Reduced neutron widths in the nuclear data ensemble: Experiment and theory do not agree

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Reduced neutron widths in the nuclear data ensemble: Experiment and theory do not agree

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Abstract

I have analyzed reduced neutron widths (Γ_n^0) for the subset of 1245 resonances in the nuclear data ensemble (NDE) for which they have been reported. Random matrix theory (RMT) predicts for the Gaussian orthogonal ensemble (GOE) that these widths should follow a χ^2 distribution having one degree of freedom ($\nu = 1$) - the Porter Thomas distribution (PTD). Careful analysis of the Γ_n^0 values in the NDE rejects the validity of the PTD with a statistical significance of at least 99.97% ($\nu = 0.801 \pm 0.052$). This striking disagreement with the RMT prediction is most likely due to the inclusion of significant p-wave contamination to the supposedly pure s-wave NDE. When an energy dependent threshold is used to remove the p-wave contamination, the PTD is still rejected with a statistical significance of at least 98.17% ($\nu = 1.217 \pm 0.092$). Furthermore, examination of the primary references for the NDE reveals that many resonances in most of the individual data sets were selected using methods derived from RMT. Therefore, using the full NDE data set to test RMT predictions seems highly questionable. These results cast very serious doubt on claims that the NDE represents a striking confirmation of RMT.

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I. INTRODUCTION

The nuclear data ensemble (NDE) [1, 2] is a set of 1407 resonance energies consisting of 30 sequences in 27 different nuclides. The ensemble was assembled to test predictions of random matrix theory (RMT) [3]. Fluctuation properties of resonance energies in the NDE were found to be in remarkably close agreement with RMT predictions for the Gaussian orthogonal ensemble (GOE). Although there have been several other successful tests of RMT using nuclear resonances (e.g., [4, 5]),the NDE is perhaps the most important because, as stated in Ref. [3], "As a result of these analyses, it became generally accepted that proton and neutron resonances in medium weight and heavy nuclei agree with GOE predictions." Hence, the NDE routinely is cited as providing striking confirmation of RMT.

Reduced neutron widths (Γ_n^0) have been reported for a subset of 1245 resonances in the NDE of Ref. [2], consisting of 14 to 178 measurements for 24 nuclides. If the GOE correctly describes the data, RMT predicts these widths should follow a χ^2 distribution having one degree of freedom ($\nu = 1$) - the Porter Thomas distribution (PTD). It has be argued that the PTD is more generally valid [6] and more robust [7] than other RMT predictions. Also, it has been demonstrated [8] that a much more reliable analysis of spectra fluctuations can be performed using neutron widths than using resonance energies. In addition, it has been shown [9] that resonance width fluctuations are more sensitive than energy fluctuations to the degree of chaos in model quantum systems. In addition, it is straightforward [10] to account for experimental effects such as missed or spurious resonances, and to use the statistically efficient maximum-likelihood (ML) method while using width data, but the same cannot be said for energy data. For these reasons, a test of the PTD using the NDE neutron widths could be very valuable.

In Ref. [2] such a test on a subset of the NDE data was described from which it was concluded that there was satisfactory accord between theory and experiment. However, there are several problems with the analysis of Ref. [2], which I will describe below. I find that when the data are analyzed more carefully, they do not agree with the PTD.

I will show below that the NDE suffers from significant p-wave contamination. For such resonances, reported reduced neutron widths Γ_n^0 are actually only "effective" reduced widths $g\Gamma_n/\sqrt{E_n}$, where $g = \frac{2J+1}{(2I+1)(2j+1)}$ (with J, I, and j being spins of the resonance, target, and neutron, respectively) is the spin statistical factor. All s-wave resonances for NDE nuclides included in my analysis have $J = \frac{1}{2}$ and hence $g = 1$ and $g\Gamma_n^0 = \Gamma_n^0$. However, there are two resonance spins possible for p-wave resonances for these nuclides; $J = \frac{1}{2}$ or $\frac{3}{2}$, and hence $g = 1$ or 2. As a reminder of these facts, I will use $g\Gamma_n^0$ when referring to reduced neutrons widths in the remainder of this paper.

II. RECONSTRUCTING THE NDE

To my knowledge, the resonance energies and neutron widths in the NDE have never been published as a set. It should be possible, however, to reconstruct this information from the nuclides and corresponding number of resonances given in the NDE papers [1, 2] and the primary references cited therein. Unfortunately, several of the references listed in the NDE papers are ”private communication”, there are a few errors in the number of resonances reported in Refs. [2], and this reference does not explain why some of the nuclides in their NDE were excluded from their analysis of the neutron widths. For example, in Ref. [2] it is stated that 1182 neutron widths in 21 nuclides were included in the analysis. However, the sum of the number of resonances in these 21 nuclides reported in this same reference is actually 1194. One resonance in ^{182}W was reported without a neutron width in the primary reference [11], so only 40 of the 41 resonances for this nuclide can be included in the width analysis. Therefore, the total number of widths is actually 1193, not 1182. Also, 19, 54, 47, and 21 resonances have been reported [8, 12] for ^{154,156,158,160}Gd, respectively, not 19, 47, 21, and 54, respectively as stated in Ref. [2]. In addition, there are three nuclides (^{160}Dy, ^{164}Dy, and ^{186}W, with 18, 20, and 14 resonances, respectively) in the NDE of Ref. [2] that were not included in their width analysis, even though neutron widths were available [11, 13].

