γ-charge correlation in heavy ion collisions: Various approaches

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Study of γ-charge correlation in heavy ion collisions, various approaches.

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Event-by-event $\gamma - \mathrm{ch}$ correlation is studied for systems going through QCD chiral phase transition. In this paper, various methods for measuring $\gamma - \mathrm{ch}$ correlation in heavy ion collisions have been discussed. Dynamical fluctuation due to the formation of domains of DCC that can affect $\gamma - \mathrm{ch}$ correlation has been addressed. We have studied known detector and statistical effects involved in these measurements and suggest suitable robust observables $\Delta\nu_{dyn}$ and $r_{m,1}$ sensitive to small $\gamma - \mathrm{ch}$ correlation signal. These observables are constructed based on moments of multiplicity distributions of photons and charged particles. Estimations of measurable signals from various available models such as ideal Boltzmann gas of pions, monte-carlo models based on transport and mini-jets have been discussed. Collision centrality dependence of the observables has been estimated from Central Limit Theorem and found to be consistent with the model predictions. Observables are found to be highly sensitive to the fraction of DCC events and have nonlinear dependence on fraction of pions carrying DCC signals. The variation of $r_{m,1}$ with order m of its moments is sensitive to the nature and strength of $\gamma - \mathrm{ch}$ correlation.

I. INTRODUCTION

Based on decades of experimental searches and theoretical studies it is widely believed that high energy heavy ion collisions produce realistic scenario for studying the phase transition from hadronic matter to quark gluon plasma(QGP). It is predicted to be associated with the de-confinement transition and the restoration of QCD chiral symmetry. Fluctuation of conserved quantities has been proposed [1] to be an important experimental signature for such a phase transition. In heavy ion collisions produced particles are mostly pions which would globally show isospin conservation. Experimentally when a limited phase space is probed, the event-by-event fluctuation of isospin could be an interesting observable. The QCD chiral phase transition is associated with melting of 4-vector condensates. An interesting phenomena like the formation of metastable domains of "Disoriented Chiral Condensate" (DCC) has been predicted to occur due to the orientation of this condensate relative to the direction of its scalar component. Such a phenomena is possible to occur in a scenario of rapid cooling like quenching [2–5] of a system while going from chiral symmetric to a broken phase. Formation of DCC domains causes anomalous production of charged or neutral pions depending on the orientation of vacuum towards its pseudo scalar component. Such a phenomenon might survive final state interactions and appear in the form of anti-correlation between charged and neutral pions[4]. As in heavy ion collisions the detected charged particles and photons are mostly from the charged pions and the decay of neutral pions respectively, signals of DCC would appear in the form of $\gamma - \mathrm{ch}$ anti-correlation.

Signals of DCC have previously been searched in p+p collisions[8], cosmic ray events[9] and in heavy ion collisions[10, 14]. There are several theoretical predictions for a hot medium described by the linear sigma model [6, 7, 15]. Ref [15] predicted that for central Pb-Pb collisions at SPS energies, the likelihood of the DCC events is less than 10^{-3}. Experimental searches at SPS WA98 experiment [10–13] at $\sqrt{s}$=17.3 GeV estimated an upper limit of DCC event fraction to be 3×10^{-3}. It has been argued [16] that in case of rapid cooling like quenching scenario, higher collision energies corresponding to lower chemical potential (e.g. $\mu_{RHIC} < \mu_{SPS}$) can provide faster cooling rate ($|dT/dt|$). This suggests that collisions at RHIC and LHC provide a more favorable conditions for DCC domain formation than at the SPS. In view of varied opinions [6, 7, 15–17] about the observability of DCC in heavy ion collisions, experiments at RHIC and LHC provide an unique opportunity to address the issue and test theoretical predictions.

From an experimental point of view, such a study is associated with the simultaneous measurement of photons and charged particles in common phase space with a very high sensitivity at low momentum. The decay of domains of DCC are final stage phenomena of the evolution of heavy ion collision and the pions carrying signals are therefore expected to be of low momentum. A combination of pre-shower Photon Multiplicity detector(PMD)[18] and Forward Time Projection Chamber(FTPC)[19] at the STAR experiment at RHIC and Photon Multiplicity detector(PMD) and Forward Multiplicity Detector(FMD) at the ALICE experiment[20] at LHC have the required criteria for such measurements.

The main aim of this paper is to highlight the issues associated with the $\gamma - \mathrm{ch}$ correlation analysis by proposing methods that will be used in heavy ion collision experiments. It should be noted that, the measures we discuss here could be used as generalized quantities for $\gamma - \mathrm{ch}$ correlation analysis and not limited to the specific case of DCC formation. Experimental observables used for $\gamma - \mathrm{ch}$ correlation measurement suffer from various detector effects. It is therefore important to construct suitable quantities and study their dependencies on experimental parameters. We use the generating function approach to calculate different observables and include various detector effects like efficiencies, mis-identification etc. These observables by construction should be able to disentangle dynamical fluctuation from statistical contributions. In the context of DCC, we will not discuss the dynamical origin of such phenomenon. We assume the formation of DCC domains to be one of the probable sources of dynamical signal of isospin fluctuation of pion production. This would lead to a distribution of neutral pion fraction very distinct from that of generic production of pions under isospin symmetry [3, 5]. We discuss the sensitivity of the observables to the fraction of DCC events and the fraction of DCC candidates in an event. Relevant to the heavy ion collisions, we address the centrality dependence of the observables. We have estimated the $\gamma-\mathrm{ch}$ correlation from various models and implemented a DCC-model based on HIJING event generator.

In section II we have outlined the method of construction of the observables and their values for DCC events of varying fraction. Section III, IV describe detector effects like mis-identification and the role of resonances. The centrality dependence of the observables has been discussed in section V. In section VI we have estimated the sensitivity of the variables on fraction of pions carrying signals in a DCC event. We have studied various non-DCC models in section VII and implemented DCC in a Monte-Carlo event generator in section VIII. We summarize in section IX.

II. METHOD

Fluctuation of particle ratios has been addressed previously in case of conserved quantities like net strangeness in terms of kaon-to-pion ratio and for net baryons in terms of proton-to-pion ratios. Relevant to the context of isospin, we would like to study photon to charge particle multiplicity ratio. Observables used in such cases should be robust against detector inefficiency. Using a simple implementation of detector efficiency in terms of a binomial probability distribution function say of the form $P(n,N,\varepsilon) = {}^{N}\mathrm{C}_n\, \varepsilon^n (1-\varepsilon)^{N-n}$, one can show that the second moment of observed multiplicity n is not proportional to second moment of produced multiplicity N. The efficiency term ε does not factorize for quantities like variance, skewness and kurtosis. However the quantities like observed second and higher order factorial moments comes out to be proportional to the measured corresponding factorial moments like $\langle n(n-1)\rangle = \varepsilon^2\langle N(N-1)\rangle$. Thus ratios of various factorial moments with powers of mean multiplicity would simply cancel the explicit efficiency dependence. Observables based on factorial moments have been previously introduced in case of model prediction for event by event fluctuations in pion multiplicities as an observable of DCC [21]. Multiplicity correlation is further affected by other complicated detector effects like mis-identification and resonance decays. In case of heavy ion collisions observables are expected have centrality and system size dependence. Based on similar context and considering various other aspects of particle ratio-fluctuation, two observables were introduced earlier as measures of dynamical fluctuations. ν_{dyn} was introduced in Ref [22] and used by STAR Collaboration [23, 24] and $r_{m,1}$ was introduced by Minimax collaboration[25].

The observable ν_{dyn} in our context can be defined as

$$\nu_{dyn}^{\gamma-\mathrm{ch}} = \frac{\langle N_{\mathrm{ch}}(N_{\mathrm{ch}}-1)\rangle}{\langle N_{\mathrm{ch}}\rangle^2} + \frac{\langle N_\gamma(N_\gamma-1)\rangle}{\langle N_\gamma\rangle^2} - 2\frac{\langle N_{\mathrm{ch}}N_\gamma\rangle}{\langle N_\gamma\rangle\langle N_{\mathrm{ch}}\rangle}. \tag{1}$$

For purely statistical fluctuation (Poissonian case) ν_{dyn}=0.

The observable $r_{m,1}$ is defined as

$$r_{m,1}^{\gamma-\mathrm{ch}} = \frac{\langle N_{\mathrm{ch}}(N_{\mathrm{ch}}-1)..\,(N_{\mathrm{ch}}-m+1)\ N_\gamma\rangle\,\langle N_{\mathrm{ch}}\rangle}{\langle N_{\mathrm{ch}}(N_{\mathrm{ch}}-1)..(N_{\mathrm{ch}}-m)\rangle\,\langle N_\gamma\rangle}. \tag{2}$$

It is designed such that for Poisson case, $r_{m,1}$=0. Higher orders(m) of $r_{m,1}$ are expected to show larger sensitivity to signals.

