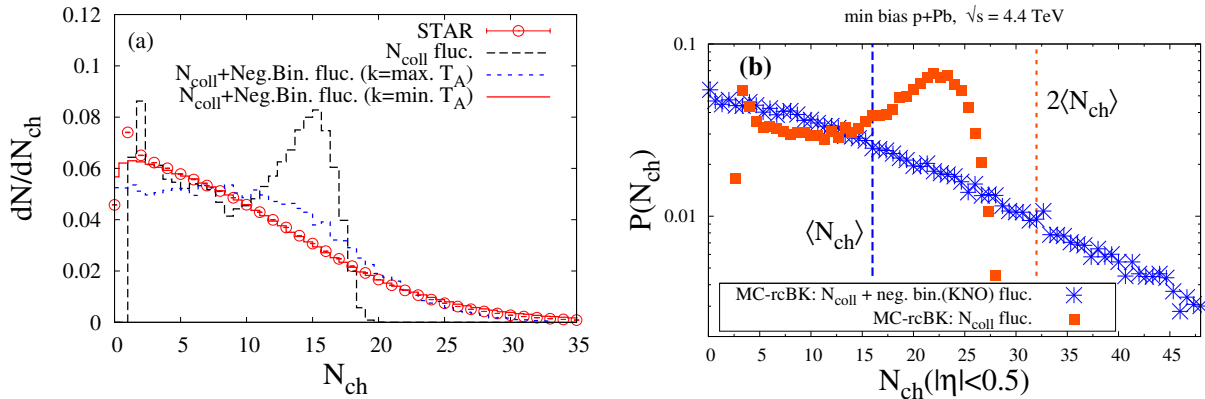


FIG. 2: (Color online) **Left:** multiplicity distribution of charged particles at $|\eta| < 0.5$ in min. bias $d + Au$ collisions at $\sqrt{s} = 200$ GeV. Various models (see text) are compared to uncorrected data from the STAR collaboration (circles) [18]. **Right:** multiplicity distribution predicted for min. bias $p + Pb$ collisions at $\sqrt{s} = 4400$ GeV.

lead to a good fit to the data; such scaling of k with the number of sources is expected due to the way that negative binomial distributions add². On the other hand, $k_{d+Au} \sim \max(T_A(\mathbf{r}_\perp), T_B(\mathbf{r}_\perp))$ produces a multiplicity distribution inbetween the above cases, exhibiting too little fluctuations. Once again, it is reasonable that the magnitude of fluctuations is determined mostly by the dilute source (as also assumed in ref [17]). Our prediction for $p + Pb$ collisions at LHC is shown in fig. 2 on the right; this corresponds to $\langle N_{ch} \rangle \simeq 16$ and k from eq. (3). (A prediction for the multiplicity distribution in $p + Pb$ collisions at the LHC from the “KLN model” was shown previously in ref. [20]).

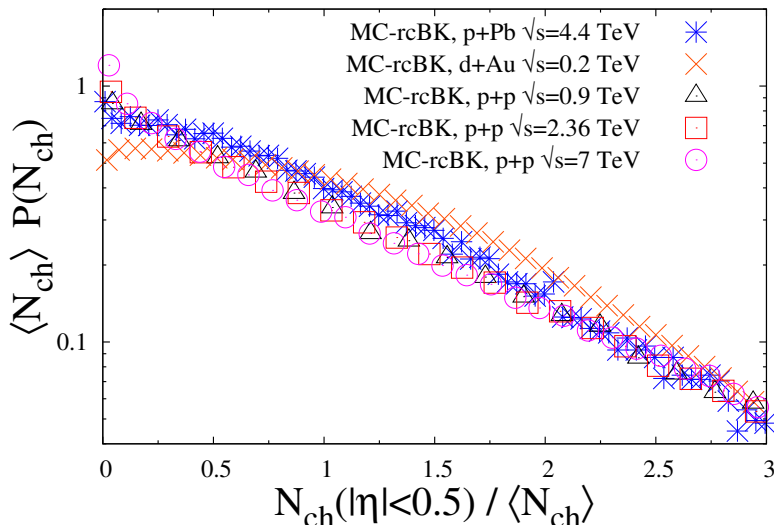


FIG. 3: (Color online) KNO scaling plot of the multiplicity distributions of charged particles at $|\eta| < 0.5$.