In total then, the NDE of Ref. [2] should contain 1245 neutron resonances in 24 nuclides for which widths have been reported. I obtained resonance energies and neutron widths for these nuclides from the primary references given in Table I, and cross checked these data with information in the EXFOR/CSISRS [14] database. Given the problems noted above, I cannot be certain that I have analyzed the same data as those in the NDE of Ref. [2]. However, using data from the primary references should minimize any differences with Ref. [2] as well as make it easier for others to reconstruct the data set I have used.

TABLE I: NDE nuclides.

Nuclide	Ref.	E_{max} (keV)	ML Results					
			Minimum Threshold			p-free Threshold		
			$T_{\max}$	N_{res}	ν_{min}	$T_{\max}$	N_{res}	ν_{pf}
^{64}Zn	[15, 16]	367.55	0.024	103	$1.35\ ^{+0.24}_{-0.22}$	0.05	99	$1.54\ ^{+0.29}_{-0.26}$
^{66}Zn	[15, 16]	297.63	0.025	65	$0.68\ ^{+0.23}_{-0.21}$	0.05	61	$0.74\ ^{+0.27}_{-0.25}$
^{68}Zn	[15, 16]	247.20	0.019	45	$0.75\ ^{+0.27}_{-0.25}$	0.05	41	$0.95\ ^{+0.36}_{-0.32}$
^{114}Cd	[17]	3.3336	0.137	17	$0.35\ ^{+0.54}_{-0.34}$	0.45	11	$2.0\ ^{+1.5}_{-1.2}$
^{152}Sm	[18]	3.665	0.025	70	$1.14\ ^{+0.27}_{-0.25}$	0.1	62	$1.55\ ^{+0.40}_{-0.38}$
^{154}Sm	[18]	3.0468	0.036	27	$0.76\ ^{+0.38}_{-0.36}$	0.1	22	$1.32\ ^{+0.65}_{-0.55}$
^{154}Gd	[12]	0.2692	0.15	19	$0.44\ ^{+0.58}_{-0.43}$	0.2	18	$0.49\ ^{+0.64}_{-0.48}$
^{156}Gd	[8]	1.9908	0.009	54	$1.22\ ^{+0.27}_{-0.26}$	0.2	46	$1.44\ ^{+0.51}_{-0.49}$
^{158}Gd	[12]	3.9827	0.012	47	$0.75\ ^{+0.25}_{-0.22}$	0.2	35	$1.17\ ^{+0.54}_{-0.47}$
^{160}Gd	[12]	3.9316	0.013	21	$0.55\ ^{+0.34}_{-0.33}$	0.2	16	$0.83\ ^{+0.75}_{-0.65}$
^{160}Dy	[13]	0.4301	0.07	18	$0.83\ ^{+0.57}_{-0.51}$	0.2	14	$1.41\ ^{+1.0}_{-0.83}$
^{162}Dy	[13]	2.9572	0.046	46	$1.02\ ^{+0.33}_{-0.32}$	0.2	40	$0.99\ ^{+0.47}_{-0.43}$
^{164}Dy	[13]	2.9687	0.04	20	$0.82\ ^{+0.51}_{-0.44}$	0.2	16	$2.3\ ^{+1.2}_{-1.0}$
^{166}Er	[19]	4.1693	0.02	109	$0.85\ ^{+0.18}_{-0.17}$	0.3	78	$1.85\ ^{+0.49}_{-0.45}$
^{168}Er	[19]	4.6711	0.04	48	$0.80\ ^{+0.30}_{-0.27}$	0.3	37	$1.32\ ^{+0.62}_{-0.55}$
^{170}Er	[19]	4.7151	0.05	31	$0.36\ ^{+0.34}_{-0.31}$	.03	17	$3.6\ ^{+1.6}_{-1.3}$
^{172}Yb	[20]	3.9000	0.05	55	$0.71\ ^{+0.27}_{-0.26}$	0.06	54	$0.70\ ^{+0.30}_{-0.26}$
^{174}Yb	[20]	3.2877	0.02	19	$0.80\ ^{+0.44}_{-0.39}$	0.06	16	$1.29\ ^{+0.68}_{-0.58}$
^{176}Yb	[20]	3.9723	0.015	23	$0.04\ ^{+0.29}_{-0.03}$	0.06	15	$1.05\ ^{+0.65}_{-0.55}$
^{182}W	[11]	2.6071	0.069	40	$0.76\ ^{+0.37}_{-0.35}$	0.15	34	$1.50\ ^{+0.62}_{-0.55}$
^{184}W	[11]	2.6208	0.04	30	$0.62\ ^{+0.34}_{-0.31}$	0.15	26	$0.99\ ^{+0.54}_{-0.48}$
^{186}W	[11]	1.1871	0.07	14	$1.23\ ^{+0.78}_{-0.62}$	0.15	13	$1.32\ ^{+0.93}_{-0.75}$
^{232}Th	[21]	2.988	0.016	178	$0.76\ ^{+0.13}_{-0.12}$	0.26	123	$1.78\ ^{+0.36}_{-0.34}$
^{238}U	[21]	3.0151	0.0045	146	0.79 ± 0.12	0.47	84	$1.02\ ^{+0.39}_{-0.34}$
W.A.		-	-	1245	0.801 ± 0.052	-	978	1.217 ± 0.092