In this section we would like to discuss the applicability, robustness and sensitivity of these two observables ν_{dyn} and $r_{m,1}$ for studying $\gamma-\mathrm{ch}$ correlation. Since we are interested in fluctuation of ratio of multiplicities, let us consider $f = N_{\pi^0}/(N_{\pi^0}+N_{\pi^\pm})$ to be the neutral pion fraction. The idea is to choose proper combination of moments such that the efficiency dependence is eliminated and observables are expressed in terms of the fluctuations of the fraction f. Various detector effects can be incorporated in different moments of multiplicity by using the generating function approach [25]. We define

$$G(z) = \sum_{N=0}^{\infty} z^N\, P(N) \tag{3}$$

as the generating function with $P(N)$ being the distribution of parent multiplicity. Here , $N = N_{\pi^0}+N_{\pi^+}+N_{\pi^-}$ denotes sum of all neutral and charged pions. Different moments are calculated by taking derivatives of $G(z)$ with respect to the variable z evaluated at $z=1$. Considering the fact that the neutral pions are distributed according to the probability $\mathcal{P}(f)$ the generating function has to be modified accordingly

$$G(z_{\mathrm{ch}}, z_0) = \int_0^1 df\, \mathcal{P}(f) \sum_N P(N)\, [f z_0\, +\, (1-f) z_{\mathrm{ch}}]^N\,. \tag{4}$$

Here $\mathcal{P}(f)$ is the distribution of neutral pion fraction. Isospin symmetry for a system pions corresponds to a generic case of pion productions for which $\mathcal{P}(f) = \delta(f-1/3)$. In case of DCC like events[3, 5] this distribution is modified to $\mathcal{P}(f) = 1/2\sqrt{f}$. For propagation of generating function to include the decay of neutral pions to observed photons we apply the "cluster decay theorem" [26]. We can express the overall generating function as

$$G_{obs}\left(z_{\rm ch},\, z_\gamma\right) = G\left(g_{\rm ch}\left(z_{\rm ch}\right),\, g_0\left(z_\gamma\right)\right) \tag{5}$$

where $g_0(z_\gamma) = z_\gamma^2$ and $g_{\rm ch}(z_{\rm ch}) = z_{\rm ch}$ considering the fact that every neutral cluster decays into two photons and the charge particles do not decay. To make the scenario more realistic and taking the advantage of same theorem, one can include detection efficiencies in the final form of generating function. We consider the observing and non-observing as different decay modes with probability equal to the detection efficiency. So for charged and neutral clusters we redefine

$$\begin{aligned} g_{\rm ch}(z_{\rm ch}) &= (1-\varepsilon_{\rm ch}) + \varepsilon_{\rm ch} z_{\rm ch} \\ g_0(z_\gamma) &= ((1-\varepsilon_\gamma) + \varepsilon_\gamma z_\gamma)^2 \end{aligned} \tag{6}$$

Here $\varepsilon_{\rm ch}$ is the efficiency of charge particle detection and ε_γ is the efficiency of detecting a photon coming from decay of a neutral pion. Various factorial moments of multiplicity are expressed in terms of derivatives of final generating function. We can define a generalized factorial moment as

$$\mathrm{f}_{m,n} = \left.\frac{\partial^{m,n} G_{\rm obs}(z_{\rm ch}, z_\gamma)}{\partial z_{\rm ch}^m \, \partial z_\gamma^n}\right|_{z_{\rm ch}=z_\gamma=1} = \left\langle \frac{N_{\rm ch}!\; N_\gamma!}{(N_{\rm ch}-m)!\,(N_\gamma-n)!} \right\rangle \tag{7}$$

It is convenient to express our observables given in eq.1 and eq.2 in terms $\mathrm{f}_{m,n}$ as

$$\nu_{dyn}^{\gamma-{\rm ch}} = \frac{\mathrm{f}_{20}}{\mathrm{f}_{10}^2} + \frac{\mathrm{f}_{02}}{\mathrm{f}_{01}^2} - 2\frac{\mathrm{f}_{11}}{\mathrm{f}_{10}\,\mathrm{f}_{01}} \quad , \quad r_{m,1}^{\gamma-{\rm ch}} = \frac{\mathrm{f}_{m1}\,\mathrm{f}_{10}}{\mathrm{f}_{(m+1)0}\,\mathrm{f}_{01}} \tag{8}$$

Using eq. 4, eq. 5 and eq. 7 we can express different factorial moments in terms of efficiency and moments of neutral pion fraction as

$$\begin{aligned} \mathrm{f}_{10} &= \langle 1-f\rangle\, \varepsilon_{\rm ch}\, \langle N\rangle \\ \mathrm{f}_{01} &= \langle f\rangle\, 2\varepsilon_\gamma\, \langle N\rangle \\ \mathrm{f}_{11} &= \langle f\,(1-f)\rangle\, 2\varepsilon_\gamma\, \varepsilon_{\rm ch}\, \langle N\,(N-1)\rangle \\ \mathrm{f}_{20} &= \left\langle (1-f)^2\right\rangle \varepsilon_{\rm ch}^2\, \langle N\,(N-1)\rangle \\ \mathrm{f}_{02} &= \left\langle f^2\right\rangle 4\varepsilon_\gamma^2\, \langle N\,(N-1)\rangle + 2\varepsilon_\gamma^2\, \langle f\rangle\, \langle N\rangle\,. \end{aligned}$$

Substituting these in eq.1 we get

$$\nu_{dyn}^{\gamma-{\rm ch}} = \left(\frac{\langle (1-f)^2\rangle}{\langle 1-f\rangle^2} + \frac{\langle f^2\rangle}{\langle f\rangle^2} - 2\,\frac{\langle f(1-f)\rangle}{\langle f\rangle\langle 1-f\rangle} \right) \frac{\langle N(N-1)\rangle}{\langle N\rangle^2} + \frac{1}{2\langle f\rangle\langle N\rangle}. \tag{9}$$

We note here that for the generic case($\mathcal{P}(f) = \delta(f-1/3)$) the term inside the bracket is zero and we have

$$\left.\nu_{dyn}^{\gamma-{\rm ch}}\right|_{generic} = \frac{1}{2\,\langle f\rangle\,\langle N\rangle}. \tag{10}$$

Using proper combination of factorial moments and doing a simple method of event mixing one can extract the generic value of $\nu_{dyn}^{\gamma-{\rm ch}}$ (see appendix-X A for details). Subtracting the generic value of $\nu_{dyn}^{\gamma-{\rm ch}}$ one can get rid of the last term in eq.9. So we propose a modified variable $\nu_{dyn} - \nu_{dyn}^{generic}$ given by

$$\Delta\nu_{dyn}^{\gamma-{\rm ch}} = \left(\frac{\langle (1-f)^2\rangle}{\langle 1-f\rangle^2} + \frac{\langle f^2\rangle}{\langle f\rangle^2} - 2\,\frac{\langle f(1-f)\rangle}{\langle f\rangle\,\langle 1-f\rangle} \right) \times \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}. \tag{11}$$

In ideal scenarios when all the particles are detected one can approximate $g_0(z_\gamma) = z_\gamma^2$ and $g_{\rm ch}(z_{\rm ch}) = z_{\rm ch}$. In that case one can show using eq.5 and eq.7 that

$$\left.\nu_{dyn}^{\gamma-{\rm ch}}\right|_{generic} = \frac{1}{2\,\langle N\rangle\,\langle f\rangle} \approx \frac{1}{\sqrt{\langle N_{\rm ch}\rangle\,\langle N_\gamma\rangle}}. \tag{12}$$

So in that case the observable $\Delta\nu_{dyn}$ is expressed as

$$\Delta\nu_{dyn}^{\gamma-{\rm ch}} = \nu_{dyn}^{\gamma-{\rm ch}} - \frac{1}{\sqrt{\langle N_{\rm ch}\rangle\,\langle N_\gamma\rangle}} \tag{13}$$

Following similar approach the variable $r_{m,1}$ is expressed as

$$r_{m,1}^{\gamma-{\rm ch}} = \frac{\langle f(1-f)^m\rangle\,\langle 1-f\rangle}{\langle (1-f)^{m+1}\rangle\,\langle f\rangle}. \tag{14}$$

Once can now study the sensitivity of $\Delta\nu_{dyn}$ and $r_{m,1}$ to a given fraction of DCC like signal. If x-fraction of events have DCC domain formation, in simplistic case one can assume the distribution of neutral pion fraction to be a combination of generic and DCC probability distributions given by

$$\mathcal{P}(f) = x\,\frac{1}{2\sqrt{f}} + (1-x)\,\delta\left(f - \frac{1}{3}\right). \tag{15}$$

So for $\Delta\nu_{dyn}$ we get

$$\begin{aligned} \Delta\nu_{dyn}^{\gamma-{\rm ch}} &= \left.\left(\frac{\langle (1-f)^2\rangle}{\langle 1-f\rangle^2} + \frac{\langle f^2\rangle}{\langle f\rangle^2} - 2\,\frac{\langle f(1-f)\rangle}{\langle f\rangle\,\langle 1-f\rangle} \right)\right|_{signal} \times \frac{\langle N(N-1)\rangle}{\langle N\rangle^2} \\ &= \frac{x}{5/9}\,\frac{\langle N(N-1)\rangle}{\langle N\rangle^2}, \end{aligned} \tag{16}$$

which is proportional to the fraction of DCC events. $\Delta\nu_{dyn}$ shows very high sensitivity to DCC like signal but it is dependent on the parent multiplicity and consequently to the collisions centrality. In a later section we would discuss this issue in detail. In case the parent distribution is Poisson, the fluctuation term $\langle N(N-1)\rangle / \langle N\rangle^2$ would be equal to 1 giving $\Delta\nu_{dyn}^{\gamma-{\rm ch}} \sim x/(5/9)$.