Due to the presence of N_{coll} fluctuations our multiplicity distribution for $p + Pb$ does not exhibit exact KNO scaling, as seen in fig. 3. Nevertheless, for $|\eta| < 0.5$ and $N_{ch} \lesssim 3\langle N_{ch} \rangle$ we predict relatively small deviations from the KNO scaling function determined from $p + p$ collisions. This is an important check for the presence of strong intrinsic particle production fluctuations (at fixed N_{part} and N_{coll}) for a heavy-ion target.

We now proceed to discuss the relevance of particle production fluctuations for various harmonic moments of the “eccentricity” of gluons produced in the initial state of heavy-ion collisions. We define moments of the initial density

² If x and y are two random variables with a negative binomial distribution with mean μ and fluctuation parameter k then $z = x + y$ also follows a negative binomial distribution with mean 2μ and $k_z = 2k$. Hence k is an extensive quantity proportional to volume, just as \bar{n} .

distribution (preceding the hydrodynamic expansion in $A + A$ collisions) in terms of the eccentricities [21, 22]

$$\epsilon_n = \frac{\sqrt{\langle r^2 \cos n\phi \rangle^2 + \langle r^2 \sin n\phi \rangle^2}}{\langle r^2 \rangle}. \quad (6)$$

Other definitions are sometimes also used in the literature, see for example [23–25]. $\langle \cdot \rangle$ denotes an average over the distribution of produced gluons in the transverse plane, $dN/d\eta d^2\mathbf{r}_\perp$; and $\mathbf{r}_\perp = r(\cos \phi, \sin \phi)$.

The eccentricities ϵ_n are of interest because through hydrodynamic response they generate the flow harmonics and angular correlations in the final state of heavy-ion collisions [21–33]. (Fluctuations in small- x evolution may also lead to detectable azimuthal momentum anisotropies in high-multiplicity pp collisions at the LHC [6] which are not due to “flow”). Flow harmonics in heavy-ion collisions have been published by the PHENIX [34] and ALICE [35] collaborations.