III. IMPORTANCE OF THRESHOLD IN ML ANALYSIS OF NEUTRON WIDTHS

Because the PTD is a special case (degrees-of-freedom $\nu = 1$) of the family of χ^2 distributions, it is assumed that the data are distributed accordingly and the ML method is used to estimate the most likely value of ν. Ideally, the data should be complete (no missing resonances) and pure (all resonances have the same parity). Unfortunately, all experiments from which neutron widths have been determined have finite thresholds for detecting resonances and for separating small s- from large p-wave resonances. Not properly accounting for these effects can result in substantial systematic errors in ML estimates of ν. Given the shape of the χ^2 distribution as a function of ν, neglecting the effect of missed s-wave resonances below threshold will, in general, lead to a falsely large value of ν from the ML analysis. Conversely, including even just a few p-wave resonances in an s-wave set will, in general, lead to a falsely small value of ν. This potential problem is especially important for many of the NDE nuclides because they are near the peaks of the p- and minimum of the s-wave neutron strength functions, and therefore neutron width is a much less reliable indicator of resonance parity.

Below I will show that the NDE is seriously contaminated by p-wave resonances. It is also almost certainly incomplete, as illustrated in Figs. 1 and 2. Reduced neutron widths for all 1245 resonances in the NDE are shown as a function of resonance energy in Fig. 1, from which it can be seen that their are fewer small widths as the energy increases. This is just what is expected from well-known [22] instrumental effects that decrease sensitivity as the energy increases; hence, more small resonances are missed at higher energies. This is further illustrated in Fig. 2 in which distributions of reduced neutron widths are shown for the 100 lowest- and highest-energy NDE resonances. As can be seen in this figure, the distribution of the highest-energy set is substantially narrower (and hence is in better agreement with a χ^2 distribution having a larger ν) than the lowest energy one. Again, this is just what is expected if more resonances are missed as energy increases.

Typically, these difficulties have been surmounted by using a energy-independent threshold as an integral part of the ML analysis, implicitly assuming that all s-wave resonances above threshold have been observed. We recently have shown [23] that an energy-dependent threshold (on $g\Gamma_n^0$) of the form $T = a\,E_{\rm n}$, where a is a constant factor, offers three advantages compared to using a energy-independent threshold. First, p-wave contamination is eliminated equally effectively at all energies. This is because the penetrability factor for p-waves differs from s-waves by (to good approximation) a factor of E_{n}. Second, experiment thresholds have approximately this same energy dependence; thus, possible diffusiveness of the instrumental threshold can be surmounted equally effectively at all energies. Third, statistical precision of the analysis is maximized by allowing the largest p-wave-free set of s-wave resonances to be included.

Analyzing a data set comprised of many different nuclides such as the NDE involves at least two additional potential pitfalls. First, as is evident from Fig. 3, the apparent sensitivities of the various experiments from which the NDE was derived differ by several orders of magnitude. Therefore, if the entire NDE were analyzed as a single set (as was done in Ref. [2]), the threshold must be at least as high as the highest apparent individual threshold. But doing this will substantially reduce the statistical precision of the result and at least partially negate the reason for assembling the NDE in the first place. Second, for a given set of data and threshold, the ML-estimated average reduced neutron width might be substantially different from the one estimated from the data (e.g., from a simple average or by assuming $\nu = 1$). Therefore, it is important to include the average reduced width as a parameter in the ML analysis. Furthermore, the difference between these two estimated values will likely vary from one NDE nuclide to the next. For these two reasons, it is important that separate ML analyses be made for each NDE nuclide.

To minimize the effects of the above problems, I have done separate ML analyses for each nuclide in the NDE using (a range of) separate energy-dependent thresholds, and then combined these individual results for comparison to theory.