The observable $r_{m,1}$ expressed in eq.14 would have a very particular x dependence given by

$$r_{m,1}^{\gamma-{\rm ch}} = 1 - \frac{mx}{(m+1)} F(m,x) \tag{17}$$

where the function $F(m,x)$ is given by

$$F(m,x) = \frac{1}{x + (1-x)\frac{2}{\sqrt{\pi}} \left(\frac{2}{3}\right)^{m+1} \frac{\Gamma(m+5/2)}{\Gamma(m+2)}}. \qquad (18)$$

For ideal DCC case (x=1), the function $F(m,x)$=1 for all values of m giving $r_{m,1} = 1/(m+1)$. For generic case($x = 0$), $r_{m,1}$=1 for all m. Fig.1 shows the sensitivity of $r_{m,1}$ for small signals of DCC. The functional form given in eq.17 can be used to extract x from a fit of $r_{m,1}$ with m. In the derivation of eq.16 and eq.17 we have assumed that the parent multiplicity distribution are similar for both the generic and DCC cases. The efficiency factors are assumed to be constant and independent of multiplicity and other kinematic parameters.

III. EFFECT OF MIS-IDENTIFICATION

There are additional complications in realistic scenarios that have not been taken care of in the above prescriptions. The study of $\gamma-$ch correlation is often complicated by mis-identification of charge particles as photons and vice versa. High energy loss of charged hadrons can form a cluster in photon detector. Similarly photon conversion can show up as single or doubly detected tracks or clusters in charge particle detectors. In both the cases the measurements are affected. Following the approach of the application of cluster decay theorem discussed in previous section, we obtain the modified forms of the generating functions

$$\begin{aligned} g_{\rm ch}(z_{\rm ch}, z_\gamma) &= (1-\varepsilon_{\rm ch}-\varepsilon_{{\rm ch},\gamma}) + \varepsilon_{\rm ch} z_{\rm ch} + \varepsilon_{{\rm ch},\gamma} z_\gamma \\ g_0(z_{\rm ch}, z_\gamma) &= \left((1-\varepsilon_\gamma - \varepsilon_{\gamma,{\rm ch}} - \varepsilon_{\gamma,2{\rm ch}}) + \varepsilon_\gamma z_\gamma \right. \\ &\quad \left. + \varepsilon_{\gamma,{\rm ch}}\, z_{\rm ch} + \varepsilon_{\gamma,2{\rm ch}}\, z_{\rm ch}^2\right)^2, \qquad (19) \end{aligned}$$

where we view neutral pions decay with 100% "efficiency" into two photons which themselves "decay" with a few modes. $\varepsilon_{\rm ch}$ and ε_γ are the efficiencies of detecting a charged particle and a photon, respectively. $\varepsilon_{{\rm ch},\gamma}$ is the probability of a charged particle being identified as a photon. $\varepsilon_{\gamma,{\rm ch}}$, $\varepsilon_{\gamma,2{\rm ch}}$ are the probabilities of a photon being identified as one or two charged particles, respectively. Substituting these in eq.5 one can calculate different factorial moments as,

$$\begin{aligned} {\rm f}_{10} &= \langle (1-f)\varepsilon_{\rm ch} + 2f\left(\varepsilon_{\gamma,{\rm ch}} + 2\varepsilon_{\gamma,2{\rm ch}}\right)\rangle \langle N\rangle \\ {\rm f}_{01} &= \langle (1-f)\varepsilon_{{\rm ch},\gamma} + 2f\varepsilon_\gamma\rangle \langle N\rangle \\ {\rm f}_{11} &= \langle N(N-1)\left((1-f)\varepsilon_{\rm ch} + 2f\left(\varepsilon_{\gamma,{\rm ch}} + 2\varepsilon_{\gamma,2{\rm ch}}\right)\right) \\ &\quad \times \left((1-f)\varepsilon_{{\rm ch},\gamma} + 2f\varepsilon_\gamma\right) + 2Nf\varepsilon_\gamma\left(\varepsilon_{\gamma,{\rm ch}} + 2\varepsilon_{\gamma,2{\rm ch}}\right)\rangle \\ {\rm f}_{20} &= \Big\langle N(N-1)\left((1-f)\varepsilon_{\rm ch} + 2f(\varepsilon_{\gamma,{\rm ch}} + 2\varepsilon_{\gamma,2{\rm ch}})\right)^2 \\ &\quad + 2Nf\left(2\varepsilon_{\gamma,2{\rm ch}} + (\varepsilon_{\gamma,{\rm ch}} + 2\varepsilon_{\gamma,2{\rm ch}})^2\right)\Big\rangle \\ {\rm f}_{02} &= \Big\langle N(N-1)((1-f)\varepsilon_{{\rm ch},\gamma} + 2f\varepsilon_\gamma)^2 + 2Nf\varepsilon_\gamma^2\Big\rangle. \qquad (20) \end{aligned}$$

This would lead to very complicated (see appendix-X B) dependencies of $\Delta\nu_{dyn}$ and $r_{m,1}$ on various efficiency factors. However a relatively simple form can be obtained in the limit of small values of $\varepsilon_{\gamma,{\rm ch}}$ and $\varepsilon_{\gamma,2{\rm ch}}$. So in case of small photon conversion in the charged particle detector one can express $\Delta\nu_{dyn}$ as

$$\Delta\nu_{dyn}^{\gamma-{\rm ch}} = \left(\frac{\langle (1-f)^2\rangle}{\langle 1-f\rangle^2} + \frac{\Big\langle \left((1-f)\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 2f\right)^2\Big\rangle}{\Big\langle (1-f)\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 2f\Big\rangle^2} - 2\,\frac{\Big\langle (1-f)\left((1-f)\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 2f\right)\Big\rangle}{\langle 1-f\rangle \Big\langle (1-f)\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 2f\Big\rangle} \right) \frac{\langle N(N-1)\rangle}{\langle N\rangle^2} \qquad (21)$$

where the the generic value of ν_{dyn} will be given by

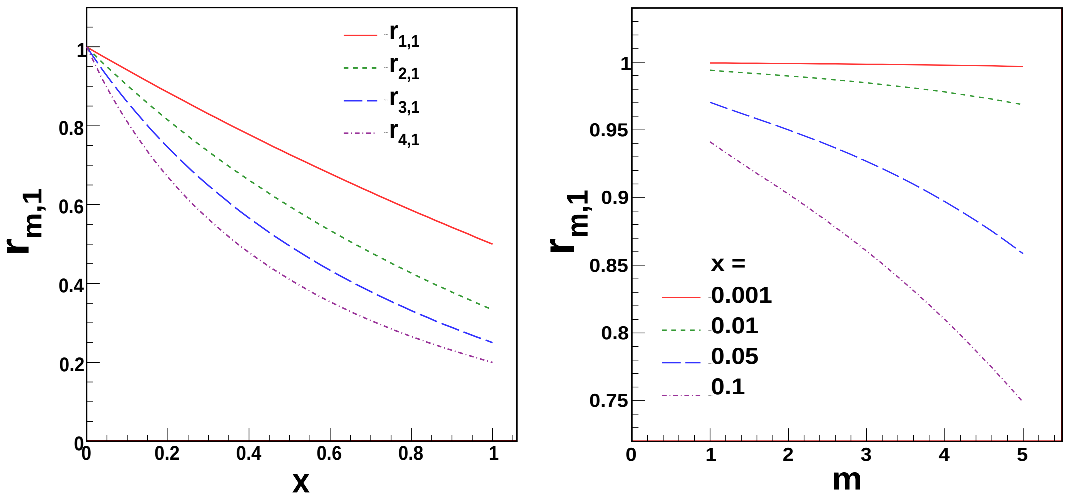
$$\nu_{dyn}^{\gamma-{\rm ch}}\Big|_{generic} = \frac{1}{2\langle f\rangle \langle N\rangle \left(\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 1\right)}. \qquad (22)$$