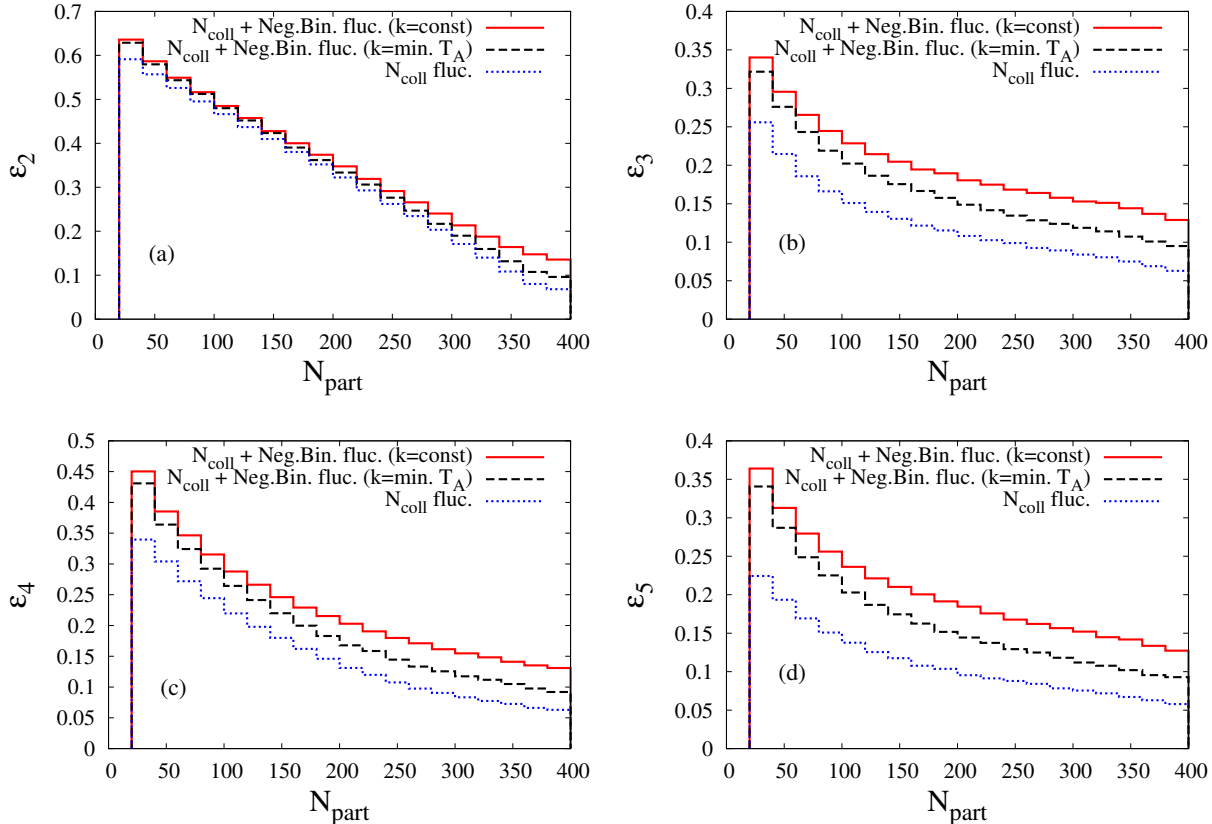


FIG. 4: (Color online) Centrality dependence of various moments ϵ_n of the eccentricity in $Au + Au$ collisions at $\sqrt{s} = 200$ GeV. Dotted lines correspond to local fluctuations of N_{coll} (“geometry fluctuations”) only; dashed and full lines add particle production fluctuations according to a negative binomial distribution with $k = k_{pp} = \text{const}$ or $k \sim \min(T_A, T_B)$, respectively.

In fig. 4 we compare the centrality dependence of $\epsilon_2 - \epsilon_5$ for three different models. In all cases, the lowest curve corresponds to the model with “geometry fluctuations” only, as usually considered in the literature. If particle production fluctuations according to a negative binomial distribution are added then in general ϵ_n increases. The effect on ϵ_2 at $N_{\text{part}} \lesssim 250$ is small since the ellipticity of the overlap zone is of course dominated by the “almond shaped” geometry of heavy-ion collisions at finite impact parameter. On the other hand, we observe a large increase of ϵ_n for all $n \geq 3$ over the entire range of impact parameters, as well as of ϵ_2 for $N_{\text{part}} \rightarrow 2A$, since these observables are fluctuation dominated. Most importantly, the ratio ϵ_3/ϵ_2 in mid-central collisions increases significantly. The largest increase is obtained if the fluctuation parameter $k = k_{pp}$ does not increase with the density of sources. Even for the more realistic case where $k \sim \min(T_A, T_B)$, higher-order eccentricities can increase by as much as 50%. We mention also that simulations using the DIPSY Monte-Carlo which performs the small- x dipole evolution stochastically have predicted a large ϵ_3 [36], although the relation to KNO scaling in pp and pA collisions at the LHC had not been pointed out.

To summarize our main results: we found that in order to reproduce the measured multiplicity distribution in $d+Au$ collisions at RHIC within the CGC approach it is important to take into account particle production fluctuations (according to a negative binomial distribution). We predict that these dominate over Glauber fluctuations also for $p+Pb$ collisions at the LHC, resulting in a multiplicity distribution which is close to the KNO scaling function measured in $p+p$ collisions. The effect of particle production fluctuations can be large also for some observables in heavy-ion collisions, such as for higher-order eccentricities. It will be interesting to see how this reflects in higher-order flow coefficients predicted by viscous hydrodynamics or in the centrality dependence of the jet quenching parameter $R_{AA}(p_{\perp})$ [37].