IV. AN IMPROVED ML ANALYSIS OF THE NDE NEUTRON WIDTHS

The analysis technique was briefly described in Ref. [23]. Each resonance λ has an energy E_λ and reduced neutron width $g\Gamma^0_{\lambda\mathrm{n}}$. The probability density function (PDF) $f(x|\nu)$ for a χ^2 distribution is given by,

$$f(x|\nu)dx = \frac{\nu}{2G(\frac{\nu}{2})}(\frac{\nu x}{2})^{\frac{\nu}{2}-1}\exp(-\frac{\nu x}{2})dx, \quad (1)$$

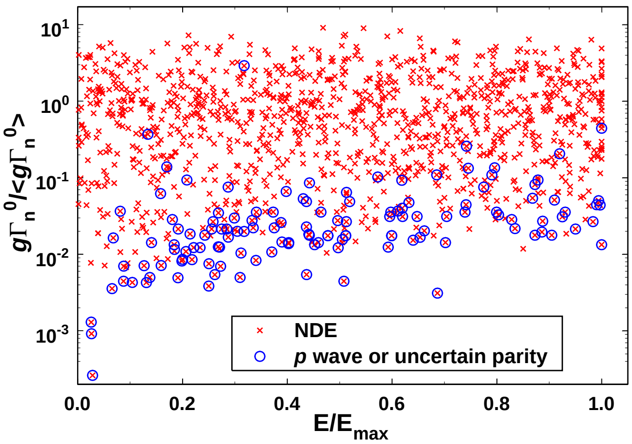

FIG. 1: (Color online) All 1245 reduced neutron widths in the NDE (red X's). Data for each nuclide have been normalized to their respective average reduced neutron widths and maximum energies. Blue circles depict those resonances which have been identified as being p wave or of uncertain parity.

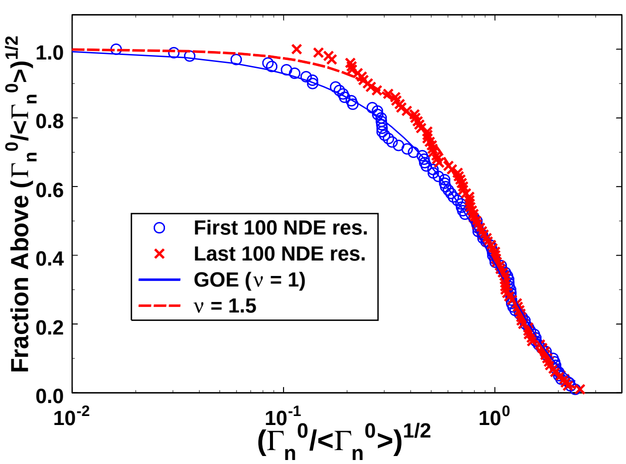

FIG. 2: (Color online) Reduced neutron width distributions for the 100 lowest- (blue cirles) and highest- (red X's) energy resonances in the NDE. The blue solid curve depicts the RMT prediction for the GOE (the PTD) whereas the red dashed curve is for a χ^2 distribution having ν=1.5. See text for details.

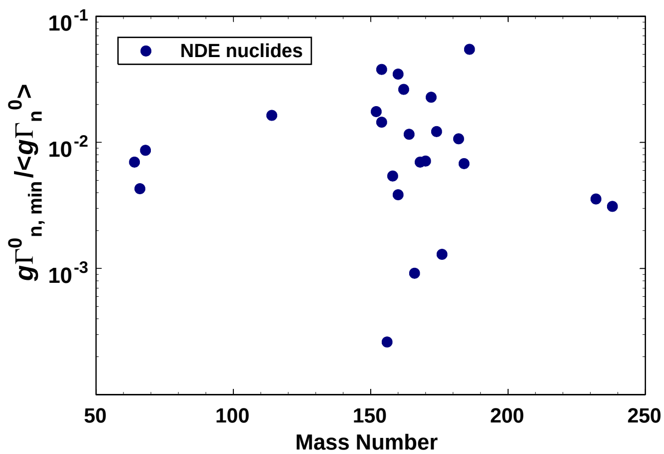

FIG. 3: (Color online) Blue circles depict minimum reduced neutron widths, normalized to their respective averages, for each of the nuclides in the NDE, versus mass number.

where $G(\frac{\nu}{2})$ is the gamma function for $\frac{\nu}{2}$, and $x \to g\Gamma^0_{\lambda n}/\mathrm{E}[g\Gamma^0_{\lambda n}]$, where $\mathrm{E}[g\Gamma^0_{\lambda n}]$ is the expectation (average) value of the reduced neutron width, with $\mathrm{E}[\bullet]$ denoting the expectation value operator. The ML method is used to estimate most likely values of ν and $\mathrm{E}[\Gamma^0_{\lambda \mathrm{n}}]$ as well as their uncertainties.

To facilitate comparison of the various nuclides in the NDE (e.g., Figs.1, 2, 3, and 5, and Table I), I have normalized data for each nuclide to their respective average reduced widths $< g\Gamma^0_n >$ (as reported in the primary references or in Ref. [24]) and maximum energies E_{max}. Hence, thresholds (on $g\Gamma^0_n / < g\Gamma^0_n >$) can be expressed as $T' = T_{\mathrm{max}}\, E_{\mathrm{n}}/E_{\mathrm{max}}$ and thus, T_{max} is the maximum value of the threshold relative to the average reduced neutron width. Values of T_{max} used in the analyses are given in Table I.