The robust variable $r_{m,1}$ can be expressed as

$$r_{m,1}^{\gamma-{\rm ch}} = \frac{\Big\langle (1-f)^m \left((1-f)\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 2f\right)\Big\rangle \langle 1-f\rangle}{\langle (1-f)^{m+1}\rangle \Big\langle (1-f)\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 2f\Big\rangle}. \qquad (23)$$

Unlike previous case it is not possible to eliminate the efficiency factors in eq.21 and eq.23. For x-fraction of DCC signals eq.16 and eq.17 will be modified to

$$\begin{aligned} \Delta\nu_{dyn}^{\gamma-{\rm ch}} &= \frac{x}{5/9} \frac{1}{\left(\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 1\right)^2} \frac{\langle N(N-1)\rangle}{\langle N\rangle^2} \\ r_{m,1} &= 1 - \frac{mx}{m+1} \frac{1}{\left(\frac{\varepsilon_{{\rm ch},\gamma}}{\varepsilon_\gamma} + 1\right)} F(m,x) \qquad (24) \end{aligned}$$



(a)Variation of $r_{m,1}$ with fraction of DCC signal

(b)Variation of $r_{m,1}$ with higher order moments

FIG. 1: Sensitivity of the observable $r_{m,1}$ to DCC like signals. Higher orders of $r_{m,1}$ show more sensitivity to small signals of anti-correlation.

where $F(m,x)$ is given by eq.18. We can see that mis identification of charged particles as photons reduces the effective fraction of DCC events. The contamination factor in eq.24 appears as a ratio of $\varepsilon_{\mathrm{ch},\gamma}/\varepsilon_\gamma$ keeping the functional form of the observables (eq.16, eq.17) unchanged. We also note here that $\Delta\nu_{dyn}^{\gamma-\mathrm{ch}}$ has a quadratic dependence on contamination factor whereas $r_{1,1}$ is affected only by a linear factor. This is because $\Delta\nu_{dyn}^{\gamma-\mathrm{ch}}$ contains an extra photon fluctuation term which is absent in $r_{m,1}$.

IV. RESONANCE EFFECT

Resonance decays like $\rho \to \pi^{\pm}\gamma$ is equivalent to artificial increase of pions and photons from generic case. Decays like $\omega \to \pi^0 + \pi^{\pm}$ would give rise to correlation in the pions. The effect of resonance leading to increase in photon and charged particle multiplicity is equivalent to event-by-event increase in efficiency of photon and charged particle detection. For event-by event fluctuations of efficiency would affect the observables, for e.g. the variable $r_{m,1}$ given in eq.14 will be modified as

$$r_{m,1}^{\gamma-\mathrm{ch}} = \frac{\langle f\,(1-f)^m\rangle\,\langle 1-f\rangle}{\left\langle (1-f)^{m+1}\right\rangle\,\langle f\rangle}\,\frac{\langle \varepsilon_\gamma \varepsilon_{\mathrm{ch}}^m\rangle\,\langle \varepsilon_{\mathrm{ch}}\rangle}{\left\langle \varepsilon_{\mathrm{ch}}^{m+1}\right\rangle\,\langle \varepsilon_\gamma\rangle}. \quad (25)$$

It is difficult to conclude the behavior of the variables from the above expressions without putting a realistic number for the efficiencies. To study the effect of resonances in a more detailed way (sec.VII, VIII) we have used Monte-Carlo models in which resonance productions are included.

V. CENTRALITY DEPENDENCE

In heavy ion collision experiment, observables are commonly studied with respect to the centrality that is related to the number of participating nucleons. It is therefore necessary that we study the nominal effect of the superposition of nucleons on the observables. In this section we would like to study the centrality dependence of the $\gamma - \mathrm{ch}$ correlation using an approach based on the "Central Limit Theorem" (CLT). Importance and applicability of CLT in the context of correlation analysis in heavy ion collision has previously been discussed in detail in ref.[27]. In a heavy ion collision, let us consider N_S number of identical sources are responsible for particle production. If N_i is the number of particles produced from i-th source, any variable $V(N_i)$ will have a distribution identical for all the sources. If we assume heavy-ion collision to be a linear superposition of many identical nucleon-nucleon collisions, under identical source approximation we can calculate the centrality dependence of the variable using CLT [28]. From CLT it follows that mean and variance of multiplicity would be given by

$$M(N) = M\left(\sum_i^{N_S} N_i\right) = \sum_i^{N_S} M(N_i) = N_S\,M(N_i)$$

$$\sigma^2(N) = \sigma^2\left(\sum_i^{N_S} N_i\right) = \sum_i^{N_S} \sigma^2(N_i) = N_S\,\sigma^2(N_i). \quad (26)$$

Since we have already assumed a collection of identical sources we can take $M(N_i) = \alpha$ and $\sigma^2(N_i) = \beta$ to be constant numbers same for all emission sources. So from CLT we have the dependence $M(N) = \alpha N_S$ and $\sigma(N) = \beta\sqrt{N_S}$. In our case N could refer to total number of produced pions, photons or charged particles. In that case similar argument also holds for $M(N_\pi, N_{\rm ch}\ or\ N_\gamma) \sim \alpha_{\pi,{\rm ch},\gamma} N_S$ and $\sigma(N_\pi, N_{\rm ch}\ or\ N_\gamma) \sim \beta_{\pi,{\rm ch},\gamma}\sqrt{N_S}$ where (α_π, β_π), $(\alpha_{\rm ch}, \beta_{\rm ch})$ and $(\alpha_\gamma, \beta_\gamma)$ are sets of constants corresponding to pion, charged particle or photon multiplicities for identical sources respectively.

Let us assume N to be equal to the total number of produced pions where we have $N_\pi = aN_{\rm ch} + bN_\gamma$. Where a and b are the fraction of charged pions and decay photons respectively[1]. Using eq.26 one gets the mean and variance of pions as

$$\begin{aligned} \langle N_\pi\rangle &= \alpha_\pi N_S \\ \sigma^2(N_\pi) &= \left(\langle N_\pi^2\rangle - \langle N_\pi\rangle^2\right) \sim \beta_\pi^2 N_S \\ \langle N_\pi^2\rangle &= \langle (aN_{\rm ch} + bN_\gamma)^2\rangle \sim \beta_\pi^2 N_S + \alpha_\pi^2 N_S^2 \end{aligned} \quad (27)$$

and if we express pion multiplicity in terms of charged and photons we get,

$$\begin{aligned} \langle N_{\rm ch}^2\rangle &\sim \beta_{\rm ch}^2 N_S + \alpha_{\rm ch}^2 N_S^2 \\ \langle N_\gamma^2\rangle &\sim \beta_\gamma^2 N_S + \alpha_\gamma^2 N_S^2 \\ \langle N_{\rm ch} N_\gamma\rangle &\sim \beta_{\gamma-{\rm ch}}^2 N_S + \alpha_{\gamma-{\rm ch}}^2 N_S^2 \end{aligned} \quad (28)$$

where $\alpha_{\gamma-{\rm ch}}$ and $\beta_{\gamma-{\rm ch}}$ are constants expressible[2] in terms of $a, b, \alpha_{\pi,{\rm ch},\gamma}$ and $\beta_{\pi,{\rm ch},\gamma}$. Using above relations, eq.1 and eq.2 we can calculate the centrality dependence of the observables. For ν_{dyn} we have

$$\nu_{dyn}^{\gamma-{\rm ch}} \sim A + \frac{B}{N_S} \equiv A' + \frac{B'}{\sqrt{\langle N_\gamma\rangle \langle N_{\rm ch}\rangle}}. \quad (29)$$

All three terms in eq.1 have similar centrality dependence . Here we note that the constants A' and B' (or A and B) could be either positive or negative depending on which term in eq.1 is dominant. The variable $\Delta\nu_{dyn}$ would have the similar centrality dependence which is evident from the form of eq.13. In heavy ion collisions, the number of sources participating in particle production can also be assumed to be proportional to number of participants ($N_S \sim N_{part}$) of the collision. In that case $\nu_{dyn}^{\gamma-{\rm ch}}$ is expected to show a scaling behavior of the form $A + B/X$ with X being either observed multiplicity or a Glauber variable N_{part}. However in case of experimental measurements it is more convenient to express fluctuation variables in terms of measured multiplicities.

Based on similar approach one can extract the centrality dependence of $r_{m,1}$. In the most general case one has

$$r_{m,1} = \frac{\sum_p^m \alpha_p N_S^p}{\sum_p^m \beta_p N_S^p} \quad (30)$$

[1] Note that $N_\pi = N_{\pi^+} + N_{\pi^-} + N_{\pi^0} \approx N_{\rm ch} + 0.5N_\gamma$; $a \sim 1, b \sim 0.5$.