Acknowledgements

We gratefully acknowledge the kind hospitality of the Institute of Physics at the University of Tokyo; our stay was supported by grant KAKENHI(22340064). A.D. also acknowledges support by the DOE Office of Nuclear Physics through Grant No. DE-FG02-09ER41620; and from The City University of New York through PSC-CUNY Research grant 64132-00 42. The work of Y.N. was partly supported by Grant-in-Aid for Scientific Research No. 20540276.

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Appendix A: Eccentricities for heavy-ion collisions at LHC energies

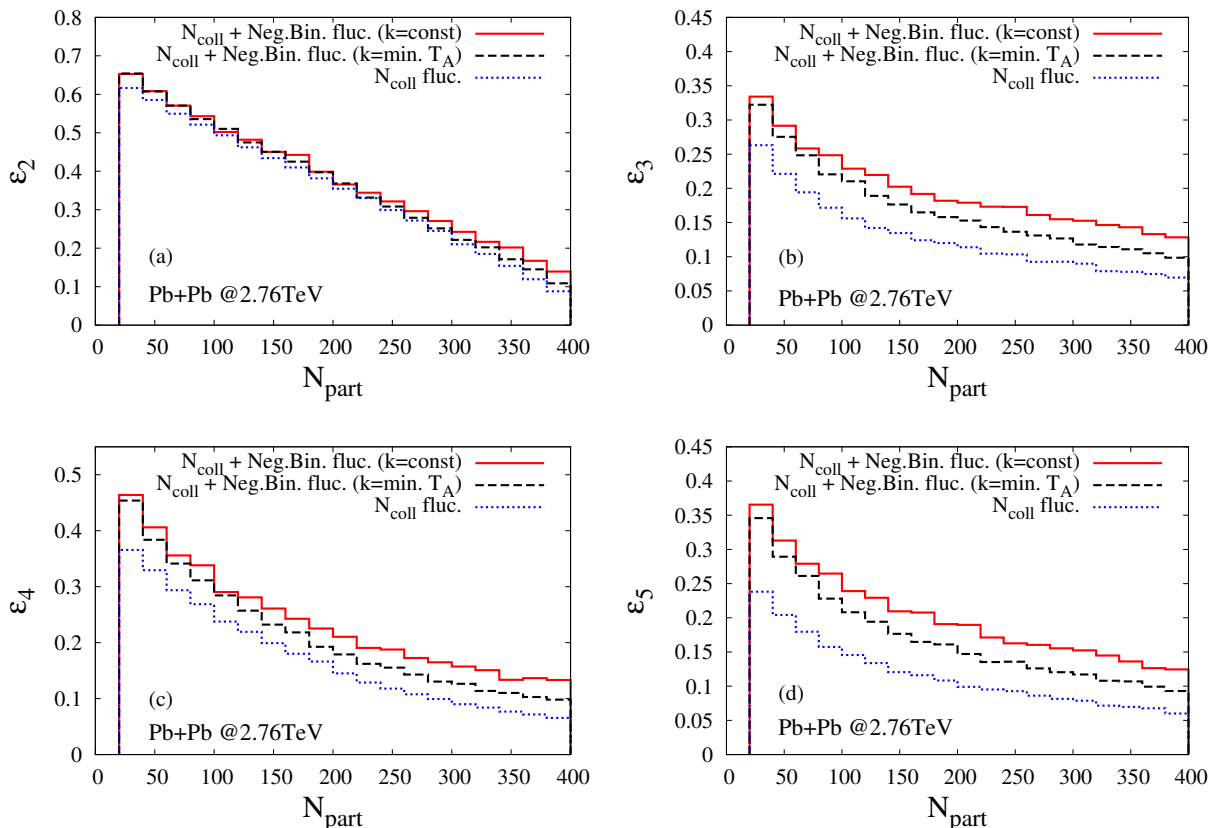


FIG. 5: (Color online) Centrality dependence of various moments ϵ_n of the eccentricity in $Pb+Pb$ collisions at $\sqrt{s} = 2.76$ TeV. Dotted lines correspond to local fluctuations of N_{coll} (“geometry fluctuations”) only; dashed and full lines add particle production fluctuations according to a negative binomial distribution with $k = k_{pp} = \text{const}$ or $k \sim \min(T_A, T_B)$, respectively.