The joint PDF for statistical variables $g\Gamma^0_{\lambda \mathrm{n}}$ and E_λ is defined in a 2D region $\mathcal{I}$ given by inequalities $E_\lambda < E_{\mathrm{max}}$ and $g\Gamma^0_{\lambda \mathrm{n}} > T(E_\lambda)$. The expression for this PDF reads

$$h^0\big(E_\lambda, g\Gamma^0_{\lambda \mathrm{n}}\,|\,\nu, \mathrm{E}[g\Gamma^0_{\lambda \mathrm{n}}]\big) = C f\left(\frac{g\Gamma^0_{\lambda \mathrm{n}}}{\mathrm{E}[g\Gamma^0_{\lambda \mathrm{n}}]}\middle|\, \nu\right). \tag{2}$$

The factor C, ensuring a unit norm of h^0, is ν- and $\mathrm{E}[g\Gamma^0_{\lambda \mathrm{n}}]$-dependent. The ML function was calculated from all n_0 pairs $\left[E^{\mathrm{exp}}_{\lambda_i}, g\Gamma^{\mathrm{exp}}_{\lambda_i \mathrm{n}}\right]$, obtained from the experiment, which fall into the region $\mathcal{I}$. Specifically,

$$L\left(\nu, \mathrm{E}[g\Gamma^0_{\lambda\mathrm{n}}]\right) = \prod_{i=1}^{n_0} h^0\left(E^{\exp}_{\lambda_i}, g\Gamma^{0\,\exp}_{\lambda_i\mathrm{n}} \,|\, \nu, \mathrm{E}[g\Gamma^0_{\lambda\mathrm{n}}]\right) . \tag{3}$$

For the initial analyses, thresholds just below the smallest observed resonance for each nuclide were used. ML results with these thresholds are given in column $\nu_{\min}$ of Table I. Because experiment thresholds might not be precisely sharp, it is expected that the resulting $\nu_{\min}$ values would be systematically a bit large. However, almost all $\nu_{\min}$ values are less than the PTD value of 1.0 .

Contour plots of ML functions for ^{232}Th, calculated at two different thresholds, in the form

$$z\left(\nu, \mathrm{E}[g\Gamma^0_{\lambda\mathrm{n}}]\right) = 2^{\frac{1}{2}} \left[\ln L_{\max} - \ln L\left(\nu, \mathrm{E}[g\Gamma^0_{\lambda\mathrm{n}}]\right)\right]^{\frac{1}{2}} \tag{4}$$

are shown in Fig. 4. Here, $L_{\max}$ is the maximum of the ML function. Contours at fixed $z = k$ encircle approximately the $k\sigma$ confidence region, and were used to derive the 1σ uncertainties given in Table I. Careful statistical analysis in Ref. [23] verified that these contours are reliable. The weighted average of results at minimum threshold for the 24 nuclides in the NDE is $\nu = 0.801 \pm 0.052$, which is 3.8 standard deviations smaller than the predicted result of $\nu = 1$. Hence, these data reject the PTD with a statistical significance of 99.98%. To check this result, the combined probability for the 24 NDE nuclides was calculated using Fisher's and Stouffer's (both weighted and unweighted) techniques [25]. These methods yielded combined confidence levels of 99.97%, 99.98%, and 99.99%, respectively, in good agreement with the weighted-average result. Hence, at minimum thresholds the NDE data reject the PTD with high confidence and indicate that ν is significantly less than 1.0. A ν value significantly less than the PTD could be a sign of interesting physics [9, 23, 26–29]. However, a more likely explanation is that the NDE contains sizeable p-wave contamination.

V. CLEANSING THE NDE OF *P*-WAVES

Results of the maximum-likelihood analyses for three NDE nuclides are shown in Fig. 5. On the left of this figure, normalized reduced neutron widths are plotted as functions of normalized resonance energy. On the right side of this figure, ν values from the ML analyses are plotted versus threshold coefficients $T_{\max}$.