[2] it can be shown that $\alpha_{\gamma-{\rm ch}}^2 = \left(\alpha_\pi^2 - a^2\alpha_{\rm ch}^2 - b^2\alpha_\gamma^2\right)/2ab$, $\beta_{\gamma-{\rm ch}}^2 = \left(\beta_\pi^2 - a^2\beta_{\rm ch}^2 - b^2\beta_\gamma^2\right)/2ab$

which shows that both numerator and denominator have identical dependence on N_S. So according to CLT, behavior of $r_{m,1}$ with multiplicity depends on the coefficients α_p and β_p.

It must be noted that breakdown of scaling from CLT would have several implications. The picture of identical source emission may not be valid in the case for formation of domains of DCC where one might observe deviation from proposed scaling.

VI. EFFECT OF MIXTURE OF PION SOURCES

In this section we would like to discuss the effect on the observables when event-wise pion sources are independent of each other. So far we have considered that in a DCC event, all the pions detected in a given coverage are coming from the decay of the domains of DCC. This assumption might be valid when the detector coverage is same as the combined size of DCC domains. The realistic scenario is when the size of the domain of DCC is smaller than the detector coverage. Also DCC pions are dominantly from lower part of the momentum distribution. In both the cases of considering bulk multiplicity for correlation analysis, the candidates carrying actual signal would be a fraction total pions considered. Let us consider a case when x-fraction of events analyzed has DCC like fluctuation carried by y-fraction of total pions. So for DCC pions we have $\langle N\rangle_D = y\langle N\rangle$ and for generic pions we have $\langle N\rangle_G = (1-y)\langle N\rangle$, N being the total number of pions. The probability to find N_D pions carrying DCC signal will be given by $P(N_D, N, y) = {}^N{\rm C}_{N_D}\, y^{N_D}(1-y)^{N-N_D}$, which would give $\langle N(N-1)\rangle_D = y^2\langle N(N-1)\rangle$. Now in this case the generating function of eq.4 will be replaced by

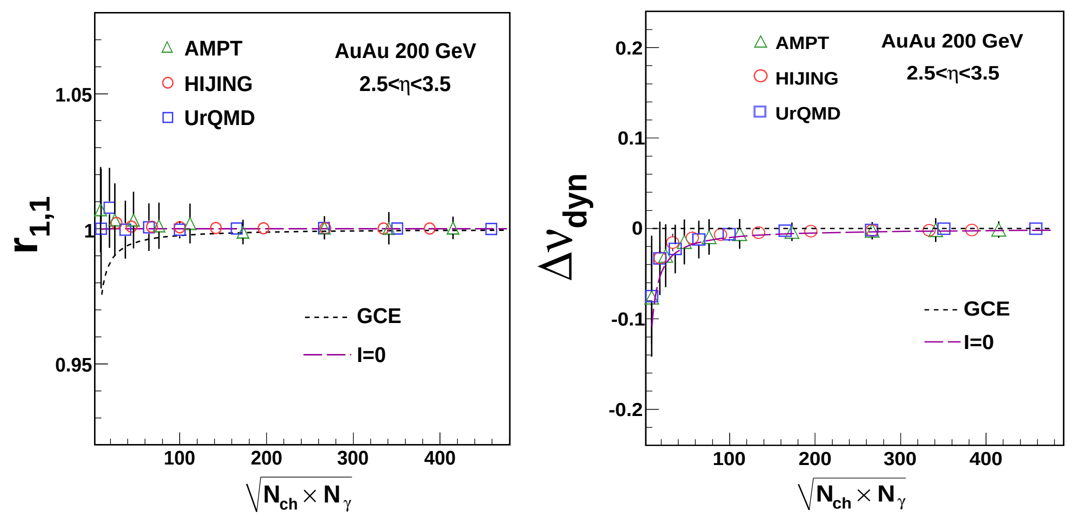
$$G_{obs} = x' G_{\rm DCC} + x\, G_{\rm DCC}\, G_{generic} + (1-x-x')G_{generic} \quad (31)$$

in which we view cases with 100% DCC production (x' fraction of events), 100% generic production and a mixture of two as three "decay modes" of a super cluster. Here $G_{\rm DCC}$ includes probability distribution $\mathcal{P}(f) = 1/2\sqrt{f}$ and $G_{generic}$ includes $\mathcal{P}(f) = \delta(f - 1/3)$. Since we consider the case of 100% DCC production is the least realistic, in the following we simplify our expression by taking $x' = 0$. Now different factorial moments will become functions of x and y (see appendix-X C for detail). In this case the observables are modified accordingly , for $\Delta\nu_{dyn}$ eq.13 gives,

$$\Delta\nu_{dyn} = \frac{x}{5/9}\, y^2\, \frac{\langle N(N-1)\rangle}{\langle N\rangle^2} \quad (32)$$

which consistent with the expression eq.16 for $y = 1$ case. For Poisson like parent distribution $\Delta\nu_{dyn}$ can be expressed as

$$\Delta\nu_{dyn} = \frac{x}{5/9}\, y^2. \quad (33)$$



(a)Variation of $r_{1,1}$ with multiplicity

(b)Variation of $\Delta\nu_{dyn}$ with multiplicity

FIG. 2: Multiplicity dependence of observables $r_{1,1}$ and $\Delta\nu_{dyn}$ as predicted from different models. The curves represent the results for different ensembles of Boltzmann gas of pions from eq.36 and eq.38 as described in the text. The markers are from different Monte-Carlo models. The error-bars are statistical.

We note here that $\Delta\nu_{dyn}$ still shows the proportionality with the fraction of DCC events x. And the interesting fact is that quadratic dependence on y means $\Delta\nu_{dyn}$ is more sensitive to the change of fraction of pions carrying DCC-like signals.

In similar approach we can express $r_{1,1}$ to be

$$r_{1,1} = \frac{5 - 2xy^2}{5 + xy^2}. \quad (34)$$

This expression is consistent with the approximate expression of $r_{1,1}$ given in Ref.[29] for small values of x. The higher order moments will have corrections from higher powes of y which will have smaller contributions. To the lowest order approximation(appendix-X C), the expression given by eq.17 is still valid with fraction x replaced by xy^2.

$$r_{m,1}^{\gamma-\mathrm{ch}} \approx 1 - \frac{mxy^2}{(m+1)} F(m, xy^2) \quad (35)$$

A functional fit of $r_{m,1}$ with m to experimental data by the above expression can restrict the contours of x and y.

VII. MODEL PREDICTION

In this section we would like to study the behavior of observables from different models available to describe heavy ion data. There are theoretical predictions of isospin fluctuation for a statistical system of pions[30, 31]. It can be shown that a system of Boltzmann gas of pions in the grand canonical ensemble (GCE), gives $\langle N_{\pi^0}\rangle = \langle N_{\pi^\pm}\rangle = \zeta$, where ζ is the single particle partition function[3]. In that case the mean-square of pion multiplicity and charge-to-neutral pion correlation are related to mean multiplicities as

$$\begin{aligned} \left\langle N_{\pi^0}^2 \right\rangle &= \langle N_{\pi^0}\rangle + \langle N_{\pi^0}\rangle^2 \\ \left\langle N_{\pi^\pm}^2 \right\rangle &= \langle N_{\pi^\pm}\rangle + \langle N_{\pi^\pm}\rangle^2 \\ \langle N_{\pi^0} N_{\pi^\pm}\rangle &= \langle N_{\pi^0}\rangle \langle N_{\pi^\pm}\rangle \end{aligned} \quad (36)$$

In ref[31] it was shown that for an ideal scenario where one assumes the total isospin of the system to be zero, above mentioned relationships would become complicated. An ensemble of the total isospin I=0 as shown in [31] would give

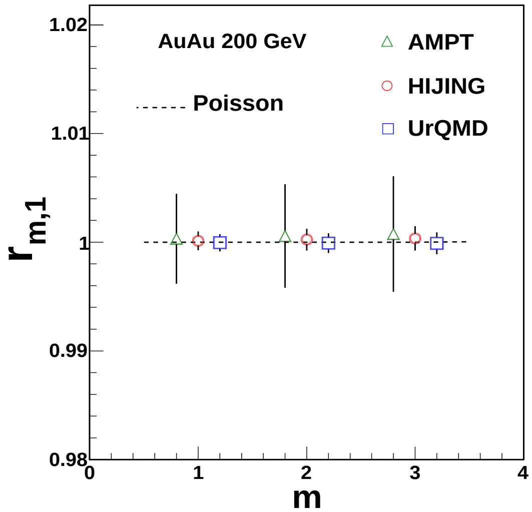
$$\langle N_{\pi^0}\rangle = \langle N_{\pi^\pm}\rangle = \frac{\zeta^2}{3} + \frac{\zeta^3}{6} \quad (37)$$

and in that case the mean-square pions multiplicities are modified as

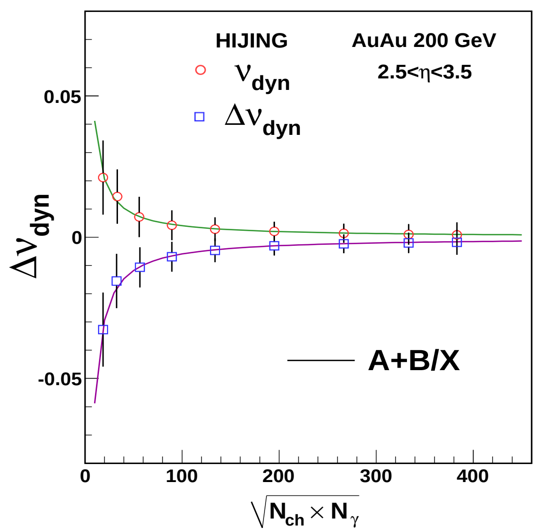
$$\begin{aligned} \left\langle N_{\pi^0}^2 \right\rangle &\approx \langle N_{\pi^0}\rangle + \frac{\zeta^2}{3} + \frac{\zeta^4}{15} \\ \left\langle N_{\pi^\pm}^2 \right\rangle &\approx \langle N_{\pi^\pm}\rangle + \frac{\zeta^4}{10}. \end{aligned} \quad (38)$$