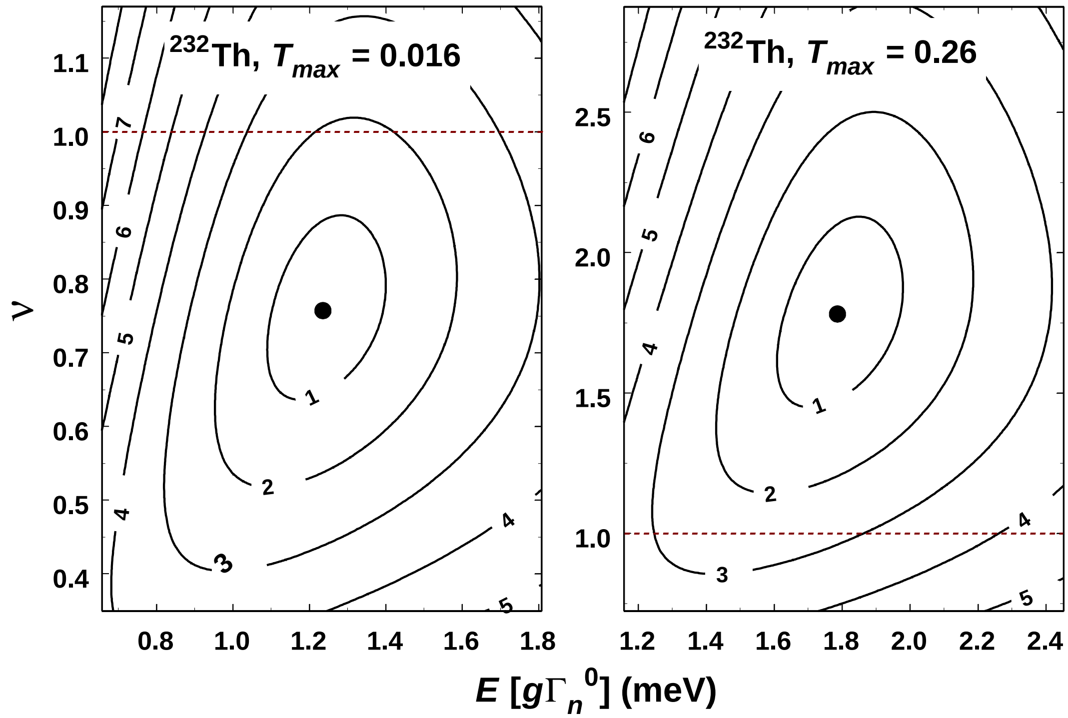

FIG. 4: Plots of $z(\nu, E[g\Gamma^0_{\lambda n}])$ constructed from ML analyses, at minimum (left) and p-wave-free (right) thresholds, of the NDE ^{232}Th data. On each plot, a filled circle indicates the location of $L_{\max}$, and a dashed horizontal line is drawn at $\nu = 1$, the PTD value.

That the NDE is contaminated by p-wave resonances is evident from this figure in two ways and from Figs. 1 and 6. First, resonances in the NDE that have been identified (in Refs. [24, 30] and references contained therein) as p-wave or of uncertain parity are shown as open circles in Figs. 1 and 5. For example, in Ref. [31], 58 p-wave resonances in ^{232}Th were assigned on the basis of γ-cascade information, 13 of which are in the NDE. One of these 13 resonances also is known [32, 33] to be p wave by its observed parity-violating asymmetry. Percentages of p-wave or unknown-parity resonances for NDE nuclides, as a function of mass number, are shown in Fig. 6. For over half (14/24) of the NDE nuclides analyzed herein, these resonances account for 5% or more of the total, for 10 of the 24 they are at least 10%, and in the three worst cases about 35%.

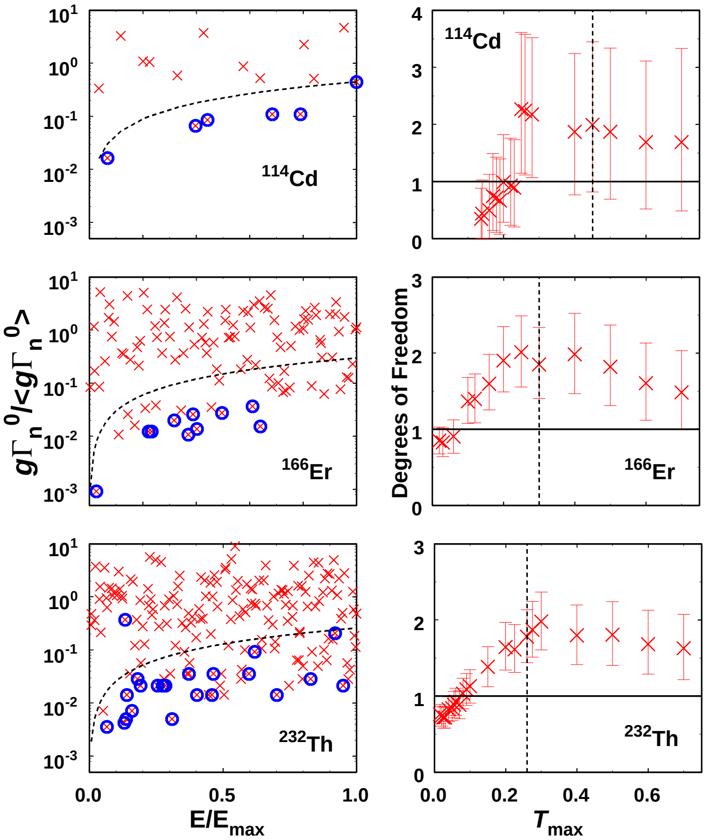

FIG. 5: (Color online) Left: normalized reduced neutron widths versus normalized resonance energies for ^{114}Cd, ^{166}Er, and ^{232}Th resonances in the NDE. Red X's depict all resonances in the NDE whereas blue circles show resonances previously identified as being p wave or of uncertain parity. Right: Red X's depict ν values from ML analyses versus thresholds used, for the same three nuclides. Error bars represent 1σ confidence levels. Black dashed vertical lines correspond to thresholds depicted by black dashed curves in the left part of this figure. See text for details.