We can generalize these results and apply in case of our observables of $\gamma - \mathrm{ch}$ correlation. The dependence on ζ

[3] $\zeta = \frac{V}{2\pi} \int_0^\infty p^2 dp \exp\left(-\frac{\sqrt{p^2+m^2}}{T}\right)$, V, m and T begin volume, pion mass and temperature of the system



(a)Variation of $r_{m,1}$ with m from different models. The m values of the points for AMPT and UrQMD have been shifted horizontally for better clarity.



(b)Centrality dependence of variable(markers) ν_{dyn} and $\Delta\nu_{dyn}$ and fits(lines) predicted from CLT. Here $N_{\rm ch}$ and N_γ refer to the mean multiplicities of charged particles and photons.

FIG. 3: Prediction of variables from different models.

can be eliminated and final observables can be expressed in terms of experimentally observed quantities like measured multiplicity (say $\sqrt{\langle N_{\rm ch}\rangle \langle N_\gamma\rangle}$). In this case one has $\langle N_\gamma\rangle = 2\langle N_{\pi^0}\rangle$ and $\langle N_{\rm ch}\rangle = \langle N_{\pi^+} + N_{\pi^-}\rangle = 2\langle N_{\pi^\pm}\rangle$. Also for decay of neutral pions we have used the relation $\sigma_\gamma^2 \approx \mathcal{C}\sigma_{\pi^0}^2$, and we have used $\mathcal{C}$=2 for our calculation [4]. Choice of $\mathcal{C}$ mostly affects the observables at low multiplicity. With these assumptions we can express the mean-square multiplicities to be

$$\langle N_\gamma^2\rangle = 4\langle N_{\pi^0}^2\rangle \;,\; \langle N_{\rm ch}^2\rangle = 2\langle N_{\pi^\pm}^2\rangle + 2\langle N_{\pi^+}N_{\pi^-}\rangle \tag{39}$$

and the correlation term will be given by $\langle N_\gamma N_{\rm ch}\rangle = 4\langle N_{\pi^0} N_{\pi^\pm}\rangle$. Now we have

$$\begin{aligned}
\frac{\mathrm{f}_{20}}{\mathrm{f}_{10}^2} &= \frac{1}{2}\left(\frac{\langle N_{\pi^\pm}(N_{\pi^\pm}-1)\rangle}{\langle N_{\pi^\pm}\rangle^2} + \frac{\langle N_{\pi^+}N_{\pi^-}\rangle}{\langle N_{\pi^\pm}\rangle^2}\right)\\
\frac{\mathrm{f}_{02}}{\mathrm{f}_{01}^2} &= \frac{1}{2}\left(\frac{\langle N_{\pi^0}(N_{\pi^0}-1)\rangle}{\langle N_{\pi^0}\rangle^2} + 1\right)\\
\frac{\mathrm{f}_{11}}{\mathrm{f}_{10}\,\mathrm{f}_{01}} &= \frac{\langle N_{\pi^0}N_{\pi^\pm}\rangle}{\langle N_{\pi^0}\rangle\langle N_{\pi^\pm}\rangle}
\end{aligned} \tag{40}$$

So using eq.36, eq.38 and eq.40 we can estimate $\nu_{dyn}^{\gamma-\rm ch}$ and $r_{1,1}$ for GCE and I=0 systems. For GCE we get from eq.36 and eq.40, $\nu_{dyn} = 1/\sqrt{\langle N_{\rm ch}\rangle \langle N_\gamma\rangle}$, which gives correct multiplicity dependence as predicted from CLT. So from eq.13 we have $\Delta\nu_{dyn}^{\gamma-\rm ch} = 0$ for GCE. The system of I=0 gives $\Delta\nu_{dyn}^{\gamma-\rm ch} \sim -0.98/\sqrt{\langle N_{\rm ch}\rangle \langle N_\gamma\rangle}$ which also agrees with the CLT predictions as shown in fig.2(b). In case of GCE $r_{1,1}$ is predicted to be $2/(1+1/\sqrt{\langle N_{\rm ch}\rangle \langle N_\gamma\rangle})$ which becomes 1 for large values of multiplicity. For a system of I=0, $r_{1,1} \sim 1$ for all values of $\sqrt{\langle N_{\rm ch}\rangle \langle N_\gamma\rangle}$ as shown in fig.2(a).

We have also estimated various observables and their centrality dependences using different monte-carlo event generators like HIJING[32], AMPT [33] and UrQMD[34] for top RHIC energy. For our calculation we choose one unit of rapidity in forward direction [5] but no cut off has been applied on transverse momentum. We do the centrality selection based on putting cuts on impact parameter following Glauber model calculation. Fig.2 shows the centrality dependence of the observables. The variable $r_{1,1}$ shows flat centrality dependence within error bars. The results from different monte-carlo models are consistent with each other and the values from the statistical model of Boltzman gas are consistent with other models towards higher multiplicity as shown in fig.2(a) and fig.2(b). At lower multiplicities they have qualitatively different nature, probably due to presence of various other effects in the monte-carlo models.

Fig.3(a) shows the variation of $r_{m,1}$ with its order m. Results from all the models are consistent with the generic case of pion production. Fig.3(b) shows the centrality dependence of ν_{dyn} and $\Delta\nu_{dyn}$ predicted for HIJING. For comparison of centrality dependence predicted from CLT, we have fitted the points with functional form of $A + B/\sqrt{\langle N_{\rm ch}\rangle \langle N_\gamma\rangle}$. This yields a value

[4] For Poissonian case $\sigma_\gamma = \sqrt{\langle N_\gamma\rangle} = \sqrt{2\langle N_{\pi^0}\rangle} = \sqrt{2}\sigma_{\pi^0}$ gives $\mathcal{C}$=2 ; incase one detects all photons from π^0 one has a maximum value of $\mathcal{C}$=4 which is not in accordance with CGE picture where limited phase space of a system is probed.

[5] both STAR and ALICE experiments has the setup of simultaneous measurements of charged and photon in one unit of rapidity.

of $A \approx 5 \times 10^{-5}$ and $B = -0.6$ for $\Delta\nu_{dyn}$. We also note here that the sign of $\Delta\nu_{dyn}$ is negative for low multiplicity. This shows that HIJING includes some intrinsic $\gamma - \text{ch}$ correlation making the last term of eq.11 to dominate over individual fluctuation. This can be attributed to the resonance decays present in HIJING model. For DCC like signal sign of $\Delta\nu_{dyn}$ should become positive for all centralities.