Second, that many of the NDE resonances are in fact p-wave is reinforced by the behavior of the ν values from the ML analyses as functions of threshold, as shown in the right side of Fig. 5. In all three cases shown, ν systematically increases with threshold before gradually stabilizing. This is just the behavior expected for a population of s-wave resonances contaminated by p-wave resonances. Similar fractions of previously identified p-wave resonances and trends in ν with threshold are seen for several of the other NDE nuclides.

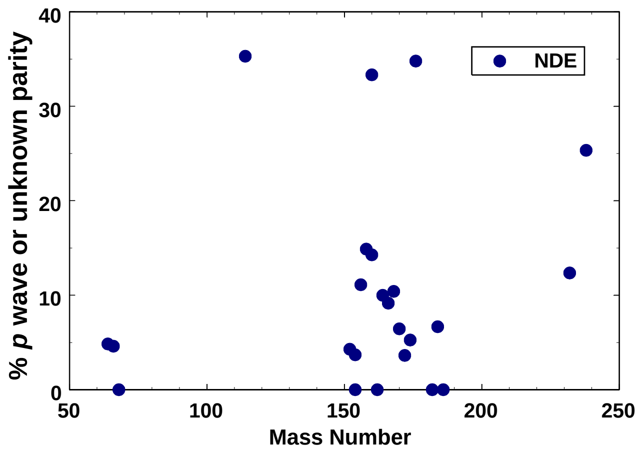

FIG. 6: (Color online) Blue circles depict percentages of NDE resonances for each nuclide which are known to be p wave or are of uncertain parity, as a function of mass number.

Removing effects of these p-wave resonances from the NDE ML analysis is a simple matter of raising thresholds until they are above the largest previously-identified p-wave resonance and/or ν stabilizes as a function of threshold. Typical "p-wave free" thresholds for three NDE nuclides are shown as dashed curves in the left-hand part of Fig. 5, and corresponding values of $T_{\max}$ are depicted by dashed vertical lines in the right-hand part of this figure. Degrees-of-freedom values for each of the NDE nuclides at these "p-wave free" thresholds are given in column five (ν_{pf}) of Table I. The resulting weighted average for the NDE is still in conflict with the RMT prediction for the GOE, albeit in the opposite direction from the result using the lowest thresholds: $\nu = 1.217 \pm 0.092$, corresponding to a confidence level of 98.17% for excluding the PTD. Fisher's and unweigthed and weighted Stouffer's confidence levels are somewhat higher; 99.15%, 99.81%, and 99.37%, respectively. Hence, when the NDE is cleansed of p-wave resonances, the data still reject the PTD with high confidence.

VI. DISCUSSION

I have shown that when neutron widths in the NDE are analyzed carefully and in such a way as to eliminate p-wave resonances, the data exclude the PTD with fairly high confidence ($\nu = 1.217 \pm 0.092$). At the same time, it has been shown [1, 2] that the NDE as a whole agrees remarkably well with energy-level fluctuations predicted by RMT for the GOE. Given the greater sensitivity of widths over energies to the degree of chaos in the system [9], and to effects related to the ”openess” of the system [27–29, 34, 35], perhaps this dichotomy can be resolved. However, I am not aware of any published model which does so while at the same time yielding $\nu > 1$.

For example, in the calculations of Ref. [9], collective effects sometimes resulted in regions of model space where transition-strength distributions deviated strongly from the PTD but, at the same time, energy-level fluctuations were consistent with GOE predictions. However, transition-strength distributions in this model deviated from the PTD in the direction opposite ($\nu < 1$) of the NDE.

Reaction effects also should be considered to explain the dichotomy. For example, it recently was proposed [35] that the standard transformation of measured to reduced s-wave neutron widths, $\Gamma_n^0 = \Gamma_n/\sqrt{E}$, should be modified for nuclides near the peaks of the s-wave neutron strength function. This newly proposed transformation, $\Gamma_n^0 = \Gamma_n/\sqrt{E} \times \pi(\frac{\hbar^2}{2m})^{1/2}(E+|E_0|)$, where E_0 is the energy (relative to threshold) of the s-wave single-particle state responsible for the peak in the neutron strength function, could affect (relative to the standard transformation) the shape of the Γ_n^0 distribution for values of E_0 near the resolved-resonance energy range. However, as most of the NDE nuclides are not near the peaks of the s-wave neutron strength function, this proposal should not affect the NDE, and therefore cannot explain the dichotomy.

Several explanations related to the ”openess” of the system [27, 28, 34] and to correlations between incoming and outgoing channels [29] have been proposed for the observation [23] that Γ_n^0 distributions for ^{192,194}Pt resonances are significantly broader ($\nu \approx 0.5$) than the PTD. However, width distributions [28, 29] predicted by these theories are better characterized by $\nu < 1$ rather than $\nu > 1$ as observed for the NDE.