VIII. DCC MODEL

We have tried to implement DCC like anti-correlation signals in HIJING events. In a given event we change the neutral pion fraction to follow $1/2\sqrt{f}$ distribution by flipping π^0 to $\pi^{\pm}$. And finally we decay the neutral pions to photons. In the process of flipping we make sure that the charge and isospin conservations are maintained. Fig.4 shows the f-distribution after the implementation

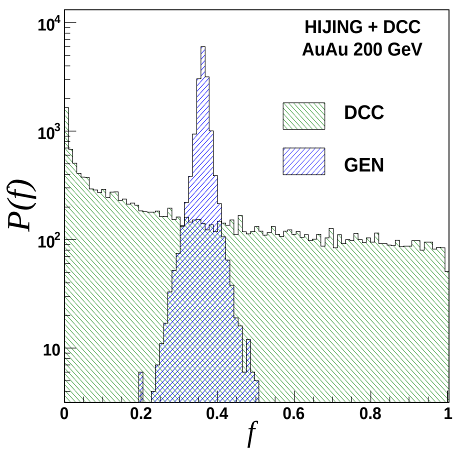
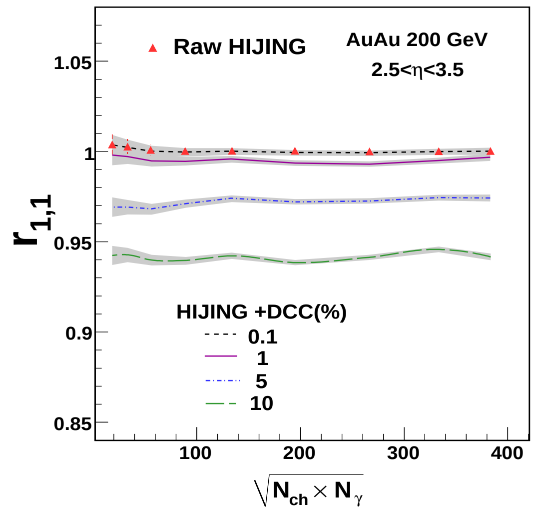
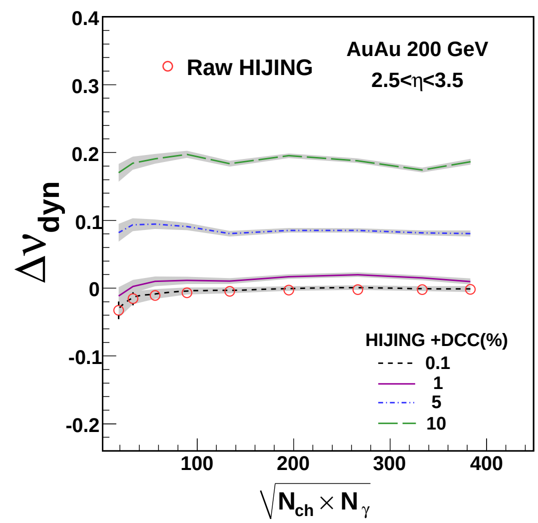


FIG. 4: Histograms showing distribution of neutral pion fraction for generic and DCC events from HIJING

of DCC in HIJING. For generic event the neutral pion fraction is peaked at 1/3 and for DCC events it has a long tail. Since the variation of DCC like domain formation with rapidity and azimuthal angle is not known, we perform this flipping for all the particles. This produces uniform $1/2\sqrt{f}$ like distribution over all phase space. To make the scenario more realistic we do the calculation of the final variables using total number of detected photons and charged particles rather than considering only pions. Other dominant sources of photons and charged particles include η, charged kaons and protons respectively. It is difficult to extract the fraction of primordial pions on which the DCC-like probability distribution could be implemented. HIJING has minijet like environment in which the production mechanism is "string fragmentation" and the abundance of particles are weighted by the spin giving large fraction of pions coming from decay of resonances. The primordial pions coming directly from string fragmentation are much smaller. Alternative environment like hydro models where the massive resonances are exponentially suppressed would give large fraction of soft pions. The difference between the two models of string fragmentation and hydro is recently contested in ref.[35]. We therefore randomly choose pions produced in HIJING events, treat them to be thermal and implement $1/2\sqrt{f}$ distribution. Fig.5 shows the centrality dependence of the two observables and their sensitivity to different fraction of DCC events. $r_{1,1}$ shows almost flat dependence on multiplicity and we also find similar dependence for all higher moments of $r_{m,1}$. Absolute values of $r_{1,1}$ are consistent with the prediction

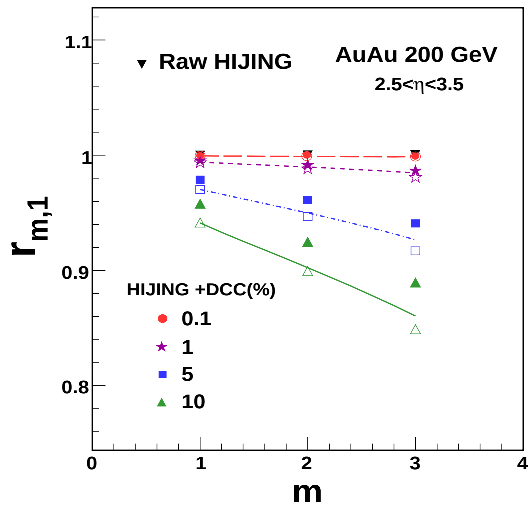


(a)Variation of $r_{1,1}$ with multiplicity

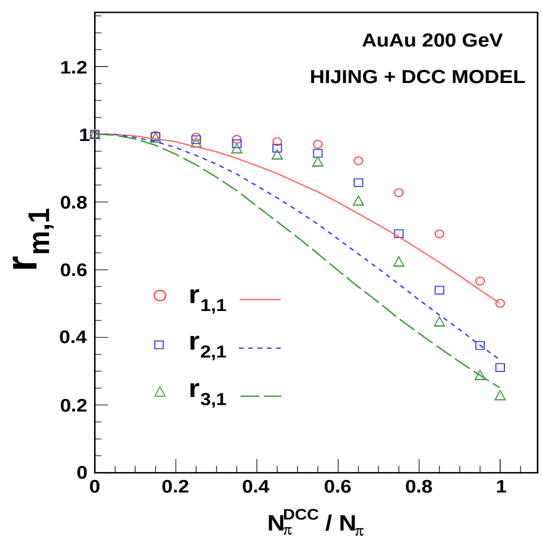


(b)Variation of ν_{dyn} with multiplicity

FIG. 5: Multiplicity dependence of observables $r_{1,1}$ and ν_{dyn} as predicted from DCC implemented HIJING model. Here N_{ch} and N_{γ} denote the mean multiplicities of charged particles and photons for various centralities. The gray band shows the statistical error in model calculation.



(a)Value of $r_{m,1}$ with m for various fractions of DCC events. The solid markers are when $N_{\rm ch}$ and N_γ includes all the charged particles and photons and the hollow markers are when only pions are source of charged particles and photons. The curves are estimations from eq.17 and points are from DCC implemented HIJING.



(b)Variation of $r_{m,1}$ in DCC events with fraction of pions coming from decay of DCC domains. Curves are estimations from eq.35.

FIG. 6: Sensitivity of $r_{m,1}$ to DCC like signals.

$(r_{1,1} = (5-2x)/(5+x))$ from eq.17. For higher fraction of DCC events the centrality dependence has slight non-monotonic behavior. This is also seen in $\Delta\nu_{dyn}$. As expected from eq.16, the values of $\Delta\nu_{dyn}$ show proportionality with the fraction of DCC events. The absolute values of $\Delta\nu_{dyn}$ are also very close to $\approx x/(5/9)$ as predicted in eq.16. The centrality dependence causes $\approx 15\%$ variation of the values of most central to peripheral events for $\Delta\nu_{dyn}$. Fig.6(a) shows the variation of $r_{m,1}$ with m. The results from DCC model match the theoretical curve (eq.17) when one considers only pions as source of charged particles and photons, however when all other sources are considered the results are slightly off towards Poissonian expectations. A more detailed study of the sensitivity to fraction of DCC pions is shown in fig.6(b) where we have shown the sensitivity of $r_{m,1}$ with the fraction of detected pions carrying DCC-signals. In fig.6(b) we also plot the curves obtained from eq.35. The effect of resonances present in HIJING seems to be resulting in reduced sensitivity of $r_{m,1}$ for lower fraction of DCC pions.

Scenarios	$\Delta\nu_{dyn}^{\gamma-{\rm ch}}$	$r_{m,1}^{\gamma-{\rm ch}}$
Generic pion production	0.	1
GCE for Boltzman pion gas	0.	$\sim$1 (m=1, higher multiplicity)
System of total $I=0$	$\frac{-0.98}{\sqrt{\langle N_{\rm ch}\rangle\langle N_\gamma\rangle}}$	1 (m=1)
HIJING, AMPT UrQMD (resonances)	negative	1
DCC (anti-correlation)	$\approx \frac{x}{5/9}y^2$	$\approx 1-\frac{mxy^2}{(m+1)}F(m,xy^2)$

TABLE I: Summary of our estimation of observables $\Delta\nu_{dyn}^{\gamma-{\rm ch}}$ and $r_{m,1}^{\gamma-{\rm ch}}$ under different scenarios relevant to heavy-ion collisions. $\Delta\nu_{dyn}^{\gamma-{\rm ch}}$ is either 0 or negative except for DCC case which gives positive value depending on the fraction x and y. $r_{m,1}^{\gamma-{\rm ch}}$ shows a particular functional dependence on m for DCC case which is distinct from all other scenarios.