Considering how data in the NDE were selected suggests another solution. It often has been stated that Δ_3 is very sensitive to missing or misassigned resonances (see e.g., [36–38] for recent work on this subject). Therefore, if the NDE was pure but incomplete, or vice versa, it would not be expected to agree well with the spacing statistics used in Refs. [1, 2]. However, it seems plausible that because the NDE is both impure and incomplete, it can be made to agree with these statistics, especially considering that there are expected to be many more p- than s-wave resonances at the small widths where it was not possible (using means independent of RMT) to differentiate the two parities, and hence an extremely large number (see below) of possible "s-wave" sets can be constructed from the observed resonances.

Many different selections were applied to obtain the NDE. For example, in Ref. [2] it is stated that "The criterion for inclusion in the NDE is that the individual sequences be in general agreement with the GOE." Furthermore, data from many of the included nuclides were selected, at least in part, using measures derived from RMT for the GOE.

Data for all but three of the 24 nuclides considered herein were obtained by the group at Columbia University. According to their publications (e.g., Ref.[19]), they had "...no specific tests for s vs p levels, so there may be errors in these assignments." Therefore, they relied on theoretical guidance, specifically measures derived from the GOE, to perform these separations. For example, for six of the 24 nuclides considered herein, including the two having the largest number of resonances, separation of p- from s-wave resonances was accomplished [39] by i) assuming all resonances having neutron widths larger than a certain (unspecified) size were s wave, ii) calculating the number of s-wave resonances below this size by assuming the PTD is correct, and iii) deciding which of the resonances below the threshold defined in the first step and needed to achieve the total number calculated in the second step, were s-wave by requiring good agreement with four spacing statistics (the Wigner nearest-neighboor spacing distribution, $\rho(S_j, S_{j+1})$, Δ_3, and the Dyson F test) derived from the GOE. Separation of s- from p-wave resonances for several of the other NDE nuclides followed the first two steps described above followed by a Bayesian analysis to decide which of the resonances below the threshold defined in the first step to assign to the s-wave set. The Bayesian analysis again assumes that the PTD is correct (for both s- and p-wave resonances) and furthermore requires the average widths for s- and p-wave resonances. The latter quantity usually is known only approximately, if at all. Such Bayesian analyses are known to be unreliable. For example, several neutron resonances in ^{64}Zn [15] are known to be definitely p wave by their symmetrical shape in transmission (total cross section) data, but nevertheless have a Bayesian probability of >99 % of being s wave.

Given these selection procedures then, it is perhaps understandable that the NDE appears to agree well with the spacing statistics examined in Refs. [1, 2] despite that fact that the data are neither complete nor pure. Consider, for example, the case of ^{232}Th, the NDE nuclide with the largest number of resonances. The authors of the primary reference from which these data were taken [21, 39] state that 80% of the s-wave set (i.e. the set included in the NDE) were chosen to be s wave because they were too large to be p wave. Hence, 36 of the 178 ^{232}Th resonances fall below this size and had to be selected as s wave using measures derived from the GOE, as noted above. In this same reference, 62 resonances are assigned as p wave (and hence by definition are below the s-wave threshold). Therefore, there are 98 resonances from which to choose the 36 needed to complete the s-wave set. Therefore, the number of possible s-wave sets is astronomically large ($\tilde{}10^{27}$), and hence it should not be surprising that at least one set can be found to agree with the various RMT measures used in Ref. [39]. Similar numbers apply to the other data sets from Columbia.

Finally, given that a significant fraction of the NDE data were selected using measures derived from RMT for the GOE, it seems highly questionable to use the full NDE for any test of this same theory. In contrast, the "p-free" results presented herein circumvent this problem by eliminating these problematical resonances from the test.

VII. CONCLUSIONS

I have shown that neutron widths in the NDE [2], when analyzed carefully, reject the PTD with high confidence. Given that reported deviations from the PTD for individual nuclides [21, 23, 40–44] have occurred on both sides of $\nu = 1$, the statistical tests used herein likely underestimate the confidence with which the PTD can be rejected for the combined set of nuclides in the NDE. Furthermore, I have shown that the NDE is not pure and very likely incomplete. Also, measures derived from RMT for the GOE often were used in deciding which resonances to include in the NDE. These facts cast very serious doubt on repeated claims that the NDE constitutes a striking confirmation of RMT.

Measurement techniques have improved considerably since the data in the NDE were acquired. In particular, new methods for determining resonance spins [44, 45] hold the promise of surmounting the difficulty of separating small s- from large p-wave resonances, which has been one of the most troublesome barriers to obtaining better data. Unfortunately most experimentalists continue to use RMT to correct their data for instrumental effects rather than use their data to test RMT. Exceptions to this practice are recent tests [23, 44] which have revealed significant disagreements between new neutron data and the PTD. These new data, together with previously reported disagreements [21, 40–43], which have largely been ignored, are potentially very interesting. Therefore, I urge experimentalists obtaining new and improved data to use them, when possible, to test RMT.

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