IX. SUMMARY

We have developed a procedure for generalization of methods for studying γ-charge correlation in heavy-ion collisions. One of the primary motivations of this study could be the search for DCC-like anti-correlation signals relevant to the ongoing heavy ion program at RHIC and LHC. We have discussed the robustness of two variables $\Delta\nu_{dyn}$ and $r_{m,1}$ and have studied their centrality(multiplicity) dependence. Observables have been estimated from different models relevant to heavy-ion collisions that do not include the physics of DCC. DCC-like anti-correlation signals are expected to be carried by pions in limited kinematic range in both co-ordinate and momentum space. Relevant to such a context, the sensitivity of the variables has been studied with the fraction of DCC type events(x) and the event wise fraction of DCC pions(y). We summarize our estimations for different scenarios in table I. We have also developed a Monte-Carlo model where DCC domains have been implemented using inputs from HIJING event generator to study the sensitivity of those variables with DCC signals. Our results show that the model predictions of the variables are consistent with the theoretical predictions using generating function approach. Various detector effects like efficiency of detection, mis-identification have been implemented in this approach. We have shown that the mis-identification reduces the effective signal strength for which an approximate expression has been derived in generating function approach. The observable $r_{m,1}$ has been found to be more robust towards mis-identification of photons as compared to $\Delta\nu_{dyn}$. The resonance decay can induce correlation which can suppress the anti-correlating DCC signal. A quantitative idea of resonance can be obtained from DCC implemented Monte Carlo model. We have studied the sensitivity of $r_{m,1}$ for varying fraction of DCC candidates. We have seen that the variable $\Delta\nu_{dyn}$ is highly sensitive to the fractions x and y. In a given centrality $\Delta\nu_{dyn}$ is proportional to xy^2. For generic case of particle production from CLT, it is predicted to be inversely proportional to multiplicity. The sign of $\Delta\nu_{dyn}$ would indicate the dominance of correlation over anti-correlation.

The variation of the observable $r_{m,1}$ seem to be flat with centrality. Higher orders of $r_{m,1}$ show higher sensitivity to x and can have contribution up to y^{m+1}. A simplified form of the functional dependence of $r_{m,1}$ with m has been calculated in generating function approach for lowest order of y^2. This would be useful to restrict the signal strength xy^2 by fitting the experimental data.

Acknowledgements

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X. APPENDIX

A. Mixed events

While analyzing data sample to calculate $\nu_{dyn}^{\gamma-\mathrm{ch}}$, one can estimate the generic term by doing a mixed event analysis. A simple method we prescribe is to take total number of photons and total number of charge particles from different events. This would only effect correlation terms like f_{11} in $\nu_{dyn}^{\gamma-\mathrm{ch}}$ keeping other factorial moments unchanged. In such case we must have

$$\left.\frac{\mathrm{f}_{11}}{\mathrm{f}_{10}\,\mathrm{f}_{01}}\right|_{mixed} \approx \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}. \tag{41}$$

Taking a particular combination of factorial moments we can calculate the generic value for ν_{dyn} we need to calculate $\Delta\nu_{dyn}$ For example one can show that

$$\left.\left(3\frac{\mathrm{f}_{11}}{\mathrm{f}_{10}\,\mathrm{f}_{01}} - 4\frac{\mathrm{f}_{20}}{\mathrm{f}_{10}^2} + \frac{\mathrm{f}_{02}}{\mathrm{f}_{01}^2}\right)\right|_{mixed} = \frac{1}{2\langle f\rangle\langle N\rangle} \tag{42}$$

which is equal to $\nu_{dyn}^{generic}$. But in case of contamination effects present in the data sample one cannot apply this simple method since in that case the efficiency terms cannot be eliminated from ν_{dyn}. A full GEANT simulation with a known event generator which doesn't include the physics of DCC is suggested to estimate the generic value of ν_{dyn}.

B. Mis-identification

In case of mis-identification of photon as charge particles and vice-versa the fractorial moments are modified as given in eq.20. The observables $\Delta\nu_{dyn}$ and $r_{m,1}$ will be given by

$$\Delta\nu_{dyn}^{\gamma-\mathrm{ch}} = \left(\frac{\left\langle((1-f)\varepsilon_{\mathrm{ch}}+2f\varepsilon_{\gamma,\mathrm{ch}})^2\right\rangle}{\langle(1-f)\varepsilon_{\mathrm{ch}}+2f\varepsilon_{\gamma,\mathrm{ch}}\rangle^2} + \frac{\left\langle((1-f)\varepsilon_{\mathrm{ch},\gamma}+2f\varepsilon_{\gamma})^2\right\rangle}{\langle(1-f)\varepsilon_{\mathrm{ch},\gamma}+2f\varepsilon_{\gamma}\rangle^2} - 2\frac{\langle((1-f)\varepsilon_{\mathrm{ch}}+2f\varepsilon_{\gamma,\mathrm{ch}})\,((1-f)\varepsilon_{\mathrm{ch},\gamma}+2f\varepsilon_{\gamma})\rangle}{\langle(1-f)\varepsilon_{\mathrm{ch}}+2f\varepsilon_{\gamma,\mathrm{ch}}\rangle\,\langle(1-f)\varepsilon_{\mathrm{ch},\gamma}+2f\varepsilon_{\gamma}\rangle}\right)\frac{\langle N(N-1)\rangle}{\langle N\rangle^2} \tag{43}$$

$$r_{m,1} = \frac{\langle N(N-1)\,((1-f)\varepsilon_{\mathrm{ch}}+2f\varepsilon_{\gamma,\mathrm{ch}})^m\,((1-f)\varepsilon_{\mathrm{ch},\gamma}+2f\varepsilon_{\gamma}) + 2Nf\varepsilon_{\gamma}\varepsilon_{\gamma,\mathrm{ch}}\rangle\,\langle(1-f)\varepsilon_{\mathrm{ch}}+2f\varepsilon_{\gamma,\mathrm{ch}}\rangle}{\left\langle N(N-1)\,((1-f)\varepsilon_{\mathrm{ch}}+2f\varepsilon_{\gamma,\mathrm{ch}})^{m+1} + 2Nf\left(2\varepsilon_{\gamma,2\mathrm{ch}}+\varepsilon_{\gamma,\mathrm{ch}}^2\right)\right\rangle\,\langle(1-f)\varepsilon_{\mathrm{ch},\gamma}+2f\varepsilon_{\gamma}\rangle} \tag{44}$$

In case of $\varepsilon_{\gamma,\mathrm{ch}} = \varepsilon_{\gamma,2\mathrm{ch}} = 0$ one recovers eq.21 and eq.23.

C. Pion mixture

In case of x-fraction of DCC events containing y-fractions of pions carrying DCC signal, different factorial moments are given by

$$\begin{aligned} \mathrm{f}_{10} &= \langle 1-f\rangle\,\varepsilon_{\mathrm{ch}}\,\langle N\rangle \\ \mathrm{f}_{01} &= \langle f\rangle\,2\varepsilon_{\gamma}\,\langle N\rangle \end{aligned}$$

which is same as the case corresponding to $y = 1$. But higher order moments are modified to be

$$\begin{aligned} \mathrm{f}_{11} &= \Big(2xy(1-y)\,\langle f\rangle\,\langle 1-f\rangle\,\langle N\rangle^2 + ((1-xy(2-y))\,\langle f(1-f)\rangle_G + xy^2\,\langle f(1-f)\rangle_D)\,\langle N\,(N-1)\rangle\Big)\,2\varepsilon_{\gamma}\,\varepsilon_{\mathrm{ch}} \\ \mathrm{f}_{20} &= \Big(2xy(1-y)\,\langle 1-f\rangle^2\,\langle N\rangle^2 + ((1-xy(2-y))\,\left\langle (1-f)^2\right\rangle_G + xy^2\,\left\langle (1-f)^2\right\rangle_D)\,\langle N\,(N-1)\rangle\Big)\,\varepsilon_{\mathrm{ch}}^2 \\ \mathrm{f}_{02} &= \Big(2xy(1-y)\,\langle f\rangle^2\,\langle N\rangle^2 + ((1-xy(2-y))\,\left\langle f^2\right\rangle_G + xy^2\,\left\langle f^2\right\rangle_D)\,\langle N\,(N-1)\rangle\Big)\,4\varepsilon_{\gamma}^2 + 2\varepsilon_{\gamma}^2\,\langle f\rangle\,\langle N\rangle \end{aligned} \tag{45}$$

which gives

$$\begin{aligned} r_{1,1} &= \frac{5-2xy^2}{5+xy^2} \\ r_{2,1} &= \frac{35-xy^2(21-4y)}{35+xy^2(21-2y)} \end{aligned} \tag{46}$$

and so on. The general formula for $r_{m,1}$ is given by

$$r_{m,1} = 1 - \frac{mxy^2}{(m+1)}F(m,xy^2) + \mathcal{O}(xy^3)\cdots \tag{47}$$

in which $r_{m,1}$ will have contribution up to xy^{m+1}. Since $y \leq 1$ higher order contribution of y are smaller and the approximate form of the above expression would be given by

$$r_{m,1} \approx 1 - \frac{mxy^2}{(m+1)}F(m,xy^2) \tag{48}$$

where $F(m,xy^2)$ is given by eq.18.

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